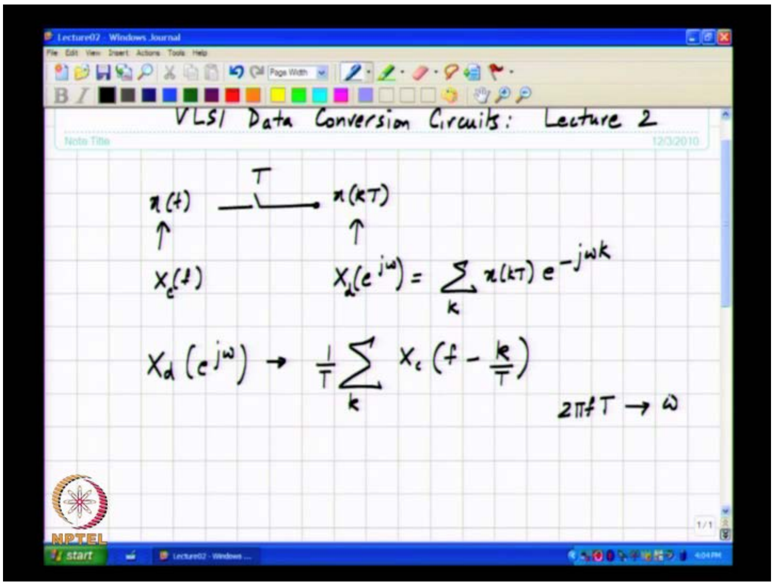
VLSI Data Conversion Circuits
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 02
Sampling - 1

So, in the last class, we were reviewing some basics of sampling, so let us quickly recap what we did in the last class.

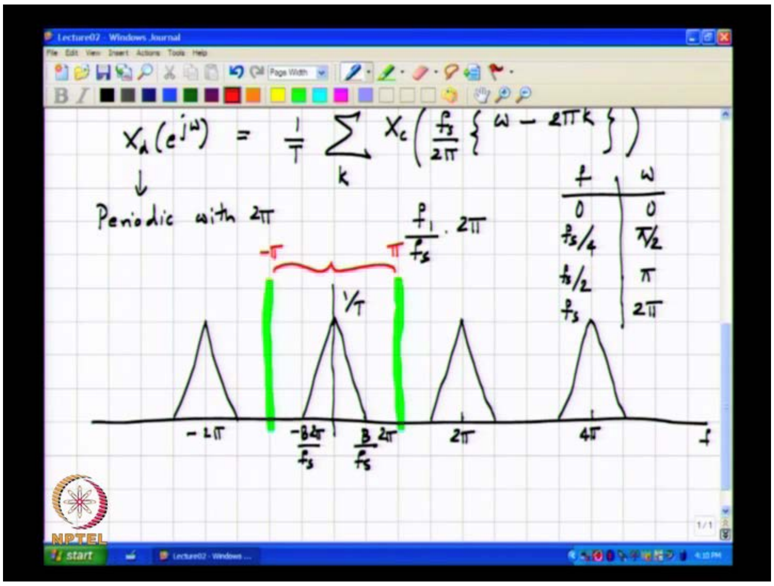
(Refer Slide Time: 00:21)



So, let us say we had a continuous time signal x of t and we sample it, at a uniform rate every T seconds, then we get a discrete time signal where time is quantized, but amplitude is continuous. And we were trying to relate the spectrum of the continuous time signal to the spectrum of the discrete time signal. And we said that, if the continuous time signal had a spectrum x of a, and we denote the discrete time Fourier transform of the sequence x of K T by x of e to the j omega, which is sigma overall k x of K T e to the minus j omega k.

And to remind ourselves that, this signal here corresponds to continuous time, I will label this chap with a subscript x c of f, and to remind ourselves that this is a discrete time spectrum, I will call that x d of e to the j omega. And yesterday we saw that x d of e to the j omega, can be simply obtained by forming the sum over all k of x c of f minus k by T times 1 over T and then, we replace 2 pi f times T with omega.

Refer Slide Time: 02:47)



And therefore, x d of e to the j omega is simply sigma overall k x c of omega by T times 2 pi minus k by T. And instead of writing 1 by T all over the place, if I denote 1 by T by the symbol f s stands for sampling frequency, then x d of e to the j omega can be written as 1 over T times sigma over all k x c of f s by 2 pi times omega minus 2 pi k. And one must remember, that x d is periodic with 2 pi, and when a continuous time sinusoid of a frequency f 1 is sampled at a rate f s. What kind of sinusoidal will that result in, in the discrete time domain, let us say you had a continuous time sign wave with the frequency f 1.

When you sample it at a rate f s, it will become a discrete time signal, what do you think the frequency of that discrete time signal would be.

Student: ((Refer Time: 05:02))

It will be f 1 by f s times 2 pi, so in particular D C, will transform to D C I am just putting that down here again D C will transform to D C f s by 4 will transform to f s corresponds to 2 pi. So, f s by will correspond to 2 pi by 4, which is pi by 2 f s by 2 will correspond to pi and f s corresponds to 2 pi, and a very easy way of drawing the spectrum is to do the following. If the continuous time signal had this spectrum, how do you draw the discrete time spectrum, the first thing would be to make copies of this with the period, let us say this bandwidth was B Hertz, I will make copies of this at f s.

So, let me just do that here, the next step is to multiply all of this with 1 over T and then, replace

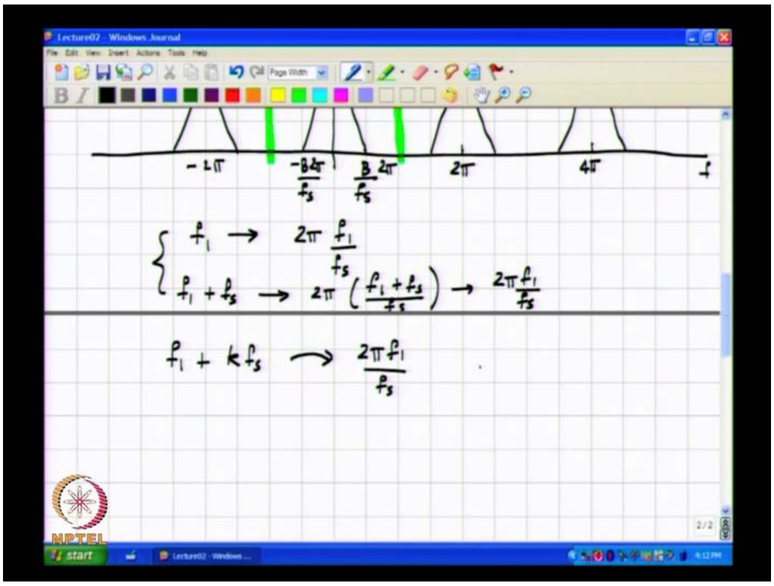
Student: Scale

Scale the x axis where B becomes B by F s times 2 pi, and f s becomes 2 pi 2 f s becomes 4 pi and so on, and as we all agreed yesterday, the discrete time spectrum is periodic with a period 2 pi. So, it does not really matter which particular period you choose, and it is common to choose the range from...

Student: Minus pi

Minus pi to 2 pi, is that clear.

(Refer Slide Time: 08:37)



Now, couple of things I would like to bring to your attention, the first thing is that if you had a continuous time signal at a frequency f 1, it will transform to as we discussed just 1 2 pi times f 1 plus f 1 by f s. Now, what if you had a continuous time signal which was f 1 plus f s, the frequency of the sinusoid was f 1 plus f s, if you sample it at f s what do you think it will look like in the discrete time domain.

Student: ((Refer Time: 09:11))

Please notice that it, may say is 2 pi times f 1 plus f s

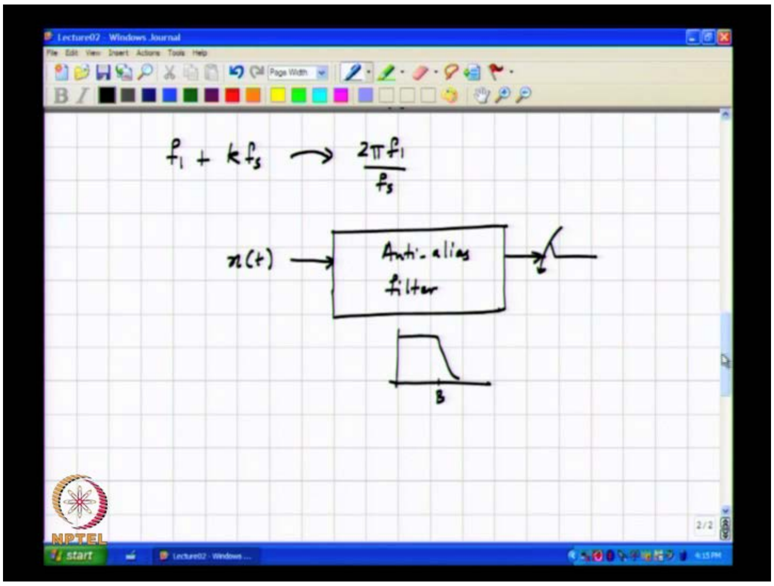
Student: ((Refer Time: 09:20))

By f s right which is 2 pi into f 1 by f s plus 2 pi, but we know that the discrete time spectrum is periodic, so everything is modulo 2 pi, so you can remove the 2 pi. So, what this is telling you is that, a frequency f 1 and a frequency f 1 plus f s when sampled will look like the same frequency in the discrete time domain this is nothing but, aliasing. And this makes a sense also, because from the Nyquist theorem we know that, if the input frequency is greater than f s by 2, then it will result in aliasing.

This is any illustration of that, it just says that if you have a frequency f 1 plus f s, it will look after sampling just like f 1. Now, by the same token f 1 plus k times f s, after sampling will also look like 2 pi f 1 by f s, when this is assuming that f 1 lies between 0 and f s by 2. So, in other words when you sample a continuous time signal, there are many frequencies that can masquerade like the, mean there are many signals which kind of map down to the same discrete time sinusoid.

So, if you are not careful before sampling, then you can be thoroughly mistaken as to what the continuous time signal is, please note that the idea in the whole the idea behind sampling is to be able to eventually reconstruct the input signal in some fashion. If you have lost information while sampling or you made errors during sampling, it is very difficult to recover.

(Refer Slide Time: 11:49)



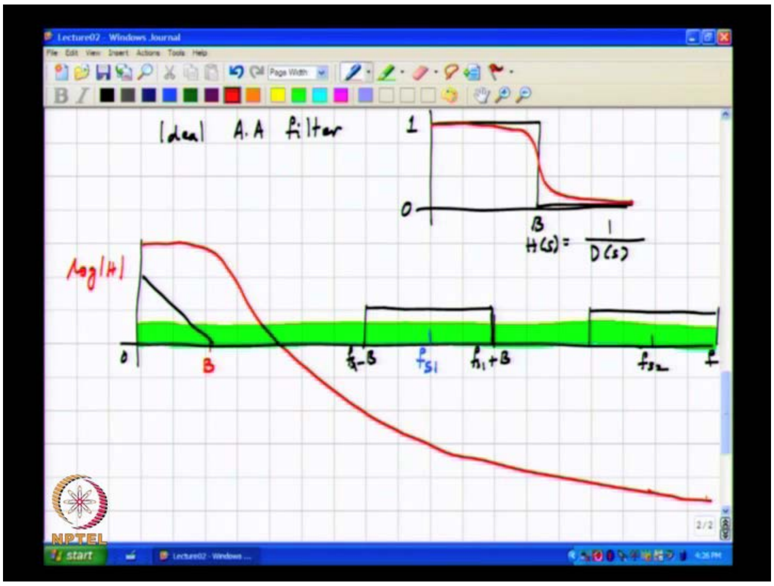
So, which is why as we were discussing yesterday we said that, while it is true that the desired signal may be of only a small bandwidth, there is always accompanying noise, whose bandwidth could be much wider than the, the bandwidth of the desired signal. Now, we should not get into the mistaken notion that, the desired signal is only got a bandwidth B, so I will only sample at 2 B. So, if we do not do anything, then noise components which are much broader band than to be, as we just saw will all alias to in band, there by degrading the in band signal to noise ratio.

So, the way around this problem is do not sample the signal directly, but to put a filter, call the anti alias filter which make sure that the bandwidth of the signal is what you think, it should be. And then the output of the anti alias filter is sampled and thanks to the anti alias filter that, the signal bandwidth does not exceed B, which means that I am safe if I sample at 2 B this is what you have seen in the communication text books.

And when we are dealing with mathematical abstraction, it is often very convenient to assume brick-wall type filters and so on. So, in your communication classes very likely that you have seen a block diagram, where you have an ideal low pass filter, whose bandwidth is B, in other words it allows everything to pass through below B, and cuts off everything beyond a bandwidth B. And by now you should know that this is not possible, any practical filter that you build with a finite order will only have a, only thing you can do is make the transition band narrower and narrower.

You cannot make the pass band absolutely flat, you cannot make the stop band at attenuation infinite, which is what one would tend to believe; if we saw the ideal block diagram of a anti alias filter, it would look like a brick-wall filter with one say ideal anti alias filter.

(Refer Slide Time: 14:31)



You would expect the frequency response to look like this, where this is 0 and this is 1 this is the bandwidth B, a practical filter can only approximate this in some sense. So, you will find that a the pass band is not as flat as you would want, the transition band is not as sharp as you would want and the stop band attenuation is not as large as you would want. Now, given this information can you comment on a other any considerations you think for the sampling rate, in other words we know that if a signal has a bandwidth B, we need to at least sample at 2 B.

And we also know that we must put an anti alias filter, now the question I am asking you is, do you think it makes sense to sample at not at 2 B, but at 4 B, 2 B satisfies an Nyquist criterion, 4 B satisfies the Nyquist criterion, 40 B satisfies the Nyquist criterion. It is natural to wonder whether should I just sample at Nyquist, after all Nyquist is telling me that, if I sample it at least the twice the bandwidth, I will be able to reconstruct no problem.

The question is should I sample only at 2 B is, 2 B good enough or I am doing better, if I sample at 4 B or am I doing even better if I sample at 8 B, do you have any comments, is there anything we gain, it seems like it is more difficult to sample at a higher rate. So, question is do we gain anything at all in this bargain.

Student: ((Refer Time: 16:58))

Very good, so the suggestion is the following, so let me draw spectrum let us say this is 0 I will only draw the positive half, this is f I will draw the characteristics of the anti alias filter in red here ((Refer Time: 17:22)). And let me draw this on a log scale, so this is log magnitude of the anti alias filter response and because, the filter has got a finite order as you move away from the band edge, this frequency below B is the, so called pass band of the filter.

The portion beyond B is the stop band of the filter, because the filter is a practical 1, it is a real 1, the order is finite which means that you can never have infinite sub band rejection, over a band of frequencies. You can have it at one frequency or two frequencies, but not over a over a band, and if you have an all pole filter and all pole filter is 1 where, the numerator is 1, H of s is of form 1 by D of s. So, if you have an n-th order filter for large frequencies, how does the how do you think the attenuation will go.

Student: ((Refer Time: 18:47))

For frequencies far away from the band edge, it must go down as 1 by omega to the power n which is 20 N D B per decay correct. So, if this is a linear scale on the x axis and a log scale on the y axis it will look like this, if I plot a log log plot it will be a straight line with a slope of minus 20 N D B per decay. Now, let us consider two situations, one the signal frequency is the desired signal is here, let us consider two sampling rates, one where the sampling rate is f s, and other one where the sampling rate is let me call this f s 1.

And the other one where the sampling rate is f s 2, so now, can you comment on the consequences of these choices f s 1 versus f s 2, what do you think becomes more simplified, if you chose f s 2 versus f s 1.

Student: ((Refer Time: 20:26)) noise component will the higher frequency component, the aliasing of the i

So, the first thing we need to understand is that, if there was no noise at all we would not have to worry about the anti alias filter, because the sampling rate evidently is much higher than twice the bandwidth. The problem or the reason why we need to have an anti alias filter in the first place is to filter of noise, and where do we want to specifically get rid of noise, what is the job of the anti alias filter.

Student: Within the bandwidth of the same thing

See you cannot

Student: ((Refer Time: 21:09))

Where outside the required band, the question is it important to get rid of noise all outside, I mean completely outside the signal band or at their specific locations where you really want to get rid of noise.

Student: B and f s

Between

Student: B and f s

No, it will be a little more specific B and f s of course, it is good but

Student: ((Refer Time: 21:35))

Where do we want to get rid of noise, we want to get rid of noise at all those frequencies which can potentially alias to the in band frequencies, which is that in band frequency 0 to B. So, now we need to figure out which all frequencies will alias to the range 0 to B, let us start with 0, which all frequencies were alias to 0.

Student: ((Refer Time: 22:03))

Obviously 0 will translate the 0, then

Student: f s 1

F s 1

Student: F s 2

Twice f s 1

Student: 3 f s 1

All my integer multiples of f s 1 will all alias to

Student: D 0

So, we need to definitely get rid of noise at f s 1, now what frequencies will alias to B

Student: ((Refer Time: 22:31))

F s 1

Student: Plus B

Plus B 2 f s 1 plus B and so on, must remember also that there is minus B, so what will alias to minus B

Studded: ((Refer Time: 22:40))

Minus B plus f s 1 minus b plus 2 f s 1 and so on, so now we know what all frequencies alias to 0, what all frequencies alias to B. So, we know I mean now all the range between also will alias to something between 0 and B, so f s 1 plus B, this is f s 1 plus B and f s 1 minus B is this range here. So, this is a band of frequencies where signal will alias to minus B to B you understand, while it is true that it will be great, if we had an anti alias filter which would just cut off everything beyond B, we know that in practice it is not possible.

So, the next question is where do we really want to cut off noise, and we really want to cut off noise at frequencies which can alias down to the range.

Student: Minus B to

Minus B to B and that will be in the if we chose f s 1 as the sampling rate it, will be f s 1 minus B to f s 1 plus B, now if instead we had chosen f s 2 as the sampling rate, what do we see which all frequencies will now alias to base band...

Student: F s 2 minus

F s 2 minus B to f s 2 plus B, now can you comment on which of these is a better choice of sampling frequency and why?

Student: ((Refer Time: 24:36))

F s 2, why is f s 2 is a better choice

Student: ((Refer Time: 24:38))

So, if we see that if we chose f s 2, then the rejection, for a given anti alias filter, the rejection of the anti alias filter

Student: Is higher

Will be higher for a higher choice of sampling frequency, so even though it appears that it is harder work to do to sample at a higher rate, while Nyquist just dictates it twice of the bandwidth is enough. We see that choosing a sampling rate, which is much higher than what was dictated by Nyquist is advantageous, in terms of the design of the anti alias filter. Now, let us take this argument the other way, if I insisted that I want to sample at Nyquist, in other words my sampling rate is 2 B, what do you think the characteristics of the anti alias filter must be.

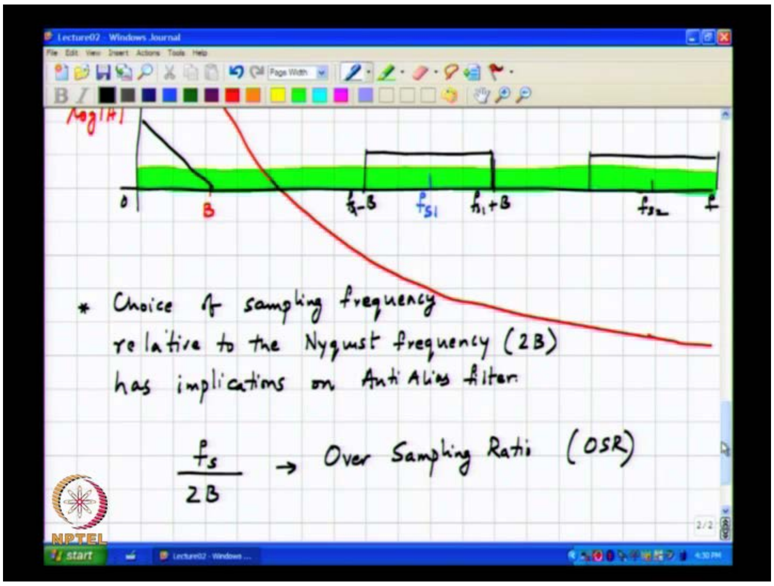
Student: ((Refer Time: 25:48))

It has to be

Student: Sharper

Lot sharper than what we have now does it make sense.

(Refer Slide Time: 26:07)



So, choice to sampling frequency relative the Nyquist frequency, what is the Nyquist frequency?

Student: ((Refer Time: 26:36))

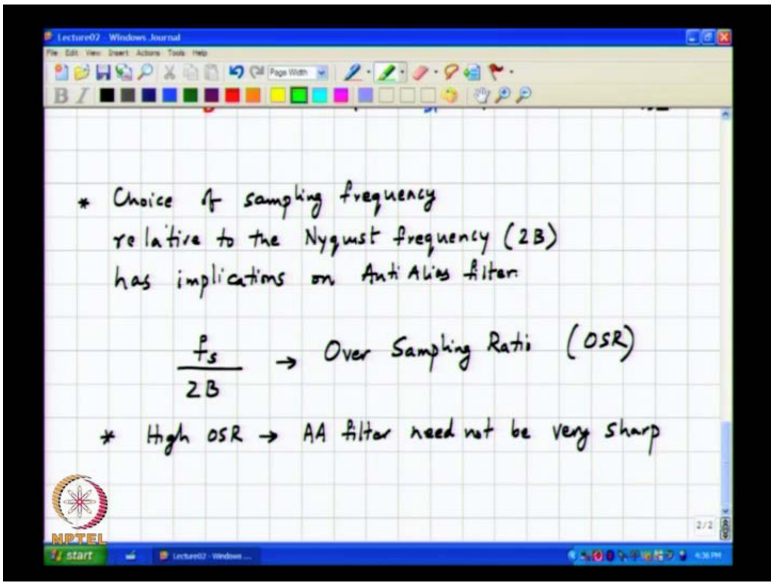
The Nyquist frequency is the minimum sampling rate you require, to sample a signal of bandwidth B, so the Nyquist frequency in this particular example is 2 B. So, the choice of the sampling frequency related to the Nyquist frequency, has implications on the design of the anti alias filter. So, to compare different sampling rates, it makes sense to only compare it with respect to the Nyquist rate, so 2 B is the minimum sampling rate required to be able to reconstruct a signal with bandwidth B.

If you are sampling at a rate over and above 2 B, it means that your, because sampling at Nyquist your sampling, if you are sampling at a rate higher than Nyquist your oversampling over. So, the ratio f s by 2 B is called the oversampling ratio or the abbreviated as the OSR, so we will repeatedly keep using these things, this abbreviation later in the course, you understand why this definition makes sense. So, now to rephrase all that we have discussed with this new jargon, if my oversampling ratio is high, what does it mean is my anti aliasing filter design easier or more difficult.

Student: Easier

It is easier, and other thing we need to bear in mind is that, it is only in the text books that you have an ideal filter, it is only in the text books where you say you have a filter with the bandwidth B, and the bandwidth is actually B. In practice whenever you say I am going to have a filter B, you must be prepared to take variations in the bandwidth. Because, no real system will have a bandwidth which is absolutely fixed, it is bandwidth will vary perhaps, because of temperature variation, because of manufacturing tolerances and so on. So, in practice it is never possible to ensure a bandwidth which is exactly what you want.

(Refer Slide Time: 29:34)



So, one high OSR means that the anti alias filter, need not be very sharp why, because if you want to reject the alias band to a certain level, if you increase the oversampling ratio, which is the ratio of the sampling rate to twice the signal bandwidth. Then, your anti alias filter must satisfy, in the pass band you want your anti aliasing filter to have a gain of 1, in the alias band you want it to have a gain of 0. Of course, we do not really expect that gain to be 0, we want it to be some small number, so if we say we want the gain in the alias band to be at least smaller than in at most some number like this.

Then the filter design problem becomes a lot simpler, because the filter needs to have a transition band which is doing this. On the other hand, if my oversampling ratio is small, then what happens I need to have the same rejection in the stop band, and I need to have the same transmission in the pass band. So, this means that my transition band from pass band to stop band must be very sharp, and designing sharp filters is more difficult than designing filters which roll off gently, without good getting into the theory, I mean while this seems at least intuitively satisfying.

Not only that can you comment on the effect of variation of this filter corner, as you change OSR, in other words do you think I can tolerate a bigger variation of the band edge, the motivation being that no practical filter will have a band edge, which is fixed in frequency, it will vary. The question now is, does it make any difference if I increase the

OSR, in other words can I tolerate the larger variation of band edge frequency with the higher OSR.

Student: Yes

Any do you understand the question no, see we know that the anti alias filter bandwidth cannot be fixed, there will be some tolerance it will move. And you want to make sure in spite of this the band edge moving, you want to make sure that it rejects the alias band to some degree. Now, the question is will you be able to tolerate a larger variation of this band edge, when the oversampling ratio is small or when the oversampling ratio is large, so it is a its very straight forward.

So, if the oversampling ratio is large, not only is the filter design easier from a point of view of the width of the transition band, it will be lot more tolerant to variations in the band edge frequency to see this, imagine what happens if you have Nyquist sampling, the filter must be really really sharp. And now if the bandwidth even moves a little bit, if the bandwidth reduces what happens, it will get rid of the alias for sure, but it will also cut off some of the desired signal.

On the other hand, if the bandwidth increases, then what will you see some of that alias band is not properly rejected, so while it is true that Nyquist sampling will is all that is necessary to be able to reconstruct the signal properly, there are some very very practical reasons, why you would want to actually over sample the input signal. So, and typically you would never have a system where the signal bandwidth is say B Hertz and you sample exactly at 2 B.

This would make the job of the filter designer very very difficult of course, you can say some of my problem is somebody else problem, but you could I mean, in the next project you could be that somebody else you understand. So, it is very common to have little bit of oversampling, so that the job of the filter designer is made easy. Of course, pushing the sampling rate high is also not at all trivial effect, that basically means that your circuits have to work that much faster.

But, system design is trade off between these possibilities, if you try to make your job easy somebody else’s job becomes a lot more difficult to do, if you want to make that guys job easy then your job becomes very difficult to do. So, both of you sit together and figure what works best for both of you.

Student: ((Refer Time: 35:39))

Meaning

Student: On the transition band what we are talking about, suppose we are having the low sampling rate

Correct.

Student: And transition has

Will be very sharp

Student: So, whether any chance of that stability criteria

Well, it is true that if you have the comment we made was, that if your oversampling ratio is very small then the filter has to be extremely sharp. And the comment he made was that, if the filter has to be extremely sharp, then that response must only be possible by poles whose quality factors are extremely high, only then you can get a sharp roll off. And yes, that is indeed a challenge, once you are very high cube poles it turns out that the sensitivity of the circuit to component variations also becomes high.

As you might have seen in perhaps your digital filter design class, the same thing also holds for analogue filters. So, whenever you want to make something very rapidly in the frequency domain, sensitivity to component tolerances, noise etcetera at those frequencies becomes large. And therefore, you would like to try and avoid very sharp filters if possible.

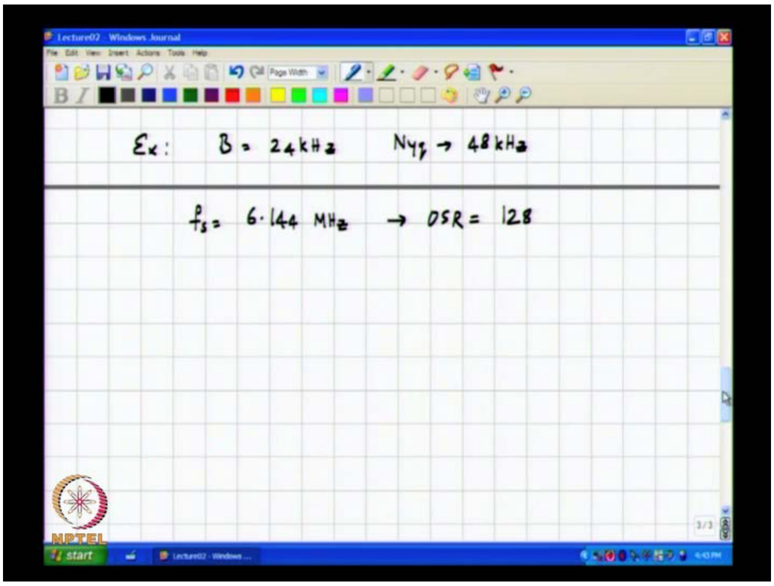
Student: Sir

Yes

Student: Bandwidth criteria whatever we are talking tolerance, how much percentage we will take in this, suppose we are having the say for your audio signal 20 kilohertz suppose we are talking, how much we can go for the sampling ratio.

The question he asked was, if you have a signal with the certain bandwidth how will you choose your oversampling ration, so will you choose a very small number or will you chose a very large number, this is very situation specific. When you are dealing with low signal bandwidths for example, audio it is very easy to sample at a higher rate given today's technology constraints. For example, 24 kilohertz audio signal. a common sampling rate to use is say 6 Megahertz. 6.144 Megahertz.

(Refer Slide Time: 37:52)



So, if let me just take an example since he has brought it up, so the signal bandwidth is 24 kilohertz, the Nyquist bandwidth is what Nyquist rate is 48 kilohertz and if the sampling rate is 6.144 Megahertz, what is the oversampling ratio.

Student: Around 120

It is 128, and this turns out that is a fairly common thing to do, as you can see the alias requirements given that the oversampling ratio is 128, the anti alias filter is to have can be actually very very gentle. Because, it needs to pass it needs to have a flat gain up to 24 kilohertz and it needs to have an attenuation at 6 point something Megahertz. So, very often a simple R C or a combination of RC, a passive RC filters all that is needed and one might also ask given that, I am doing all this extra work I am, I need to sample only at 48 kilohertz, but now I am sampling at much higher rate.

Can I exploit this to improve circuit properties, can I exploit this to a larger degree than simply saying anti alias filtering becomes easy, it turns out that this is the subject or what is called the oversampling form in the family of weighted e convertors, called oversampling analogue digital convertors. Where we exploit the fact that, there is your oversampling significantly, oversampling significantly means that you are sampling at a much higher rate than is necessary which means, there is a lot of correlation between successive samples of the signal.

In other words, if you are watching a movie it is like watching slow motion there is or if you are one of those T v soap box, it is like [FL] you watch today and you watch tomorrow and it will looks like the same thing. There is no difference between successive samples, which is basically telling us Nyquist state is very low you come back 2 years, later and watch and you will know perfectly well what is happening.

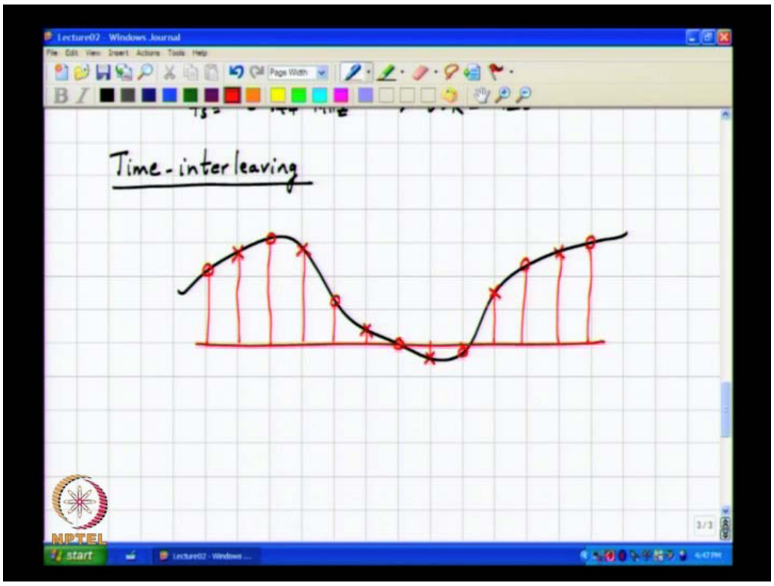
So, later on this course, we will see how oversampling can be exploited, not only to simplify the bandwidth of, I mean simplify the requirements of the anti alias filter, you can also use it to like to improve the performance of the A to D converter. So, of course, now if the signal bandwidth becomes very high, it may become impractical to be able to sample it at in an oversampling ratio this large. In which case you have to settle for more modest values of oversampling ratio and then, simply because you are not able to build circuits, which can sample this fast.

Now, the next thing I wanted to talk about is related to what I just said, sometimes you want to sample a signal at a very high rate, that could be A, because you want to oversample or B, simply because the signal bandwidth is extremely high. A case in point being front ends of oscilloscopes, today to test your high speed circuits you need an oscilloscope, which is much higher in speed. Only then, you will be able to test something which is high speed to begin with, so oscilloscope for front ends have been a big application area for requiring, higher and higher and higher sampling speeds.

Unfortunately device technology is may not be advancing at the rate you require. let us say you want to build a sample and hold which samples at say 40 Gigahertz, because you want to test something. Then, it may not be possible first of all to be able to build sample and hold, which can operate at such high speed. So, one way around that what do you think you can do, if one fellow cannot do the job quickly enough, what do you think you will do, you put two guys on the job, so it is the same thing here.

So, you can have many sample and holds which are working at a lower rate, and put them together and make it look like a single sample, and hold working at a higher rate.

(Refer Slide Time: 43:25)

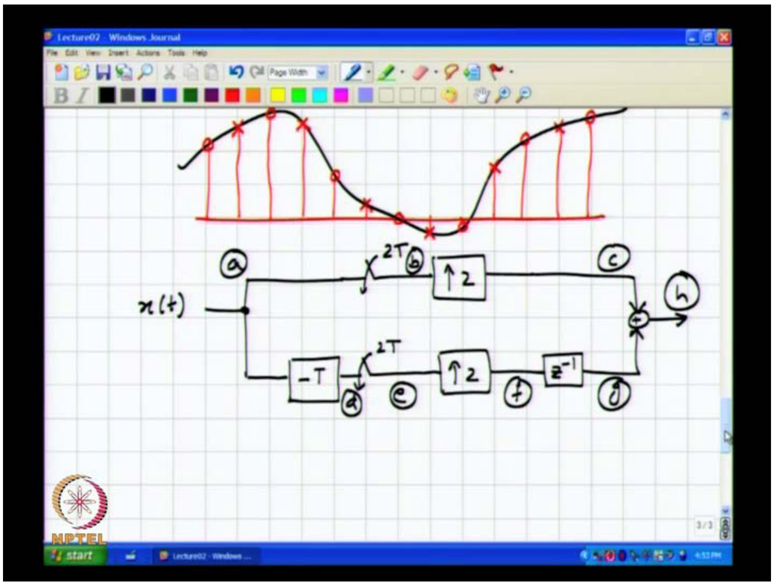


So, let us take and this technique is called time interleaving, where you have many sample and holds working parallelly at lower speeds making, a single unit which appears as if it is working fast. So, now you say then, I do not know how to design a really high speed sample and hold, however I know how to design a low speed sample and hold, if I put many of the sample and holds together in some fashion. I will be able to hopefully combine the outputs, in such a way as to make a sample and hold which looks like a high speed, it seems like a reasonable idea.

So, this is the principle of what is called time interleaving and the basic idea is like this, let us say we had a signal here something like this, and you want to sample this at some rate. However, you are not able to build a sample and hold which travels at that high rate, so the simplest case I am going to use two sample and holds, one which samples all the even samples. And one which samples the odd one, so the circles and the crosses represent, outputs coming from in different sample and holds.

So, please note that, even though the effective sampling rate is f s each individual sample, and hold is working at the rate in this particular case, f s by 2 this can be extended to n sample and holds sampling systems operating in parallel. So, let us try and first analyse this also serves as a good way of seeing, if we understand this continuous time, to discrete time conversion and spectrum and all that properly.

(Refer Slide Time: 46:07)



So, if I had a continuous time signal, one equivalent way of representing this system mathematically, is to say I have two samplers operating at a sampling rate of f s by 2 which means that, they are sampling at 2 T. But, mathematically I can get the samples if I take the signal advance it by T and sample it at 2 T, so both the sample and holds of these sampling systems the switches are being closed simultaneously. It is equivalent to it you can either skew the input signal, or you can skew the stamping clocks, I have chosen simply for mathematical convenience.

I have chosen to advance one of the signals, it does not I mean have a advanced version of the signal, it does not as long as mathematically equivalent it does not matter. So, now here I have a discrete time signal, which is the samples of the continuous time signal taken at even instance of time. And here what do I have, I have again a discrete time signal, where the samples are taken at odd instances of time. Now, how do I reconstruct when I want to use these two signals, to make the output look like a sample and hold which was operating at f s.

So, what do you think I should do to these two output discrete time signals, no please if you if you simply add these two signals what will happen, you need to switch before I what is the rate of these samples f s by 2, if I simply add the two sequences what will be the rate be

Student: ((Refer Time: 48:48))

It will still be

Student: F s by 2

F s by 2, but I need to eventually get to a rate of

Student: F s

F s, so what do you think I should do

Student: Multiply, during one time period and during the second, set at the next second signal of some sample

See these samples are

Student: ((Refer Time: 49:17))

No, once you sampled only I have list of samples there is no more time, so only sequence of numbers how will you generate the

Student: ((Refer Time: 49:40) interpolate it, one signal interpolate

I am understand what you are saying, but what technically needs to be done is you must first insert you must increase the sampling rates of

Student: Individually

Individual strips that means, you must

Student: Inside 0

Up sample each of these sequences, so now up sampling means you insert zeros, every your up sample were factor of 2, so you insert zeroes and then, what should you do, you must put a delay here and now what should you do, you simply add these two. So, now, if I put all of this in a big box this should look exactly like

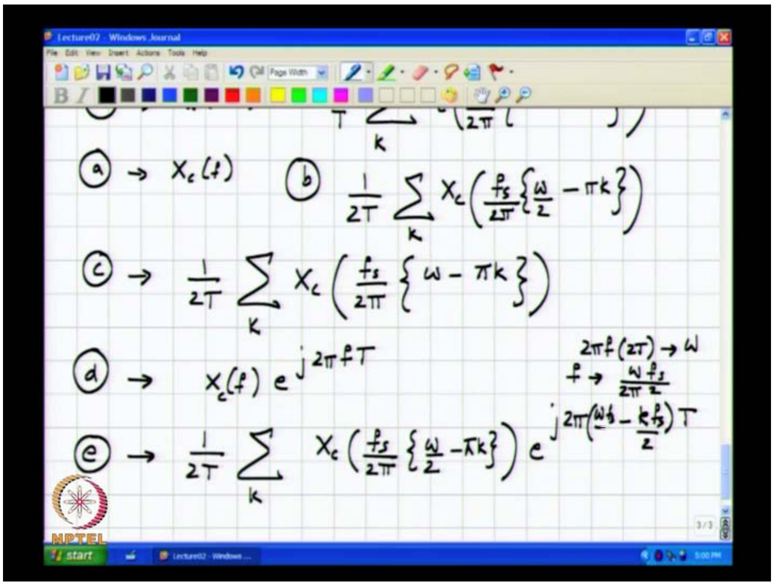
Student: Sample

A sample and hold

Student: F s

Operating at f s, so obviously, the first question that would come to you is apart from here ((Refer Time: 51:00)), why would you want to do this, and the answer is that the sampling operation which I claim to be difficult to implement is now happening at half the rate. And so let us try and figure out the spectra at various places in the signal chain, this is also good exercise to do to see if we understand, I mean you already know the final answer, we know what the spectrum must be at h.

(Refer Slide Time: 51:55)



Let us remind ourselves with that the x d of e to the j omega at h must be 1 by T, it looks mind you like a sample and hold which is running at f s, so it must look like 1 over T sigma k x c of what did we see just now f s by 2 pi times omega minus 2 pi k. This is what we must get at h, so the spectrum at a is simply, the signal at a discrete time or continuous time?

Student: Continuous time

Continuous time, so this is x c of f at B how do the signal look like, please note that the signal at B is sampled at f s by 2, so all we need to do is replace T with 2 T. So, is that at b is it a discrete time signal or a continuous time signal, after sampling it is discrete, so this is nothing but, sigma k x c of what should I do now, f s by 4 pi times omega minus 2 pi k, which I will write as f s by 2 pi. And pi times k, but actually I think I made a mistake, I think it is. Now, what is the spectrum at C, C mind you is this signal here, what do you think that is, it is up sampled so you are inserting 0, so what is.

Student: ((Refer Time: 55:04))

It is simply scaling of the

Student: Axis

Student: Frequency axis

Frequency axis

So, what should I do I replace,

Student: ((Refer Time: 55:14))

F s by 2 pi I must replace

Student: ((Refer Time: 55:26))

No

Student: ((Refer Time: 55:29))

Omega with

Student: 2 omega

2 omega

So, what must this become

Student: ((Refer Time: 55:48)

Omega minus

Student: Pi k

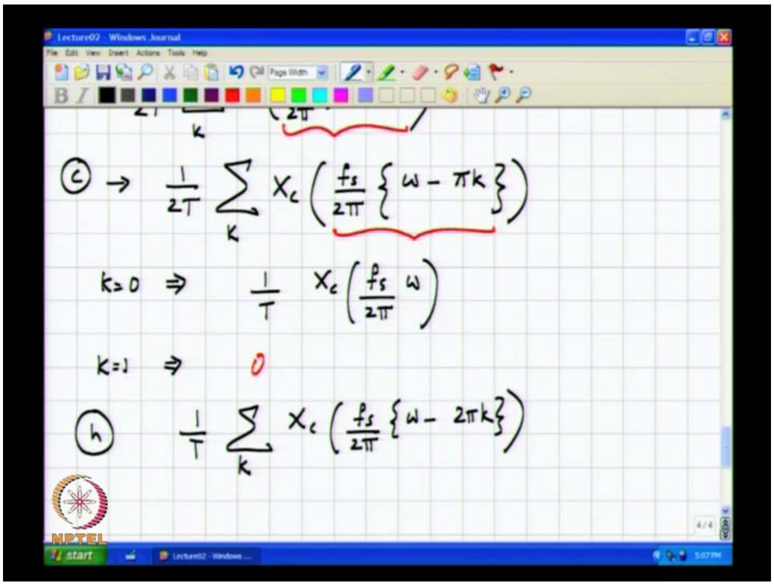
Now, what about the spectrum at d, d is this signal here is it continuous time or discrete time?

Student: Continuous time

It is continuous time, so what is the spectrum at d, x of f e to the x e of f e to the minus, e to the j 2 pi f times T, because it is advancing it is plus T, now what is the spectrum at e, e is nothing but, the sampled version of the continuous time spectrum. So, it must be of the form 1 by 2 T sum over all k x c of f s by 2 pi times omega by 2 minus pi k times e to the j 2 pi. And what should be do, mean how do we get the discrete time expression, you now you form the term f minus k times f s, and replace f with please note that you must replace 2 pi f T with here, the sampling rate is 2 T.

And therefore, sampling rate period is 2 T, so sampling rate is 1 by sampling rate is f s by 2, the delay is T, and I have f and how do I what must I do here I must replace 2 pi f times 2 T with omega. So, how does this look like f must be replaced with omega by 2 pi times f s by does it make sense, it is the same expressions I am manipulating them with f s by 2.

(Refer Slide Time: 59:11)



Rather than, and which simplifies to recalling that f s times T is 1, we simply see that this expression is omega by 2 the first term minus k pi. So, that is the spectrum at e what would be the spectrum at f, at f it is simply 1 over 2 T sigma k x c of f s by 2 pi times omega minus pi k times e to the j omega minus k pi. Now, what are we doing, we are delaying this by one sample, so this is what how does that look like, g is simply a stuff at f e to the minus j omega.

So, what goes away the plus j omega and the minus j omega go away that makes intuitive sense, because we have advanced the signal here by 1 by a factor T. And we are delaying it by 1 sample, which is also I mean in time it is time T, I mean at c and f what is the sampling rate?

Student: F s

So, that goes away, so this simply becomes e to the minus j k time times pi and what are we doing finally, we are adding the stuff at c with the stuff at f, let me just copy and paste this is what we had per things at c. So, now if we add I mean what is so funny about e to the minus j k pi, it is minus 1 to the power n, so for even values of k it is 1 for odd values of k it is minus 1. So, k equal to 0 how does this look 1 by T x c of f s by 2 pi times omega, for k equal to 1 what happens?

Student: ((Refer Time: 64: 10))

For k equal to 1, this argument is the same as this argument, this is 1 where as this is minus 1, so when you add the 2, the 2 go away and you get 0, so going the same way for all odd k, these things simply vanish you understand. So, this is only valid for, these things will be non zero only for, so that h it will be of the form 1 by T times sigma over all k, all even k which I can simply say as f s by 2 pi times omega minus 2 pi k, which is the same as what we get for a sample and hold, operating at the pole rate.

Well this is where the maths stops from the communication point of view, or the signal processing point of view, when you make a practical system like this, there are a whole bunch of problems. For instance, there might be an offset here, which may not be there in that path, in other words there is a D C offset which is different in both parts. The next thing is that the gains of both the paths may not be exactly the same, the third thing is that, in practice you are not going to implement it like this, you are going to one switch is going to be sampling at 2 T, 4 T and so on.

The other switch is going to be sampling at 1 T, 3 T, 5 t and so on, but in practice what happens is that 2 independent switches, so the exact sampling instance of both these switches may not be may not be exactly T apart. So, you ideally want the first sample and hold to sample the second sample and hold must exactly sample the time T later, in other words, both of these are sampling at a sampling rate of f s by 2 that is every 2 T, but the offset between their sampling instance, must be exactly T.

Otherwise, it will be like this is a small skew, then it will be like one is sampling here, the other one is sampling here, this guy is sampling again here, the other one is sampling here and so on. So, this corresponds to some kind of non uniform sampling of the input, all these will have will cause artifacts in the discrete time spectrum which must be addressed, you understand. So, we will continue with the effect of these artifacts which are fundamental things, which will happen every time you implement time sample sampling system, we will see this in the next class.