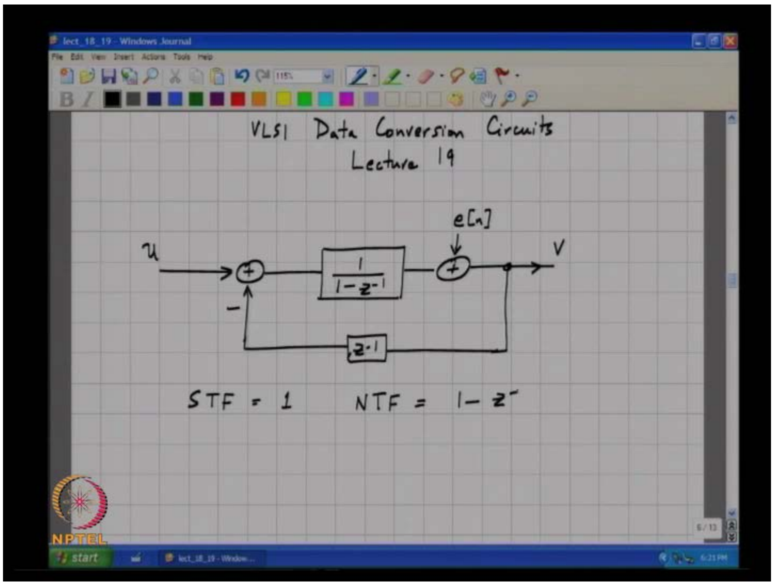
VLSI Data Conversion Circuits
Prof. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture - 19
Delta-Sigma Modulation – 1

This is VLSI data conversion circuit lecture 19.

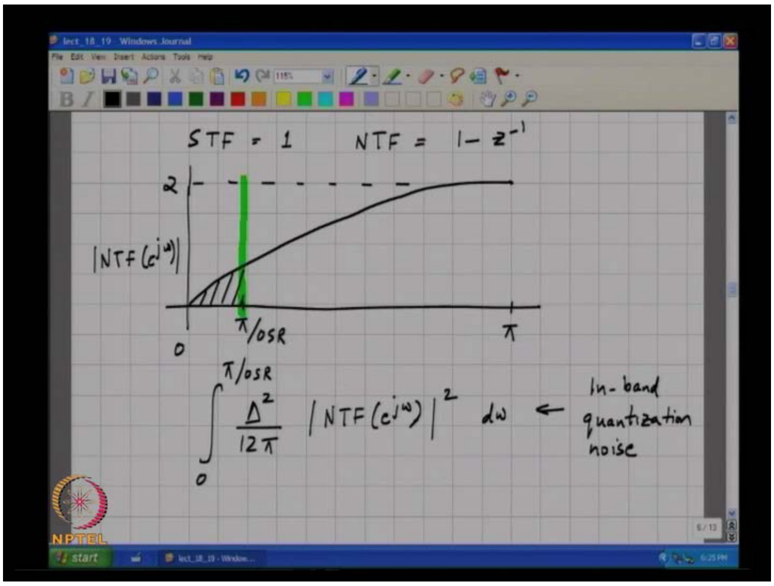
(Refer Slide Time: 00:17)



In the last class we saw that using a high gain block means I mean in front of the quantizer when we place the quantizer inside a negative feedback loop did not really work; because the poles of the closed loop system lying at z equal to minus a. And, if you want good quantization noise separation; we wanted a to be large which will in turn mean that the pole is way outside the unit circle which means that the system is unstable. So, then we said we know that the signal is confined to low frequencies. So, we would be quite happy; if the so called amplifier there had only a large gain at low frequency we do not really care what it does at frequencies in other than low frequency.

And, as a case in point we said let us try and make or choose an amplifier with an infinite gain at D C right. And, the first thing that comes to our mind is an integrator where the gain at D C is in another words it has a pole at z equal to 1 all right. So, when we replaced a with 1 by 1 minus g inverse; we saw that the signal transfer function is given by 1. And, the noise transfer function is given by 1 minus z inverse, the noise transfer function.

(Refer Slide Time: 01:55)



The noise transfers function; therefore relates the effect, the quantization noise has on the output sequence. And, in the frequency domain if we need to plot the magnitude response of this noise transfer function what would you do? You evaluate the noise transfer function at on the unit circle as you go from 0 to pi correct. So, if you evaluate mod NTF of e to the j omega; at D C z equal to 1. So, magnitude of the noise transfer function is 0; at z equal to I mean at omega equal to pi what will it be? At omega equal to pi as z is equal to minus 1 which means that this will go to 2.

So, in other words the noise transfer function varies from 0 to 2 like this all right. And, where is our signal band, the band of interest? Is only from 0 to pi by OSR; and we know that after this system in other words after we do this. So, this mind you is the quantizer all right; we have just taken a quantizer and put it in a feedback loop that is all. So, this output sequence V of (n) is quantized is a digital sequence. And, what are we suppose to do after this; we need to low pass filter this with a very sharp filter whose cut-off frequency is at pi by OSR.

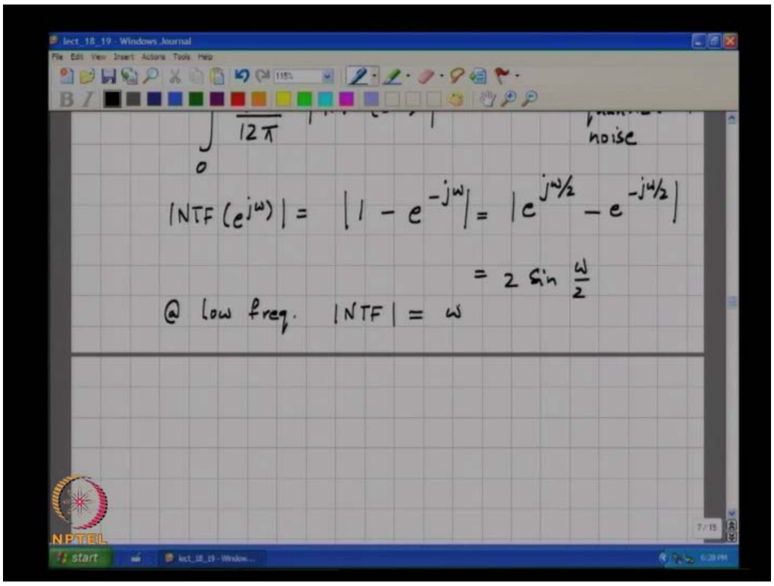
So, in other words with that understanding therefore we do not really worry about the fact that this gain here is much greater than 1; we are only interested in that portion of this noise transfer function which lies between 0 to Pi by OSR. So, in other words we are interested in integrating the noise after this quantization noise has been passed through a transfer function which looks like this. So, we are only interested in this part correct. So, what is the spectral density of the quantization noise?

The quantization noise we have assumed to be white. And, therefore it is spectral density must be delta square by 12 spread across all the way from 0 to pi. Now, this is passed through a filter with some transfer function correct. So, what do you think the spectral density of the noise will be after it passes through the transfer function?

Student: ((Refer Time: 05:32)).

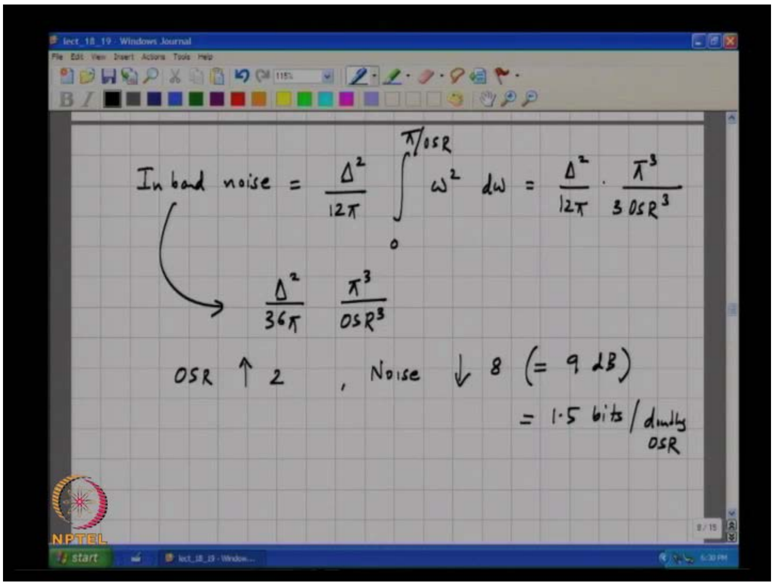
A delta square by 12 times mod NTF of e to the j omega; the whole square will be the spectral density of the quantization noise at the output correct. But we know that we are going to take this and filter it with the brick wall filter going from 0 to Pi by OSR. So, the noise power will therefore be please note this is a function of frequency now. So, what should you do? You integrate this from 0 to Pi by OSR. So, this gives you the in band quantization noise does it make sense all right. So, let us pull on.

(Refer Slide Time: 07:13)



And, what is NTF of e to the j omega; the modulus of NTF of e to the j omega is nothing but1 minus e to the minus j omega; which is e to the j omega by 2, minus e to the minus j omega by 2 which is e to the minus j omega by 2 times sin omega by correct. The magnitude of this please note this going to erase these thing; this is nothing but the magnitude of e to the j omega by 2 minus e to the minus j omega by 2 which is sin of omega by 2 multiplied by correct; of course at low frequency what is mod NTF is approximately goes as omega correct.

(Refer Slide Time: 09:01)

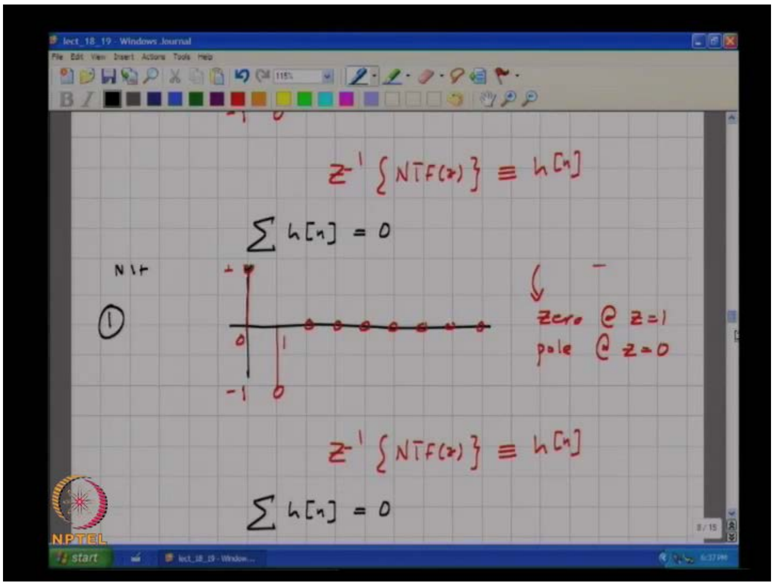


So, when we integrate the in band noise power. So, in band noise is nothing but delta square by 12 pi; it is by 0 to pi over OSR times omega square b omega. And, this is delta square over 12 pi times pi cube by OSR the whole cube times one-third all right. So, as you can see the in band noise is delta square by 36 deltas square by 36 pi times pi cube by OSR all right. So, if OSR goes up by a factor of 2; noise within the signal band goes down by a factor of 8 which is how many d B?

Student: ((Refer Time: 10:37))

9 d B all right; which is how many bits? 1 and a half bits; for doubling the OSR. So, definitely we seem to be doing better than blindly low pass filtering the over sampled signal right. So, by use a negative feedback; where we take the quantizer and put it inside a feedback loop. And, where the so called forward amplifier has got only high gain at low frequency; we see that in the signal band which corresponds to 0 to pi by OSR a lot of the quantization noise has got a shaped or being pushed out of the signal you understand.

(Refer Slide Time: 12:07)



So, this kind of convertor is not only exploits oversampling; it also what is called noise shaping I mean which is nothing but a fancy way of saying as far as the noise is concerned is the transfer function from the noise to the output is what kind of filter if you think about it? It is rejecting low frequency and letting high frequency through. So, this is basically a high pass characteristic all right. So, there the use of feedback has essentially made the quantization noise go through a high pass filter.

And, this is I mean say and this structure basically exploits not only noise shaping I mean not only over sampling but also negative feedback to give you noise shaping; where we get a I mean significantly higher benefit to oversampling when compared to simply low pass filtering the over sampled sequence which is being quantized right. And, doubling the OSR gives us 1 and a half bits rather than simply half a bit that we are getting earlier is that clear; which means therefore, that if you want to make a 6 bit quantizer look like a 10 bit quantizer what do you need to do? How much do you need to oversample by now?

4 by 1 and a half for us roughly 3. So, 2 power 3 you know 8 x over sampling is good enough; if you implement the noise I mean if you implement, if you place the quantizer inside a negative feedback loop where you also shaping a noise away from the signal band. A couple of things I would like to draw your intention to 1; what is the impulse response of the noise? I mean clearly there is a noise transfer function which means that

in the time domain there is an impulse response correct. So, what is the impulse response of the NTF sequence?

It is 1 minus z inverse; what is the impulse response?

Student: ((Refer Time: 14:38))

1 and.

Student: ((Refer Time: 14:47))

And, sorry 1 and minus 1 and after that it becomes 0; the sum of the samples must be 0 why?

Student: ((Refer Time: 15:14))

So, what?

Student: ((Refer Time: 15:17))

The sum of the samples of the NTF impulse response must be 0 and why.

Student: It is area 0.

Ah.

Student: It is area is 0 come.

No, why is the area 0?

Student: Sum of samples.

Yeah, the sum of samples is 0; I am asking you why is the sum of sample 0?

Student: ((Refer Time: 15:54))

The D C gain of the noise transfer function is 0. Because of the way we have implemented the loop correct. If the D C gain of a transfer function is 0 it means that if you evaluated at z equal to 1; it will be 0 in other words the sum of the samples is 0. So, in other words this can be written as 1 minus or rather z minus 1 by z. So, where are the zeroes of the noise transfer function?

Student: ((Refer Time: 16:45))

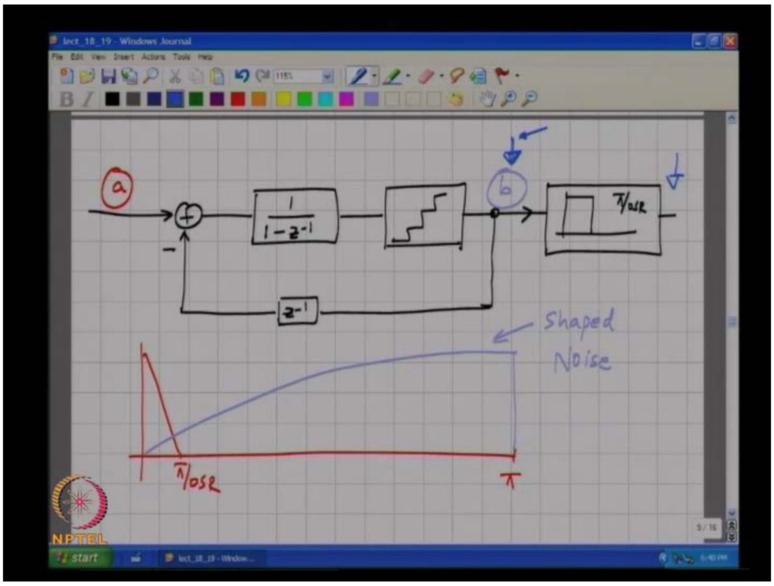
This acts as 0 at z equal to 1 and a pole at.

Student: ((Refer Time: 16:51))

Z equal to 0; if the inverse transform of NTF of z is denoted by h of n then sigma h of n is 0 does make sense all right this is 1 thing I want to point out. The next thing I want to point out is that fine within the signal band; we know that the quantization noise is much smaller than what we had before. But can we comment on the total quantization noise; do you think we are doing better or worse I mean you might argue that it hardly matters because we are only interested finally in the in-band quantization noise.

But just as a matter of curiosity the total quantization noise which is I mean what is the spectrum of the quantization noise before we get to the decimation filter? We just now said that you know if we have this kind of noise shaping convertor; how will the spectrum look at the let me just draw that diagram again.

(Refer Slide Time: 18:34)



So, let me refresh your memory and this is the input signal which is being over sampled then what we have now; therefore, is not a high gain. But we have only a high gain at low frequencies and we have the quantizer here which we have modelled by an additive noise source. And, there is a delay here all right this is our so called noise shaped quantizer right. And, after this the entire system how does it look what we must have a low pass filter whose corner is at pi by OSR. And, then you can drop samples and so on; what I am trying to ask you is what is the spectrum at b? Let us say the spectrum at a was something like this. So, this is pi by OSR and this is pi; what does the spectrum at b look like?

Student: ((Refer Time: 20:07))

It will consist of 2 quantities; one is the signal which is gone through a signal transfer function which is 1. And, the quantization noise which is gone through a high pass filter. So, the spectrum will look like this; this is the spectrum of the shape noise all right. The question I want to ask you is can we comment on the mean square noise here I mean here we know that the mean square noise is very small right; you calculated that already you get some delta square by 36 pi times you know pi cube by OSR cube. But can you comment on the mean square noise here; that is if I took this sequence subtracted the input from it what will be left?

Student: ((Refer Time: 21:18))

Only the shaped quantization noise; if we measure the mean square value of that do you think it will be larger than delta square by 12 or smaller than delta square by 12; larger why?

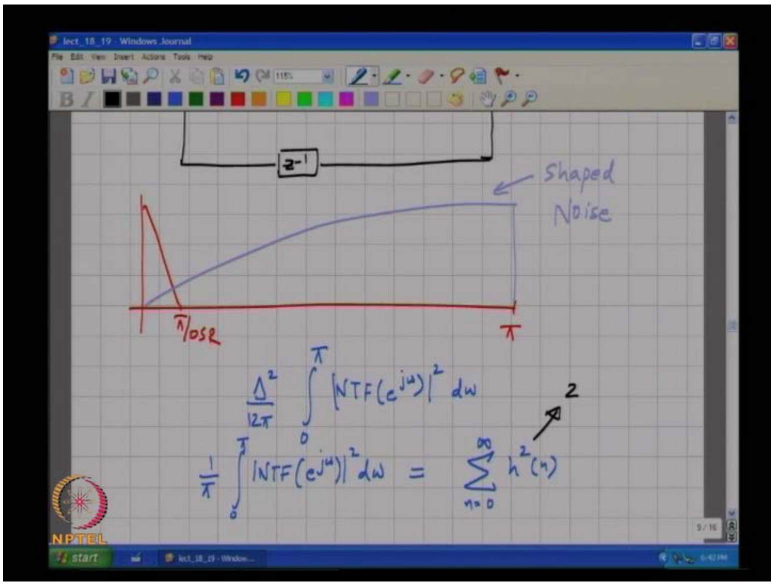
Student: ((Refer Time: 21:42))

So, what?

Student: ((Refer Time: 21:45))

Ok.

(Refer Slide Time: 22:00)



So, then therefore if we want to measure the mean square noise at the output of the noise shaped convertor; what we must do is therefore integrate delta square by 12 pi.

Student: Integral.

Integral 0 to pi N T F.

Student: Instead of small.

E to the j omega, the whole square d omega correct; but can somebody use some clever theorem.

Student: ((Refer Time: 22:22))

Parseval is basically saying that 1 by pi integral 0 to pi mod NTF whole square must be the same as sigma.

Student: Sigma h of the h of n.

h square of (n) correct. So, what must this mean which is what? Which is 2 all right So, the mean square value of the quantization noise here at b is in fact

Student: ((Refer Time: 23:09))

Greater than what you had without noise shaping all right. In this particular example it turns out to be twice what we had without noise shaping; without noise shaping you would get delta square by 12. Now, you are seeing twice that and that need not bother us; because anyway we are going to pass this through a low pass filter which is going to clean up all the stuff outside. The signal band which is 0 to pi by OSR does it make sense.

Student: Sir why it is? So, actually where we actually adding noise in the path that it is increased or we are just delta square by 2 or delta square by 6.

No, you are taking I mean you. So, as somebody pointed out right; the transfer function is not I mean if we did not have noise shaping the transfer function for this quantization noise would be 1 throughout the. And, the integrated noise is proportional to the square of the transfer function correct. Now, what are we having the noise transfer function has got a gain of 0 at D C; I mean I am actually my diagram is not representative. Because sin must look like that must look like something like I think it look like this; no, I am sorry let us write still find no. But this is like this and the gain is actually, the gain is 2 right. But what matters is the square of the gain.

Student: ((Refer Time: 25:18))

Correct.

Student: You want to see that if the physical picture; we are not actually adding noise anywhere.

No, we are right; because this is I mean this is where we are adding noise all right. And, if you selectively amplify it over a certain region; it means that it will get amplified more that is all. Because we are just taking this flat spectral density and passing through a high pass filter; where the gain of the high pass filter in some frequency bands is much greater than 1 and what matters actually the square of the gain right. And, when you integrate all of it just turns out to be 1.

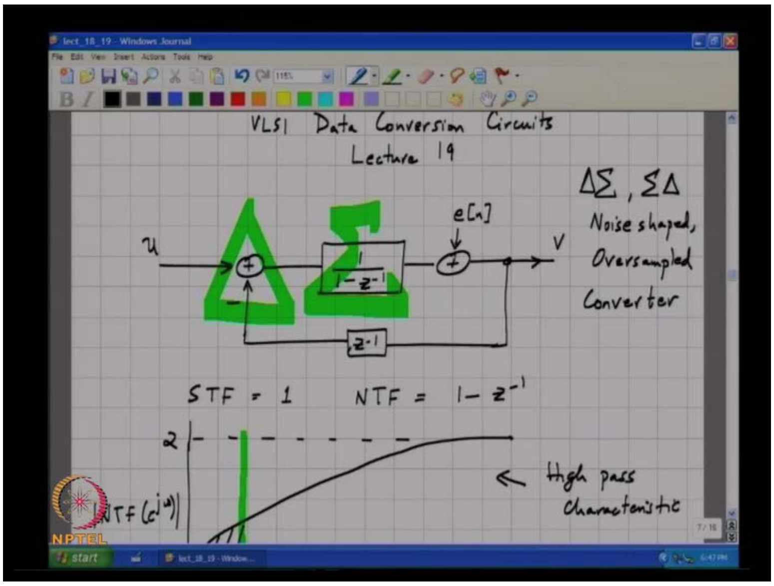
Another way of thinking about it in the time domain is that the impulse response is 1 and minus 1.So, if we had a component of noise at omega equal to pi; omega equal to pi corresponds to a sinusoid where the next sample is the inverse of the previous sample correct. So, noise frequencies close to omega equal to pi; when you convolve it with an impulse response 1 comma minus 1 right will actually.

Student: ((Refer Time: 25:38))

It takes the difference between successive samples those. So, there will conceptual interfere giving a power which is actually 4 times; because the conceptual interfere right those frequency components. So, it will give you a gain of 4 and it is of course a long why not way of saying you multiply NTF; e to the j omega whole square and integrate you understand great. So, I mean. So, basically you can see that it is taking garbage from your house and then you dump it on in your neighbour's place that is all right.

As long as your signal band is clean you really do not care what happens outside; because you are hoping that you are decimation filter will clean up everything beyond the desired signal band.

(Refer Slide Time: 27:32)



And, now if you think about it all right; so, this is an integrator. So, what is the integrator doing it is summing. So, this is sigma all right and what is the, what is this guy doing? It is difference and that is delta. So, this is also called a either a delta sigma or a sigma delta or a noise shaped or an oversampled you understand. So, you know sigma delta, delta sigma are all used interchangeably. And, since because there is a feedback loop you call it as a delta sigma loop sometimes; all these refer to the same basic structure. And, basically the delta sigma loop is associated with the signal transfer function and a noise transfer function; it is also associated with an oversampling ratio that is the band over which you would like to integrate the quantization noise. Again, mind you with the understanding; that we have an ideal decimation filter after the delta sigma convertor ok.

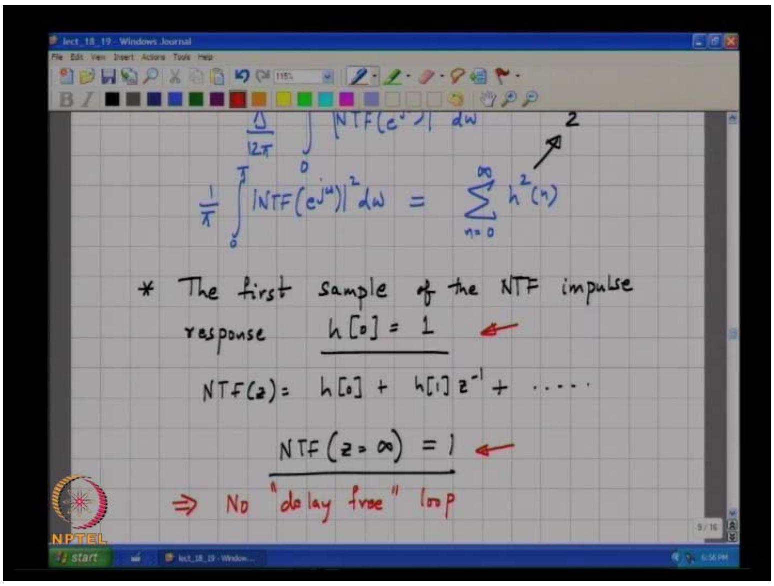
Now, yet another observation I would like to bring out at this point is the fact that if you look at, if you found; if you want to find the impulse response from the quantization noise port to the output port what will you do? I mean apart from writing the math if you are for example, given this box in a lab and you have 2 input port; one was the signal, one was the this port e. And, you wanted to find that the impulse response of the transfer function from the input port to the output port what will you do?

Student: ((Refer Time: 30:03))

I will put an impulse and then measure the sequence which comes out. So, if I did that to find the noise transfer function; if I injected an impulse here all right what do you think the output sequence will be it will be 1 minus 1. The minus 1 I mean how do you get the minus 1; because the first sample that comes out must be 1 why? What I am trying to point out is that the first sample that comes out of v; at v must be the impulse that you injected into the port e why?

Because you know the rest of the feedback loop is only going to see that impulse in the next clock cycle or the next sampling instant. So, this output must be 0 in the beginning correct you understand. So, regardless of what transfer function I have here; the first sample of the impulse response of the noise transfer function will always be 1; does it make sense all right.

(Refer Slide Time: 31:52)



So, this is a very important aspect; h of (0) must always be equal to 1 ok.

Student: ((Refer Time: 32:22))

Pardon.

Student: ((Refer Time: 32:25))

Well, we will come to that a little going forward. But we have I mean at this point one thing I can say is that I can say that h of 0 equal to 1; regardless of what there is in that regardless of what I put here h of 0 is always equal to 1 correct. And, please note that h of (0) corresponds to NTF evaluated at z equal to I mean NTF of (z) is h of (0)plus h of (1) z inverse plus bla, bla, bla correct. So, h of (0) equal to 1 means what?

Student: ((Refer Time: 33:33))

Evaluating NTF of z equal to infinity this must be equal to h of (0) which is, which must be 1 does it make sense all right. So, this is I mean and why is this happening? The only fundamental constraint that results in this equation is the fact that this loop must have at least 1 delay correct you understand. And, that is because the quantizer output is not available immediately right. So, the rest of the circuitry can only process that output in the next sample; this is exactly analogous to what you have been used to with finite state machines right. There is combinational logic and there is some set of states right ok.

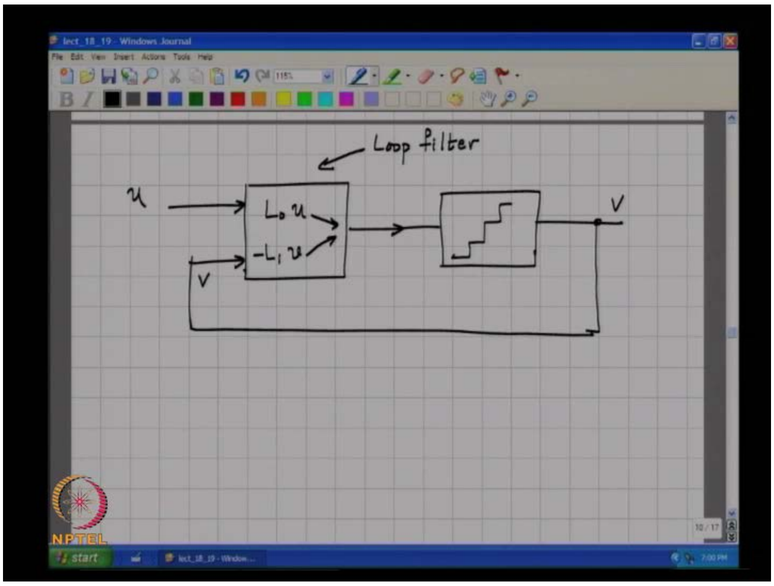
So, the next state is nothing but present state is some function of present state and the input. The next state cannot be a function of next state and input you understand is that clear. So, this in other words this feedback loop must have at least 1 delay. And, this constraint which quantifies that is equivalent to saying that there are no in the literature this is what is called no delay free loops in this case; do you follow h of (0) must be equal to 1. And, in the frequency domain this corresponds to NTF evaluated at z equal to infinity being equal to 1 and what kind of filter is the N T F? It is a high pass filter.

So, in other words if somebody gave you a high pass filter transfer function and told you that this is an NTF; how will you check what is 1 sanity check you must do?

Student: ((Refer Time: 36:32))

If somebody gave you NTF of z; and I mean some high pass filter of transfer function right. And, told you that this is a noise transfer function of a delta sigma modulator; 1 sanity check is to make sure that when you evaluate that transfer function at z equal to infinity you must get 1 right you understand. And, this is coming because of the delay free loop not being tenable all right.

(Refer Slide Time: 37:29)

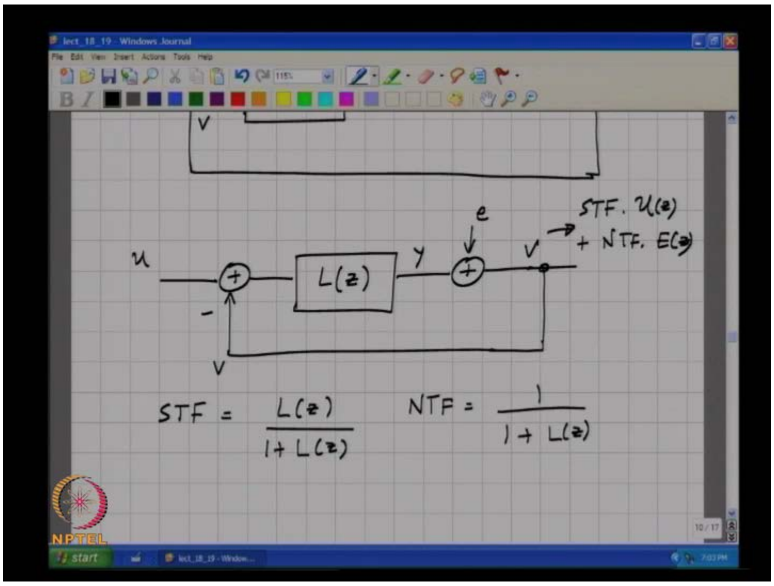


And, let me now also introduce you to some more jargon or rather some standard names for the various quantities involved this is u. And, going forward it is likely that we replace this 1 by 1 minus z inverse by a more complicated block. I mean see once you tested blood you know, you only get greedier right; you say hey by putting an this in a feedback loop you seem to have got an we are putting 1 integrator; no, integrator half a bit 1 integrator1 and a half bits 2 integrators may be god knows right. Hopefully, it is much better than 1 and a half bits per doubling OSR right. So, you bound to get greedy.

So, in other words this is not we are not going to sit quite with what we have. So, in general you have some L of (z) right. And, this is the quantizer this is the output of the quantizer which is denoted by v. And, this quantity at the input of the quantizer is denoted by y; I mean the pneumonic to figure out what is y and what is v is that you take y. And, then you screw it up oh sorry; and the quantizer basically messes it up and the output is v ok. So, this is the good way of remembering the stuff. So, the input to the quantizer is y; the output of the quantizer is v all right.

And, then one of you may argue that why should. So, what is going into this block L of (z) is nothing but u minus v all right. And, one of you might argue that why should u and v see the same transfer function correct. It is perfectly legal for instance to have this where you have some contraption which takes u, another contraption which takes v and generates L naught times u plus L 1 times v; in this fashion or you say I mean because you want to include the inversion here you put minus L 1 times. And, this is just more general form of that integrator right. So, this is often called I mean this is basically a filtering network. So, this and this sits in the loop. So, this is called the loop filter ok.

(Refer Slide Time: 41:19)



And let me take a couple of examples of loop filter; one of course is the special case if I had L of (z) here, this is u, this is v the quantization noise is modelled by an additive sequence e this is y, this is v. So, what is the STF of the signal transfer function?

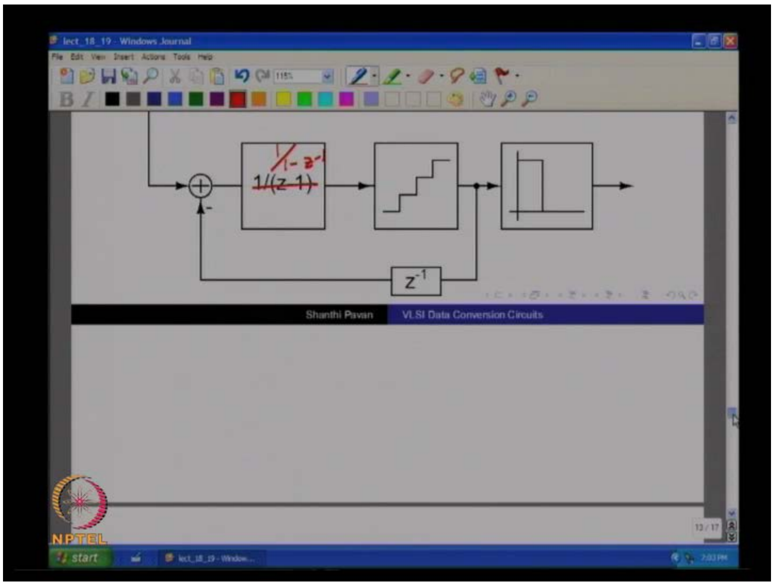
I mean. So, one thing I have forgotten to do here is to insert the z inverse. But then one might say I might as well push this z inverse into the I mean I have pushed this into this block right. And, call this the loop filter instead all right; this is a z of z inverse is already pushed in is the assumption. So, in this example what is the S TF? L of (z) by 1 plus L of (z) the NTF is 1 by 1 plus L of (z) correct and the output here will be STF times u plus NTF times e.

So, in terms of time domain sequences the STF is 1; because you want the input signal to pass through unaffected you only want to high pass filter the noise. So, this time domain sequence will basically look like the input plus.

Student: ((Refer Time: 43:35))

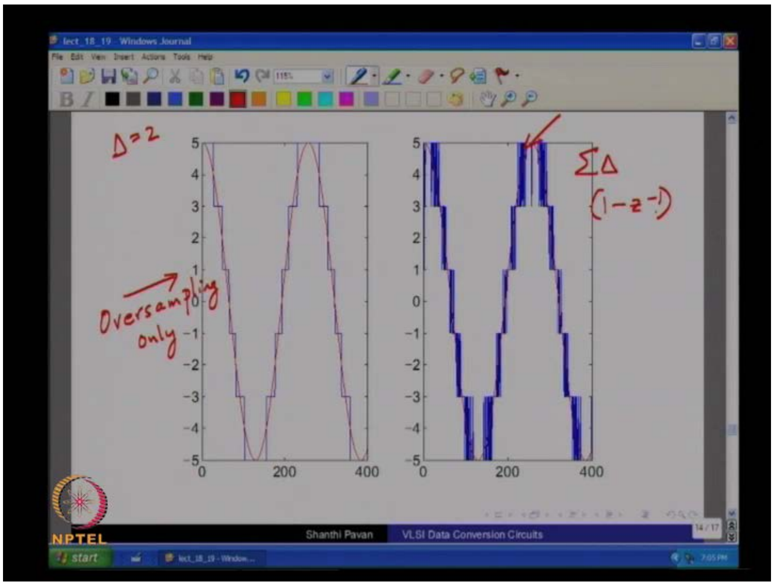
Plus shaped noise you understand e is I mean, I am sorry this must be all capital use correct. So, this must be the output sequence must look like the input plus some noise which is being high pass filtered, shaped high pass filter; high pass filter means that there is a lot of gain at high frequency. So, the output wave form you can expect to be rapidly jumping up and down correct. So, let us take a look at couple of waveforms and compare it with what we have done before.

(Refer Slide Time: 44:31)



So, this is the entire system level diagram; I think I made an error here this must be 1 by 1 minus z inverse.

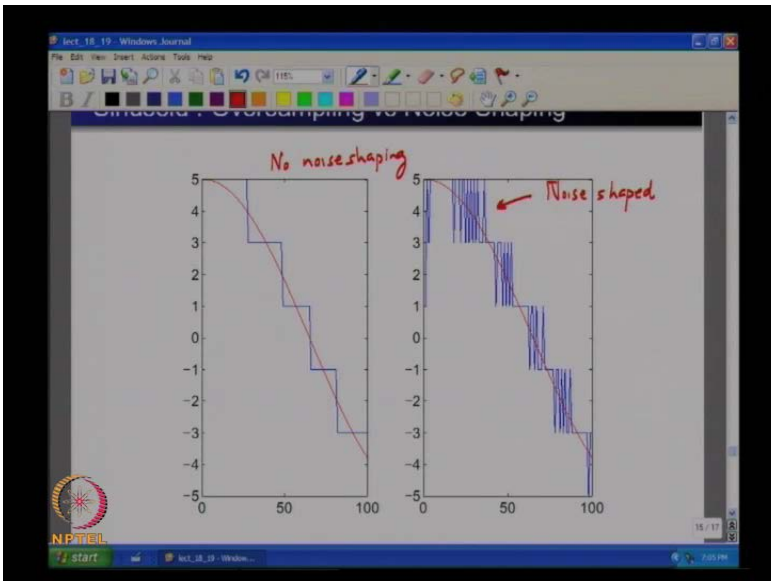
(Refer Slide Time: 44:49)



And, this shows a sinusoid this is only the step size is 2 in this example the red colour shows the input sinusoid, the blue step like waveform shows the quantized sinusoid. And, as you can see the approximation is rather poor correct; the this is over sampling only whereas here on the right side the input signal is still marked in red I do not know if you are able to see it but there is also noise shaping. And, as we expected because the noise shaping involves high pass filtering frequency components at omega equal to pi have a gain of 2 remember.

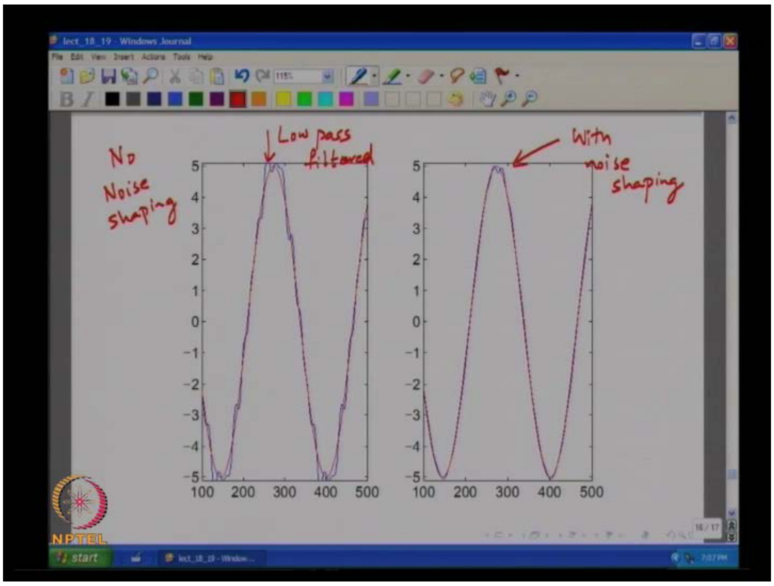
So, you can expect the output to wiggle a lot faster than what we saw in the case without noise shaping; is that clear in both cases there is over sampling. But this is oversampling with noise shaping.

(Refer Slide Time: 46:28)



In other words this is the output sequence of a delta sigma modulator with an NTF of 1 minus z inverse and let us takes a closer look all right. So, this is again no noise shaping; while this is noise shaped is this clear.

(Refer Slide Time: 46:54)

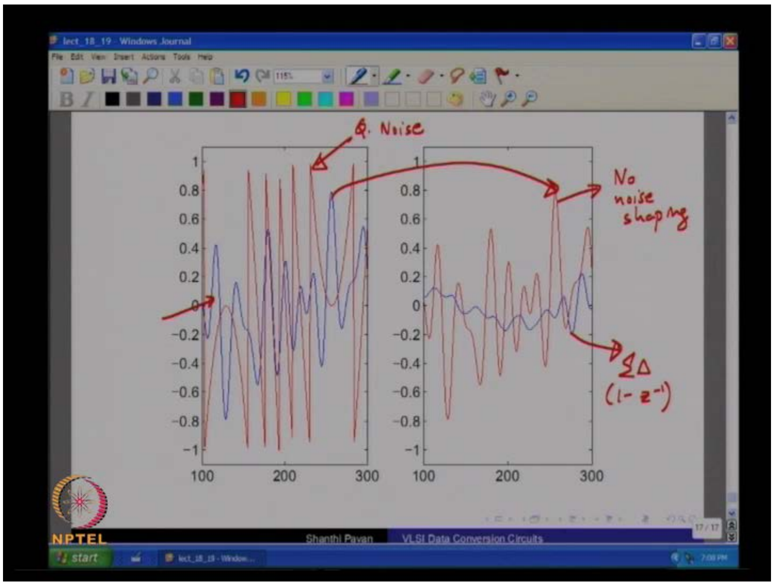


And, it is being instructive also to compare the reconstructed waveforms after low pass filtering. So, again red is the input which has been passed through the same low pass filter. So, that I can do an a to b comparison without any delay right. And, blue is the this have been low pass filtered this is again no noise shaping all right. And, this is with noise shaping and you notice anything or what do you noticed rather?

Student: ((Refer Time: 48:57))

When you use noise shaping the error is visibly smaller. And, you can actually see also plot it.

(Refer Slide Time: 48:10)



So, in this case this is the raw quantisation noise this is before low pass filtering all right; after low pass filtering you see the blue curve this is just oversampling without any noise shaping. And, as we saw last time the mean square value of the noise is much smaller than what you would get with I mean without doing the low pass filtering all right. Now, the same curve has been plotted here. So, this is no noise shaping while the blue curve or blue wave form is the error between the input and the filtered output of the noise shape convertor.

And, as you can see the mean square value is definitely much smaller than the converter that we had I mean with only oversampling and no noise shaping correct all right. I mean I guess at a pinch it would be difficult to imagine that you know such a waveform; has actually a smaller error within the signal band when compared to say this waveform all right. But it is true. So, in fact when we go to better and better delta sigma modulators you will find that the output waveform can look very very messy that make sense; because the gain at omega equal to pi can be.

Student: High.

Very high; so, it will make the waveform look very messy. But the moment you smooth then it properly the approximation to the input will be very very good right. And, as I told you last time this is a way of making a very bad quantizer look like a very good quantizer. So, you can turn the argument upside down and say if I wanted a quantizer with a certain resolution I can do it in 2 ways; one I can actually build a quantizer with that resolution or I can use the principles of oversampling and noise shaping; and make a very bad quantizer look like a like a quantizer whose resolution I am shooting for.

In the absolute limit you can you can go on increasing the oversampling ratio and do noise shaping and make the loop quantizer which is the quantizer inside the delta sigma loop poorer and poorer and poorer right. This does not mean eventually that you remove the poorer quantizer the easiest you can do is make a 1 bit quantizer. And, it turns out that this is routinely done in systems where you take a 1 bit quantizer use oversampling noise shaping with appropriate loop filters. And, make it look like an incredibly precise quantizer right may be 14, 15 bits. So, it is like saying I have a scale 15 bits is 1 part in 32000 correct.

So and 1 bit is you know 1 part in 2 correct. So, it is like saying I have a scale which is only precise to 1 meter right. But I want to use it to measure a distance of 1 by you know 1 by 32000 meters; which is 0.43 millimetres you understand. How can you do this? If I had multiple instances of this small quantity I want to measure; one thing I could do is stack up many of these guys right use the ruler to measure it. And, then I will you know. And, stacking up if I had multiple instances of the same unit that I want to measure is equivalent in signal processing terms to have a sequence; which does not change or changes very slowly. So, we will get into that now that you have a basic idea; we will get into the details in the next class.