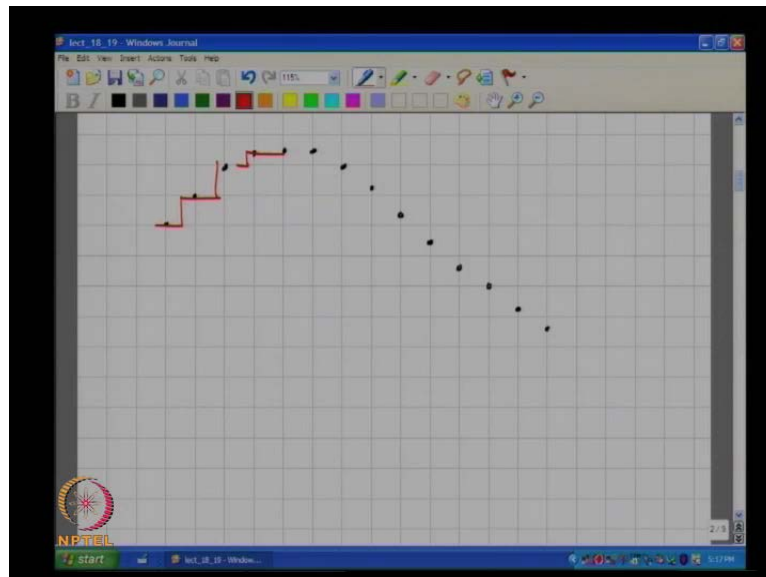


**VLSI Data Conversion Circuits**  
**Prof. Shanthi Pavan**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture - 18**  
**Oversampling and Noise Shaping**

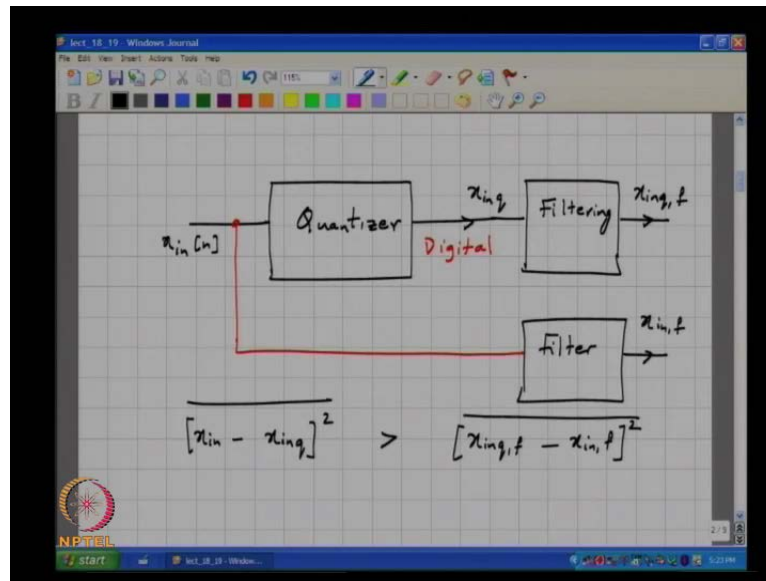
This is VLSI data conversion circuits lecture 18; in the last class we were looking at the benefits of oversampling as regards in our quantization noise reduction in the signal band. And, we saw that apart from the usual benefit that over sampling gives us in terms of relaxing the specifications of the anti alias filter; one can also take a look at the oversampled sequence right.

(Refer Slide Time: 00:45)



For example, if this was the input sequence and we knew that it is changing very slowly with respect to the sampling rate; which means that between successive samples the signal is not changing very much; the moment you quantized it with a very coarse quantizer the waveform is probably going to get a in approximation to the waveform is something like that. And, even visually you can see that; if you knew that the signal is changing very slowly. Then, you can create a signal from the quantized samples; where the error between the input and the signal that you have generated; which is a smoothed version of the quantized signal is much smaller than the quantized signal itself; you understand?

(Refer Slide Time: 01:58)



In other words you have a signal let us say it is a sinusoid let us call that  $x$  in of  $n$  this is now bunch of samples; assuming  $x$   $n$  is over sampled highly; in other words  $x$   $n$  is very correlated from sample to sample. If you quantize it and look at the quantized signal  $x$  in  $q$  right intuitively we saw last time that simply smoothing this quantized signal gets rid of I mean when you smooth something what you do?

Student: ((Refer Time: 02:41)).

I mean you get rid of all high frequency components. And, that does not affect our desired signal because we know that the signal is limited to.

Student: ((Refer Time: 02:52)).

Low frequencies only right. So, if you think of the quantizer as being a combination of I mean as being an additive noise source. Then, what we are in principle doing is taking this output sequence; which is the input sequence plus some quantization noise which we assumed to be white. And, you know uniformly distributed and all that stuff. And, that smoothing is basically getting rid of the high frequency components of the quantization noise; while it does nothing to.

Student: The signal.

The signal because a signal lies within the pass band of the filter. In other words I mean it is a very loose way of saying that if the signal is I mean varying very slowly; it is so smooth that smoothing it will not change it at all right smoothing is only going to get rid of its.

Student: ((Refer Time: 03:56)).

Of the rough edges right if the signal is smooth to begin with; then smoothing it any further you will hardly make any difference. So, smoothing is nothing but filtering. So, all that I am saying is that by taking this quantized sequence. And, banking on a knowledge that the input is varying very slowly from sample to sample; the simple act of smoothing which is a very loose term for filtering. Now, what kind of filtering must we do?

Student: Digital filter.

I mean of course it is a digital filter because this output sequence is a.

Student: Digital.

This is a digital sequence; why is it a digital sequence? Because it is of course already sampled in time now the number of levels are also.

Student: Discrete.

Discrete therefore this can be represented by a finite number of.

Student: ((Refer Time: 04:56)).

Of bits per sample. So, if the output of filter is basically a smoothed version of the quantized signal. And, last time around we saw we actually plotted the error between the smoothed or the filtered quantized signal. And, the true input and we saw that it is much smaller than the difference between  $x_{in}$ . So, let me if I call this  $x_{in}$  filtered right the error or the mean square value of  $x_{in}$  filtered; I mean with what signal must you compare  $x_{in}$  filtered?

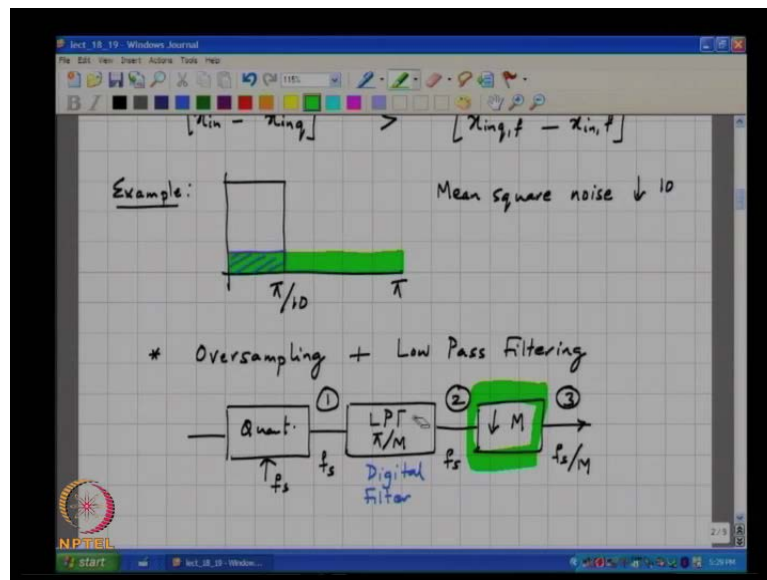
Student:  $x_{in}$ .

I mean ideally you cannot compare it to the  $x$  in because you know  $x$  in there is a lot of delay because of the filter. So, in order plot these error quantities what you must do is pass  $x$  in through the.

Student: Same.

The same filter; I mean this is just a mathematical construction to measure the errors correct. So, if you if  $x_n$  is very deep inside the pass band of the filter which is what it means to say the signal is very low frequency. Then, this will be  $x$  in filtered and the error the mean square error between  $x$  in and  $x$  in  $q$  must be larger than the mean square error between  $x$  in  $q$  filtered and  $x$  in filtered; does it make sense?

(Refer Slide Time: 07:12)



And, the last time around our filter was a brick wall filter with a band edge of  $\pi$  by 10. So, and the quantization was noise assumed to be uniformly distributed between  $0$  to  $\pi$ . So, we can calculate the in band noise that is after filtering the mean square noise must be.

Student: Point.

You will only get that component of the noise within the pass band of the filter; assuming the filter is a brick wall type filter we get only that much. So, the mean square noise must go down by a factor of 10, right. So, from this what we see is that oversampling and filtering it is very important to not to forget the filter; it does not make

any I mean it would not really help. If you oversample and then you look at the unfiltered sequence. If you simply quantize it the mean square error will still be equal to  $\frac{\Delta^2}{12}$ ; where  $\Delta$  is the LSB of the or the step size of the quantizer correct.

So, oversampling plus smoothing or low pass filtering will result in a smaller mean square noise. And, if this filter is a perfect brick wall filter after the quantizer; so the system basically looks like this you have the quantizer; then you have the a low pass filter. Let us say the corner of the low pass filter is  $\frac{\pi}{M}$ ; where  $M$  is some large number; it follows that the signal must be.

Student:  $\frac{\pi}{M}$ .

I mean the maximum frequency content of the signal must be less than.

Student:  $\frac{\pi}{M}$ .

$\frac{\pi}{M}$  and then the spectrum at this point at the output of the low pass filter will only be restricted to.

Student:  $\frac{\pi}{M}$ .

$\frac{\pi}{M}$ . So, in principle even if you down sample it by a factor of  $M$  correct; you are not going to lose any information right. Because there is no frequency content given that this is a brick wall filter all its frequency content lies between 0 and  $\frac{\pi}{M}$ . So, in principle one can down sample this by a factor  $M$  and you will therefore get samples at a rate. So, this is sampling at a rate  $f_s$ . So, what is this the rate of samples coming out at the output of the quantizer at  $0.1f_s$ ?

Student:  $0.1f_s$  because.

$f_s$ ; what about the rate of samples coming output at  $0.2f_s$ ?

Student: ((Refer Time: 11:11)).

It is  $f_s$  and what is the rate of samples coming out at?

Student:  $\frac{f_s}{M}$ .

Its  $f_s$  by  $M$  all right. And, please note that this is a digital filter all right. So, it is operating on the digital numbers generated by the quantizer. And, giving out a digital sequence which can be I mean where you can drop samples; and eventually take samples out at the rate of  $f_s$  by  $M$ ; does it make sense?

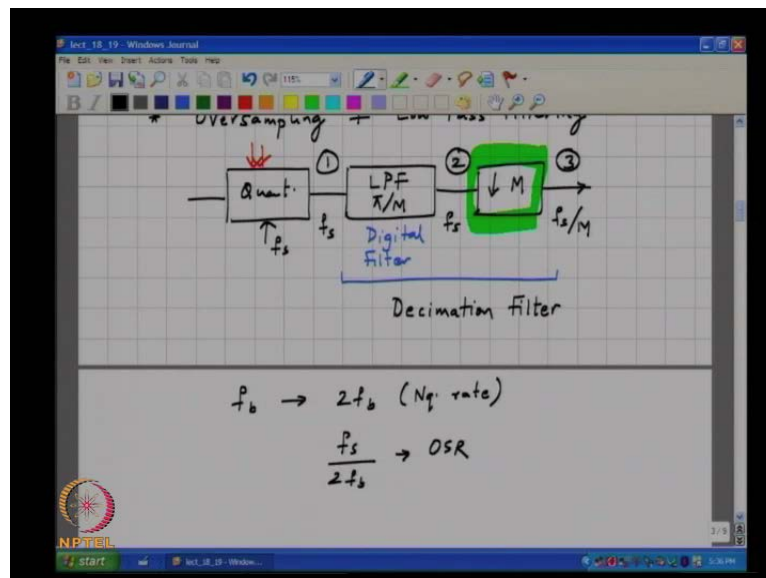
Student: While we are doing it comes out.

Well, I mean so if there is if the signal is very low in frequency; it means that it would not need to collect all samples; you can actually.

Student: Drop samples.

Drop samples. So, that you still satisfy ((Refer Time: 12:35)) at the end of the whole process; I mean please note that without the low pass filter; if this low pass filter was not there. Then, you cannot down sample because there is quantization noise going all the way from 0 to  $\pi$ . And, if you down sample this quantization noise from outside the signal band will all aliases back into the signal correct.

(Refer Slide Time: 13:15)



So, effectively this whole system is looking like a quantizer whose mean square noise is.

Student: ((Refer Time: 13:27)).

Is delta square by 12 divided by?

Student: M.

M; where M is some number; if somebody gave this whole system in a box to you and told you look this a quantizer; obviously because the output is digital, input is analog go and tell me what the resolution is what will you do?

Student: ((Refer Time: 14: 03)).

I mean what one would be to put in a sine wave.

Student: Can measure the mean?

Measure the mean square error equate that to delta square by 12; from that you find delta which will give you the resolution of the quantizer; given that you know the full scale correct. If you did that experiment on this box where instead of having a real quantizer you had this arrangement. Then, you would be fooled into thinking that the resolution you have inside the box is much higher than the actual resolution of this quantizer, this guy here; is this clear? In other words, this way of signal processing which is oversampling, quantization.

Student: Low pass.

Low pass filtering and dropping samples is a way of making a poor quantizer look better all right; you understand?

So, let me now mention some jargon. So, this digital filter followed by dropping samples is what is called the decimation filter; decimation is just the act of dropping samples. So, you are also filtering and not very surprisingly it turns out that instead of filtering at a high digital rate. And, then dropping samples correct; does somebody see any problem with this, do you think this is smart thing to do from a implementation point of view.

Student: Reverse the orders.

You cannot reverse the order because.

Student: They are ((Refer Time: 16:11)).

Or rather I mean I am not asking you for what the smartest way of doing it is. But at least can you comment on if this is a smart thing to do at all; I mean what kind of digital filters are you familiar with?

Student: FIR.

Say FIR So, if you want to get a very sharp filter a what do you think; how many do you think there will be a lot of taps in the FIR or there will be very few taps?

Student: Lot of taps, lot of taps in the FIR.

There will be a lot of taps in the FIR because you want a very selective filter correct. So, you have a lot of taps I mean an FIR filter is nothing but tap delay line. So, you delay and multiply and add. So, you have a lot of taps it means that you have a lot of multipliers right. And, the power consumption of a digital filter is proportional to the number of gates times the rate at which they are operating, correct? So, if you look at what you're doing here you have a low pass filter where you are doing this FIR filter its operating at the.

Student: ((Refer Time: 17:21)).

Digital rate  $f_s$ . And, then after you do all this computation your saying I do not need I need only one in  $M$  samples right I do not need the other  $M$  minus 1 samples. I mean this I mean then the guy who is designing the filter say you know if you need only one out of  $M$  samples I mean you should have told me that before right that is your making me work unnecessarily hard by making me compute all the  $M$  samples right. And, then.

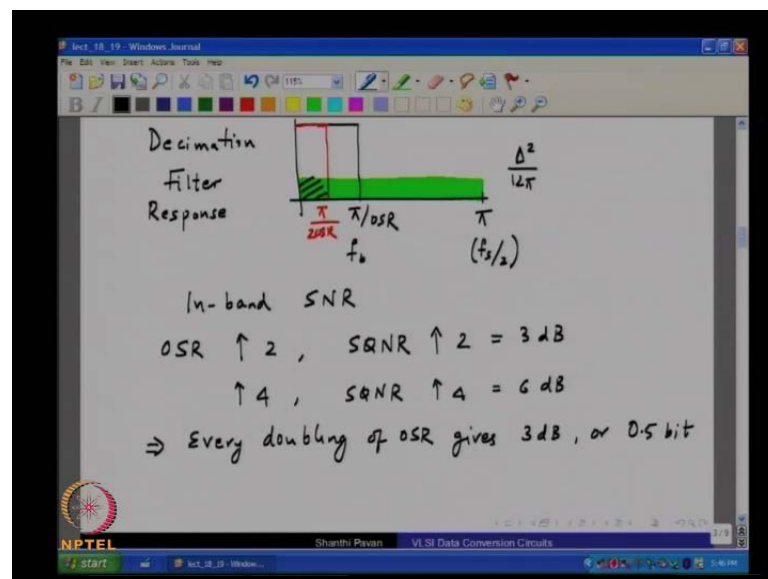
Student: Dropping  $M$ .

Dropping  $M$  minus 1 of them; it is like giving you a assignment I mean 100 assignments. And, then saying I am going to only take one and you know every 10 right without actually telling you before beforehand; I am sure you would not be very happy with that right. So, well while it is not clear what the smartest way of doing it is definitely doing this as is not a very par efficient way of doing things. So, it turns out that you; I mean you can actually combine this operation of decimation and low pass filtering into one power efficient unit; where both of them are accomplished in stages ok.



If we have time we will go into that but otherwise that is where I will let it stand. And, this is called the decimation filter there are well established techniques for doing this; where you do sampling rate conversion as well as low pass filter. So, and also the as I said before the ratio of this signal bandwidth to the sampling rate in other words; if  $f_b$  is the signal bandwidth the nyquist rate is  $2 f_b$  right is the nyquist state. And, the actual sampling rate is  $f_s$ ; so  $f_s$  by  $2 f_b$  is called the over sampling ratio or the OSR all right.

(Refer Slide Time: 19:51)



So, therefore the digital filter therefore will have ideally a cut of rate corresponding to  $f_b$  mean turns out its new jargon.

Student:  $\pi$  by 2.

$\pi$  corresponds to  $f_s$  by.

Student: 2.

2 the signal corresponds to  $f_b$  correct which in radian corresponds to.

Student:  $f_s$ ; ((Refer Time: 20:24)).

$f_s$  by 2 is  $\pi$  correct; which means that  $f_b$  must corresponds to.

Student:  $\pi$  by.

Pi by o sampling ratio OSR. So, this is the decimation filter response I mean in principle; of course yeah once you actually start designing the digital filter you know that no brick wall kind of response can be realized. Then, you start you can start worrying about how sharp should I make this and so on; what order will I need right. And, there I mean that is all left to the guy who is the digital guy who is designing the decimation filter. But as far as we are concerned we can say digital filter design is a mature thing everybody knows what to do ok. So, all that we are going to do is we assume that there is a brick wall filter which goes from 0 to.

Student: Pi by OSR.

Pi by OSR. So, when we integrate noise we are only interested in that part of the spectrum which is in the band 0 to pi by OSR. And, that is call the in band signal to noise ratio or an in band signal to quantization noise ratio. Now, if I double the over sampling rate. So, OSR goes up by a factor of 2; what you think will happen to my in band quantization noise? So, in other words I have the same signal band width I double the over sampling ratio which means that my sampling rate has.

Student: Increased, doubled.

Has doubled correct. So, the question I have is what happens to my in band signal to quantization noise ratio?

Student: Decrease, double, more uniform.

What will happen to the mean square noise after I low pass filter it?

Student: ((Refer Time: 23:01)).

Pardon.

Student: It will increase by 3.

Increase or decrease I mean if you are oversampling ratio has increased; do you expect to do better or worst?

Student: Better.

We expect to do better so the noise must.

Student: Over noise reduce.

Reduce; by how much do you think it will reduce?

Noise power, noise becomes power becomes ((Refer Time: 23:28)).

I mean if the OSR goes up by a factor of 2 then what happens as you can see here is that; if you think about it in terms of radiance only all the time. Then, your low pass filter band width has gone to.

Student: Pi by 2 OSR.

Pi by.

Student: 2.

2 OSR right the quantization noise spectrum is flat from.

Student: 0 to.

0 to.

Student: Pi.

Pi. And, what will be the density; if it is flat what will be the height of this spectrum noise spectrum; assuming its white and all this stuff?

Student: Quantized to.

It will be delta square by.

Student:  $12 \pi$ .

$12 \pi$  that is the spectral density correct. So, within a band width of  $\pi$  by 2 OSR obviously the noise power is now gone down by a factor of.

Student: 2

2. So, increasing the OSR by a factor of 2 increases the in band SQNR by.

Student: 3 times.

Student: ((Refer Time: 24:54)).

So, the SQNR goes up by a factor of 2 which is.

Student: 3 d B.

3 d B signal power remains the same quantization noise power has gone down by a factor of 2; which means that signal to quantization noise ratio has gone up by a factor of 2 which in d B terms is 3 d B.

Student: Sir, that should be LSR by 12 or delta is I mean pi or 2 pi noise.

No, no, no, no is.

Student: On 1 side it is.

Is it is on side all right; I mean if you want to take 2 sided then you have to integrate this I mean this also the filtered noise also from minus pi by. So, you can take this further and say if I increase the oversampling ratio by 4 the SQNR goes up by.

Student: 6 D b.

Yeah by 4 which is 6 d B. So, in other words every doubling of the OSR gives 3 d B or I mean per bit is how many d B? If you increase the resolution of the quantizer by 1 bit SQNR goes down by.

Student: 6 d B.

6 d B. So, this is giving you 3 d B. So, in other words doubling the OSR is equivalent to improving the resolution of the quantizer by.

Student: Half a bit.

Half a bit or half a bit. So, in other words if you take a bad quantizer and quantize an oversample version of the input signal; you can make it look like a.

Student: Better.

Better quantizer than the bad one you used. And, every doubling of the OSR will make that improvement greater by.

Student: Half.

Half a bit say if you want to make a 6 I mean a 6 bit quantizer look like a 10 bit quantizer right; you should increase the oversampling ratio by a factor of.

Student: ((Refer Time: 27: 24)).

6 bit to 10 bit.

It is like 26 bit 2 power ((Refer Time: 27:32)).

It is not 8 it is.

Student: 16 bits.

You are your gaining 4 bits.

Student: 16.

Half a bit for every.

Student: 3 d B.

Half a bit for every doubling of the OSR. So, which is?

Student: 2 raise to.

2 raised to.

Student: 8.

8 you understand which is.

Student: 256.

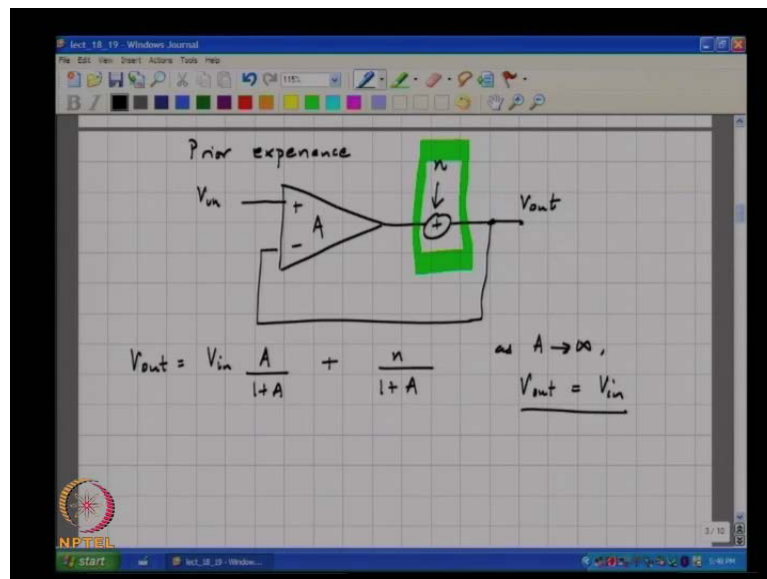
256 Correct all right. So, in other words based on our simple model for the quantizer which is an additive noise model; if we take a signal quantize it to 6 bits right. But we over sample with our OSR of 256; and then we put a digital brick wall filter which cuts off everything above  $\pi$  by OSR. And, then we throw away samples right we decimated by 256. Then, the final sequence that we get right in terms of the quantization noise will be indistinguishable from a 10 bit quantizer; does it make sense? So, we can see

therefore, that oversampling does give benefit and makes the quantization noise look lower; in other words make a poor quantizer look great. But now we get greedy and say we are only getting half a bit for every.

Student: Doubling.

Doubling of the OSR and natural question is it possible to get much more than this is it not; it means a natural question you say I am oversampling. And, doing some digital filtering I seem to be getting some benefit right this is good. But will I will be able to do can I do better than this; correct so all right. So, let us kind of think a little bit and see what we can do in other words we try to pick of our old experience with analog circuits right.

(Refer Slide Time: 30:03)



And, one thing I had like to remind you is that we know that if we had an opamps. So, this is prior experience from negative feedback right. If we know we had an op amp and we added some noise here this is  $V$  in, this is  $V$  out let us say the gain of the op amp is  $A$  which is a large number. Now, what is  $V$  out?

Student: ((Refer Time: 30:50)).

$V$  in times.

Student:  $A$  by  $1$  plus  $A$ .

A by.

Student: 1 plus.

1 plus A plus.

Student: ((Refer Time: 30:59)).

N by.

Student: 1 plus A.

1 plus A this is well known stuff correct you look at this and say hey; if now think of this is the quantizer. Because after all we have replaced the quantizer by.

Student: ((Refer Time: 31:14)).

How we are interpreting the quantizer as an additive noise source whether it is justified or not is an a completely different matter right; we are assuming that the quantizer can be replaced by an additive noise source. And, we say hey from my prior experience I know that if I take an opamp based negative feedback circuit. And, if I inject noise at the output of the opamp the noise basically.

Student: ((Refer Time: 31:41)).

Does not come through to the output at all. Because negative feedback is doing what it takes to keep the output equal to.

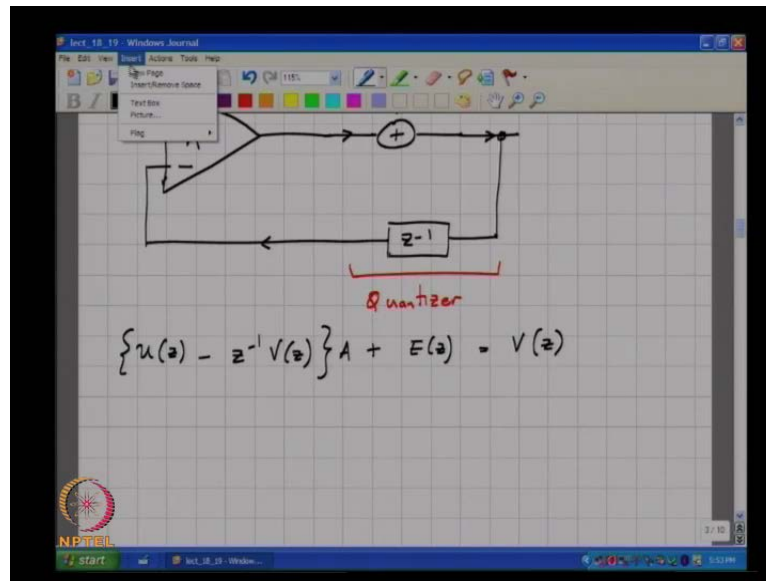
Student: Input.

Input correct and clearly as a tends to infinity V out equals.

Student: V in.

V in and noise is completely gone away correct which rings a bell. And, I say I know that my quantizer is nothing but an additive noise source maybe I can do this with a with a quantizer 2 seems like a fair thing to do is it not.

(Refer Slide Time: 32:36)



In other words, I have a quantizer whose output is related to the input by some additive noise source. And, your taking the q from this I just say now I am dealing with samples mind you; I put a big gain here the output of my gain goes to the quantizer. And, since we are dealing with samples; please note that any physical quantizer that we eventually planned to build will have some non zero delay correct. In other words if an input to be quantized was given at an instant or at a clock tic n; please note that we are dealing with samples again. So, the I mean time as we know it is only in.

Student: Discrete.

Discrete increments. So, at the rising edge of clock n for example; if you give an input to a quantizer the output is only going to produced.

Student: ((Refer Time: 33:54)).

Somewhere between in physical time somewhere between the n'th clock cycle and the n plus 1 clock cycle. But if I am only processing samples the next time I can look at it is only at the.

Student: Next rising edge.

The next rising edge right this you must be familiar with this from digital state machines correct. So, you have some combinational logic you have flip flops right; the flip flops



clock at say the rising edge. And, give you some logic that goes and propagates throughout the combinational logic. And, the next time you clock it is when your able to see the effect of the previous decision going through the combinational logic correct. In other words the output of the quantizer is not instantaneous; since we are dealing with this in the with the sampled data domain it means that there is a delay associated with it of one clock cycle we cannot get the quantized output at this same instant correct. So, this therefore is the quantizer and based on our old experience we just do this. And, we say the quantizer I am going to replace by an additive noise source; let me call that  $e[n]$  this I will call  $u[n]$ . And, this I will call  $V[n]$ ; does it make sense so far all right. So, now let us go and calculate  $V[n]$  in terms of  $u$  and  $e$ . And, since the  $z$  inverse it makes sense to work with  $z$  transforms. So, what does this mean  $u(z)$  minus  $z$  inverse  $V(z)$  multiplied by a plus  $e(z)$  equals?

Student:  $V(z)$ .

(Refer Slide Time: 37:18)

The image shows a handwritten derivation on a grid background. The equations are as follows:

$$\{u(z) - z^{-1}V(z)\}A + E(z) = V(z)$$

$$\Rightarrow V(z) \{Az^{-1} + 1\} = Au(z) + E(z)$$

$$\Rightarrow V(z) = \frac{A}{1 + Az^{-1}} u(z) + \frac{1}{1 + Az^{-1}} E(z)$$

Below the first fraction, the text "STF" is written in red, with "Signal Transfer Function" written below it. Below the second fraction, the text "NTF" is written in red, with "Noise Transfer Function" written below it.

$V(z)$  which means  $V(z)$  times  $Az^{-1}$  plus 1 equals  $A$  times  $u(z)$  plus  $e(z)$ ; which means the output  $z$  transform is nothing but  $A$  by  $1 + Az^{-1}$  times  $u(z)$  plus  $1$  by  $1 + Az^{-1}$  times  $e(z)$  all right.

Student: Sir, in that model which is saying that quantization ideal without any.

Yeah this is a just a.

Student: ((Refer Time: 38:37)).

Correct.

Student: ((Refer Time: 38:38)).

So, this is the actual quantizer because the quantizer output is available only.

Student: After 1 cycle.

It is I mean the output is not available immediately; which means that the rest of the circuitry can only process it.

Student: Process in the next.

In the next cycle.

Student: So, that  $V(n)$  its only come from after the  $n$ .

Yeah. So, I mean  $V(n)$  you know you can not  $V(n)$ ;  $V(n)$  minus 1 is the output of the.

Student: Oh sorry  $V(n)$ .

Yeah. So, yeah a delay does not make any difference.

Student: Just a model 2.

Correct; yes all right. So, is this clear? And, therefore there is a transfer function from the input signal to the output; there are 2 inputs, 1 output. So, there must be 2 transfer functions; this corresponds to the signal that we are interested in and this is call the signal transfer function. And, this corresponds to the transfer function from the quantization noise to the output. And, is therefore called the noise transfer function; and clearly as you can see as  $u$  tends to infinity it seems like  $u$  will tend to.

Student: ((Refer Time: 40:24)).

I mean  $1/z$  inverse which is.

Student:  $z$  inverse.

$z$  right which means the output comes before the.

Student: Input.

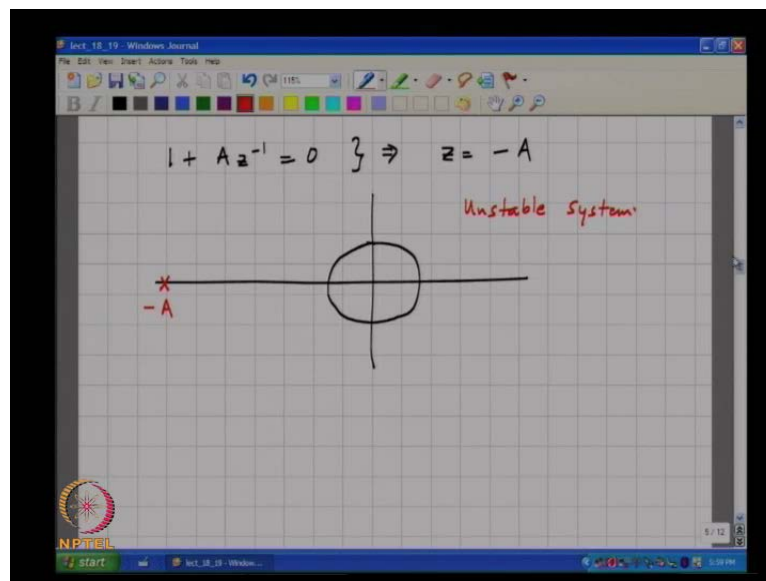
Input is it not; which; obviously cannot happen which means that there is something seriously wrong with assuming a tending to infinity; of course one thing I would like to bring to attention is that both the transfer functions the signal transfer function as well as the noise transfer function have the same denominator that make sense. Because in a linear system all transfer functions will have the.

Student: Same.

Same poles. So, that is no surprise the numerators are different because the inputs are injected different points in the system. So, where is the pole located?

Student: ((Refer Time: 41:18)).

(Refer Slide Time: 41:19)



The location of the pole is obtained by solving finding the root of the denominator of the transfer function that is this which means that z equal to.

Student: Minus A.

Minus A right. So, if you have a discrete time system and if you want this system to be stable; where would you want the location of the poles?

Student: Inside the unit circle.

They must lie.

Student: Inside the.

Within the unit circle right. And, where on earth is this pole, this is the unit circle right and where is this pole?

Student: ((Refer Time: 42:05)).

If you want this to work as we expect; if you in other word if you wanted to reject quantization noise  $A$  must be large which means that the pole is sitting at minus  $A$  which is way outside the.

Student: Unit circle.

Unit circle and the only way to make it work is to choose.

Student:  $A$  less than.

$A$  less than.

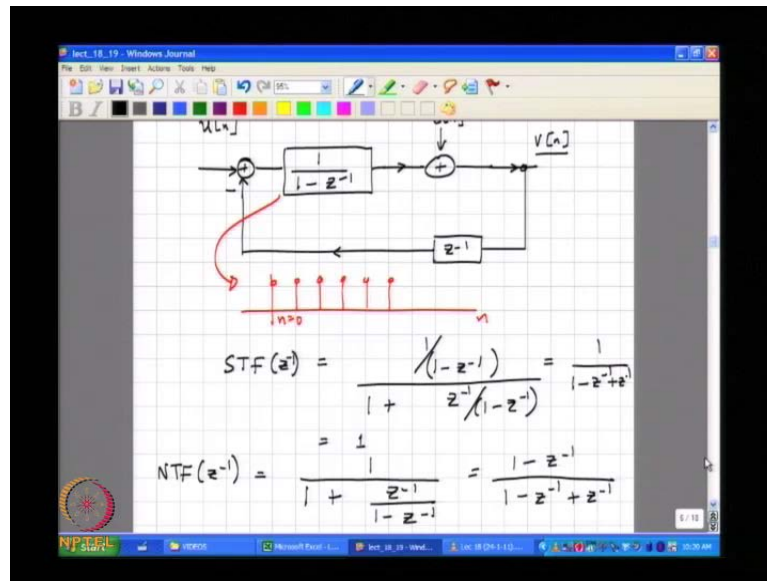
Student: 1.

1 and sure enough this system would be stable but.

Student: ((Refer Time: 42:37)).

It is not doing what we'd like it to do; is this clear. So, this is a unstable system in other words if we just did this the system is unstable; so then the question.

(Refer Slide Time: 43:18)



Let me redraw this in another way; basically what I am doing is subtracting the input from the output rather the output from the input. And, then amplifying it by a factor A and closing the loop and this is does not work. And, then you say hey the wait minute I want to make this work but what is the fact that we have not exploited in this whole approach; what is one factor that we have not exploited at all?

Student: Now, required signal is in the lower frequency joint but we have gain throughout.

Very good. So, one key thing to observe is that if we just choose A to be very large. And, by definition is independent of frequency; what is happening is that this is indeed becoming 0 but across.

Student: All frequencies.

All frequencies. But we exploited the fact that the our desired input signal is confined only to.

Student: Low frequencies.

Low frequencies and in other words frequency is close to D C. So, therefore it is only sufficient to have.

Student: Large.

A large gain at low frequencies and we do not really care.

Student: ((Refer Time: 45:23)).

What?

Student: ((Refer Time: 45:25)).

You know what this does at.

Student: High frequency.

High frequency you understand; obviously, if you make a very large that is a seems like a desirable situation. Because then you will be able to eliminate quantization noise across all frequencies. But unfortunately that system is unstable unless you choose a very small  $A$  in which case it does not work at all right; in other words it its stable but you have a system which does not reject quantization noise. So, then you say hey my signal is confined only to.

Student: Low frequencies.

Low frequencies. So, why would I want to worry above if  $A$  is falling with frequency; as long as the gain  $A$  is high at low frequency that is all I am worried about. So, given this intuition and the reason why this is acceptable is because the input signal itself is at low frequencies; does it make sense? So, we know now that this must not be chosen to be a large number at all frequencies; we only need it to be infinite at.

Student: Low frequency.

Low frequency meaning infinite at  $D C$  for example correct. So, low frequency corresponds to what  $z$  equal to.

Student: ((Refer Time: 47: 03)).

This is the unit circle.

Student: ((Refer Time: 47:05)).

$D c$  corresponds to  $z$  equal to.

Student: 1.

1. So, in other words our signal is hovering around  $z$  equal to 1 all right. So, what we need is a box here; where the gain is high at  $z$  equal to 1 ideally we had like the gain to be.

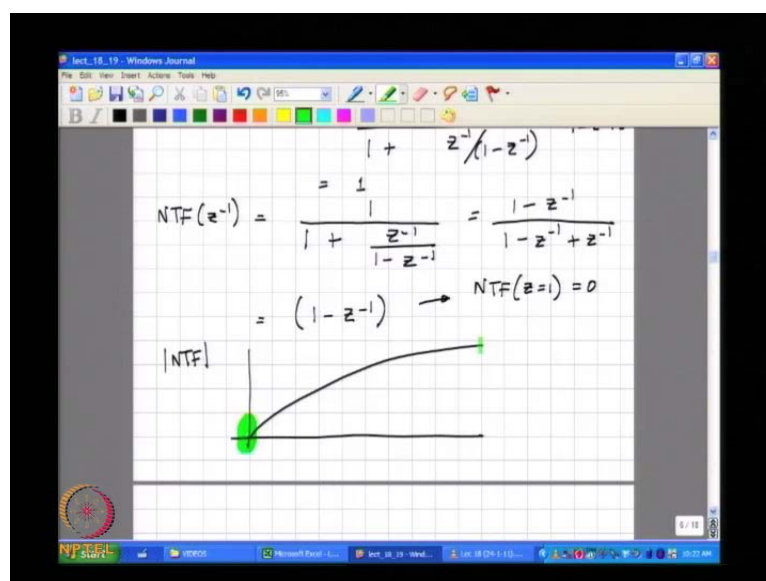
Student: Infinity.

Infinite at  $z$  equal to 1. So, can you think of a simple lowest first order transfer function where the gain is infinite at  $z$  equal to 1?

Student: 1 over.

So, this must be 1 over 1 minus  $z$  inverse and why? This is got a pole  $z$  equal to 1 and if you think of it the impulse response of this structure; this is an integrator it is got an impulse response of all one's starting at  $n$  equal to 0. Now, let us try and evaluate the signal and noise transfer functions of the modulator with this integrator in place; the signal transfer function of  $Z$  or  $z$  inverse is nothing but 1 over 1 minus  $z$  inverse divided by 1 over  $z$  inverse by 1 minus  $z$  inverse. And, this is equivalent to 1 divided by 1 minus  $z$  inverse plus  $z$  inverse which means the signal transfer function is 1. The noise transfer function on the other hand is 1 by 1 plus  $z$  inverse by 1 minus  $z$  inverse. And, this must be equal to 1 minus  $z$  inverse divided by 1 minus  $z$  inverse plus  $z$  inverse or in other words the noise transfer function is 1 minus  $z$  inverse.

(Refer Slide Time: 49:47)



So, we therefore see that the noise transfer function has a response which goes to 0; when evaluated at  $z$  equal to 1. And, this makes sense because the gain of the forward amplifier. So, to speak which is now an integrator is infinite when  $z$  equal to 1 or else at D C. So, for frequency is around a small range around D C the magnitude of the noise transfer function is close to 0. And, at  $z$  equal to plus 1 the magnitude response goes to 2 all right. So, where are the poles located now?

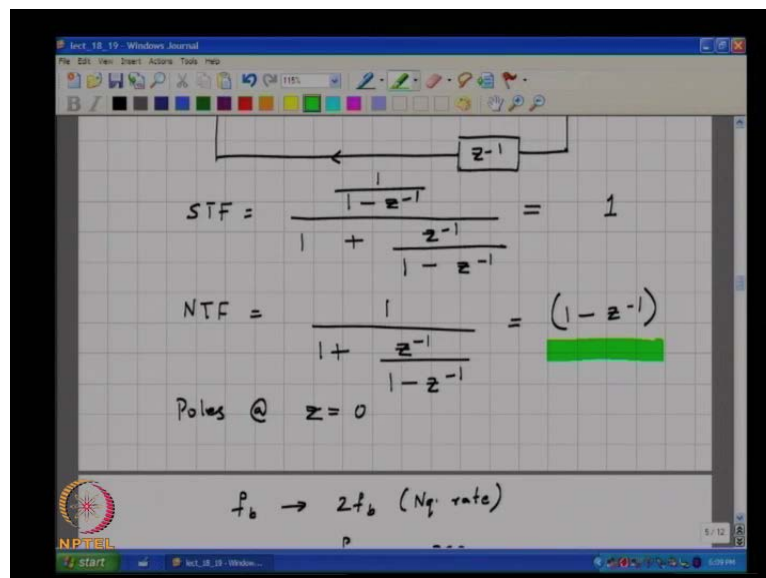
Student: ((Refer Time: 51:02)).

$Z$  is equal to.

Student: 0.

0.

(Refer Slide Time: 51:06)



Poles are at  $z$  equal to 0; is this clear? So, does this mean that this a stable system?

Student: ((Refer Time: 51:26)).

I mean you cannot get any you know any farther from the edge of the unit circle right. So, this is a definitely a.

Student: Stable.



Stable system all right. Now, therefore by exploiting the fact that the signal is confined only to low frequencies and which in turn means that our forward amplifier. So, to speak need only have high gain at.

Student: Low.

Low frequency. And, the frequency at I mean and the gain at high frequencies is this not particularly of concern at this point; we saw that you can in fact realize a.

Student: ((Refer Time: 52:11)).

A closed loop system which is actually stable. And, at D C the signal transfer function will be is 1; and more importantly the noise transfer function is; what is the noise transfer function?

Student: ((Refer Time: 52:31)).

The noise transfer function is.

Student: 1 minus z inverse.

1 minus z inverse. And, what is the value of the noise transfer function at D C?

Student: 0.

Evaluate N T F at z equal to 1 which gives 0; this makes sense because we saw that if a was infinite the NTF would go to.

Student: 0.

0 correct; unfortunately or fortunately here this is only happening at D C; where the gain is infinite. If we move away from D C what to you what you think will happen?

Student: It will increase.

To the noise transfer function.

Student: Increases.

It increases from D C because the gain is falling off from infinity to something lower than that. And, therefore the noise transfer function increases all right.