VLSI Data Conversion Circuits
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Lecture - 16
Quantization Noise – 1

This is VLSI data conversion circuit's lecture 16. In the last class what we saw was how on one could characterize the static input output characteristics of a Quantizer. All right we saw a several non idealities that could occur in practice which seem perfectly reasonable one. Of course, is that we had like in principle a uniform staircase. The first thing that we can think about is that the staircase is shifted parallel to its to the ideal one we want. And, this in mathematical terms can be modeled as an ideal Quantizer within offset upfront. The next thing one could think of is that the staircase starts of at the right place. But the step size even though it is the same for all steps is different from the ideal step size. And, then we saw that this would lead to a staircase whose average slope is different from what you want; and this can be modeled by a gain error right.

The third one of course, is when the step sizes are all different. And, are not the same as the ideal one that you want; and this will lead to so called differential integral nonlinearities. And, we saw last time that a differential nonlinearity is trying to give you an indication of the local slope, right. Because; it is giving information about the deviation of the actual step size from the ideal step size and, normalizing it to the average L S B of the quantizer. The integral nonlinearity on the other hand is giving you an idea about how far 1 core transition lies from the ideal transition. Of course, none of these non idealities occur in isolation; any practical converter a to d converter that we take and measure will have all these together.

It will be affected by offset, it will be affected by gain error; and of course, the step sizes will not be uniform. And, the last time around we saw that differential nonlinearity and integral nonlinearity are 2 ways of conveying the same information given one we can find the other; however these are ways of representing the transition points of each of these codes in a visually appealing way in the sense that they immediately give us an idea of what is wrong with this characteristic. As we discussed last time it does not help too much if you simply draw the characteristics themselves right.

All that you will see is you will see you know something which looks like a staircase right with the levels slightly shifted apart. And, that is not of you know it is very difficult to discern what is really going on? What we had be more interested in is? What the deviations are from the ideal values which is why D N L curve and a I N L curve are important and are coated on all Nyquist rate a to d converter data sheets ok, all right. So, the next thing to do is to look at the spectral properties of quantize signals.

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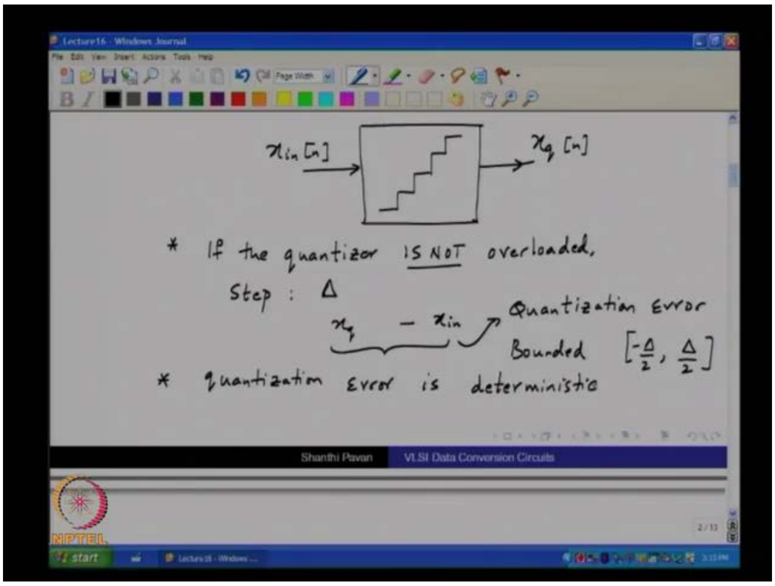

So, we will do that today ok. So, as we have seen the input output characteristic of the quantizer is a horribly non-linear affair, because of the steps in the characteristics. And, whenever you have discontinuities in the characteristic you know that mathematically this going to be a very difficult beast to deal with right. So, let us do this anyway. So, this is the input and this is the quantized input. And, clearly, if the quantizer is not overloaded. Then, what happens; we can say that the difference between the input and the output is; let us assume that we have a quantizer which is ideal. In other words the step size is uniform there is no offset there is no gain error all that stuff.

So, each let us further denote each step by delta in other words the width of each code is the same and equal to delta. Now, if you are in the no overload region of the quantizer in other words we are in that part of the stair case which is not flat ok. Then, what can we say about the input and the output first of all is there a loss of information.

Student: Yes.

Lecturer: Yes, why?

Student: ((Refer Time: 06:30))

Lecturer: I mean.

Student: Several value same code.

Lecturer: Several values of the input result in the same output.

So, given the output we cannot find uniquely what the input is? So, there is definitely a loss of information right. The next thing to find I mean then the next observation is that if we are not overloading the quantizer then clearly the output is not the same as the input. So, there is some error which we call the quantization; let me call this since we are dealing with samples; let me denote them by N rather than t. So, x q minus x in is nothing but the quantization error. And, can we make any comment about this quantization error; if we know that we are not overloading the quantizer.

Student: ((Refer Time: 07:38))

Lecture: Clearly, The quantization error if we have not overloading the quantizer is bounded by the?

Student: Step size.

Lecturer: Step size correct. So, x q minus x in is bounded in the region minus delta by 2 to dealt by 2.

Student: ((Refer Time: 08:04))

Lecturer: Correct. Then, let me also draw your attention to the following is this quantization error correlated with the input.

Student: ((Refer Time: 08:25))

Lecturer: And, what we understand loosely by correlation?

Student: ((Refer Time: 08:34))

Lecturer: In other words given the input can we predict quantization noise a quantization error at this point.

Student: You had given the step size and.

Lecturer: If we know the characteristic of the quantizer which I am assuming we will know right if we know the characteristics of the quantizer and if we know the input.

Student: ((Refer Time: 09:09))

Lecturer: Clearly, x q minus x in is for known quantity, correct. Because given x in we can find x q which means that you can find x q minus x in; which means that the quantization error is a deterministic signal it is not random, you understand. So, quantization error is definitely deterministic in nature all right can you?

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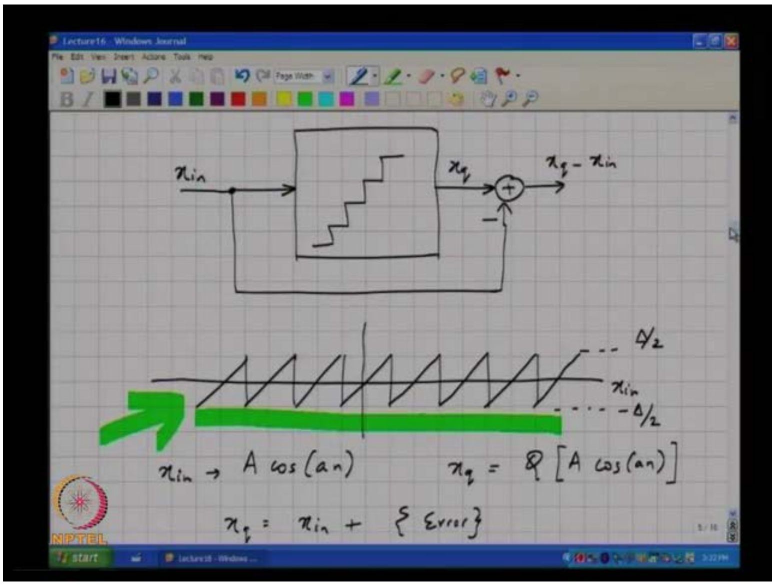

So, if you plot the transfer function of the error. So, this is the transfer function of the quantizer or the transfer curve of the quantizer. If I plot the error which is x in subtracted from x q; this is the quantization error. And, how do you think this curve would look like. So, if we are in the no overload region. This must look like a saw tooth type signal bounded from above by delta by 2 and bounded below by minus delta by 2 correct. So, if one wants to find the spectrum of the output of the quantizer when the input is a sinusoid; how would one go about it?

Student: ((Refer Time: 12:08))

In other words if x in was a sinusoid or the form say some A cos 2 pi or let me call this some cos a times n is the sequence. And, I would like to find the spectrum of x q of n which is a quantized version of A cos of a times n. How would we go about it? And, what should we expect? Before we actually go and do this we know that x q is nothing but x in plus quantization error. And, this quantization error can be formed by simply taking the input and passing it through a box whose transfer curve looks like this is not it. So, if you now take a sinusoid and pass it through a block do you can you comment on the block is linear or non-linear.

Student: Nonlinear.

Lecturer: This is a black box the input output transfer curve of the black box looks like this. Is this a linear system or a non-linear system? It is clearly?

Student: Nonlinear.

Lecturer: Nonlinear right. So, what must you expect if you take a pure sinusoid and pass it through such a transfer curve?

Student: Transfer curve.

The output would be will have a lot of harmonics. So, one thing we must expect and especially given that the transfer curve is got lot of discontinuities. You must expect that the quantize sinusoid will have many many harmonics of the input. Does it make sense? And, before we actually take a look at the spectrum finding the harmonic content of a sinusoid when passed through as quantizer turns out is quite understandably not a very easy problem to tackle. Because of the; you know big discontinuities in the transfer curve.

So, you will it turns out that you end up with Bessel functions and you know the math is actually quite complicated. So, a lot of you know. So, in other words the going really gets very tough, because of the mathematical difficulties involved with first of all I mean a non-linear system is bad enough. And, now we have non-linear system with discontinuities. So, you can imagine that finding the Fourier series coefficients of a sinusoid transformed by such a curve is not going to be very simple. So, when the going gets tough as I say.

Student: Tough gets going.

Lecturer: The tough get going right. So, when the going gets tough you know engineers begin to make all sorts of approximations; which you may or may not be valid right. But at least it is a way to move forward without dealing with lot of heavy math. Surprisingly, in the I mean there is not some approach that encourage you to take an all situations right. But in this particular case it turns out. That the assumptions which I will show you which are quite shocking actually; they actually work in practice. And, you know every time you make a phone call you know this assumption is being put to test you know many times all right. So, given the difficulty of dealing with quantization error given the whole process is so non-linear even for a sinusoid. It is actually quite messy you can imagine what will happen when you have a real signal which is not a sinusoid.

If I had a sinusoid is if that flows all I had to convey then there is no information the sinusoid itself right. So, if we begin to make a couple of assumptions. The first thing is we treat quantization noise a quantization error as noise. So, in other words we say that oh there is a yeah sure enough there is a step shape discontinuity in the input output characteristic. But we do not really worry about it; we say the input after getting through this box is getting corrupted by?

Student: Noise.

Lecturer: Some noise and since we call this box a quantizer we call this noise a.

Student: Quantization.

Lecturer: Quantization noise. So, we say alright this is so complicated I do not understand it let me just call it noise. And, it is once I call it noise you know I am justified at least in saying that the output is input plus some wave form that wave form is what I term as noise.

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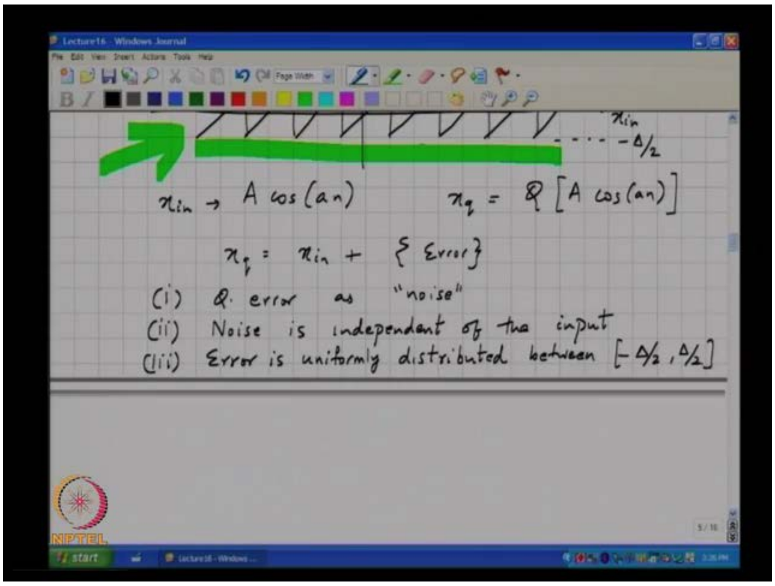

Then, I make the first of my bold assumptions which are at the noise is independent of the signal. I mean this a correct assumption or it is not a correct assumption?

Student: Not a correct assumption.

Lecture: It is clearly not correct, because we just discussed a few minutes ago that given the input and the nature of the quantizer I can.

Student: Find out the.

Lecturer: Find the output; which means that I can find the error; in other words the quantization error the last thing about it is that it is independent of the input you understand. So, we the first assumption is we say noise is independent of the input. And, well at least you can say when you look at the output wave form it looks. So, messy that it does not demand of the input anyway all right. And, then you say it does not the next thing is you say the error is bound between we know for sure that the error is bounded between.

Student: Plus or minus.

Lecturer: Plus or minus delta by 2 then we say the error is bounded, but we also assume that the error is uniformly distributed between minus delta by 2 and Delta by 2.

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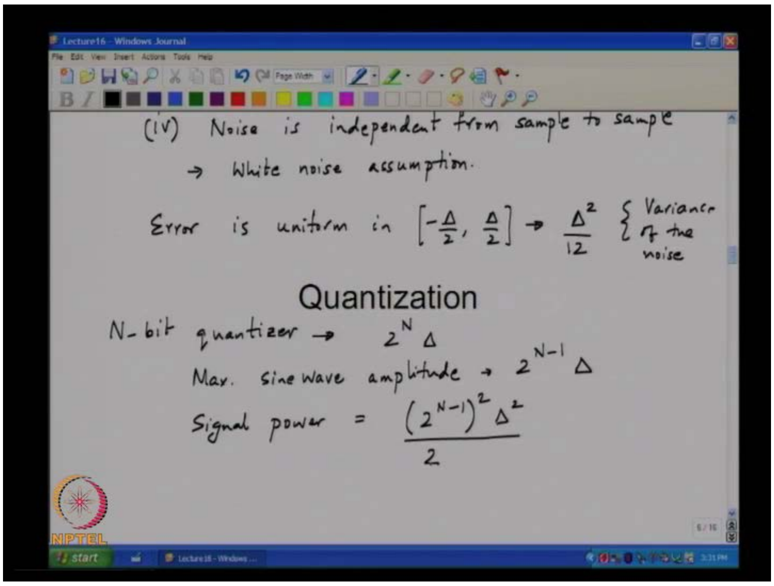

Please, note that this is also a very big assumption without too much of a mathematical basis right. I mean it turns out with the with many real signals this is a fairly decent one; for at this point I mean it seems like a big stretch all right. Because you can find any number of p d f's between minus delta by 2 and delta by 2. Apart from making the math and our lives more simple there seems to be no particular motivation for choosing a uniform p d f for the error. The third assumption given that we have made 2 glaring glaringly wrong assumptions.

It does not hurt to make one more right; which is the quantization error spectrum is white; in other words the quantization noise is independent from or this is the so called white noise assumption of the. You must bear in mind that these are again I said engineering approximations which work very well in practice justifying their use all right. Of course, the more mathematically inclined of my colleagues will squirm when I start making lose statements like this ok. But you know it works in practice that is the only justification we all have one can of course, go and do the math's as I say. But it basically all that it boils down to is it leads to a lot of you know many pages of algebra.

And, finally it is not clear whether you know it makes a big difference in practice. You with the fact that if you know that the error is not uniformly distributed is not clear. If it helps in practice is knowing it that it is not does not really help in practice what I am trying to say. So, these are the ground rules by which we simply work. And, as I said the justification for this is that they work in practice. Now, if the quantization error is uniform in minus delta by 2 and delta by 2; what does this mean for the so we say that it is noise its independent of the input it is white it is uniform in minus delta by 2 to delta by 2 which means that the mean square value of this quantization noise process is therefore.

Student: Delta square.

Lecturer: Delta square by 12; is the variance of the noise. And, if we have an N bit quantizer how many steps are there?

Student: The original minus 1.

Lecturer: There are 2 raised the range of the quantizer 2 raise to the N; which means what is the maximum sinusoidal amplitude that I can have which fits the entire range of the quantizer.

Student: Maximum.

Lecturer: Very good. So, the maximum amplitude with the sine wave is 2 to the power of N minus 1 delta. So, the entire range of the quantizer is 2 to the power of N delta. So, what is the maximum signal power that you can pump in without saturating the quantizer? For a sinusoid you know that amplitude square by 2 is the power. So, 2 to the power of N minus 1 whole square times delta square by 2 is the signal power. What about the quantization noise power?

Student: Delta.

Lecturer: Delta square by 12.

So, the signal to quantization noise ratio. So, called the S Q N R is the ratio of the signal power to the quantization noise power. And, that is 2 to the power of N minus 1 whole square by 2 times 12; which is 6 times 2 to the power of 2 N minus minus 3; does it make sense?

Student: ((Refer Time: 26:49))

Lecturer: Am I missing something here.

Student: ((Refer Time: 26:55))

Lecturer: Oh sorry yeah sorry, sorry.

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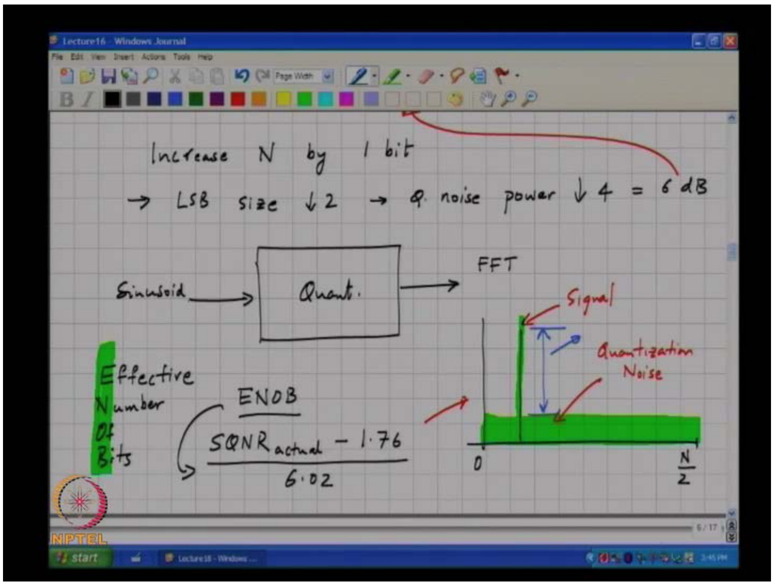

Now, so S Q N R is 6 times 2 to the power of 2 N minus 2. And, in d B this turns out to be?

Student: ((Refer Time: 27:38))

Lecturer: 6.0 2 N plus 1.76 d B since we are I had like to remind you that since you are taking the ratio of the powers the logarithm should be to the base of 10 not 20 right. So, every increase in the bit in the number of bits will increase the signal to quantization noise ratio by.

Student: 6.

Lecturer: 6 d B. So, why does this make initiative sense 6 d B is if noise signal to noise ratio increases by 6 d B; what is it in real terms?

Student: ((Refer Time: 28:37)) Amplitude rather than (Refer Time: 28:43)

I mean 6 reduction and signal to quantization noise ratio by 6 d B is equivalent to if you if we normalize with respect to the signal power. It means that the noise power has gone down by factor of.

Student: 4.

Lecturer: 4 right. Noise power has gone down by a factor of 4. And, that makes initiative sense, because if we increase the resolution by 1 bit it means that the step size has gone down by.

Student: ((Refer Time: 29:09))

Lecturer: A factor of 2 correct. So, the step size is gone down by a factor of 2 the power of the error must have gone down by a factor of.

Student: 4.

Lecturer: 4 right; which therefore means that the noise power has gone down by a factor of 4 which is 6 d B all right. So, increase N by 1 bit. And, this means that LSB size goes down by factor of 2; which means that quantization noise power goes down by factor of 4 which is 6 d B right. And, therefore this make sense alright. So, if we take a sinusoid quantize it. And, plot the F F T of the sequence what would we expect? Means understood that the F F T must be done properly in other words the input and the sampling rate must be chosen to be proper. So, that there is no leakage in all this other stuff that we have learnt. So, far. So, let us assume the all that is in place what would you expect when we look at the F F T?

Student: ((Refer Time:31:08))

Lecturer: We should see the signal and because we have assumed that there is no leakage you will see it only in.

Student: 1.

Lecturer: In 1 bin. And, if all the assumptions that we made were true right; white noise uniformal distributor. And, all this stuff we should expect to see a quantization noise spectrum which is?

Student: Flat.

Lecturer: Flat throughout.

So, if I took an N point F F T there be exactly N by 2 bins that I am interested in right.

So, this is 0 this will be the N by 2 th bin correct. So, this should be the input signal and this is the quantization noise. So, how would 1 compute this signal to quantization noise ratio given this spectrum and nothing else? If for example, I gave you this picture and told you to go and compute the number of bits in the quantizer; what would how would you go what doing it?

Student: ((Refer Time: 32:37)) quantizer signals.

Lecturer: Yes.

Student: Because quantization of noise will (Refer Time: 32:51)

Lecturer: Very good.

So, we know that the signal to quantization noise ratio is given by this expression given the input is is sinusoidal right; we know that it is 6.02 N plus 1.76 d B. And, if the spectrum is given what one would do to find the number of levels in the quantizer is to find the signal to quantization noise ratio. And, how would one measure this is signal to quantization noise ratio given this picture? The signal power is simply the power in this bin right the noise power is?

Student: Somehow the power noise.

The power of the noise in all the other bins. You may want to get rid of the d c bin because that might be coming from offset in the converter right. But you add up the powers of all the other bins; and the ratio of the 2 will give you.

Student: S q.

Lecturer: The s q N r and you can compute that in d B. and, then correlate with that with 6.02 N plus 1.76 you understand. So, a very common misconception is to say I look at this level here. I look at that distance right in d B and I conclude that that is my.

Student: ((Refer Time: 34:25))

Lecturer: S Q N R that is clearly

Student: ((Refer Time: 34:30))

Lecturer: A very very a wrong thing to do right. Because what you need to do is actually add up the noise powers in.

Student: All the bin.

Lecturer: All the bins. And, this for a given quantizer if I keep increasing the number of bins; what will happen to this distance?

Student: Basically written somewhere is to know (Refer Time: 34:54)

Lecturer: Same quantizer if I went on increasing the number of F F T bins what do you think will happen to that distance shown in loop.

Student: Increases.

Lecturer: It will increase go on increasing, because the total power is fixed and equal to delta square by 12. If you have a larger number of bins then the power per bin will.

Student: Decreases.

Lecturer: Reduce.

In fact, if you double the number of bins the height of the noise floor will go down by about 3 d B correct, because the same power now has to be.

Student: Spread.

Lecturer: Spread across twice the number of bins. So, in each bin it will go down by 3 d B. So, this height in blue is just that can be arbitrarily increased by going on increasing the number of bins over which you compute the F T; however, for a given ideal quantizer the power of I mean noise when you compute it across all the bins must remain the same. And, equal to delta square by 12 is that clear alright. So, in practice of course, we know that the quantizer staircase is not quite as nice as we had want it to be first of all the step size is not uniform there may be gain error all sorts of things. So, when you actually take a real converter and plot its spectrum apart from the sinusoid at the input you will also see.

Student: Harmonic.

Lecturer: You will see harmonics. And, some of the harmonics the higher harmonics may actually.

Student: Alias back.

Lecturer: Alias back. And, will look like low frequency tones you understand. I mean clearly the harmonics of the sample and hold for example, will lie well beyond Nyquist right. So, when you sample it at f s they will alias back and so on. And, so when you actually take a real quantizer with also so non idealities. And, make a diagram like this. And, when you actually compute the signal to quantization noise ratio you will find that the actual S Q N R is smaller than what you expected. Let us say you designed a 10 bit converter; which means that in principle you would be expecting a signal to quantization noise ratio for about 61.76 d B or so alright. But in spite of having a 10 you know 10 bit quantizer and when you actually made it let us say you got only 45 d B you understand due to say bad design or some other non idealities.

But this is indeed a 10 bit converter in the sense that 10 wires come out of the of the box. So, clearly you know we need a metric not just I mean just, because 10 wires are coming out of a box it does not mean that this is a ten bit converter alright. So, my converter may give you 50 d B his converter may give you 51 d B all of them being nominally 10 bit converters. So, there is a need for not just I mean saying my a to d converter box has got 10 wires coming out of it right. You need the some more scientific method of qualifying the performance of the a to d converter right.

So, 1 such metric is called the effective number of bits or the ENOB of the converter. And, for this what do you suggest we do?

Student: ((Refer Time: 39:24))

Lecturer: That is all.

So, the definition is quite straight forward. The, we take the actual signal to quantization noise ratio that we measure right from this we subtract 1.76 and divide by 6.02. And, this would give the effective number of bits. And, to compute this signal to quantization noise ratio; the input signal power is clearly you know there is no ambiguity there and often the rest of it whether its distortion or quantization noise you combine all the bins. And, that is a taken as the a noise power you compute the signal to noise ratio that way and you subtract 1.76 and divide by 6.0. You will find that for Nyquist converters if you design for resolution you will always get an effective number of bits which are?

Student: ((Refer Time: 40:32))

Lecturer: Smaller than what do you design for? I mean if you design a 10 bit converter a very good 10 bit converter will probably give you say 9, 9 and half bits; whereas, a poor converter will give you know. An effective resolution which is lot smaller than 10 bits. This effective resolution also depends on the input frequency. So, when the input frequency to the converter is very small in other words the input frequency is very small compared to f s by 2. Then, you know presumably it is easier to make circuits work, because the input remains the same from sample to sample. So, circuits do not inside do not have to work quit as hard as they do when the input is changing rapidly. So, you will find in general that the ENOB of the converter or the effective number of bits of the converter keeps falling with.

Student: Input.

Lecturer: With Input frequency.

This is something to be expected of course, we have a very good design. Then, the fall will be hopefully very small. So, all these matrix are coated whenever you report a converter or when you buy a converter the manufacture will give you these numbers. And, these curves as a function of frequency. So, that you can be sure if it is good enough for the application your targeting alright. So, now again please note that this uniform this I mean the white assumption for the quantization noise is what is prompted as to assume that the quantization noise spectrum for an ideal quantizer is going to be flat ok. Now, let us take a sinusoid quantize it and take a look at the spectrum and see what we get?

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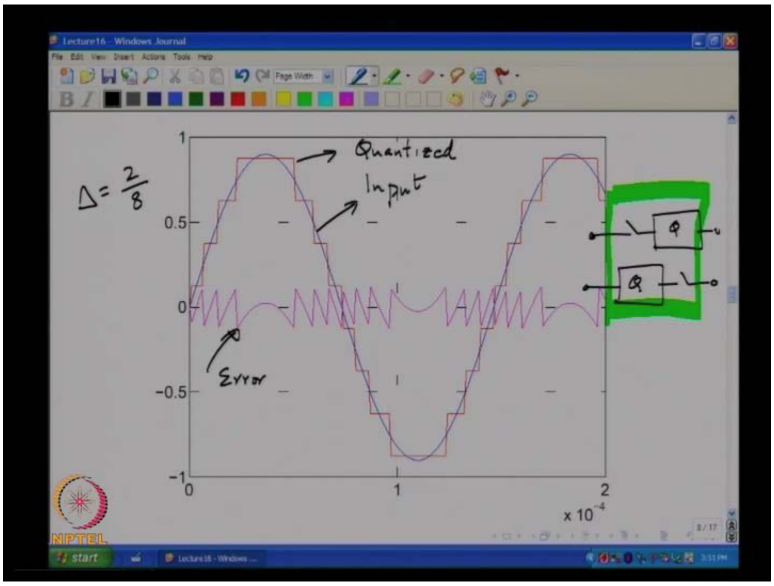

So, this is a sinusoid shown in blue, the red one is the quantize sinusoid. The picture in the curve in magenta is the quantization error. And, clearly as you can see when the input is varying rapidly you can see a softwood type waveform add the tops of the sinusoid you can see a waveform which varies a little more gently. But it does not take a genius to figure out that the frequency content of the quantization noise is when you must expected to have a a lot of high frequency content. See one useful mathematical equivalence is the following whether you take a signal sample it and quantize it or you take a signal you quantize it and sample it later. The end sequence will have the as far if I put both these inside a black box you will not be able to tell me which is which?

In other words, the sequence of the end is has the same spectral properties is identically in other words. So, sometimes it is useful to think of a quantization this way where we say we take a sinusoid we quantize it this is all now in continuous time. And, then we sample the quantized continuous time signal. Clearly, they quantize will have a whole bunch of harmonics, because of the non-linear operations preceding I mean there that are going on inside the quantize. And, clearly these harmonics will not satisfy the Nyquist criterion even if the input does. So, the moment you sample spectra from.

Student: All these harmonics.

Lecturer: And, all these harmonics will all alias back into the range 0 to f s by 2. And, we will produce some spectrum in that region. So, this corresponds. So, this particular picture corresponds to a L S B size or step size of 2 by 8. And, why I write 2 by 8 rather than 1 by 4 is that this entire signal range is assumed to be from minus 1 to 1. And, I would divided that up into 8 steps 8 bins rather all right.

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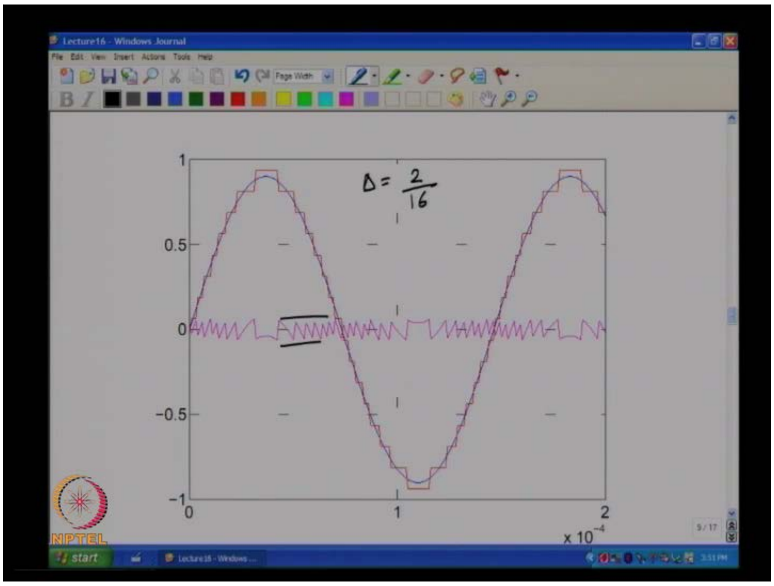

Now, I repeat the same experiment with delta is now 2 by 16 I will increased the number of levels by a factor of 2 all right. Now, one thing that you can observe is that the height of this error has gone down by a factor of 2 when I increase the resolution by 1 bit. Please, note that this is an increase in the resolution by 1 bit, because the doubled the number of levels.

(Refer Slide Time: 46:27)

Now, when I look at this spectrum of one of these guys we see that?

Student: ((Refer Time: 46:34))

I mean this quantization I mean this is the input signal. And, all this stuff is quantization noise. And, not surprisingly we see that it is not very it is not quite as flat as we expected. And, you know that make sense because we made a whole bunch of you know questionable assumptions. And, when you plot this spectrum the quantization noise spectrum is not quite as flat as we expect. And, there are we can see many spikes in the spectrum. And, this make sense because basically this is a deterministic process and you gone in I mean all these you generate a whole bunch of harmonics and some of them I mean they all alias back into 0 to f s by 2. So, these are all harmonics of the input which have alias back ok.

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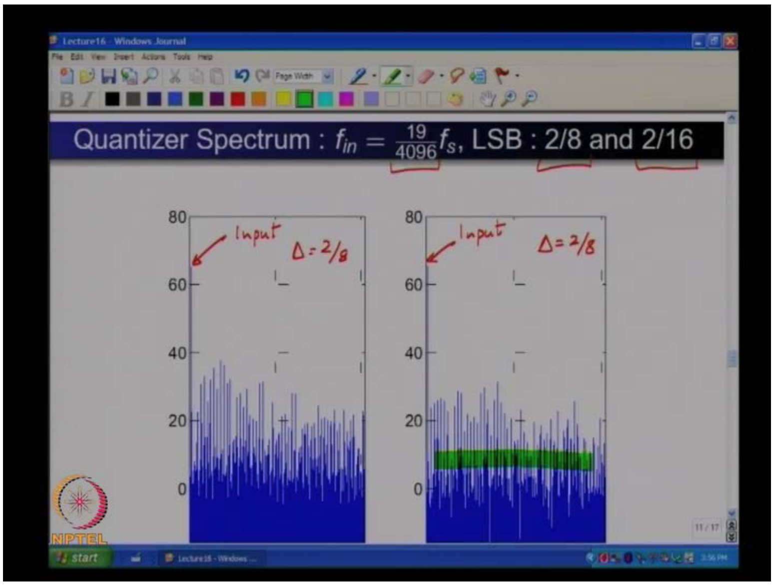

Now, it is interesting to compare the spectra of the same sinusoid pass through 2 different quantizers. Please, note that in both these cases I have taken a 4096 point F F T. And, the input signal is related to f s by a rational 19 by 4096. So, the input therefore, lies on 1 bin do I need to be worried about F F T leakage at all given that there is noise as well as signal.

Student: Mean if it is a shift noise then ((Refer Time: 48:45))

Lecturer: Why not now?

Student: Because in this case the f in by f s is ratio of integers.

Lecturer: Alright.

So, yes I mean if this was noise was shaped then discontinuities at the edges as we saw will cause filling up of those notches in the spectrum right. Another thing here is that even though we think of this as noise in reality this is a deterministic phenomenon. So, the quantization noise is actually periodic with the record lengths correct. The input is periodic which means if the quantization noise must also be periodic because the quantization noise is a deterministic signal which depends on the input. So, we do not have to worry about leakage at all you understand. Now, given that the ratio of the input to the sampling rate is 19 by 4096; which is a very small number. I do not know if you are able to discern this, but that is the input.

Can you tell me. So, 1 of them corresponds to an L S B of 2 by 8. And, the other one corresponds to an L S B of 2 by 16; can you tell me which spectrum corresponds to an L S B size of 2 by 8 and which corresponds to 2 by 16.

Student: ((Refer Time: 50:14))

Lecturer: The one of the left corresponds to.

Student: 2 by 8.

Lecturer: 2 by 8 and why would that be.

Well usually it seems as if the noise spectrum here when you integrate all the power and all these tones is larger than the noise you might get when you integrate the power in all these tones right. So, that is definitely correct in the next class we will take a look at some other important aspects of you know a couple of more things that we can intuitively find from these a spectral diagrams and see why they make sense? Ok.

Thank you.