VLSI Data Conversion Circuits
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Lecture - 14
Spectral Windows – 2

This is VLSI data conversion circuit lecture 14; let us quickly recap what we did the last time. And, wind up our discussion on the various windows that how commonly used in data converter work.

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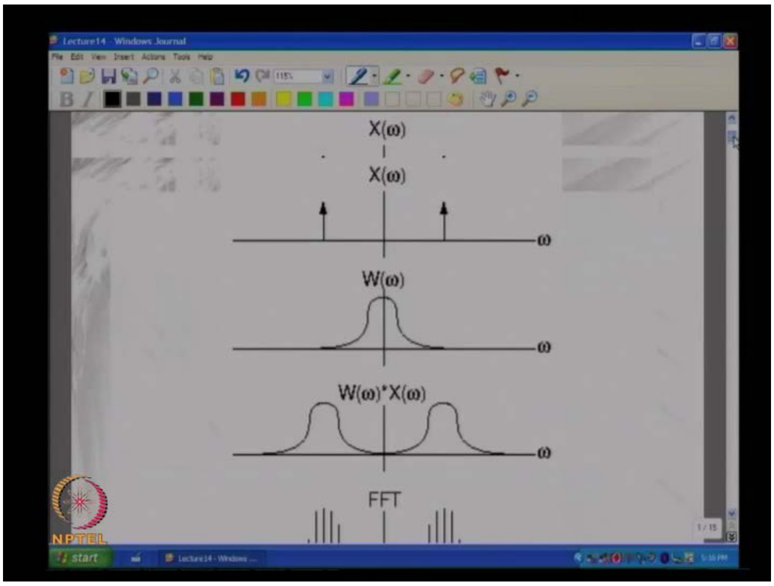

So, if this represents the spectrum of the input sine wave that you want to decipher from it is samples; we are looking at a finite record which can be thought of as an infinite record multiplied by a finite length window. So, if you have a cosine and it runs all the way from minus infinity to infinity this is the Fourier transform discrete time. Fourier transforms of the cosine sinusoid; we are interested in looking at it only over a window. The very first case we thought about was the so called the rectangular window; were all samples are weighted with equal importance.

Then, we said that there is trouble with that and we would like to weight the samples in the end, less than the samples in the middle. So, we multiplied the infinite record not with a rectangular window. But a window with a specific shape one was, one example was the raised cosine type window called the hann window and the other one was the

Blackman Harris window. And, what we said was when you multiply the sinusoid with a window then, in the Fourier domain it is convolution of the 2 Fourier transforms; this denotes the Fourier transform of the window function right. This is just a representative shape depending on the exact nature of the window you choose this shape will keep changing.

So, after convolution what you end up with is a spectrum of this shape, this is nothing but the window spectrum convolved with that of the cosine sinusoid which was just to impulses. Please, bear in mind that this represents the Fourier transform of this Now, finite length sequence; when you compute the FFT what you are doing is sampling the spectrum at you know multiples of 2 pi by m right or n were n is a number of record points in the record which means that here basically ending up with these samples whose envelope will kind of look like this right.

So, after the FFT this is what you will see; you will only see the samples. So, as you can see the choice of the window is very important in determining how much the input tone spreads in frequency right. And, depending on the choice of the window you will be able to say I mean different windows have will behave differently with respect to spreading of the input.

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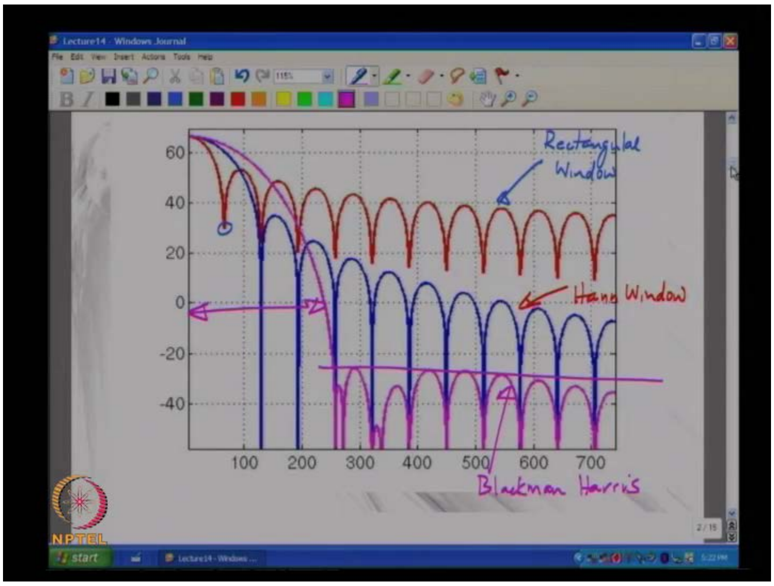

So, it is instructive to plot the discrete time Fourier transform of some popular windows; the one in red is that of the rectangular window as you can see the main lobe width is quite small right unfortunately it does not roll off very quickly which gels; well, with our experience that when we used rectangular window. And, if the input tone was not lying on a bin then you are not basically sampling the nulls of the Fourier transform right. And, you end up with a tail which is not dying down very rapidly; an improvement was the hann window and you can see that it is better than the rectangular window all right.

It turns out that the rectangular window as we discussed couple of classes back falls of in frequency only as one over omega; because it is of the form sine something by something this is sync function which goes off rolls off as one over omega and that makes sense. Because the window function has got discontinuities at it is edges; if there is a discontinuity this spectrum will follow of as 1 by omega, if there is a discontinuity in the derivative1by omega square it will follow of as 1 by omega square. And, if the there you know higher order derivatives are all going to 0 the rate at which it falls off will be much higher than you know omega square correct. The Blackman Harris window is depicted here and you can see that definitely the main lobe is much wider. But the side lobe strength is now much smaller than what you have with the rectangular or the hann windows.

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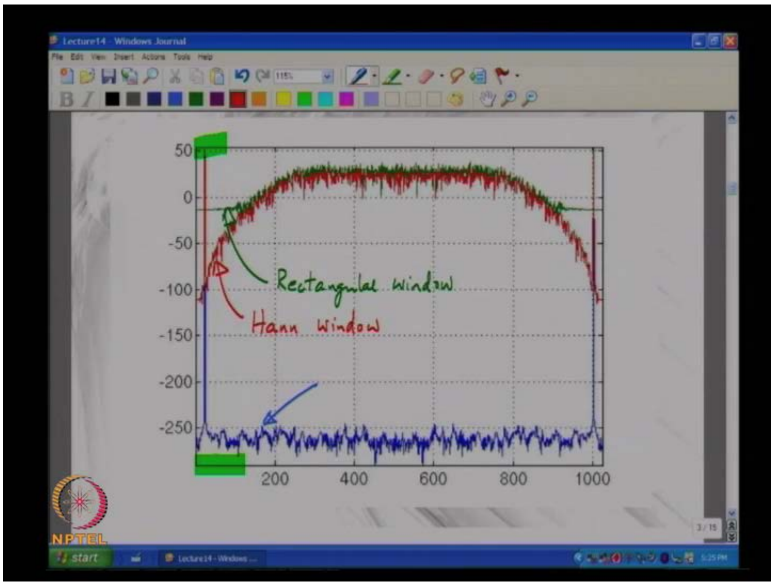

Now, this has some practical implications when we talk about data converter work before we get there I just want to again remind you the context in which we were talking about the choice of which window to use right. Please recall that if we were dealing with

Nyquist converters were the quantization noise as it turns out is mostly flat with frequency. Then, the if the input is lying on a bin then there is no need for you know any exotic window simply using a rectangular windows a good enough; because the input is lying on a bin.

And, even if there is discontinuity in the noise between the beginning and end of the record you will not be able to distinguish it that will not cause enough leakage to cause the noise spectral shape or the values to change appreciable. On the other hand as we will see going forward there are families of data converters were the output of the converter; the sequence is signal plus noise. But this noise is not flat with frequency it is high pass filter; in other words at low frequency that is here the power of the noise is actually very very small under these circumstances.

As we saw last time if we there are discontinuities in the noise at the end of the record then that small discontinuity is enough to cause spectral leakage; which will mask the true S N R that the converter is actually producing right. And, as we discussed last time around this represents what you would get with rectangular window while this represents what you would get when you take the sequence; the input mind you still lies on a bin And, to emphasize that I have shown this in blue when you just compute the FFT of the input there is no problem at all correct. The problem is occurring because of the noise leaking; because of discontinuities at the edges.

So, this represents what you would get with the hann window. And, as you can see we are doing much better with the hann window then we are with the rectangular window. And, this is again I will emphasize even if the input is setting on a bin that means that the input source and clock sources are synchronized all right.

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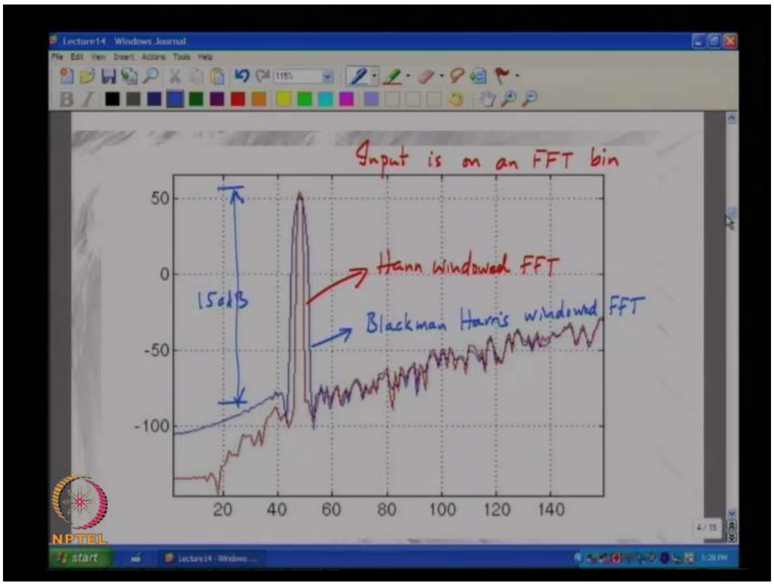

Now, going forward if I used a Blackman Harris window instead of a hann window and the input lies on a bin. So, input is on an FFT bin and I use a Blackman Harris window instead. Please, recall that with a hann window there are only if the input lies on a bin then the input will lead to exactly a total of 3 bins. The bin on which the input lies and 2 neighboring bins and all others will be 0; that is not the case with a Blackman Harris window. So, which is why you are seeing this part? So, this represents the red curve here is the hann window while the blue curve is the Blackman Harris window all right. However, notice that this level of leakage is roughly about this curt is about 100and 50 d B down not perhaps 100 and 50; but it is a reasonable approximation. So, is usually for most of the resolutions that a measured in practice this is not a problem right.

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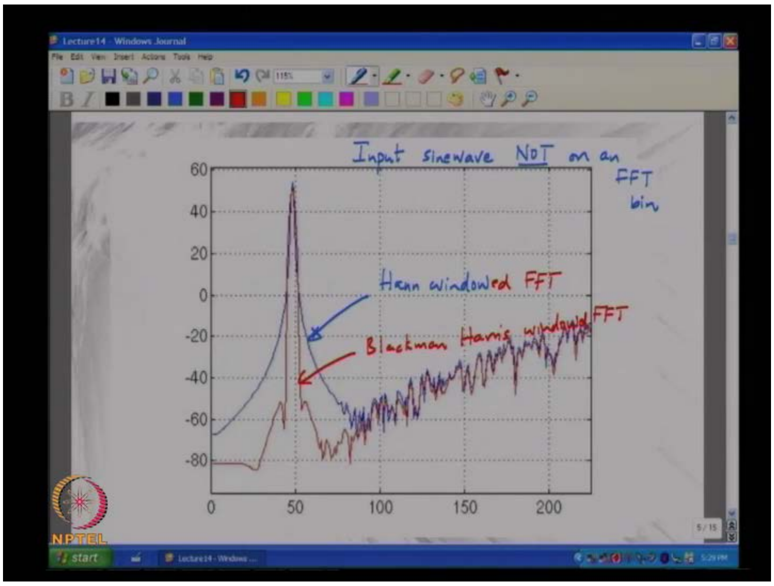

So, if you put the thing on the bin the Blackman Harris seems to be doing slightly worse than the hann window. Now, what is important is what happens when for some strange reason; let us say we are not able to synchronize the input source and the clock; in other words; the input now no longer is not sitting on a FFT bin. And, clearly as you can see which do you think is the hann window; please note that the hann window has a smaller main lobe. So, this is the hann window; however, it does not fall off quite is rapidly as the Blackman Harris window. And, this makes sense when you look at we saw this shapes of the Fourier transforms of the window functions. And, this makes sense much more rapidly than the hann window the Fourier transform of the hann window is this clear.

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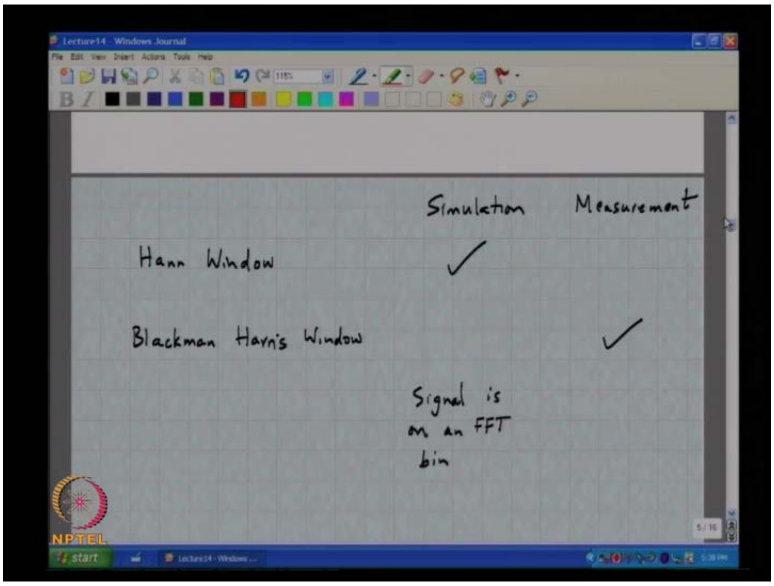

So, now we have seen the properties of various windows. So, let us discuss where you would want to use, which one right. So, when the signal is setting on a bin which is the better window to use?

Student: Pardon.

Yes, so signal is on an FFT bin. And, what scenarios do you think it is possible to put the signal on an FFT bin? Under what practical scenarios do you think is possible to do this? So, one you know possibility is when you are running simulations on circuits; we have the freedom to choose whatever input frequency you want. And, by definition on a computer; if you say sine omega t and you know sine omega n t and sine omega s t or sine 2 pi f in t and sine 2 pi f s t if f in by f s is chosen properly they will be in sink correct. So, during simulation or if it is so, possible that during measurement if it is possible to sink the clock source with the input.

Then, also you can force this input tone being on an FFT bin; under both the circumstances you will want to use the hann window with regard to simulation it is especially important to note that one of the big advantages of the hann window is that it only occupies the main lobe if the input is sitting on a bin is only 3 bins wide right. Because if you run if you want a long record length; I mean the advantage of having only 3 bin wide main lobe is that in the desired signal band which is only at low frequencies.

The number of bins are limited; obviously, the record length is covers the entire range from 0 to f s by 2 right.

But we are only interested in a part of that frequency range which means that the number of frequency bins in the desired frequency range is small. Now, out of those bins; if you within quotes waste lots of bins spreading the input sine wave around then the number of bins left to which I mean from which to estimate the noise properly is small correct. So, from that point of view it makes sense to choose a window were the main lobe is as small as possible right. Obviously, the rectangular window does not work it is main lobe is only is a smallest you can get. But we saw that there is a big problem with leakage a very good choice is the hann window right.

And, this means that if you are only prepared to lose you know 3 bins in the signal band to the signal to the input signal. Then, you have a few more bins to estimate the noise properly from right or given that you want so, many bins to estimate noise properly; it means that you can run, you can have a record lengths which is much smaller in the case of a hann window; then, you would need when you had a window which had a wider main lobe for example, the Blackman Harris is that clear right. So, and in simulation if you want a longer record length it means that you have to wait for a much longer time; because a computer takes you know so much longer to compute a data which is good enough for a larger length FFT is this clear.

On the other hand if you are not able to sync the source with the sampling clock and when does this occur? During measurement one very often comes up, comes across the situation where it is not possible to sink the input source with the clock source. In which case the input is definitely not sitting on a FFT bin which means that using a hann window is not a good idea; because there will be a lot of leakage. So, here what would you want to use; you would want to use a Blackman Harris window. The fact that a Blackman Harris window has a wide main lobe is not really a concern anymore and why do you think that is not a concern; yeah, there are 9 bins.

So, we said that is problem in the earlier case; if you spread the input signal over 9 bins then; obviously, the number of bins that you have to estimate noise from has reduced is not it. So, how many I mean how is that not anymore a problem when you are doing measurements.

Student: ((Refer Time: 20:18))

So, during measurement the constraints are very different right. So, it is very easy to capture 10 times more data exist no more memory; say, capturing digital data a longer record length simply means that you have to capture a longer stream. So, instead of taking 1 second it will take 10 seconds to capture 10 times more data right. But that is not a problem at all which means that it does not matter; if the main lobe is you know you lose 9 bins to the main input tone you collect.

Let us say you know 4 times more data you have a record length which is 4 times more which means that you have a lot more bins in the desired signal band right; which means that whether you know the main whether the input signal is spread over 3 bins or 9 bins does not really matter because you have so, many other bins is this clear. So, this is a something that is useful to bear in mind. So, during simulations you will almost always use a hann window; whereas, while making measurements you know you can use any other window which also has a smaller side lobe a good window to use is the Blackman Harris window.

And, this would be specifically using when we talk about o sample converters where turns out that the output spectrum of the A-D converter consists of the input signal plus noise; which is looks as if it is high pass field. And, we are interested in only in noise in the low frequency bytes all right. So, this kind of concludes my concludes whatever I had to say on F F T S it is and windowing and when you would do what ok.

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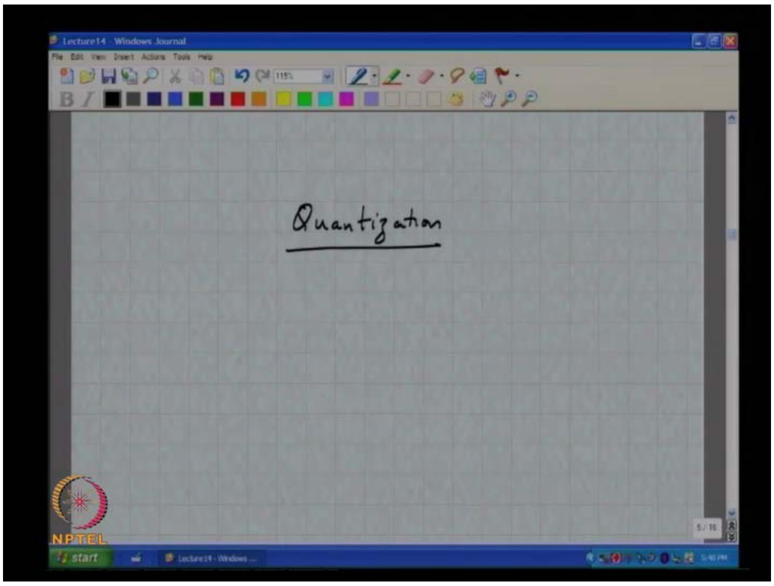

So, now let us move on to the next part which is quantization. So, in other words you have sampled; we know how to build circuits which sample an input on to the capacitor right. I am assuming all of you are taking analog a c designs simultaneously or have taken it before which means that once you have capacitors charged to some voltage that we want using operational amplifiers; we can figure out ways in which it can drive other circuitry without disturbing the voltage on the charge on these capacitors.

So, a little further down on this course hopefully I mean you will have done lot more circuit design in analog I c design. And, then will put the op amps together with the capacitors which have already sampled the input. But for the time being let us assumed that we know once we have captured the input on a capacitor; we know how to make it drive, you know other circuitry without disturbing the charge which is been held on this capacitor right ok.

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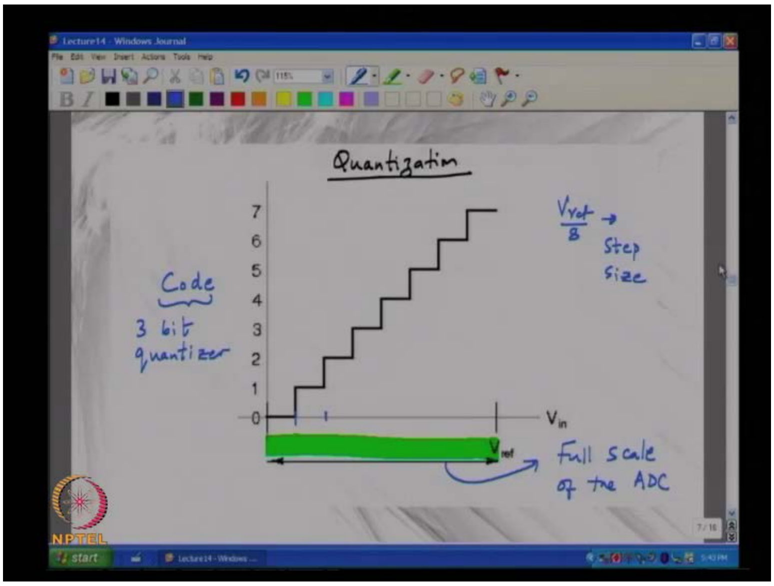

So, now let us look at quantization at a very abstract level. So, you have an input as well as we discussed before which is been sampled and held on a capacitor. Now, we want to figure out in which of these bins I mean which of these ranges the input lies in right. So, the input is assumed to occupy a range say 0 to V ref right which is called the full scale of the A D C; please note this A D C is nothing but analog to digital converter consists of both sampling system once you have sampled we need to quantize.

So, always trying to do now is figure out to the sampled value lies between in this particular example; we want to divide up the input into 8 equal ranges and the width. So, to speak of each range is V ref by 8 and this is also called the step size. So, if the input lies between say 0 and V ref by 8; we arbitrarily assign to it a digital code. So, this mind you is a code which can be represented by yeah how many? This can be represented by a 3 bit a digital word ranging all the way from say 0 0 0 to 111. So, the input lies within this range you get 0; when the input lies between 2 V ref by 8 and V ref by 8 you get 1 right. So, this is what we want to do this is what is called an ideal quantizer.

In this case this is a 3 bit quantizer which divides the input into 8 equal ranges and figures out in which of these ranges the input lines. Obviously, when you want an ideal characteristic like this in practice what you get is probably anywhere I mean now here closed to what you want. Let us now figure out what all errors can happen; the first thing is that perhaps you are lucky enough that this staircase is just shifted either towards the left or towards the right. And, the most behind kind thing of that you can expect to happen.

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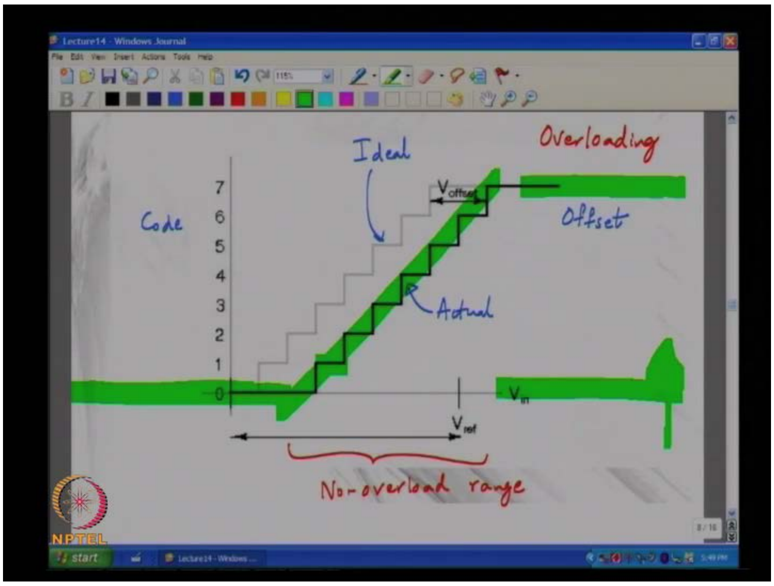

And, so this thing here represents the ideal transfer curve; whereas, last picture shows the actual transfer curve right. Both of them are staircases of the same kind is just that one is offset from the other and quite logically it seems to call I mean this kind of error is called offset all right. And, as we discussed before in many cases offset errors are not really a concern; because it can be correct it can added digitally somewhere else in the in the system you understand.

One more aspect that I would like to point out is that if the input exceeds a certain amount, the output code will either be 0 or will saturate at 7; I mean the input is here somewhere clearly the output gets is not going to exceed the output code is not is going to is not going to exceed 7. And, in the same fashion if the input is way below 0 input code is not going to exceed or not going to go below 0. And, so within codes the useful range of the quantizer is this place where this staircase; if you look at it from far away has got a slope of 1 ok.

So, if the input lies in this range then the quantizer is you know operating properly; if on the other hand if the input is within these ranges where the output of the quantizer is saturated this is a term for this is called overloading the quantizer which is just a fashionable of way of saying the quantizer is saturated. And, sometimes it is not on common to refer to this range; the normal range of operation as the no overload range all right. The next kind of error is that well may be there is no offset.

But maybe all the steps instead of being V ref by 8 or the ideal staircase might be has got equal step sizes; all the step sizes being equal to in this particular example V ref by 8. Now, one kind of error is that the step sizes may all be equal may be there is no offset. But the step sizes may not be equal to V ref by 8 may be there is they are V ref by 8.5 or V ref by 7.5; something like that you understand. And, how do you think the he characteristic will look then?

Student: ((Refer Time: 31:48))

No.

Student: ((Refer Time: 31:55))

The number of levels will remain the same.

Student: ((Refer Time: 31:57))

Pardon; if the step size, if each of these step sizes becomes each of these step widths becomes more what do you think will happen? How will the staircase look? I mean is this step becomes more the slope of the staircase becomes the step size becomes more it becomes the slope changes; it becomes smaller than what it was earlier. And, not surprisingly this is what is called a gain error; slope is often related I mean is basically the change in digital code due to a change in the analog input voltage. On the average I mean it is very difficult to talk about a slope of a staircase; because a slope is either 0 or infinity. But what when is say this slope is changed basically means if you kind of draw a straight line which fits the staircase; the slope of that straight line is changed is this clear.

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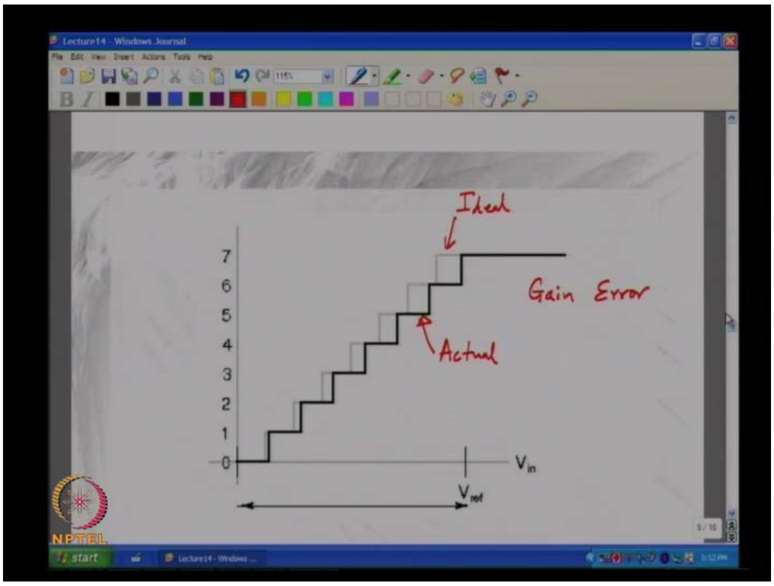

So, this is an example of an error this is the ideal staircase. And, here is a case where the step size is uniform; however, that uniform step size is not the original intended step size of V ref by 8. And, as we can see please note that there are no missing codes; all the codes exist is just that you now have to excite the input a little more in order to get the same code in this particular example; you can also have a situation where the steps are smaller in which case this slope will be larger right. And, this is what is called a gain error all right.

And, again in many systems a small gain error is usually not a problem; because there are in a system we will have some adjustment possible to calibrate out in all the small gain errors. And, please note that if you kind of discount the staircase shape this I mean if you fit a straight line I mean a straight line is within codes a linear system ok. So, in most analog systems gain error a small gain error is be 9. And, so is a small offset that does not mean that now you add of offset. And, the system will work right or you know if I multiply if I make all the steps size is infinite then the slope will become 0; that does not mean it is you understand.

So, small gain error is tolerable. And, similarly a small offset is usually there are applications were it is not in which case we have to worry about it; but most of the time you are ok. Now, the next kind of error what do you think what other errors do you think? I mean the most general thing of course would be that the step sizes are not the same throughout right. I mean in fact; if the all step size were all the same you know I will be very surprised when you try to build a system. And, the step size are all you know magically the same there must be something really a nice going on inside.

So, but if you kind of realize the system like this more often the not you would not be surprised at all if all the step sizes were non uniform. And, so in a real quantizer you will find that all these errors occur together; it is not as if you will have only offset error or only gain error or only variable step sizes, you will have you will have an offset you will can potentially have a gain error along with a non-uniform steps.

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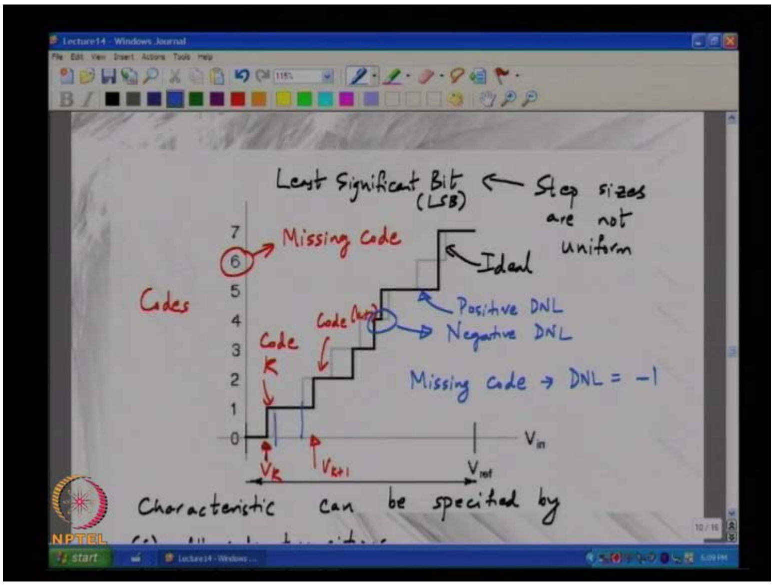

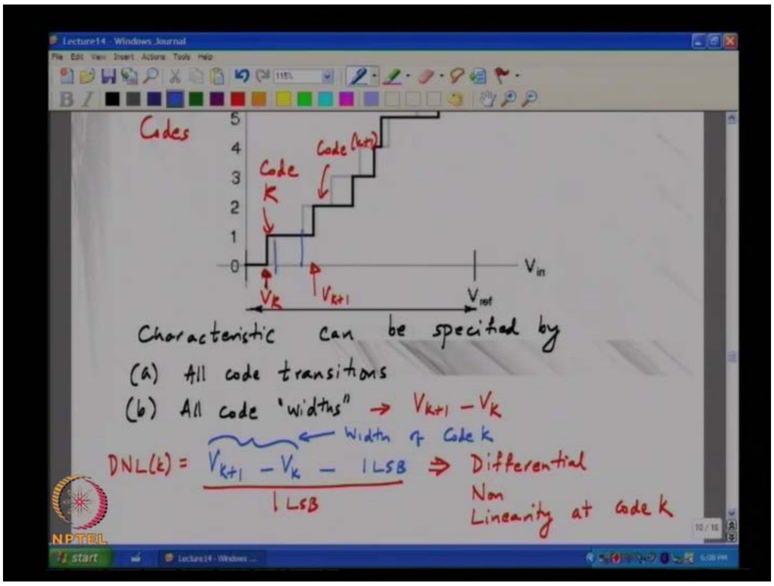

So, as to a picture and see a case where the step sizes are. So, another piece of jargon the step size is often also referred to as the least significant bit of the LSB right; I mean the idea being that the step size is the change in the input voltage required to cause the code to flip by 1 LSB right. So, in other words it is you know the size of 1 of the ranges that you are trying to classify the input into. So, often you will here in data converter literature or you say the LSB is 1 mille volt or the LSB is half a mille volt; what that means is that you are dividing up the input analog range into many steps ideally all equal steps. And, the size of each step is 1 mille volt a half mille volt as the case may be.

Now, clearly in this picture the grey shows the ideal staircase that we were expecting unfortunately the step size is a not uniform; and we get the different codes having different widths. For example, the code 1 has a width which is it greater than the normal width or smaller than then than the normal width?

Student: ((Refer Time: 39:11))

I have drawn the ideal staircase in grey code 1.

Student: ((Refer Time: 39:22))

So, the width corresponding to the input range corresponding to code 1 is this right. And, ideally what was supposed to be; it was supposed to only extend from here to here. And, as you can see this code is wider than what it was ideally supposed to be correct. Similarly, code 2 I would say from these pictures is also wider than what it supposed to be what about code 3 and code 4; code 3 is definitely smaller. And, code 4 is very definitely smaller than the ideal step size; code 5 is much wider what about code 6? Code 6 does not occur at all right.

So, code 6 is what you call a missing code you understand. So, when you want to specify a quantizer; what information would you need to give somebody a complete idea of how the characteristic of this quantizer looks like? There are many ways in which you can give this information right; please note that all this information is simply telling you at what analog input voltage does this code transition occur right. So, if I give you a list of all analog input voltages at which code transitions occur right; I have in principle giving you all that you need to know about this quantizer you understand.

Another equivalent way of doing this is to give the width of each code correct you understand. So, characteristic can be specified by (a) all code transitions, (b) all code widths. And, when I mean code widths what are the, what do you I mean by code widths? It is the analog input range over which that code is coming out of is asserted by the quantizer. And, it is simply related to the transition voltages; the width of code for example, code i let us call this if I call this code i right; let me call this V i or let me not use i. Because it get confuse with the input; let us call this code k. And, let us call this transition voltage which is the voltage at which this code first appears.

Let me call that V k then the code width is nothing but V k plus 1 minus V k; the set of all the set of all V k is to completely specifies the characteristic right which means at the if I give you V k plus 1 minus V k for all codes that also equivalently specifies the characteristic you understand all right. Now, ideally the step size is suppose to be ideally the step is supposed to be constant and equal to in this particular case V ref by 8 in general equal to 1 LSB by definition correct. So, it does not make sense to specify code width as 0.8 LSB, 1.1 LSB 1.01 LSB and so on; we know that they are nominally supposed to be 1 LSB it is enough to specify the deviation from the nominal step size you understand. So, in other words giving information in the form of V k plus 1 minus V k which is the width of code k minus 1 LSB what are the units of 1 LSB?

It will be if we are talking about a quantizer which quantizes voltage; then, 1 LSB will obviously have a dimension of volts. So, V k plus 1 minus V k is the width of the k-th code this minus 1 LSB is the deviation of the width of code k from the idea right. And, this is got a name I will come up, I will tell you why this is called a given the name it is this is called the differential nonlinearity at code k. So, DNL of k is this and as we have written it the DNL with differential nonlinearity has dimensions of volts.

And, in order to compare different inverters it may; now it may turn out that there even though they have the same number of bins. In other words they have the same you divide up the input range into the same number of partitions; the absolute value of the input range may differ from converter to converter. So, it does not make sense to talk about the absolute DNL of code right; it make sense to talk about a related to what? Relative to the nominal LSB of the converter; in other words you do not talk about it in terms of mille volts, you talk about it in terms of number of LSB of that converter correct. So, in other words the differential nonlinearity corresponding to a code k is often normalized to the LSB.

So, V k plus 1 minus V k minus 1 LSB divided by 1 LSB whatever that happens to be. Now, the question is why is it called differential nonlinearity? Why does nonlinearity itself make sense? Why do you think it makes sense to call it nonlinearity? Pardon; well, it is non-linear to begin with any you know the step size is the same; because this is non-linear is not it.

Student: Sir, can LSB different in different codes at level this for k plus 1 code it may this will be the different.

It is different yes.

Student: So, this type this is this itself is not constant?

Yeah. So, you can I mean it is a basically if the code widths keep changing; it kind of means that the slope of the curve is different at different inputs. So, definitely that is something which is nonlinear. And, differential nonlinearity gives you information about the local slope correct you understand. So, I mean DNL or differential nonlinearity is giving you information about how the converter performs, how uniform the step size is in a I mean over a narrow range you understand ok.

So, this is one way of specifying the characteristic of the quantizer the DNL I mean if I give you the DNL as a function of all the codes; if I gave you all the code I mean the DNL for all the codes; then, in principle the entire characteristic is specified no more information is necessary you understand. So, but often it is also convenient to have not only. So, differential nonlinearity; it is also interesting to see not just locally how the steps vary. But I had also like to see how far away my actual code transition is from the ideal code transition you understand that yes.

Student: Sir that DNL can 1 LSB that 1 LSB for the ideal characteristics or for I am got confused if the V k plus 1 and minus V k that is also 1 minus V k.

No, no V k plus 1 minus V k are the actual; let me say this is code k, this is code k plus 1 correct V k represents the lowest voltage that will result in a code k. So, by the same token V k plus 1 will represent the lowest voltage that will result in a code k plus 1;

please note that if the quantizer was ideal right then there should have been V k values V k and V k plus 1 should coincide with those 2 blue lines right. But due to some non idealities in the implementation the V k and the V k plus 1 are not.

Student: Coincides.

They should be or not what they should be ideally that different correct. And, we are quantifying that difference because V k and V k plus 1 are not what they should be; the width of the code k is not what it should have been which is ideally 1 LSB, right.

Student: Sir, it means that difference between the blue lines is a 1 l s b.

Correct, right I mean are you able to see the.

Student: Yes, sir.

The grey characteristic that is the ideal characteristic whereas; the black one is the actual one. So, V k plus 1 minus V k is ideally supposed to be 1 LSB. So, for an ideal converter the DNL is or the differential nonlinearity is 0. So, the actual one; of course it can either be positive or negative you understand. For example, code 4 here; do you think the DNL is positive or negative? The step size is smaller than the ideal step size. So, this is negative DNL what about code 5? Positive DNL what about code 6?

Student: ((Refer Time: 52:54))

So, for a missing code is minus 1 you understand. And, please note that the DNL as a function of code is telling you the width of that code k. So, it is you know in some sensing giving you the local slope of the staircase you understand all right. So, in the next class we will continue with this. And, another measure of, another way of specifying the same information which is called integral nonlinearity; we will continue with that in the next class.