VLSI Data Conversion Circuits
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Lecture - 13
Spectral Windows -1

So, in the last class we got some intuition about a y; when you use a rectangular window there is a lot of leakage d; what one can do to fix this problem, which is we understood that the problem is coming because of discontinuities at the edges, right. So, if we multiply the original sequence by so, called window sequence which is kind of flat in the middle, but tapers of nicely and smoothly at the ends then, one has a much smaller leakage because the n points an out totally deemphasized in the whole process. And, as you all guessed; you can come up with any number of functions which are kind of flat in the middle and taper off within quotes nicely at the edges. So, it is not surprising that there are a whole bunch of windows in the literature. So, I would like to give you a quick bit of intuition on one property of windows which is commonly coated.

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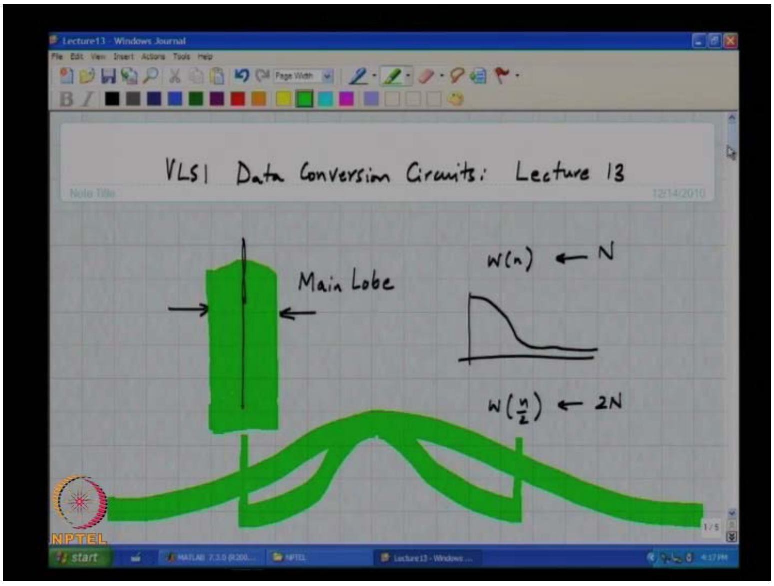

We saw that with the Hann window the input tone right, if the input is lying on a bin will spread to neighboring bins, right. So, you can kind of say that the input which was originally kind of concentrated in one bin is now, spread across not just one bin, but across a few bins. In the hann case it happens to be 3, all right. Now, it turns out that this, in other words; the glow I mean the frequency region where the input power is concentrated is not now, a single frequency bin, but many. So, this is often called another main lobe, all right. And, as we saw the last time around, the leakage is drastically reduced when compared to a rectangular window, when the input was not on not setting on a bin, right. And, that is a consequence of the discrete time Fourier transform of the window function, correct ok.

So, the main lobe width depends on the low frequency part of the window function, where as the amount to which it rejects the leakage depends on the high frequency part of the window function. Please note that a finally, it is multiplication in time of a rectangular window with whatever window function you have correct which means that in the frequency domain it is convolution of the rectangular window with whatever window the with the Fourier, I mean; convolution of these discrete time Fourier transform of the rectangular window with that of the window function, right. That will get convolved with the sinusoid and that is what you are seeing finally as leakage, all right.

So, if there was a window function W (n), right. Let us say I want to make the low frequency part twice as y, right. What is it mean to say you know twice as y, what is it mean to say I have a bigger main lobe? The input tone is spread across more frequencies, right. In other words; the band width of W (n) is lower, right. If it has to be spread across many bins, it has to be multiplied by no, let us say I mean; I limit and put it this way. If you want the leakage to reduce right this transition from being you know high in the middle to zero in the edges, must happen as smoothly as possible, right. And, you know when it is a function you know very smooth.

Student: Slow density.

When it is band within quotes when it is band width is.

Student: Low.

Is very low, right. So, in other words; if there was a window function of window sequence of length N right, which had you know some kind of main lobe, and some kind of side lobe; now, if I made this a sequence window of n by 2 with a length of 2 N, right. Do you think this is smoother than before or.

Student: ((Refer Time: 06:50)).

Or you know sharper than before.

Student: Sharp.

n by 2 is the same thing I was now stretched it.

Student: Smoother of ((Refer Time: 07:00)).

Right, so, this means what? If I stretch this in time

Student: It will compress in ((Refer Time: 07:11)).

I mean it will become what you call this thing, right. So, in other words, but if I am only interested in window sequence of length N right, I can think of this as widening it, making it smoother and chopping off the edges, right. The moment I chop off the edges what happens to the high frequency components? Earlier, the window was tapering off like this. Now, what I have done is created another window which is just this stretched in to you know twice it is span, but if I am interested in only a window of sequence N, right. What I will chop off the edges. So, you can see that if the main lobe width is wider the side lobe width is compromised because I mean it is because of the chopping it becomes.

Student: Less smooth.

Less smooth, ok.

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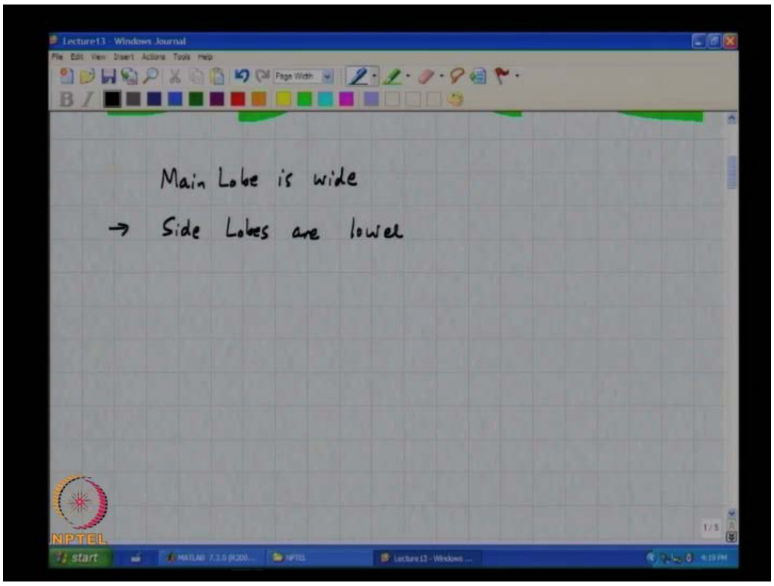

So, this is often called if the main lobe is wide, the side lobes are suppressed, but are lower. So, if the main lobe is narrow, the side lobe suppression is poor. And, this is what you see when you see rectangular window the main lobe is just.

Student: Single tone.

Single tone; when the input is one, is sitting on a bin right. You see that there is only one coefficient which is non zero, right. Unfortunately when the input is off bin because the side lobe suppression is not good enough we see that there is a lot of leakage. On the other hand; if you spread if the main lobe was very wide. In other words; if you put the input sinusoid on a single bin it would spread to several bins around the input bin. Then, you can expect the side lobe suppression to be much better, a case in point being the hann window, where the main energy is spread across 3 bins. And, we saw that the side lobe magnitudes are much smaller, when compared to rectangular window is this clear.

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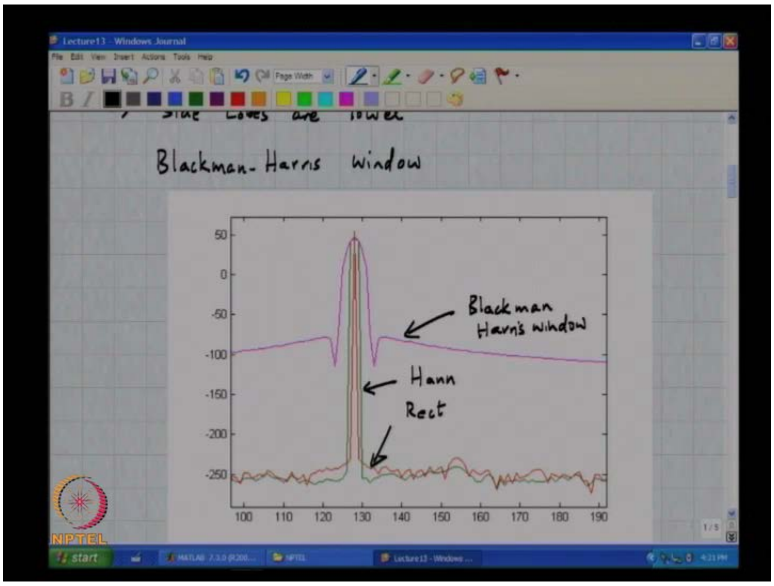

Now, having said this, the hann window is not the only window around, another popular window used in data converter work especially is what is called the Blackman Harris window. So, this picture shows the spectrum, when 3 different windows are used the red one corresponds to.

Student: Rectangular.

Rectangular window, please note that the input is sitting on a bin. The green one corresponds to a Hann window, right. And, as you can see the main lobe width is now 3 samples rather than one, the magenta one corresponds to the Blackman Harris window, all right. See all the coefficients of the Blackman Harris, I mean; when you multiplied by the Blackman Harris they will not be zero, I mean that depends on the nature of the window function. For the raised cosine or the Hann for the hann window it turned out to be like that because it was multiplied by a cosine, right. And, therefore; when you convolve this spectrum with that of the rectangle then, we see that the ((Refer Time: 12:28)) just get moved around by exactly one bin, and therefore; only the main lobe the main 3 bins are non zero, and the all the other coefficients are zero. The Blackman Harris window you know the more complicated function right, it be interesting to see what is interesting is to see what happens when the input is not on a bin.

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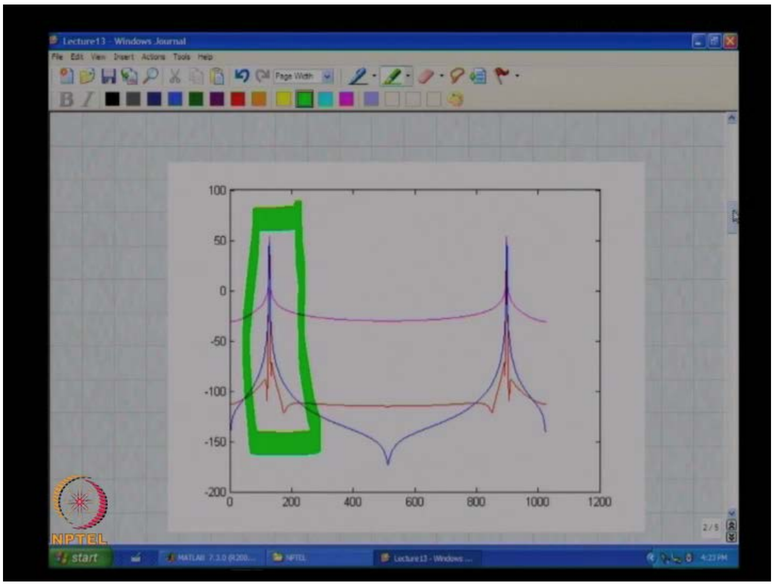

So, here again the input is moved slightly of the bin. And, one notices that the rectangular window is of course, the worst because of a lot of leakage, all right. The hann window has some amount of closed leakage, where as the Blackman Harris falls off a lot more rapidly, you understand ok. So, that completes the discussion on windows.

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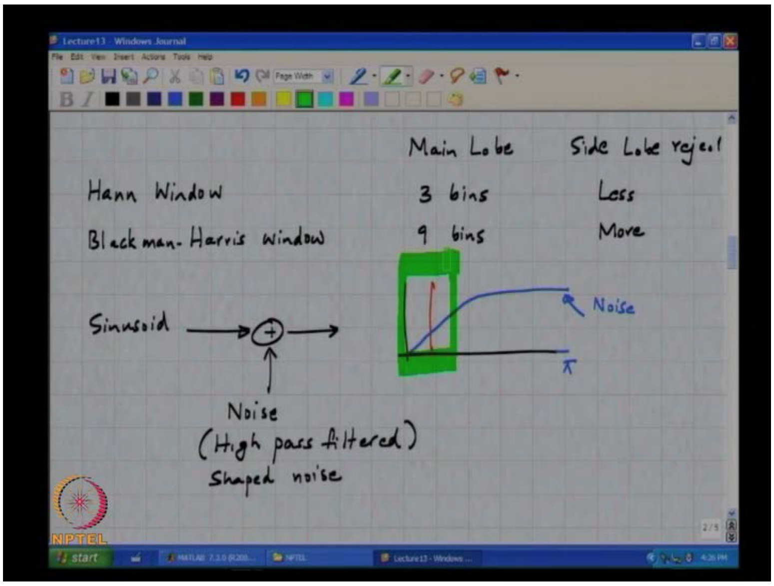

The 2 windows as far as data converter work concerned at the so, called hann window, and the Blackman Harris window, all right. And let us just quickly tabulate this main lobe width is happens to be 3 bins, as we discussed. Whereas for the Blackman Harris it happens to be 9 bins in the most common, this side lobe rejection is much higher in a Blackman Harris window. Why I am focusing on these 2 windows is the following; going forward will end up in situations, where the signal that we want to analyze consist of a sinusoid, and to this there is noise added, all right. And, when we are dealing with over sampled or delta sigma data converters, we will be dealing with a special kind of noise which is called shape noise.

In other words; the spectral density of this noise is not uniform across all frequencies, this noise happens to be high pass filter, right. So, in other words; you understand the noise is high pass filtered which is also often referred to as shape noise. In other words the true spectrum that you are looking at is got a sinusoid and then, some noise which is very small at low frequency, and kind of thus this at high frequency. Does make sense? I mean; for the time being you just assume that this is a situation in which one needs to compute say the signal power, and say also the signal to noise ratio in this frequency back.

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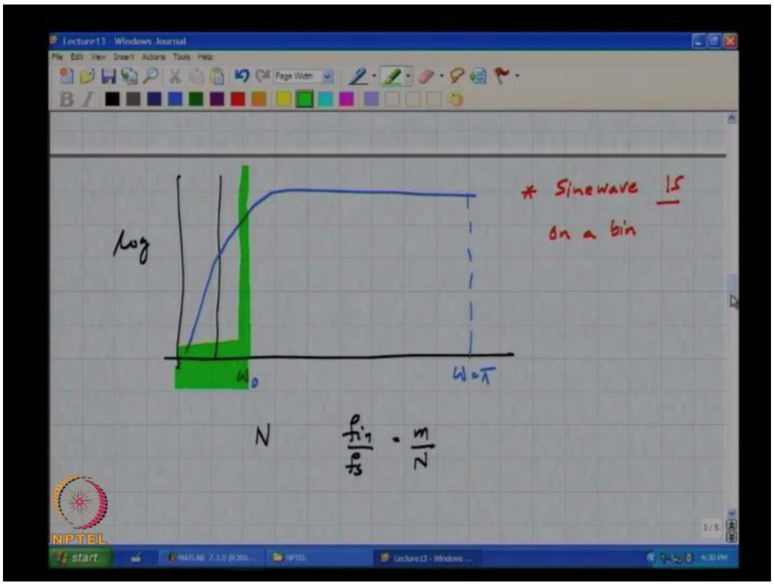

In other words; you have a sin wave embedded in high pass filtered noise, this is all analog scale, right. The noise spectrum is something like this, and you are required to find the signal to noise ratio in a certain band width omega naught. What would you do? Let us for argument sake say; the input sine wave is on a bin, what would you do?

Student: Because; in my problem and the noise problem can be integrated that you can see noise.

So, I have a sequence of n samples what will I do?

Student: At the ((Refer time: 19:32))

No I have a time domain sequence of n samples right, the record length is n as usual, and f in by f s is m by n. Now, can you tell me what I should do with this sequence?

Student: I am sure that it is a ratio of integers.

It is m, let us say; yes, I am sure that is a ratio of integers.

Student: And directly take it.

I can compute the discrete Fourier series coefficients right, and I hope to see some spectrum of this form, right. I am only interested in the ratio of signal to noise in a band width omega naught. So, to find the signal par what will I do?

Student: ((Refer Time: 20:20)) square minus.

I know, but that my signal lies on a bin. So, I know I mean looking at the spectrum they will be a big spike in one of the bins that clearly corresponds to the desired signal, right. All the other bins I add up the powers that must correspond to

Student: Noise power.

The noise power I must only add up those bins which are less than omega naught in frequency, correct. This should in principle give me.

Student: Signal.

Signal to.

Student: Noise ratio.

Noise ratio correct, let us see what happens? Right, I will run a numerical experiment.

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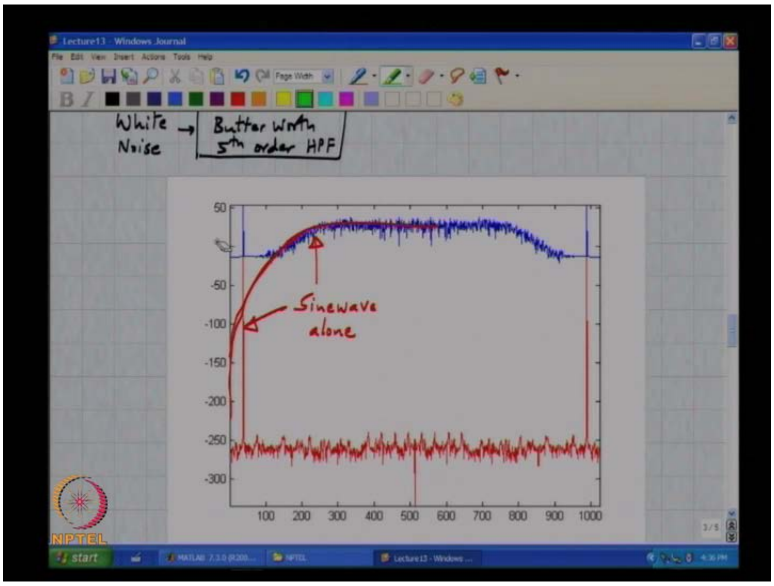

Where what I do is there is a sinusoid, and as we indicated before it does lie on a bin, right. To this I add high pass noise, and how do I generate high pass noise, and what is your easiest what is the thing that you can think of to generate high pass noise. Take white noise and, then pass it through a high pass filter here. I have chosen high order butter worth high pass filter, correct. And, this is my sequence and as usual I compute the f f t. In principle it would seem as a no window is necessary is not it, because; the input lies on a bin. So, let us see what happens. So, this is the f f t I get, there are 2 plots here, this corresponds to the sine wave alone, right. And, as we expect since the input lies on a bin, when you compute only the sine waves f f t, you can see that there is only a single spike the rest of it is all zero.

So, in the log scale you are getting about the noise being about 300 d b below, the input signal level is this clear, all right. Now, this is supposed to be high pass noise. So, how is it supposed to be you know how does it how should it looks like? If I take white noise and pass it through butter worth high pass filter, right. This is the corner of the butter worth filter must be somewhere here right, because of the high pass filter must be doing something like this. At D C, what must be the reaction of high pass filter?

Student: Infinity.

Must be infinity so, this must actually do this, correct. But what do we see is happening.

Student: ((Refer Time: 24: 12)) again frequency.

It is.

Student: Beyond.

Beyond a certain frequency, it is simply flattening out and we know this is wrong because I mean here, in this experiment everything is known, right. If I was dealing with an unknown noise then, I would say perhaps there was a low frequency component in the noise, right. But here, I know that I am generating noise, and I am generating that by taking butter worth filter, high order butter worth filter and passing white noise through it. So, definitely at low frequencies it must go as. I mean in the log I mean if you have an nth order high pass filter around D C, how will that transfer function look like?

Student: Noise sequence, noise, omega power n.

It will look like omega power n correct, which must go as on the log plot must go to minus infinity, right. Clearly we see that is nowhere close to happen, you understand. The question is; hey what is happening here right, this is this spectrum is clearly not doing what we expected to do. So, the question is what is happening, and do you have any theories, this cannot be the signal because when I plot the spectrum of the signal alone, it is clean. So, this must be something to do with.

Student: Noise.

Noise now; what do you think that something is.

Student: Because the noise can alter in any frequency it cannot be while speaking in any particular bin.

Yes. So, what? I mean all the other frequencies seem to be why is it that.

Student: Sir Aliasing.

This is sequence; there is no question of aliasing. We are all dealing with sequences; yes, why do you think this is happening. So, it turns out that the sinusoid of course, you have no trouble with at all, right. The noise sequence again I am going to show it as a continuous wave form right.

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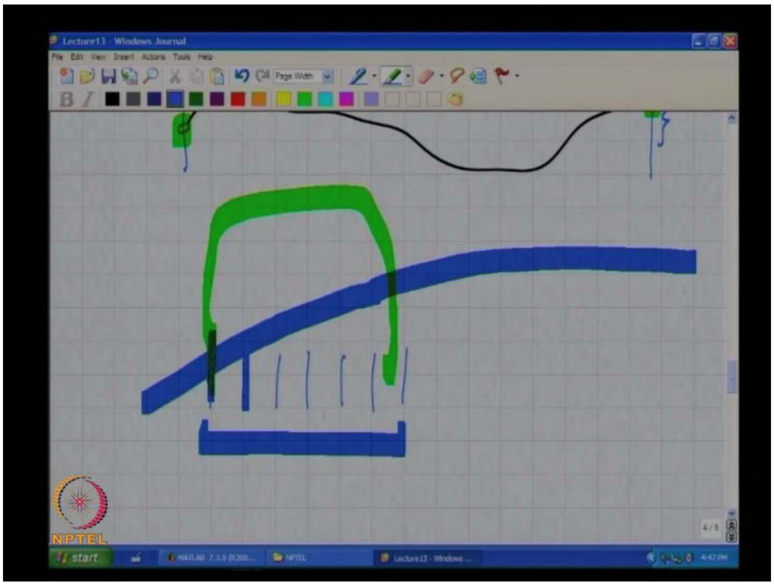

This is a very bad example of high pass shaped noise right, high pass shaped noise must be in fact very weakly, but this must be to illustrate the point. So, you have the beginning of the record, and the end of the record, correct. And, this point and this point will never be the same, correct. So, now when you compute the discrete Fourier series of this sequence, what happens? There is a discontinuity at the end points of the record, right. If the noise is small then, the discontinuity will also be small because the peak amplitude of the noise itself is very small. Then, a discontinuity is also bound to be small this discontinuity is again like an impulse, right. So, when you expand it in a Fourier series what will happen? It will consist of you know free frequencies where the fundamental period is, and samples, and all multiples, there 2 pi by n, and all multiples there off. So, what is happening to the low frequency bins? They were all supposed to be zero right, which means; that in the log scale they must all be minus infinity, right. Well, then there is a small component because of.

Student: Discontinuity, noise.

Discontinuity of these noise samples in the beginning and end of the record. So, when you compute the d f t, I mean d f f t; what you will see is that I mean it is indeed small as you can see, right. In this particular example; if the noise was small also, and you can see that there is below a certain level, the actual spectrum is something like this, but on top of that there is error due to.

Student: Discontinuity.

Which is basically; leakage again then, noise is into leaking all over the place and that is because of the rectangular window, yes. So, this the input signal consist of signal plus noise, right. The signal as we have postulated lies on a bin. So, if you compute the Fourier series coefficients of the signal itself, they should be no leakage right, which is precisely the spectrum shown in the red curve, right. We have also said that along with the signal there is high pass shaped noise right, where technically the low frequency components must go to.

Student: 0.

0, because I know that I have generated high pass noise by taking white noise and passing it through high order high pass filter. High pass filter by definition has suppresses noise at around D C to zero. So, on the log scale the spectrum must go to minus infinity correct, but hey when we do when we compute the f f t we see that at low frequencies the spectrum is flat right. So, clearly something fishy is going on that cannot be coming from the signal, it must be coming from the noise sequence. And, the reason is that noise being what it is I mean it will definitely not be periodic right, which means that they will be a discontinuity between the beginning and the end point of the sequence, right.

Now, if there is a discontinuity between the beginning and the end of the sequence then, as we discussed before this will have; when you expand this in a Fourier series will have components at 2 pi by n had all multiples there off. If the noise is small and, therefore; the discontinuity is small, right. It will follow that this flat part of the spectrum corresponding to this discontinuities Fourier expansion will be very small. The only place it will be visible is when, there are nulls or the original spectrum has got.

Student: Low energy.

Very low energy at those frequencies; I mean here, we do not see anything why in these regions everything looks why.

Student: Because it is lower the energy.

Because the discontinuity is very small, and it is energy corresponds to something like that, right. So, if the at frequencies where the high pass filter gain is sufficiently high right, the noise leakage due to windowing right, is much smaller than the spectral density of the noise itself. So, we do not see it, the only places where we will see it are at frequencies where there are nulls in the spectrum of the noise and in this particular example happens at around D C, right. So, beyond a certain point you see that this just becomes flat is that clear. So, this is again happening because of discontinuities in the beginning, and the end of the record.

So, what is the fix? Same old same old right you do not want stuff at the end, you want stuff in the middle. So, you multiply it by window. So, now the question is what window do we use? You understand the question right, I mean we have a choice of many windows. So, they must be you know some window must be better than the other, correct. As long as it is not Microsoft window the any window is all right. So, I mean any suggestions?

Student: If the windows are too wide.

Ok.

Student: Then the noise power also will be split into the signal wave.

Yes ok.

Student: So, I mean we should not use window whose main lobe is too wide.

So it is a good point. So, what he suggesting is we know that the input signal is lying on a bin, right. So, if we multiply it with a window function whose main lobe is very wide. In other words; the input signal even though it lies on a bin let us say it spills into many bins by the side then, in all those bins into which the signal spills, right. The noise is we do not know in those bins there is noise, there is signal also, and we are not quite sure how to account for it right, whether the power in those bins clearly cannot be all signal there is some.

Student: Noise.

Noise in it so, if we add the powers in all those bins; if we say they correspond to the signal then, we are underestimating the.

Student: Noise.

Noise is that clear, in other words; let us say the because of windowing these are all the bins right, because of windowing let us say the signal is spilled into 1 2 3 4 5 6 7 bins. And the noise, I mean; noise was actually something like this now, in these 7 bins the power consist of both.

Student: Inside the signal.

Signal and noise, we can do one of 2 things you can say all this corresponds to the signal. In which case; we are discounting for they getting an optimistic estimate because we are not.

Student: Counting the noise.

You are not counting noise in these bins. On the other hand; if you say all this is noise then, you finished because you know signal at all, you understand. So, common practice is to choose a window, where the main lobe is small right, and a good choice is the. I mean of course, the smallest is you can use a rectangular window, but that does not work as we just seen. So, the next best thing is.

Student: Hann window.

Is the hann window, where we know that the signal only spills to 1 bin on either side? So, total of 3 bins, in other words this accounting problem becomes less problematic ok.

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So, let us show the same thing with hann windowed spectrum. So, this is the signal, this in blue is the rectangular windowed spectrum. And, the one in magenta is.

Student: Hann window.

The hann windowed spectrum here also there is leakage, right. I mean, ideally they should be minus infinity you understand, but you can see that there is a huge improvement of about 100 d b or so on, you understand that make sense because you had basically forcing the end points to be zero. Therefore, if there is discontinuity it just gets squeezed completely.

Student: how would you account for the noise it gets filled on the 2 things?

Well, you can see that noise also gets I mean any frequency right, will spill on to 2 neighboring bins. And, in that process I mean the noise power is reduced right, but fortunately the same thing is happening to the Signal also. So, if we are only interested in computing signal to noise ratio then, this is largely transparent. You only worry about the you know the spilling happening at the ends of the bins that you are counting. That you give if there enough number of you knows bins in that you are counting usually not a problem. So, mean let me take for argument sake; if you are trying to find compute the signal to noise ratio in this frequency bin, I mean this frequency range, with a rectangular windowed spectrum, you know what will be the estimate of signal to noise ratio?

Student: d b m.

Pardon, how many d b?

Student: S n r with the rectangular window will be about 300 d b.

No. It is I mean; what is the noise spectrum, I mean with the rectangular window the blue guy is the spectrum you will see, right. You will definitely see a spike corresponding to the input signal, but because of leakage you will see that in band, in this desired band. There is you know a lot of leakage, and you know the signal to noise ratio is about may be 40 d b there are this is about 50 d b down or not 50 may be 60 yes 70 perhaps. And, they are 100 bins right, that is another 20 d b right. So, the s n r you will compute is only 40 d b, even though the true s n r is much higher, is this clear. The true s n r within the signal band width is the following within this region, you find the signal power.

And, the noise power by summing up the power in all those bins, and taking the ratio of the powers, correct. Where you do that to the rectangular window there is; obviously, a lot of leakage. Whereas, when you use a hann window to compute the signal power how many bins powers would you have to add you add; we know that the when you use a hann window and the signal is lying on a bin, it leaks on to one bin on each side. So, you need to simply compute the 3 bins one on either side of the signal, right. And, for noise all the other bins, you understand all right.

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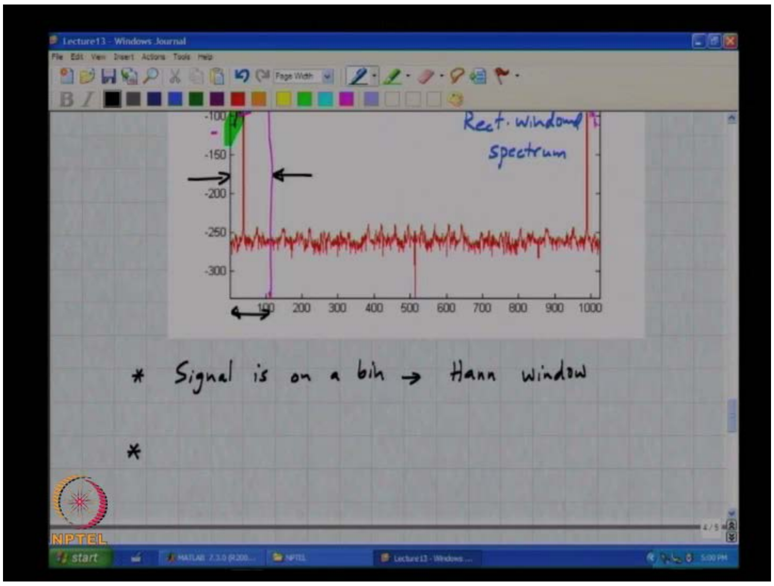

So, if the signal is on a bin then, one would tend to use the Hann window. Let me remind you again, why you want to use a hann window the reason is that the signal only spills to a total of 3 bins correct. Whereas if you used a more exotic window, what is for example, if you used a Blackman Harris window, what would you what do you think will happen.

Student: Noise spills.

It will spin to the signal power will spill to 9 bins, which is I mean in principle there is nothing wrong with the signal power spilling to 9 bins. I will simply add up the.

Student: Power.

The power in all the 9 bins to get the signal power; however, the number of bins within my signal band of interest that is the band width over which I am interested in computing my s n r is limited, correct. So, let us say I have only 100 bins in all, right if I lose 9 bins for the signal; that means that I am accounting for noise only in the other 91 bins. Whereas if I used a hann window I lose 3 bins for the signal and I have.

Student: 97 bins.

97 bins, it I mean 91 versus 97 may not seem like a big deal, but you often get into situations, where the band of interest is only say 30 bins, all right. In which case a loss of 9 bins for the signal is a significant thing whereas, losing only 3 bins is not a problem, you understand. So, if it is possible to put the signal on a bin then, it make sense to use a hann window, does it make sense? all right now.

Student: This kind of phenomena I mean using a hann window only for the high frequency noise not for when there is a low frequency noise then.

I mean if there is low frequency noise then, you I mean you are finished right. No actually I did not quite understand your question.

Student: I mean this whole discussion was because we had a high frequency noise.

Well, this whole discussion was that we had noise. So, whatever noise you have they will be discontinuities at the ends. And if you have a discontinuity at the end that will basically you know cause leakage in the sense that they will be components at multiples of 2 pi by n. So, in other words; they will be some background flat noise that you add, correct. The discussion I mean; I chose high pass filtered noise here to illustrate the fact that this is not really a problem correct, unless you are interested in finding s n r's over band widths, where the inherent spectrum of the noise is zero, you understand.

So, this kind of thing will never show up if the noise is flat, you understand. So, in Nyquisted converters for example, where the quantization is a largely flat, I mean a rectangular window is just good enough. I mean one thing is that quantization noise is periodic because it is not real noise, but even if you add small amount of white thermal noise, rectangular window is perfectly ok. If the input signal lies on a lies on a bin, but with us when you have shape noise right, I mean the phenomenon the leakage is still there, even in the earlier case.

The only thing is that now, because of shaping or high pass filtering the spectrum at low frequencies happens to be ideally 0, right. So, even if you add a small amount you will get totally misleading results; if you compute the signal to noise ratio in a small band, right. I could have chosen band pass I mean; what do you call, noise not necessarily through a high pass filter I could have chosen a band stop filter right, where in some band there is no noise. The same thing will happen there too right, because of discontinuities there will be a flat part added and I will just fill up those bins, where technically there was supposed to be no noise at all, you understand. Yes Sumit.

Student: Yes sir.

Now in practice it so, happens that again in some cases it is not possible to synchronize the input source with the sampling clock. In other words; the input sine wave is not sitting on a bin. So, in that case I mean what you would do is to use a window where the side lobe suppression is larger right. For example, a Blackman Harris window would be preferable to a hann window, and now we have to deal with the fact that the signal now occupies in a 9 bins. So, and if you have only a finite number of bins for the signal band of interest then, I mean this whole thing does not is not very accurate. So, the solution to that is simply increase the number of bins in the signal band which is equivalent to saying take a larger f f t or take a larger record length, which means; that there will be a larger number of bins overall which means that in a given frequency range there will be a much larger number of bins. So, if you lose 9 bins for the signal it perhaps does not matter, you understand. So, I will continue with this in the next class.