VLSI Data Conversion Circuits
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Lecture - 11
S/H Characterization – 2

This is VLSI data conversion circuit's lecture 11. In the last class, we were discussing how one should choose the input frequency in relation to the sampling rate. So, that the resulting discrete time output sequence is periodic. The motivation for making the output sequence or the sample sequence periodic is that we will be able to decompose it into a Fourier series which is now discrete time. And then look at the various Fourier series coefficients and identify the coefficients with the harmonics. And towards the end of last class, we saw that this f N by f s.

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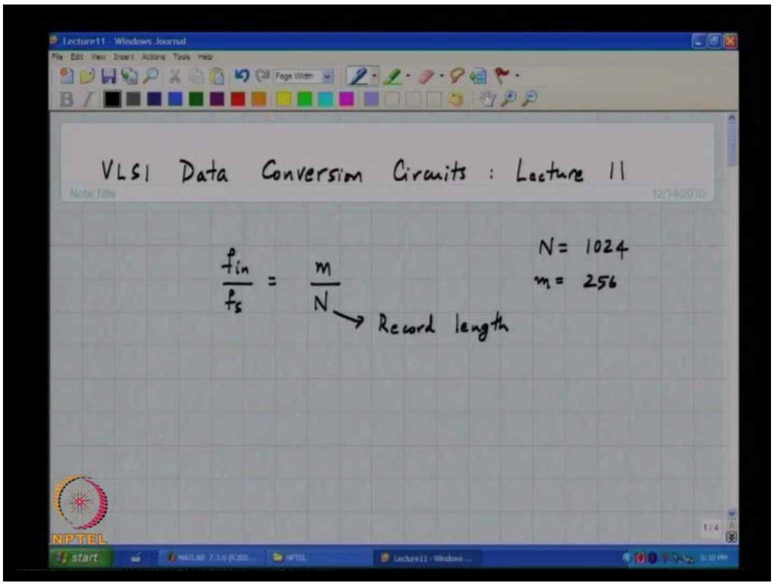

Being equal to m by N where N is the periodicity of the resulting sample sequence and m is an integer is just a necessary condition. It is not sufficient because if m and N are not chosen within course properly then one can have the situation that several harmonics alies on to the.

Same frequency bin

Same frequency bin right I mean causing us to wrongly conclude that a, you know. There are no distortion components or make erroneous judgments about the frequencies of the distortion components a case in point being the last time we said if we choose N to be 2 to the power 10 which is 1024. And m to be 256 the motivation being that we wanted to test the performance of our sample and hold around f s by 4 clearly m is an integer. And so, is N and the ratio of the 2 is 1 fourth then we saw that the second harmonic the fourth I mean the second the sixth and the tenth harmonics all alias to f s by 2. The fourth harmonic eighth twelfth all alias to d c and all they are odd harmonics alia to the fundamental.

Fundamental.

The fundamental. So, for example, if we had a sample and hold which fundamentally had no even harmonic distortion, but had only odd harmonic distortion which is a very normal thing for a class of circuits to do. For example, you can have a class of circuits where only odd harmonics are produced. Now, if you take such a sample and hold and put in a tone at exactly 256 by 1024 times f s which is f s by 4 all the harmonics will alia to.

Fundamental.

To the fundamental and we look at the spectrum it will look like wow my my sample and hold does not add any distortion at all. Whereas in reality what is happening is that all the odd harmonics are aliasing to.

Sir.

The fundamental band. So, in the time domain if you look at it 256 by 1024 is what 1 fourth right which means that the periodicity of the sequence is how many samples?

444.

Its only 4 samples even though you express it as 256 by 1024, the real periodicity of the sequence is only.

4.

4 samples

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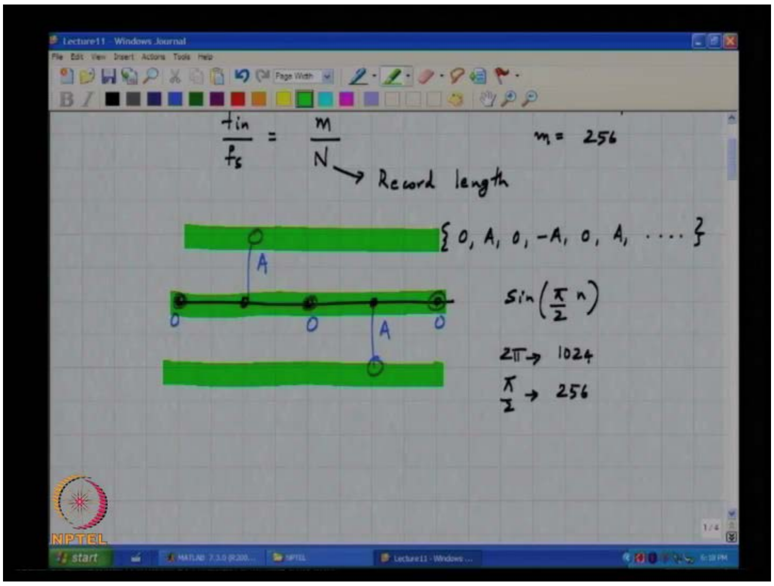

So, let us say we put in a sinusoid a clean sinusoid as the input And let me and our sample and hold is really bad. So, when you put in a sinusoid let us say it is producing a a square wave right if you cannot get any more non-linear than that. So, for argument sake if we say the the non-linear component of the sample and hold is actually the square wave which is being sampled by an ideal sample and hold. That is the model for the sample and hold that we established now if this is sampled the samples are these you understand. So, the black curve is the pure sinusoidal tone to the sample and hold the red wave is that which is being distorted. And the samples in magenta sorry of the samples in blue represent the values that obtained after sampling correct. Of course, we have no access to either the black curve or the red one we are only trying to design or reconstruct. What the input might be and what the a sample and hold is doing to the input by looking at the blue samples?

Blue.

Right. And what do we see the samples that we are seeing are this is some a this is a this is 0; this is 0; this is 0. So, we are seeing a sequence 0 a 0.

0 minus a.

Minus a 0 and so on and it does not take a genius to or even a computer to decompose this into a Fourier series. What is this the Fourier decomposition of this series of the sequence?

The sequence is 0 a 0 minus a 0 a and so on.

It is a single single sinusoidal.

It is a single sinusoidal at what frequency?

64.

Right. So, this is simply cos I am sorry sin omega N where omega is 2 pi by.

4.

4 correct. So, this is sin pi by 2 time’s n. So, if you plot the discrete Fourier series coefficients you will get a big peak. I mean see please note that 2 pi corresponds to 1024. So, pi by 2 must correspond to.

256.

256. So, at m equal to 256 you will get a spike and that wll be the only spike in the entire spectrum. And then you look at it and what do you conclude? You conclude that I have put in a clean sinusoid I look at the output spectrum and appears clean. So, my sample and hold is.

Work.

Is working you know just fine whereas, we see that what is actually happening is that the sample and hold is doing a terrible job by taking a sinusoid and distorting it and making it look like a.

Square wave.

Square wave. In spite of that by looking at the output spectrum I am not able to figure out that bad things are happening inside. And the reason for this is that all the odd harmonics are aliasing back onto the same bin as the fundamental. Another way of thinking about it is that if the input was chosen in this way that is to be equal to 256 by

1024 times f s right every time we are pretty much sampling the same 3 values. So, you are every time; you sample you are either sampling around the 0 crossings or at the peak or at the.

Negative peak.

Negative peak right. So, you are not exercising the sample and hold for other values of the.

Input.

Input it is only these 3 sequences I mean values that you are looking right. So, which is why I mean you are missing the action and other parts of the sinusoid which will happen if the input frequency is chosen wrongly. So, what you think we can do to this fix problem clearly you must make sure that the harmonics do not.

Alias.

Alias to either the input or to any other harmonic of course, if for example, you have a 1024 point record. Then how many unique harmonics do you think you can fit in the f of t?

500.

So, 512 is I mean d c and then they find I mean you know only up to 512 or unique right the rest of it is a mirror image. So, and most working a workable systems the harmonics will died on beyond about 5 or 6. So, in other words of you take the reasonable record length then you will have lot more bins than harmonics. So, it is easy to make sure that the harmonics at least the first 5 or 6 you know do not.

Alias.

Alias onto either the fundamental or any of the other harmonics and a common a simple thing to do is to choose m and N to be relatively prime.

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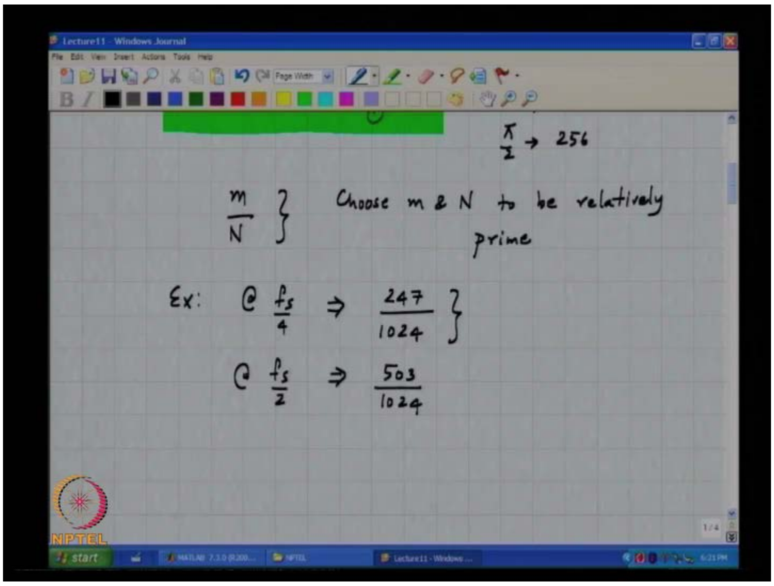

As an example in our case if you want to characterize the sample and hold at about f s by 4 the thing to do is to may chose certain may choose for example, as we did say 247 by 1024. And if you want at f s by 2 you should not put in 512 by 1024 again you will have the same problem you put 512 by 1024. You are only looking at you know the top maximum in the positive and the negative maximum. So, even if this square wave I mean the sin wave has become a square wave if you sample only at 2 points you write you will always be able to fit a sinusoid into the sequence. And therefore, will give you totally erroneous and it will give you right results. But that I will I mean you know it wll lead you to erroneously conclude that your sample and hold is perfect does it make sense.

So, for example, if you want to characterize it at f s by 2 a reasonable choice I suppose would be say 500 and 3 by 1024 or you know choose something else which is relatively does it make sense. And a caveat when you run simulations you know you must have this in decimal form to the complete precision. You should not; you do not truncate it after 2 or 3 decimal places thinking the others are don’t really matter I we will discuss what happens if you truncate. Now, but you should not do it if you do you will see where things happening in the spectrum does make sense in practice when you are making measurements in the lab.

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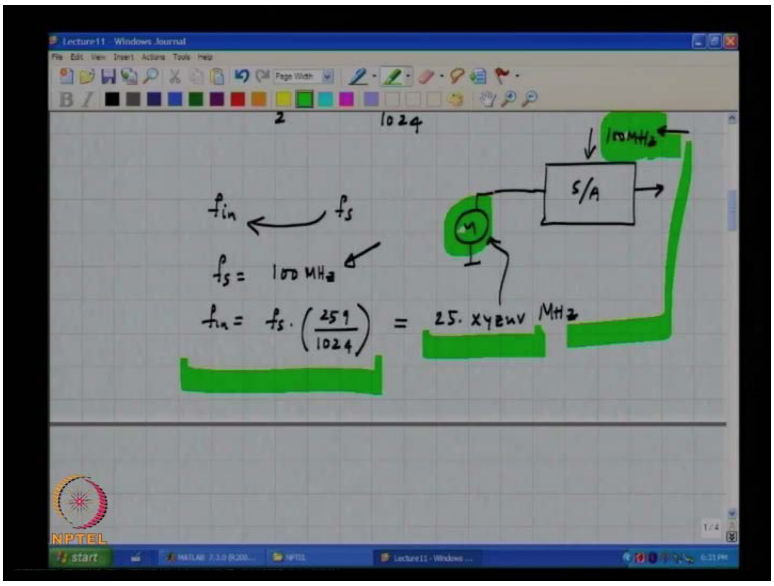

You need to therefore, have f in and f s f s is usually fixed, because you are trying to make a sample and hold which samples at this frequency. And it is your job to produce an f in which has which bears some you know rational relationship with f s. So, you want to make in other words f N f in you want to generate f in such that f in by f s is a number of the form m by N where you know N is a power of 2. And m is an integer which is not which is relatively prime with N the question is how would you do it a. You can take one oscillator let me take an example let us say f s is 100 Mega Hertz. And let us say f in is chosen to be f s times, let us say 259 by 1024 times the sampling rate which will turn out to be some number which is about 25. x y z u v Megahertz some frequency like this. So, the question is how would you generate and this is fixed what do you think you would do for this? Pardon no you need to generate a sinusoid with this frequency that is what it means right.

As per.

Pardon no do you understand the question?

Yes.

So, you have a sample and hold its sampling at f s which is 100 Megahertz. You need to characterize this sample and hold. So, you want to generate a sin wave at this frequency. So, can you suggest a way of doing this can I for instance this I mean you see in function generators in your lab right where you can have you turn the knob. And then you get whatever frequency you set. So, one way of doing this would be to take a function generator and set its knob to 100 Megahertz. And take another function generator and set it to 25 point whatever x y z u v Megahertz. Do you think this is a good way of doing things. Give that 100 Megahertz to up sample by 1 0 equal to below sample by 251 cancel it. No I want I do not want a sequence I want to continuous time sinusoid you understand? There are chances that it may tune into 258. And the one question is it possible to find us a function generator where you can set the frequency. So, accurately. So, let us say you can do, you understand the question?

Yes.

What is the question? What is the question can somebody rephrase it in your own words. And tell me whatever you are trying to solve or whatever you are trying to do.

We are trying to actually provide 2 clocks to switch to the sample and hold f in and f s and.

I mean the clock only goes to the f s.

F s.

That is the sampling clock yes.

And we want to maintain that m by N relationship between f N f in and f s.

Yes all right. So, 1 mean the first thing that would strike my mind is I need 2 frequency within course generators on which generators 100 Megahertz which I will use as a sampling clock. And the other one which reads which you know it generates 25 point you know blah blah blah Megahertz. I will you know filter it make it make sure that it is sufficiently clean it has to be a single tone mind you and plug it into the.

Sample.

Sample and hold right. Why am I interested in making the input frequency exactly 25.1 x y z Megahertz? This is 1 by m that ratio. So, that the sample discrete time sequence is.

Periodic.

Periodic. Now, any comments on this way of going about things, do you think it will work or it will not work? Because these forces are independent.

Yes.

And I mean I do not think it is practically possible to maintain this exact ratio 259 and 1024 between 2 independent source. How about if I mean so, the comment I got is it is not possible to maintain 259 by 1024. Because these are you know not does not look like do not look like very friendly numbers. How about if it was 256 by 1024? Do you think you would be able to maintain it?

Sir actually it is not the number as such it can be varying bit so.

Pardon.

It can be I mean you cannot have a fixed frequency it can keeps slightly changing with time with time or which 1 I mean both of them or 1 of them or.

Relative to each other.

Ok.

And then you can lose that ratio. So, it turns out that if you take I mean these are nothing but signal chosen nothing but oscillators, isnot it? And each of these oscillators have different frequencies or time periods. And since they are completely independent it becomes impossible to be both are them cannot be in sink. It is like saying my watch and your watch, for example, are both oscillators with nominally the same. Hopefully the same the same frequency correct. So, I mean one simply thing we can do is we synchronize our watches today. If these 2 are exactly identical then I come back you know 40 years later and then it must still be showing the same exact same time I mean clearly that does not happen why?

The frequency is set with time.

Frequencies are function of a whole bunch of things. And since the sources of noise affecting his oscillator very independent and different from those affecting mine right the 2 will drift with respect to each other you understand. So, even if you manage to kept a generator which is you know good enough to be able to specify its frequency to 6 decimal places. These 2 if f s and f in are you know 2 independent sources you will find that even though nominally the ratio is you know 259 by 1024. They will drift away with time just like it is you know as you all understand is certainly impractical to expect that we all set a watches sink our watches up to something today and expect that we can continue on for you know 10 days. And then all our watches will still show the same time. Because there are though they are all nominally identical with respect to the frequency of you know oscillation that there are differences small differences and then there is noise and so on and they were all drift off. So, if they if you do not want these watches to drift off, after you come back for 40 years, what do you think? You can do simply.

Pardon.

Observe for every year.

Every.

Some duration.

And correct it.

So, I mean if you want these independent oscillators. So, to speak you know to be in sink you periodically.

Synchronize.

Synchronize them with respect to.

A single rule; you mean either a reference which is stable or one with respect to the other. For example, all this relationship is telling you is that f in by f s is 259 by 1024 which means that there are 259 cycles of f in for.

1024 cycles.

1024 cycles of.

F s.

F s; so, you know conceptual way of sinking these 2 independent oscillators is that I will measure the number of cycles of f in 1024 cycles of f s. If I get exactly 259 I do not do anything to the input if the number of cycles are smaller than 259. I know that my input oscillator is running too slowly I do something to speed it up. And if its running too fast if there are more than 259 cycles in 1024 cycles of f s I slow down my input oscillator. So, I keep monitoring continuously and then tweaking my input oscillator such that at all times 259 cycles of the input occur in exactly 1024 cycles of the sampling clock. This is a feedback system correct I am looking at number of cycles here comparing it with what I want and.

Tweak.

Tweaking the oscillator this negative feedback system is called a phase locked loop and. So, it turns out that if you by sufficiently good quality signal sources right there can be phase lock to each other right. So that even though when you punched 25.x y z Megahertz on one and you know 100 megahertz on the other. Because they are being constantly sinked with the high quality reference you will find that this precise frequency relationship is maintained over time. So, this is what is called synchronous sampling.

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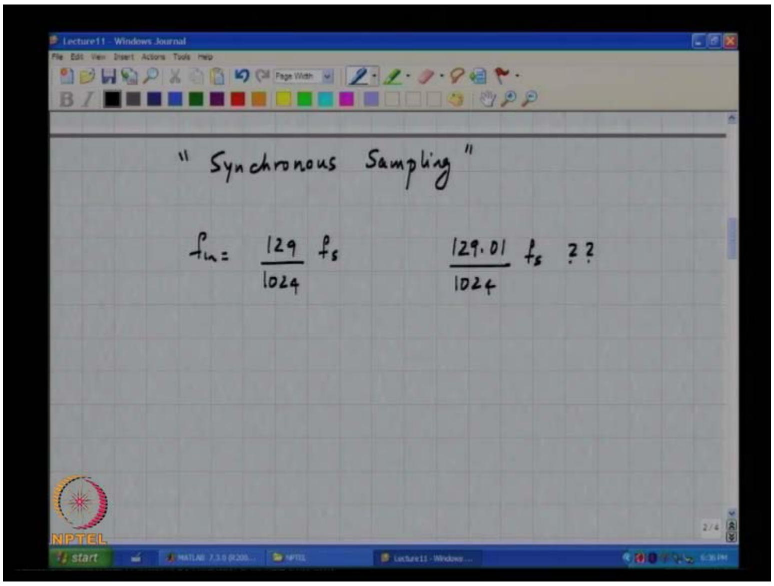

And only, when you have the input source and the clock sinked in some way. Will it make sense to talk about f in by f s is equal to you know this m by N. And all this stuff does it make sense what I want to bring to your attention is you just cannot take 2 arbitrary sources. And then you know even if you are able to set their frequencies to you know finite decimal places. You will still not be able to maintain the required ratio because the 2 oscillators will drift over time. So, that is one thing, the next thing I would like to bring to your attention is what if the input frequency of slightly off? For example, what we said was if f in was 129 by 1024 times f s, what would we see in the spectrum when you what is the period of the sequence?

1024.

1024 samples and when we decompose it as a discrete Fourier series you see a big spike in the 129 bit all the other bins are supposed to be 0. But in practice there will be you know 10 orders of magnitudes smaller than what you see in the fundamental I mean in the input bin correct. Now, if, what would you think or what should we except to happen if this was not 1024 by number 129 by 1024 by f s? But let us say 129 point say 0 1 by 1024 times f s so.

No longer periodic.

It is no longer periodic yes, but if I taken also express it as 12900 and 45 10,000 1 0 2 4 0 so correct needed is the larger. So, I agree that if you increase the record length by factor of 100 then indeed.

It will get greater sir.

It will get at 1000.

You will get it at the 12900 and 1 bin.

First bin, but it will you know let us say I still I am only interested in a 1024 point record right either unknowingly or because of truncation in my. Or because just careless somehow the frequencies 129.01 by 1024 times f s what would we expect to see.

Sir when you take a Fourier transform of a periodic sequence you will get you know impulses triggers.

Sure.

But if you take Fourier transform of 8 periodics signal.

Yes.

Then it I mean all the coefficients you will have non-zero or.

Yes.

All coefficients will be non-zero.

So almost.

Great. So, let us take a look and get some intuition about what first of all let us see what happens by doing the you know computing the series. And then we figure out why this is happening and see what to do to fix the problem? Because this is also a practical scenario where sometimes it turns out that, you may not be able to sink the clock source with the input. I mean if you are lucky for that to happen great sometimes you just do not have the equipment which will give you a sink signal. So, that the 2 can be sinked together and you are stuck with a small difference between the 2 frequencies. I have just taken an arbitrary difference of you know 1 part in about 100 1 29.011 the difference is 1 part in. In fact, say it is 1 part in about.

12000.

12000 right.

12900.

So, it is a very very small difference. So, we will say hey what difference does it make right 1? One part in 1000 you know does it really matter. So, let us see what happens?

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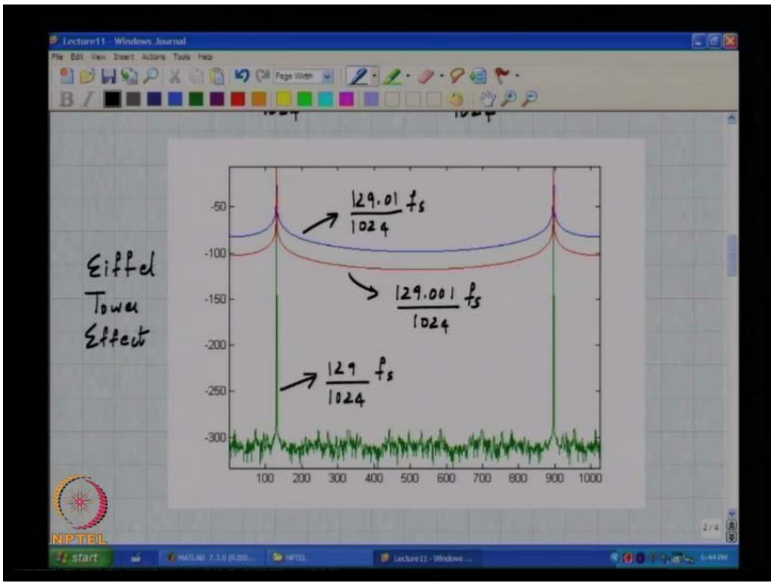

So, I have computed the discrete Fourier series coefficients for 3 signals. This in green corresponds to an input tone which is 129 by 1024 times f s. And this corresponds to 129. 0 1 by 1024 times f s and this corresponds to what do you think?

It is basically within a.

And how do you figure that out?

Sir the more it get closer to.

Sure I mean 1 thing you can say is that as this gets closer and closer to.

139 1 29.

129 I mean this should become closer and closer to the ideal one, but that is still does not tell us why this should be it 129.01. It actually turns out that it is 129.001 and it turns out that you can actually infer that from the 2 curves right. But I will get to that little later you understand the answer is indeed 1 29.001 times f s. So, clearly one of the things that we see is that the spectrum looks terribly wrong mind you there are no harmonics in the output. This is just the output is within the quotes of pure a pure sinusoid. And we would do you compute the spectrum and my god you see that all the bins have full correct. And what does this lead you to conclude?

The sample and hold is very. This is well I mean you would normally given all the experience. So, far we will say hey you know bins are bins are full I mean my sample and hold is doing all sort of crazy things you understand right. And this is mind you for 129.01 if I made it 129.1 what would you expect? I mean this thing would probably look like something like this and couple of things one if there was indeed a harmonic which was say at this level we would not be able to design it all. Because it getting mass by you know these side bands if you will you understand. And in this particular numerical experiment I know that my tone is actually clean.

And this is an artifact of the f f t by the way f f t is just a fast way of computing the discrete Fourier series coefficients. So, this is nothing but an f f t plot. So, we know now because we have run the experiment that these are nothing but artifacts of the f f t right. But if you did not know this I mean it would be tempted to concluded that there is something wrong with my system. And it also makes sense that as this frequency becomes closer and closer to 129 its seems logical that this you know this weird stuff reduces in magnitude. The first of all this effect is you know what do you think? You can call this effect some famous structure that this resembles if somebody draw something like this what would you say this is.

Eiffel tower.

This is the I mean looks like the Eiffel tower right. So, this is often called the Eiffel tower effect. I do not quite remember the spelling as double f or signal f.

Double f.

So, we need to understand a why this is happening b whatever can be done to fix this right. So, so first let us get to why this is happening. This is not correct this is the not clearly this is happening, because the input sequence is not periodic.

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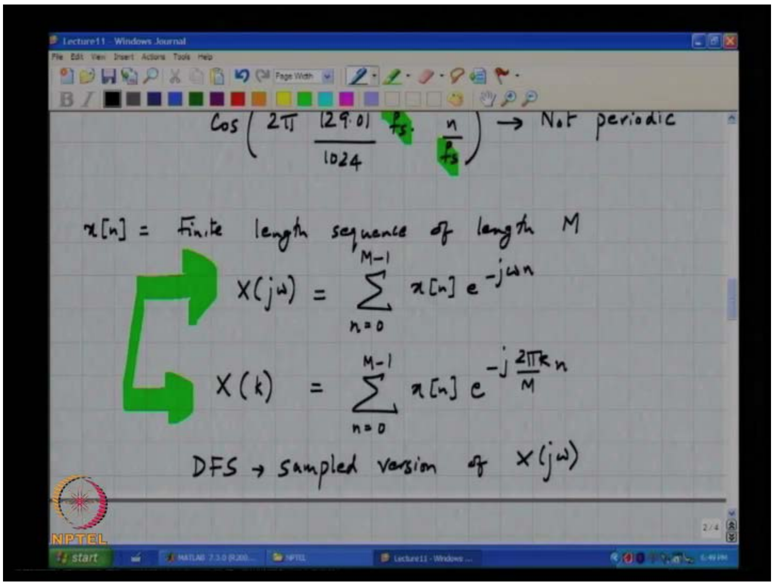

Because this cos of 2 pi 129 point o 1 by 1024 times f s times N by f s is this is not periodic all right. And so, when you compute the discrete Fourier series coefficients what are we inherently doing when we compute when you take a sequence of length l and compute its discrete Fourier series.

At the dual clock signal dual clock 1 period.

I mean. So, when we find the discrete Fourier series coefficients of a sequence of length l what we are doing is the following we are assuming that this is 1 period of a.

Periodic signal.

Periodic signal right, where the signal is basically copy paste forever right of this basic sequence is not it. Now, let me just go through some quickly revise some basics on Fourier series and Fourier transform. So, if you have a finite length sequence of length m capital M here all right that discrete time Fourier transform is defined as what?

Sigma.

Sigma.

X of n.

X of N e to the minus.

J omega N N going from.

0 to minus 1.

M minus 1 simply because the sequence is only m long correct Now, if we find if we expand this finite length sequence into a discrete Fourier series. Where the understanding is that the discrete Fourier series is nothing but a Fourier series representation of a periodic extension of the same point sequence. Then what is the expression for the discrete Fourier series?

X of N E to the minus J.

J.

2 pi.

By.

M.

M.

Into n.

Into.

A into a into.

So, x of whatever let me call this.

K.

K right.

N

N N ranging from 0 to m minus 1 because they are normalizing constants donot worry about them for the time being. So, staring at these 2 expressions what does 1 conclude?

Sampling of sampling of d to a.

So, the, this is nothing but a sampled version of.

E 2.

Discrete Fourier series is nothing but sampled version of the discrete time Fourier transform does it make sense all. I mean this I am sure you have you have learnt it in your d s p class. Now, let us try and understand why given this let us try and understand why it makes sense that if we choose the input properly if we choose the input to be of the form m by N, where m and N are integers. There is only a single spike I mean we have seen that from other considerations before. But let us try and use the correspondence between the discrete Fourier series and the discrete time Fourier transform to understand this a little better.

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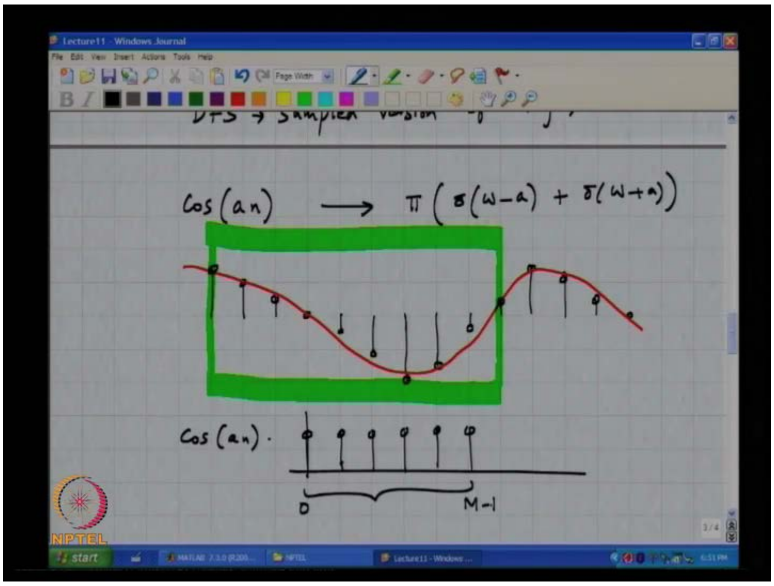

So, if you have a sinusoid cos a into N, what is its discrete Fourier transform discrete time Fourier transform?

Pi into delta pi into.

Pi into.

Delta of.

Omega minus a plus delta of.

Omega plus a.

Omega plus a. Now, if I please note that this extends all the way from minus infinity to.

Plus infinity.

Infinity; now, if I take only a chunk of this lasting m samples, so, the original sequence is there all the way from the original sequence is there all the way from minus infinity to infinity. But if I am only interested in a chunk of this sequence lasting m samples then it is in principle equivalent to taking a signal which is cos a N and multiplying it by a rectangle which last from 0 to M minus.

M minus.

1.

1 right; so, you multiply this with this is 0 and this is m minus 1. So, what you think would be the discrete time Fourier transform of such a sequence? This is now finite length sequence correct convolution.

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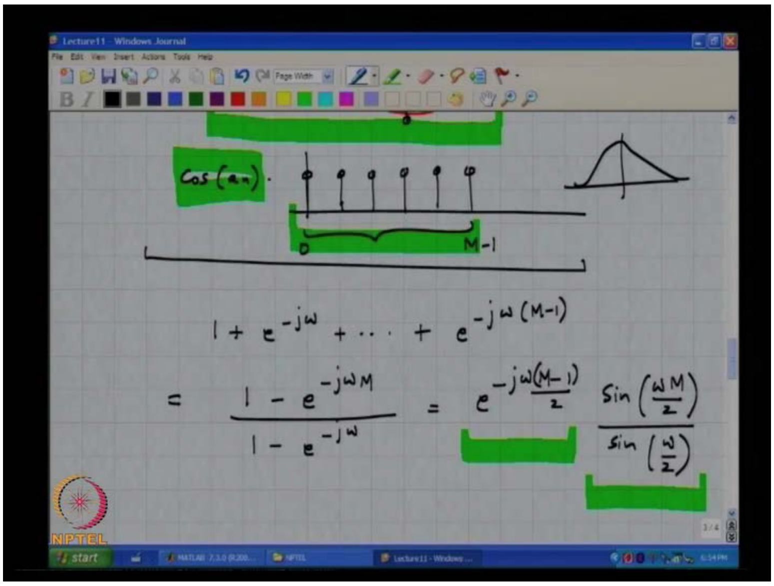

It was simply be the convolution; convolution of this Fourier transform which is a bunch of impulses with the Fourier transform of this signal. And what is the Fourier transform of this signal?

Periodic since.

It is 1 plus e to the minus j omega all the way up to E to the minus j omega into m minus 1 correct and can somebody give an expression for this. It is 1 minus e to the minus j omega into.

M

M minus 1 into m is it.

Yes sir.

I am sorry, by 1 minus.

By 1 minus.

E to the power minus j omega.

Minus j omega which is nothing but e to the minus j omega.

M minus 1.

M minus 1 by 2 times.

Sin omega.

Sin.

Omega omega.

Omega m.

By 2 divided by sin.

Omega by 2.

Omega correct; why does it make physical sense? It I mean of course, it comes out of the math, but why does that make sense that term e to the minus j omega? You know m minus 1 by 2. I mean it is square should be the linear phase b f s it is a linear. So, what can I say about this sequence? Is it symmetric is it an even function of time or an odd function of time?

Sir even.

It is an even function correct which means that its delay is. .If you have pulse like this sorry what is this delay? I mean simply strictly speaking delay is nothing but the. So, called center of gravity of the pulse correct. So, if you have an even function of time or even function of N in this case its delay is.

0.

0 right so, but what is the delay of what is the average delay of this sequence.

M minus 1 m minus 1.

M minus 1 by 2. So, this delay is 0 and this is the average delay is here correct. So, that is why it makes sense that this is e to the minus j omega times. So, on the average is sequence is delayed by m minus 1 by 2 samples that is all.

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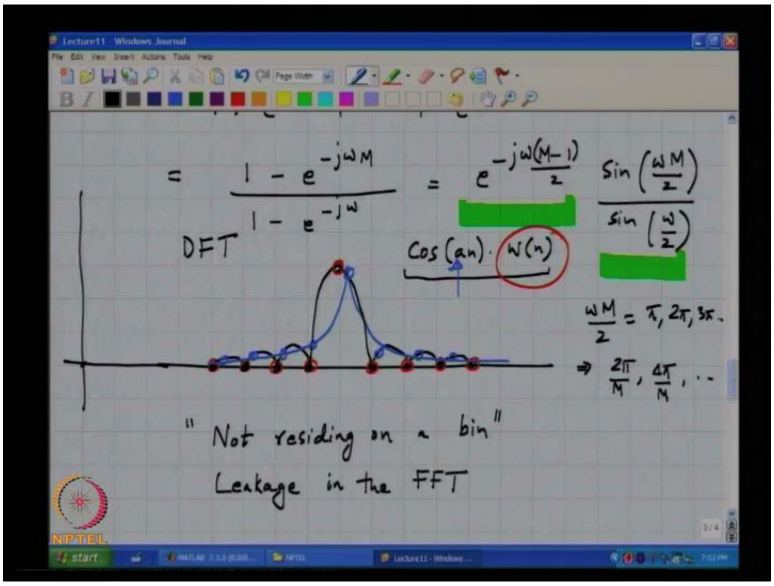

So, you we are not really worried about this part because magnitude is 1 what is the magnitude of this character?

0 to 0 to.

So, this will go to 0 at d c it must be.

M.

M all right and what happens to this as you know omega increases.

It will decrease decreases.

It will decrease and go through zeros and so on and at what frequencies will it be 0? It will be 0 at.

Omega m by 2.

Omega by 2 must be equal to multiples of.

Pi pi 2 pi of pi.

Of pi.

Pi correct must be let me at pi 3 pi and. So, on sorry pi 2 pi 3 pi and so, on which means that it will be 0 at.

2 pi by m 2 pi.

2 pi by m 4 pi by m. So, in other words it would be a zeros at multiples of 2 pi by m. Now, what we are doing in other words what is the discrete time Fourier transform of this whole thing?

Simply.

It is simply that the convolved I mean convolution of the spectrum of cos a and we just simply 2 delta functions with this sink right. So, in other words if I just draw this on only 1 side it is simply like moving the sink to a. So, this is the spectrum the discrete Fourier transform of cos a N times w of N that w of N is the. So, called window function it just selects some period of time this window is in particular is of length m. Because we are only interested in m samples of cos a N correct. So, this is the discrete Fourier I mean this is a discrete time Fourier transform of the sequence cos a N times w of N. Now, if you compute the discrete Fourier series of this sequence, what you think you will get?

Discrete.

What did we just discussed the discrete Fourier series is nothing but.

Sampled version.

A sampled version of the discrete time Fourier transform. So, and sampled at at what frequencies?

2 pi by 2 pi by 2.

Multiples of 2 pi.

2 pi by N correct. So, if this a happen to be a multiple of 2 pi by m in the first place correct. So, let us say this was k into 2 pi by m then when you sample this discrete time Fourier transform. Your samples will lie exactly here is this clear which is why when you look at the discrete Fourier series coefficients. You will see just once spike and all the others being.

0.

0 right is the somewhat explanation of why if you take a periodic sinusoid in the discrete time domain. And compute discrete Fourier series coefficients you will only find 1 spike, But the reason for going into this seemingly long winded explanation is to get intuition and has to what will happen when this a right which is the frequency of the of this cos a N. What happens when a is not a multiple of 2 pi by m what do you think will happen? Order and then basically so, we are basically if a is not a multiple of 2 pi by m then what will happen is that the samples will now be all slightly off. So, we will perhaps say that was a.

So, we are now sampling here correct and therefore, when you plot the discrete Fourier series coefficients you will get something like this right. Whereas, if the frequency was exactly if a was exactly a multiple of 2 pi by m correct you would get only a single spike. And nothing else whereas, if that is not true you will get something like this because you are not sampling the discrete time Fourier transform at the nulls. And clearly the spectrum of w of N has a bearing on how rapidly this Eiffel tower decays is this clear? So, this phenomenon where a is not exactly a multiple of 2 pi by m is often called input frequency not residing on a bin all right and this phenomenon where in a clearly. Now, the other bins were not 0 and this is called leakage in the f f t does it make sense. So, we will continue with we will get some more time domain intuition into what this means. And that will gives an idea of what to do to fix the problem, we do that in the next time.