VLSI Data Conversion Circuits
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Lecture - 10
S/H Characterization – 1

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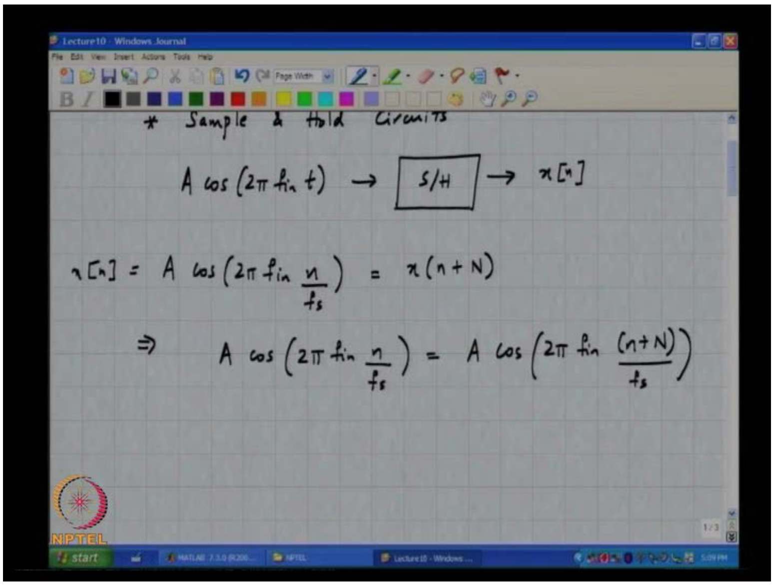

This is VLSI data conversion circuits, lecture 10. In the last class, what we saw was various realisations of sample and hold circuits suitable for continuous time and discrete time inputs. And we saw the variation issues involved like distortion or we saw how these can be fixed. But no real circuit solution will work you know up to the full degree that you expect. We attempted to linearise the sampling switch by keeping the gate overdrive fixed and independent of the signal. And if the gate over drive was truly independent of the signal then perhaps this switch would be perfectly linear. But in practice there is the body effect there is also some amount of charge sharing between the boost capacitor as well as the gate of the sampling switch. For example, what we said that we charged a capacitor to voltage V d d and hooked that up in between the source and the gate of the sampling switch transistor.

Obviously, the gate of the sampling switch transistor need some charge in order to be able to sustain a potential that charge must be coming from the boost switch. So, coming from the, I am sorry the boost capacitor. So, every time you charge the capacitor to V d d in pi 1 or phi 2 I do not remember which pair. And you hook it in between the gate and the source there will be some charge lost. So, all these second order effects lead to some small dependence of the resistance of the switch on the input signal. So, they will indeed cause distortion this distortion may be very small. But they will indeed cause distortion and so will the body effect. So, as soon as you designed a circuit like this the first thing that you would like to find it out is how good is my sample. So, where for example, the input here is continuous time. And what we are interested in is not the wave form across the capacitor all instance of time.

We are only interested in the samples of the voltage across the capacitor at the end of the sampling cycle. Or at during the hold phase technically speaking the value that is going to be processed eventually is the held value on the capacitor. So, what we are interested in is the.

Distortion.

Distortion or we are interested in the properties of the sequence of voltages which are on the hold capacitor. So, in other words let us say we have a continuous time input signal. And we pass this through our sample and hold system or track and hold system the output of interest is the sampled sequence x of N. And traditionally we have been used to analysing nonlinearity of systems or testing nonlinearity of systems using what is with called the sine wave test. The idea is to put in a pure sinusoid into a system if the system is a linear system we know that the output must only contain components at the input frequency. If you are talking about a continuous time system when the movement harmonic start showing up in the output you can say that definitely this system is not linear. So, a degree of, how linear the system is; how small these harmonic components are in relation to the fundamental component that you put in?

So, since we have been used to this way of testing continuous time systems for a long time. It makes sense to see if you can do a similar sort of thing with a sample and hold where the input is continuous time, but the output is discrete time. So, before we get into the specifics a couple of points that I would like to point out is if we have a continuous time sinusoid and we sample it at a rate f s. Then the sequence you get will be of the form cos 2 pi f N times N by f s. So, this is the let us say this is the output of the sample and hold provided it is ideal. Now, this is a discrete time sinusoid, as we know not all discrete time sinusoids are periodic correct. So, what constraints must be satisfied by f N and f s? To ensure that the discrete time sequence is periodic first of all when now, we will figure that out. But first of all why are we instructed in making the discrete time sinusoid periodic.

To analyse the analyze that.

Right basically what we have been used to is in the domain of continuous time systems. We look at the output of the system we decompose it into a Fourier series and measure the strength of the second harmonic component. And third harmonic component which are basically saying I will compute the Fourier series of the continuous time output and look at coefficients of the harmonics which are basically terms in the Fourier series. So, we would like to attempt an analogous situation here which is you compute the Fourier series of the output which here happens to be.

Discrete.

Discrete time and if you want to compute a Fourier series it only make sense if the periodic sequences, sequences periodic. So, that is where we start off and we have wondering now whether this sample sequence is periodic. And by definition what is the periodic signal? It must be identical to something which has been shifted by N samples. Because the sequence is discrete time which means that if this sequence has to be periodic cos of 2 pi f N times small N by f s must be exactly equal to cos 2 pi f N times N plus capital N by f s where the large case N denotes the period of the discrete time sequence.

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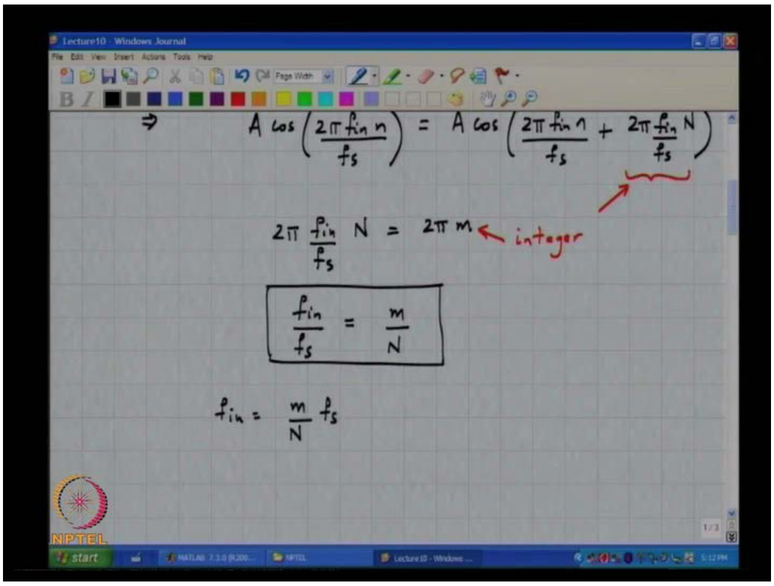

Now, what this means is that this is simply ((no audio 07:54 to 08:25)). And what this is telling you is that if one is interested in making the discrete time sequence periodic then this factor 2 pi times f N by f s time's capital N.

Integer minus

Must be m integer multiple of.

2 pi.

2 pi. So, in other words 2 pi times f in by f s times N must be 2 pi times m where m must be an integer which means that f N by f s must be of the form m by N. So, the moral of the story is that if you want that discrete time output to be periodic with a period of N samples. Then you just cannot choose any old frequency for the continuous time input it must be related to the sampling rate in some way. And that way is that the ratio of the input frequency to the sampling frequency must be of the form m by N where m is the m and N are integers.

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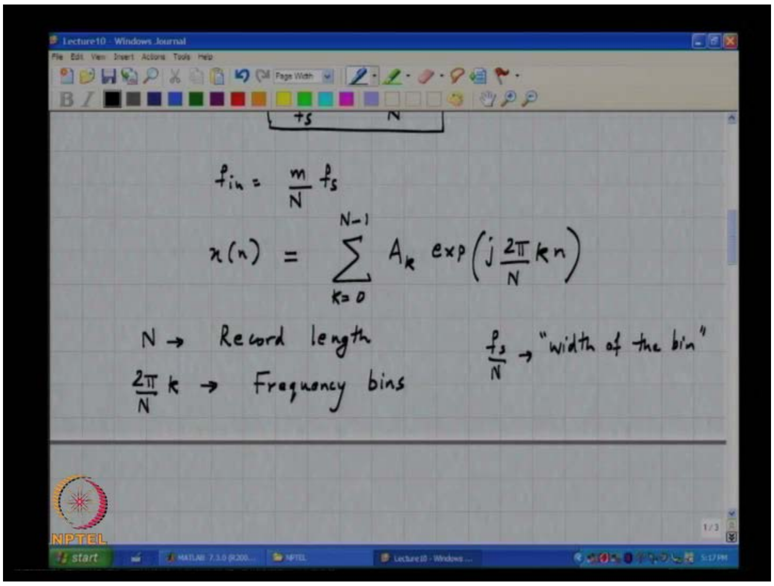

So, this is the first thing that 1 has to bear in mind. Now, suppose we choose f in this way in other words if we choose f in to be m by N time's f s. Then the resulting sequence is periodic by if the choice of f in and the period is N samples. Now, what we have at hand is a periodic signal with the period of N samples and this can be decomposed into a.

Discrete time.

Discrete time, Fourier series; this is very analogous to the continuous time Fourier series that you probably more familiar with. So, x of N can be expressed as the sum of many sinusoids in the continuous time case how many sinusoids would be necessary to.

2.10 point.

Express an arbitrary continuous time signal infinite number of Fourier coefficients would be needed; however, in discrete time you need exactly N sinusoids. And the usual thing we write them as sum of complex exponentials with the understanding that the coefficients for a real signal will be you know the magnitude will be even and phase will be odd. So, that when you add them up the imaginary terms go away I am sure you all familiar with this from your d s p classes. So, in general therefore, this must be the sum of k sinusoids each with a frequency of what is the fundamental frequency.

2 pi by N 2 pi by N.

The fundamental frequency of this decomposition will be 2 pi by.

N

N and its harmonics. So, the harmonics will be multiples by 2 pi n. So, there will be k and you have the sequence index which is N and k must run from.

0 to 0 to N minus 1.

0 to.

N minus 1.

N minus 1 very nice all. So, I will now introduce to you to some jargon, you might have already heard this N is often referred to as important. No is what is it called it is often chosen to be a power of 2 I will mean, we will come to the reasons behind that little later on. But have you heard of any I mean what do you call n? No I mean it is a. So, in the data convertor power lines this is often called the record length. It is merely the period of the discrete time sequence that you are analysed all and this each of these frequencies 2 pi by N times k. So, this will be 2 pi by N twice 2 pi by N all the way up to 2 times by N times N minus N. And these are what are called frequency bins all. So, what do you think each spacing of the bin corresponds to in continuous time? This is all the way from either 0 to f s or f s by 2 to minus f s by 2 to.

F s.

F s by 2. And there are how many bins are there?

N.

N bins so the resolution of the equivalent continuous time signal frequency resolution wise is f s by n. So, often this is called the bin width f s by N is the width of the bin all. So, this is just simply jargon the discrete time Fourier series need not concern itself at all with sampling rate. You just have a sequence; you are perfectly free to compute this discrete Fourier series, what why we bring in sampling rate is because we understand that the sequence has come by sampling.

.

A continuous time signal at a sampling rate f s which is why f s you know f s by N all this stuff makes sense for us you understand. Now, if the input sequence is periodic and in other words if it is been derived from a continuous time signal where the input frequency. And the sampling frequency have been chosen in such a manner as to make x of N periodic. And if the input happen to be a sine wave then when you compute the Fourier series, how many coefficients do you think one will be non zero.

Only one of the.

Pardon.

Sir I think 2.

You understand the question.

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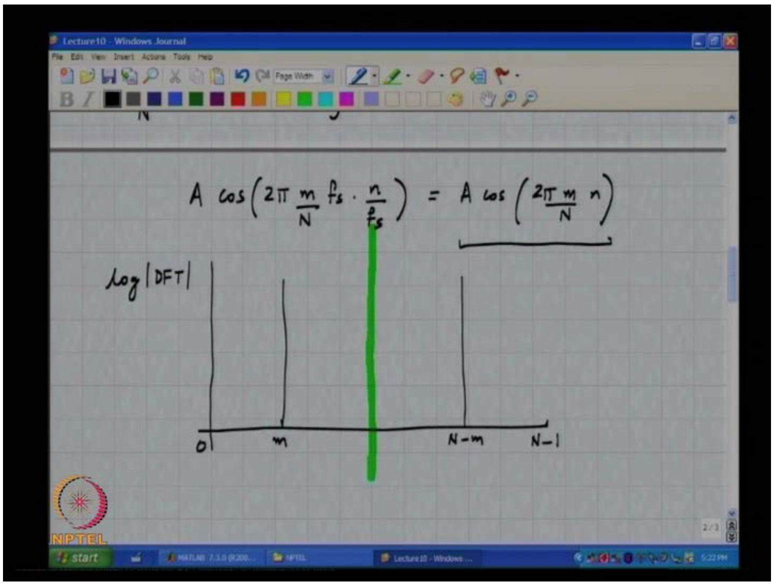

So, let us say I chose a times cos of 2 pi and f in happen to be say some m by N times f s. And this is sampled at the rate f s the resulting sequence therefore, is a cos 2 pi m by N times N correct. So, if I now compute the discrete Fourier series of this sequence which we all agree is periodic what would I get?

2.

I will get.

2 divided by 2 non 0 coefficients 1 corresponding to the n-th bin correct. And one corresponding to the minus m-th bin I mean the, you can understand the bins are being either going from 0 to N minus 1 and with a understanding that after N by 2 there will be correspond to mirror. There will be a mirror image of the components below N by 2 correct this is a consequence of the real nature of the signal that we are decomposing into a Fourier series this is clear all. So, if I if I plot these coefficients on a log scale typically you plot these on a log scale. Because you are hoping that the circuits that you design the second harmonic and third harmonic will be.

Small.

Very very small compared to the fundamental. So, that it does not make sense to plot them on a linear scale it makes sense to plot them on a log scale. So, if you plot the discrete Fourier series coefficients on a log scale with the index running from 0 to N minus 1 you will see or you expect to see 2 spikes 1 at m and 1 at N minus. So, this is sorry I am not a very good artist. So, on somewhere here. So, this must be N minus 1 minus or N minus m.

N minus m.

This is the axis of symmetry. So, apart from the n-th bin all the other bins must technically be 0. So, if you plot them on the log scale log of say modulus of the d f t you will get.

Minus infinity.

You must get minus infinity for all coefficients and only you know whatever you get this log of you know it depends on the amplitude of the signal only that bin will be non zero. So, I just let me show you an example.

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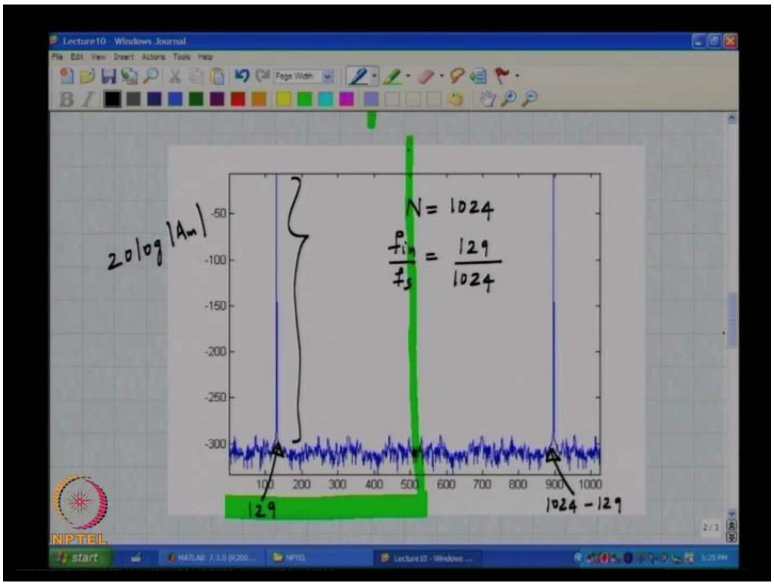

So, this is an example of a d f t that you get I have taken a sinusoid that sinusoid happens to be I have chosen in this example N to be 1020 4 and f N by f s I have chosen to be 129 by 1024. And sure enough the 120 ninth bin shows a very large magnitude in relation to all the strengths of the signals in all the other bins this is the log scale. So, this is the log of the magnitudes of the Fourier coefficients as I run index from 0 to 10 2 4. What is this telling you the difference between the input tone and all the other tones which must technically be what?

Sinusoidal.

It must technically be infinite, why because 1 is log 0 which is minus infinity. But machine precision will limit the ratio that you can get. And typically when you do this? You will find this actually 20 log mod A m. So, this difference in d b is about 300 d b that is what you are expect with a regular computer. You understand I mean there will be quantization effects when you compute the Fourier series coefficients. Because of finite precision effects and all this other stuff and this is what you get hm. And as we expected 1024 minus 129 will also show a spike since the stuff about 512 is a mirror image of the bins below 512 as far as magnitudes are concerned. We just need to worry about 1 half of this of these coefficients is this clear all now that we understand this.

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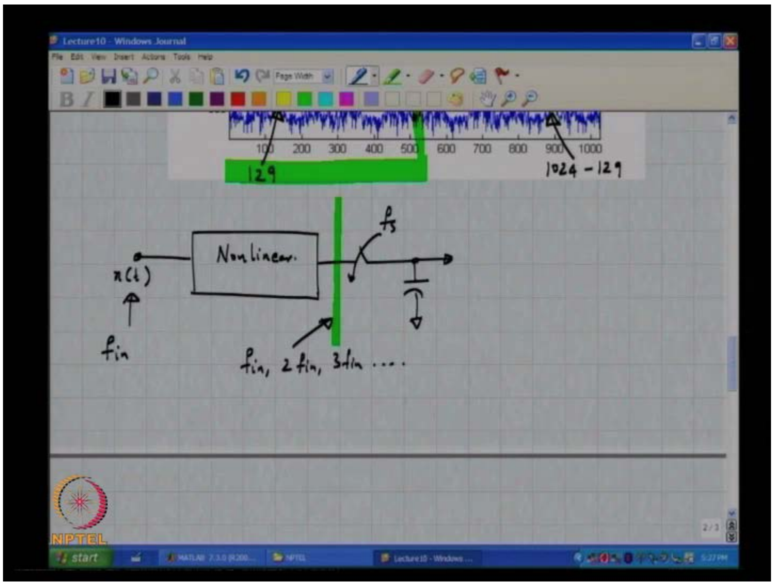

Let us move forward and see what happens in our sample and hold. What we saw last time was when the sampling switch is non-linear you can model it as the input being distorted due to the second and third order coefficients non-linear coefficients of the resistor. And then so, this is a nonlinearity and then you have an ideal switch and a capacitor that is that was the model that we arrived at for any switch where the switch resistance depends on the input voltage. So, now, because this element is non-linear if the input was a sinusoid what can you say about the components of the signal there. Please note that that is still a continuous time signal. So, what do you think if this was a f N what kind of frequency components do you expect to find at the output of the non-linear block? You will find multiples of f N and even though f N may satisfy the nyquist criterion clearly it is just a matter of counting high enough. You will find some harmonic which does not satisfy the nyquist criterion. So, if this is f s soon enough you will find a harmonic which does not satisfy the nyquist criterion. So, it will. So, when you sample it at f s, what will happen? It will alias back.

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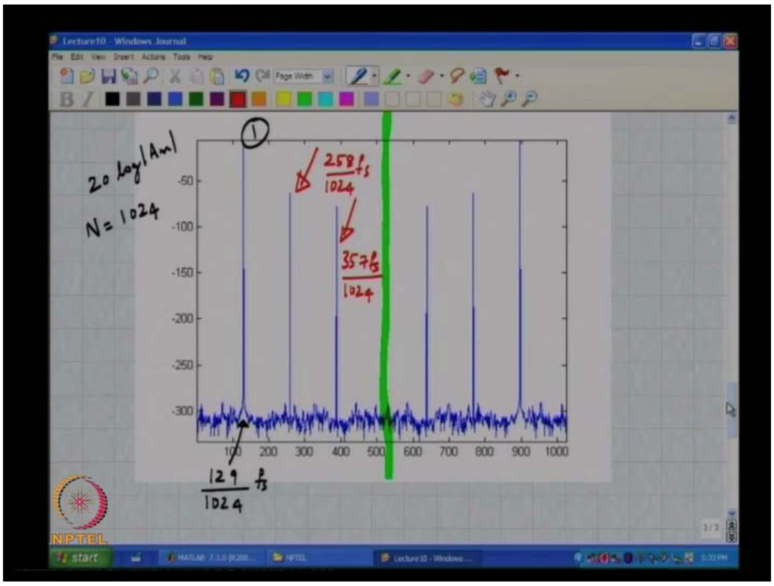

So, by looking at the discrete time sequence one should be able to carefully analyse it and reconstruct what the continuous time signal look like you understand. So, let me illustrate this with an example all how can you guess what the input signal must have been? There are how many sinusoids are there?

333 sir.

Sure enough because we have we discard half of this picture there are 3 sinusoids. Good and reasonable assumption to make is that the fundamental tone is much stronger in amplitude than.

The harmonics.

The harmonics I mean if the harmonics are bigger that the fundamental then you have big problem all. And let me also indicate to you that this is in the d b scale. So, this is 20 log magnitude of the series Fourier series coefficient. So, clearly there are 3 tones and if we say the biggest om4 is the fundamental. This is the fundamental and it is roughly this is again 129 by 1024 times f s because the peak happens to be at 129 bin. And I have already told you that N is 1024. So, the second harmonic will lie where. So, as you can see 129 by 1020 4 f s is basically means that input tone is roughly at f s by 10 correct. So, the second harmonic will be at 2 f s by 10. So, will that alias to lower frequency or will it be if the input frequencies are roughly f s by 10 the second harmonic is at what frequency?

F s by 5 f s by.

F s by 5 or 2 f s by 10. So, the question is does this component satisfy the nyquist criterion.

Yes it will satisfy the nyquist criterion.

Yes. So, and sure enough when you look at the spectrum this will be twice 129 by 1020 4 which is 258 by 1020 4 times f s. So, this will lie in the 258 bin correct what about the third harmonic? Will it alias.

No yes.

It is roughly the input tone is roughly f s by 10 third harmonic is 3 f s by 10 which is less than 5 f s by 10 5 f s by 10 is highest frequency beyond which things will alias back. So, this is indeed the third harmonic and is 357 f s by 1024 and the other half of the spectrum is simply this mirrored about the centre correct. So, in other words if the frequency is sufficiently small. Then the first few harmonics you know will be able to read them on a graph like you normally are used to doing. Now, where do you think the fourth harmonic of which will lie?

4 into.

220 times 2 is what?

516.

516 by 1024 times f s wherever it is alias or not.

No yes sir yes sir.

It will alias and it will look like what.

1024 minus 0.1.

So, it will look like 516 512. So, it will look like 580 f s by N 24 all. So, in this example since only second and third harmonics are considered we see that everything seems the way we used to in a continuous time system. There is no none of this alias is happening as you keep sweeping the frequency axis. You will find that you can go on for any number of harmonics and there would be components. Now, because of sampling some of these harmonics will fold back. And in this example there have been only second and third harmonics and there is no aliasing of the harmonic components. Now, let us look at another example this shows the log magnitude of a discrete time sequence which is periodic, first of all.

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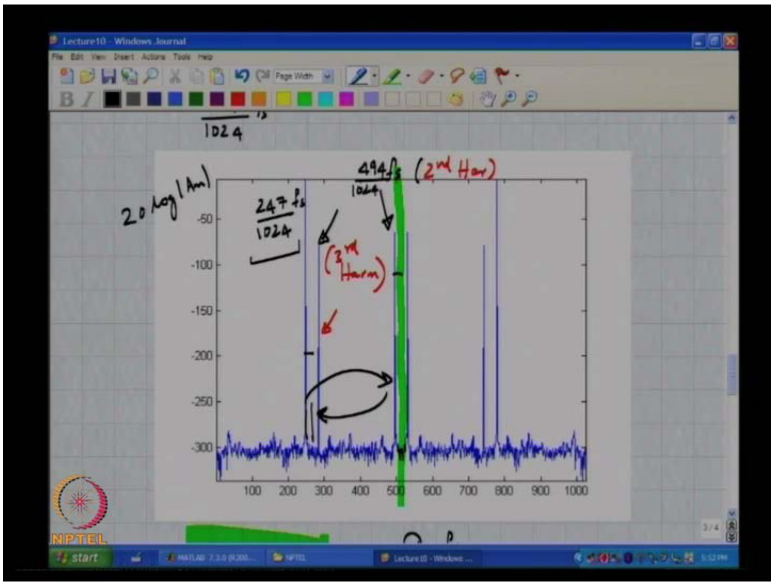

Let us try and identify the number of input tones, how many input tones are there?

6.

So, you discard 1 of the picture. So, how many tones are there? There are still 3 tones like before all. And again let us assume that the fundamental is most dominant 1 which is the fundamental.

Around 250.

Around 250 its turn out to be actually turns out to be 247 by 1025 times f s all. Now, what happens to be second harmonic?

4494.

494. So, which is the third harmonic.

494.

So, this is the.

Second harmonic.

Second harmonic all. So, this is 490 4 by 1024 times f s, what happens to the third harmonic?

That will alias.

It will alias back, because clearly third harmonic is too high in frequency roughly the input is at 1 quarter the sampling rate. So, the third harmonic is at 3 quarters the sampling rate which will after aliasing look like.

Quarter.

Roughly a quarter of the sampling rate. So, the third harmonic is actually this, you understand? So, this is the second harmonic while this is the third harmonic. So, if there are higher harmonics. Now, for example, where do you think the fourth harmonic will be?

Symmetric.

It will be I mean the input is roughly about quarter the sampling rate. So, the fourth quadratic would be roughly the sampling rate a sampling rate I mean a signal at the sampling rate will alias to. I mean clearly it does not satisfy nyquist it will alias to roughly d c. So, the fourth harmonic actually of this particular sinusoidal will probably lie somewhere somewhere here does make sense. So, when you interpret the spectrum on the Fourier series decomposition of the periodic output sequence of a sample and hold. Or going forward this is going to be quantized also you need to be careful in looking at the spectrum. And figuring out which tone is actually what you understand I mean if you very naive and looked at this the spectrum I mean you would be very confused. I mean typically you expect the third harmonic to be if at 3 times the fundamental. But you do not see any of that happening you see some tone very close to the input.

And you could mistakenly conclude that there are actually you know 2 signals 1 which is getting distorted by a second order distortion. And another tone which is close to the input tone, but if you know that this came from a sample and hold. The tone which is sitting you know next to the input tone is not really some rogue tone. But it is the third harmonic distortion which is gotten aliased, because of sampling you understand does make sense. So, while we have done this for a sample and hold when we do nyquist convertors, you will find that the same principles apply the only difference is that the sample signal is further quantised? And then you take you decompose that quantized sequence into a discrete Fourier series. And you know plot these harmonics plot the spectrum and you will see tone sticking out at various parts of the spectrum. And it is for you to sit and figure out which tone is coming from which harmonic all. The next thing I want to point out is the following.

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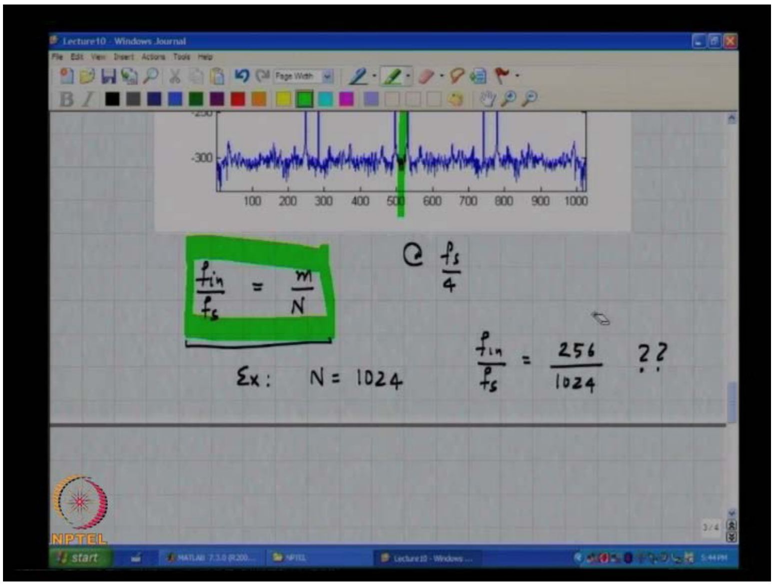

Now, we all agree that f N by f s must be chosen as a ratio m by N where m and N are integers a few of a few kind of said N must be a multiple of. 2.2 or rather a power of 2. So, why should N be a power of 2? So, they are technically speaking there is nothing illegal about decomposing any arbitrary sequence into its Fourier series components here. So, turns out that if the length of the sequence is a multiple of is of the form some 2 power something. Then you know extremely efficient fast algorithms can be I mean exist to compute these Fourier series coefficients and that is called the fast Fourier transform. Or thee f f t all then of course, this made a lot of sense in the old days where computing power was was restricted.

But for the kind of the stuff that we deal with at you know this may, you may as well call this simply legacy. We just use to using a 2 power something for the for the f f t for the record length and we continue to do. So, though there is no there is no earthly reason for doing it I mean given our data complexity. I mean let us say 1024 point sequence or a 1020 point sequence will virtually take the same time when you run it on a computer these days you understand. So, traditionally this is chosen to be a power of 2. So, let me call this 2 to the power p and in our example this happens to be 2 to the power 10. So, that 1024 point f f t or the record length of the sequence is 1024. Now, if this happens to be a power of 2 there are many ways of choosing an integer n. So, let us say you want to test your sample and hold at; obviously, several input frequencies you just do not want to put in d c.

And make sure that the output is working out you want to be able to sweep the input frequency over the entire range of operation. Or expected operation of your sample and hold and make sure that the performance of the circuits is good for wide range of frequencies. So, for example, let us say you want to check your sample and hold at roughly f s by 4. So, what do you think is a good choice for the input frequency? We know that this equality must be satisfied and let us for argument sake say N is 1024. And we want to test our sample and hold or the performance of the sample and hold needs to be characterized at a frequency of say f s by 4. So, what suggestion I mean what is your choice of m? This is the only thing to be determined for example, 1 choice could be f N by f s is 256 by 1024 being sure enough. It satisfies this constraint m is an integer N is an integer. So, can you make any comment about this particular choice for example, 256 is 255 is 257 is all of them will result in an input frequency which is about f s by 4. Please note that this constraint is only needed. So, that the resulting discrete time sequence is.

Periodic.

Periodic it is only a trick for characterisation it does not mean that the input will I mean the sample and hold will not work. If this equality is not satisfied does make sense and is as it turns out 256 or 255 or 251 or 257. All these numbers for N seem like reasonable choices because they are all integers. So, let me say I choose 256 as m. So, f N by f s is 256 by 1024, what do you think will happen you put I mean do you understand the question. The input frequency is 256 by 1024 times f s it has been passed through the sample and the hold, because of nonlinearities in the sample and hold, what is happening?

Harmonics be.

Harmonics are being generated.

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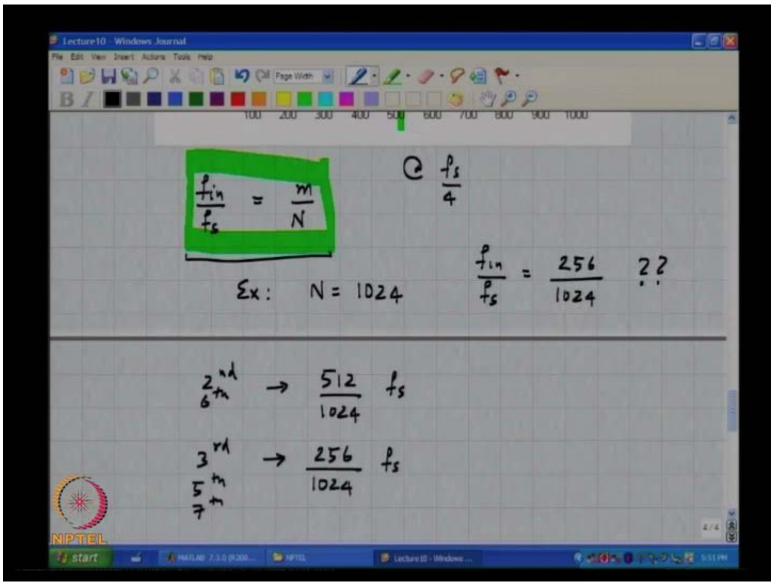

So, let us take I mean. So, what harmonics will be generated the second harmonic will be at what frequency?

512 by 1024 times f s the third harmonic will be at.

I mean which bin of the f f t will it fall on?

0.268 point.

768 is.

first.

Not first, why first?

Sampling.

Alias.

Aliasing aliasing.

It will alias. So, what frequency will it to what bin will it alias?

3 naught 6.

256 into 3 minus 512.

226 256.

Correct. So, it will alias back to 256 by correct does make sense at all, what will happen to the fourth harmonic?

It will go to the DC.

It will go to the DC, fifth harmonic.

Yes sir, suppose 18.

6 harmonic.

512 by.

What about the seventh harmonic?

256.

256. So, 357 etcetera were all alias to.

1 bin.

1 bin, the same bin there will be some components at d c which correspond to fourth the eighth and the twelfth harmonics and so on. You understand, what is happening here? So, what is the moral of the story? So, the question now, after we have gone through this

discussion is 256 by 1024 are good choice for the input frequency. If yes why if not why not?

Sir it is the sample length which is like the same within a period if you are.

No, can you know tie it to the discussion we just had what did we just discuss if the input tone was of the form 256 by 1024 times f s. Then we find that several of the harmonics are aliasing on to in this particular case the same bin and in this very particular case all the odd harmonics 357 and so on are all aliasing onto the same bin as the fundamental correct. Because 256 was where the fundamental was the third harmonic is also aliasing onto to the 256 bin. The seventh is also aliasing onto the 256 bin and so on.

So, if you naively look at the spectrum, what will you conclude? There is there will be something at d c corresponding to the fourth and all this fourth eighth and so on. But you do not you are just looking at the spectrum trying to infer what the sample and hold is doing? And if I choose this particular input frequency it looks as if there is a d c offset which I say hey d c offset is with me. Because I know it can be fixed in the system then there is a tone at F s by 2 which corresponds to the second harmonic. But there seems to be no other harmonics at all you understand if you simply look at the spectrum there will be no other harmonics because all the harmonics have.

Alias drawn to.

Same size bin.

The same bin and this particular case it is they have aliased on to the.

Same input.

The fundamental bin itself there by wrongly leading you to conclude that the performance of a sample and hold is a lot better than it actually is you understand. And we will also come to time a domain understanding of this. But in the frequency domain we can see that if you make a poor choice of m several harmonics will alias onto the same band or into the same bin there by making it very difficult to you to design various harmonics. And sometimes also they can also if in this particular choice for this particular choice of frequencies. For example, we see that all the harmonics odd harmonics alias onto the bin where the fundamental lies. And then you look at the spectrum and say wow my sample and was working great, because I do not see any odd harmonic distortion at all. So, you basically fooling yourself by a clearly inappropriate choice of input frequency the sampling frequency is fixed. So, the input frequency is 1 that you have choice over and you chosen a particularly bad value and why is this bad.

If harmonics are there.

No, I mean harmonics are there that that they are being generated by the sample and hold there is nothing you can do about it.

We will not be able to we will not able to.

This is a bad choice of input frequency because several of the components. Harmonic components are alias onto the.

Same band.

Same band all I mean you could somehow magically come up with perhaps a sample and hold somehow many harmonics alias onto the same bin. Because of a wrong choice of f in and if by chance the magnitudes are same and the phase is exactly opposite. You know they could cancel and then you say wow my sample and hold is fantastic. It is distortion free you understand and that is clearly not correct you understand. So, what do you think? We should enforce on of our choice on m not only the f in should be chosen in a way that f in by f s is of the form m by N that is necessary, but that is not sufficient. So, in the frequency domain feature, what do you think we should do we should make sure that whatever m we choose the harmonics must not be on there.

The harmonics should not fall on the.

Same bin..

On the same bins, you understand I mean a case in point is being here for example, we chose the input frequency to be 247 by 1024 times f s. And note that this is the second harmonic and this is the third harmonic if we are pushed this 247 to 256, what is happening? This guy will move to f s by 2 and the third harmonic will lie on.

256.

256 making it impossible to figure out, what is really happening in the sampling? So, the next time, we will look at what we need to do to fix this problem. In other words it is just boils down to choosing m in a way that.

.

Harmonics.

Do not alias back on to the same bins multiple harmonics do not alias into the same bins.