Semiconductor Device Modeling
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Lecture - 20
Drift-diffusion Model: Equations, Boundary Conditions, Mobility and Generation-Recombination

In the last 4 lectures, we have learned a number of things about the Drift-diffusion model. So, let us summarize all those things which we have learned.

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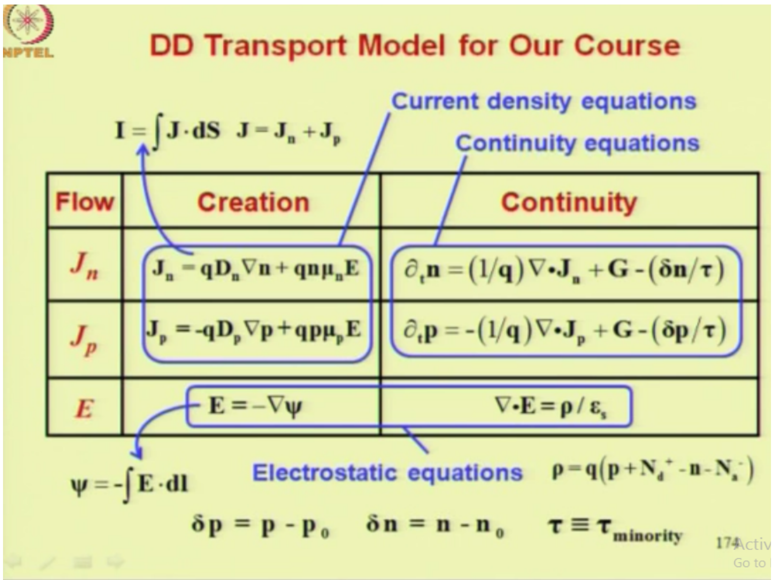

DD Transport Model for Our Course

$I = \int J \cdot dS \quad J = J_n + J_p$

Current density equations

Continuity equations

Flow	Creation	Continuity
J_n	$J_n = qD_n \nabla n + qn\mu_n E$	$\partial_t n = (1/q)\nabla \cdot J_n + G - (\delta n/\tau)$
J_p	$J_p = -qD_p \nabla p + qp\mu_p E$	$\partial_t p = -(1/q)\nabla \cdot J_p + G - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla \cdot E = \rho/\varepsilon_s$

$\psi = -\int E \cdot dl$

Electrostatic equations

$\rho = q(p + N_d^+ - n - N_a^-)$

$\delta p = p - p_0 \quad \delta n = n - n_0 \quad \tau \equiv \tau_{minority}$

So, we said that the Drift-diffusion model consists of 6 equations, 2 current density equations, 2 continuity equations and 2 electrostatic equations. You can integrate the current density obtain by summing up the electron and hole current densities at any contact, dS is the contact area. And you can get the voltage across the device by integrating the electric field from one end of the contact to the other end of the contact.

The space charge in this equation is given by q times the whole concentration + the ionized donor concentration minus electron concentration minus the ionized acceptor concentration. Similarly, the excess whole concentration is equal to the whole concentration minus the value under equilibrium and similarly for the electron concentration. We have let delta n and delta p they are different here though in most situations these 2 quantities will be approximately equal and quasi neutrality will hold.

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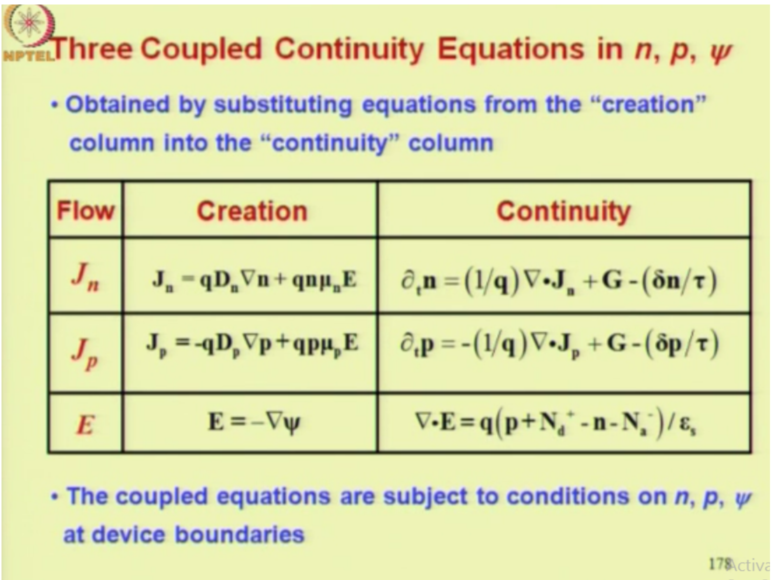

Three Coupled Continuity Equations in *n*, *p*, ψ

- Obtained by substituting equations from the "creation" column into the "continuity" column

Flow	Creation	Continuity
J_n	$J_n = qD_n\nabla n + qn\mu_n E$	$\partial_t n = (1/q)\nabla\cdot J_n + G - (\delta n/\tau)$
J_p	$J_p = -qD_p\nabla p + qp\mu_p E$	$\partial_t p = -(1/q)\nabla\cdot J_p + G - (\delta p/\tau)$
E	$E = -\nabla\psi$	$\nabla\cdot E = q(p + N_d^+ - n - N_a^-)/\varepsilon_s$

- The coupled equations are subject to conditions on *n*, *p*, ψ at device boundaries

We have said that you can reduce these 6 equations to 3 continuity equations coupled to each other and these equations will be in n, p and psi. So, you can reduce the 6 variables n, p, Jn, Jp, E and psi to just 3 variable equations in n, p and psi. So, you can do that by substituting equations from the creation column here to the continuity column. So, recall that we have said these equations can be regarded as equations describing the creation of the flow because it relates the effect to the cause.

For example, Jn is due to gradient of the concentration and the electric field. Similarly, the electric field is due to the gradient of the potential. That is why these are called equations of flow creation. So, you can substitute these equations into the equations here for the continuity and you get the 3 coupled equations in n, p and psi. Coupling denotes the fact that carriers perturb the fields which cause their motion.

The coupled equations are subject to conditions on n, p and psi at device boundaries. So, to solve these 3 coupled equations you need 3 boundary conditions, one each on n, p and psi.

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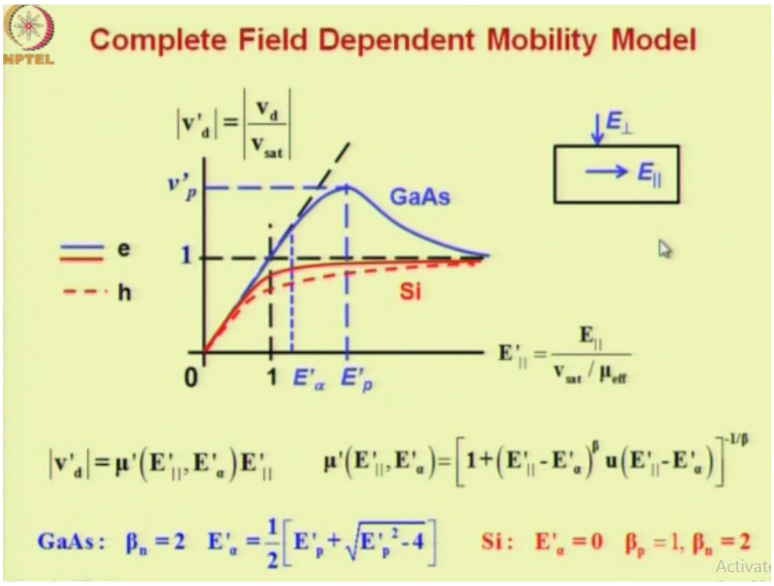

Complete Field Dependent Mobility Model

$$|v'_d| = \mu'(E'_{||}, E'_\alpha)E'_{||} \qquad \mu'(E'_{||}, E'_\alpha) = \left[1 + (E'_{||} - E'_\alpha)^\beta u(E'_{||} - E'_\alpha)\right]^{-1/\beta}$$

GaAs: $\beta_n = 2 \quad E'_\alpha = \frac{1}{2}\left[E'_p + \sqrt{E'^2_p - 4}\right]$ Si: $E'_\alpha = 0 \quad \beta_p = 1, \beta_n = 2$

Now, so the things to be discussed about the Drift-diffusion model are the boundary conditions the expression for the mobility. We do not consider the expression for the diffusion coefficient because diffusion coefficient is given in terms of mobility by Einstein relation. Then we need to consider expressions for generation process, excess generation process and recombination or life time.

So, this is what we will do in sequence will summarize the results for each of these. So, first the complete field dependent mobility model. Now we can show the results of this model on a normalize plot like this where the parallel electric field has been normalized with respect to Vsat/mu effective which we had called as E critical in our module. Similarly, the Drift velocity is normalized with respect to the saturation velocity.

The advantage of expressing the results in this normalized form is that for all the various materials the velocity field curves will follow this regular pattern namely that the linear part of the curve will start on this line with the unity slope on this normalized graph and finally all the curves will saturate at this unity line. For example, if you take silicon for electrons the velocity field curve looks like this on the normalized plot. For holes it is this dash line.

For Gallium arsenide which has a peak it would be the curve shown in the blue. Let us consider the model for Gallium arsenide because we know that this is general model which can be reduced to the model for silicon by setting some parameter to 0. So in the normalized scheme, the velocity field curve for Gallium arsenide is represented like this the Drift velocity.

The normalized Drift velocity = the normalized mobility into the normalized electric field where the normalized mobility is given by the mobility that we derived during the module discussions divided by the effective mobility. The normalized field dependent mobility is expressed in terms of the normalized parallel electric field and a parameter E dash alpha a field, parameter E dash alpha.

The E dash alpha represents the normalized field until which the curve for the Gallium arsenide remains linear and the deviation from linearity occurs beyond this point. For electrons in Gallium arsenide beta is equal 2, so this beta here is 2 and E dash alpha is related to the normalized peak electric field by this formula. For silicon, the same formula can be used by setting E dash alpha=0.

For holes in silicon beta is 1 and for electrons it is 2. The parallel electric field and perpendicular electric fields are shown here for reference the directions are shown there.

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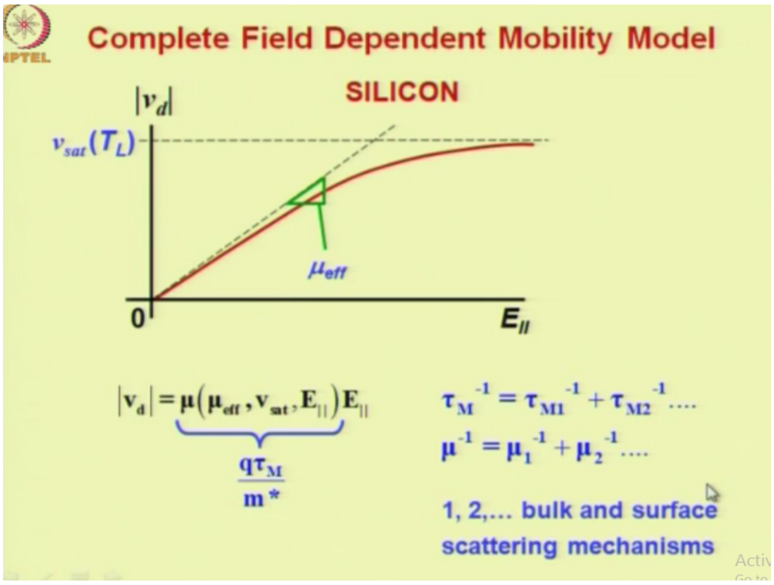

Complete Field Dependent Mobility Model

SILICON

$$|\mathbf{v}_d| = \underbrace{\mu(\mu_{eff}, \mathbf{v}_{sat}, \mathbf{E}_{||})}_{\frac{q\tau_M}{m^*}}\mathbf{E}_{||}$$

$$\tau_M^{-1} = \tau_{M1}^{-1} + \tau_{M2}^{-1} \ldots.$$

$$\mu^{-1} = \mu_1^{-1} + \mu_2^{-1} \ldots.$$

1, 2,... bulk and surface scattering mechanisms

Let us consider the detailed model for silicon which is the material that is used in all the integrated circuits. Its initial linear segment the slope is the effective mobility and the saturation velocity is the function of lattice temperature. So, this is the expression for Drift velocity versus electric field the field dependent mobility is a function of effective mobility that is this quantity saturation velocity that is this limit and the parallel electric field.

Now this mobility physically can be expressed as q times the momentum relaxation time by effective mass. The momentum relation time for several different scattering mechanisms in the bulk or at the surface can be obtained by this Matthiessen’s rule where reciprocal of the overall momentum relation time is given by some of the reciprocals of the momentum relation times corresponding to each scattering process.

And because of this kind of formula for momentum relaxation time the effective mobility, the overall mobility taking into account all the various scattering processes is given by this formula. 1 by overall mobility = 1 by mobility due to a particular scattering process + 1 by mobility due to another scattering process and so on. So, therefore, to develop the mobility expression it is important, it is useful to approach the expression from this reciprocal terms point of view.

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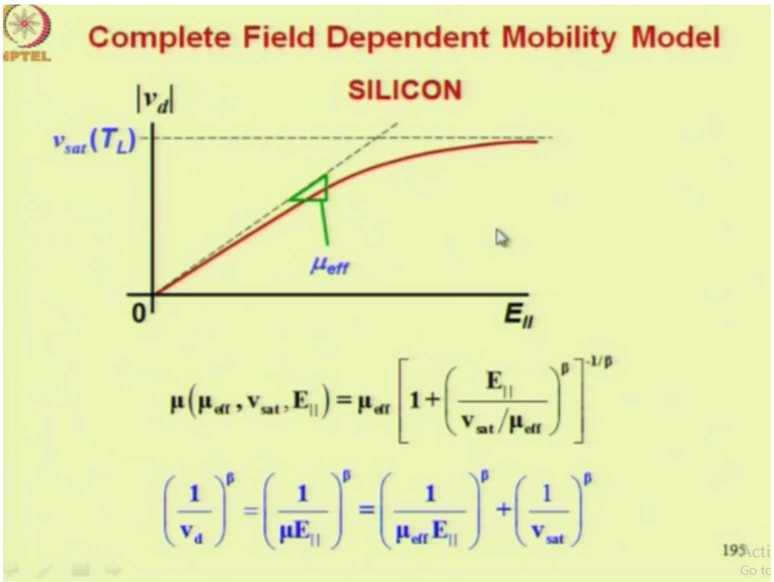

Complete Field Dependent Mobility Model

SILICON

$$\mu(\mu_{eff}, v_{sat}, E_{||}) = \mu_{eff}\left[1+\left(\frac{E_{||}}{v_{sat}/\mu_{eff}}\right)^{\beta}\right]^{-1/\beta}$$

$$\left(\frac{1}{v_d}\right)^{\beta} = \left(\frac{1}{\mu E_{||}}\right)^{\beta} = \left(\frac{1}{\mu_{eff} E_{||}}\right)^{\beta} + \left(\frac{1}{v_{sat}}\right)^{\beta}$$

For example, the field depend mobility expression is given by this formula which can be developed using this approach where the 1 by drift velocity is expressed in terms of the asymptote for small electric field that is this straight line portion and the asymptote for large electric field that is this horizontal straight line and then you raise it to the power beta to control the transition from one segment to other segment higher the beta sharper the transition.

So, this is an empirical method of fitting a curve from a knowledge of asymptotes corresponding to its 2 limits or limiting asymptotes.

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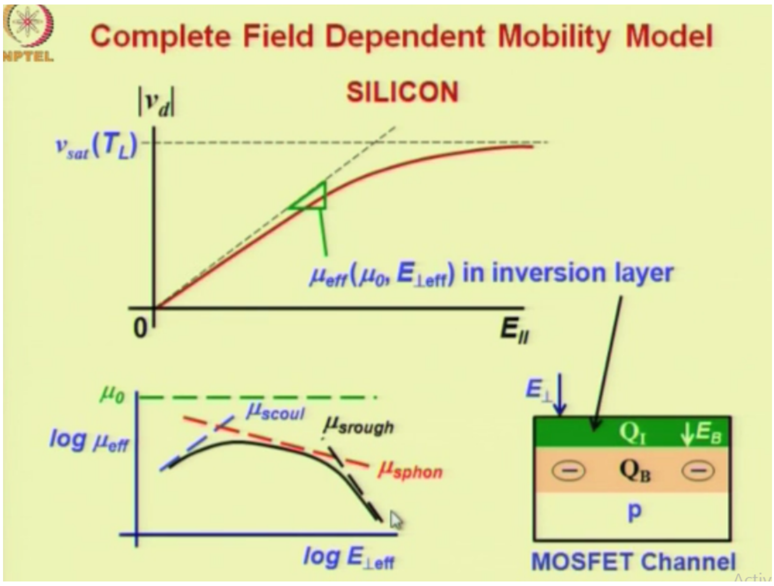

Complete Field Dependent Mobility Model

SILICON

The effective mobility is a function of bulk mobility and effective perpendicular electric field for electrons in inversion layer. Now this effective perpendicular electric field is average of 2 perpendicular fields one which is at the silicon, silicon dioxide interface and other at the end of the inversion layer where it has a boundary with the depletion charge. So, that is this field. So, average of these 2 fields is the field experienced by the entire inversion charge.

So, in terms of the charges this effective perpendicular electric field = half of inversion charge plus the depletion charge the modulus of this divided by the electric constant or permittivity of silicon. The relation between effective mobility and perpendicular effective electric field it shows several asymptotic segments, straight line segments. So the, for small E perpendicular effective it is surface coulombs scattering then you have the surface coulombs scattering and finally the surface roughness scattering.

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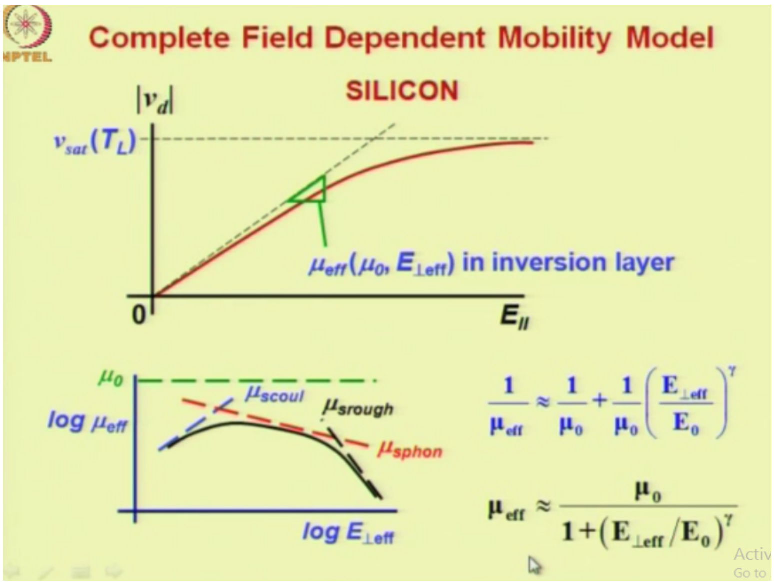

Complete Field Dependent Mobility Model

SILICON

$$\frac{1}{\mu_{eff}} \approx \frac{1}{\mu_0} + \frac{1}{\mu_0}\left(\frac{E_{\perp eff}}{E_0}\right)^{\gamma}$$

$$\mu_{eff} \approx \frac{\mu_0}{1+\left(E_{\perp eff}/E_0\right)^{\gamma}}$$

So, as a result you can write an expression for the mobility, effective mobility by using the reciprocal approach. It is =1/mu not which is the bulk mobility + the mobility because of individual scattering mechanisms. So, for each of the scattering mechanisms because on your log-log plot it is a straight line you have this kind of a power law behavior versus E perpendicular effective. So, each segment will have its own gamma and E0.

However, we work with a single E0 and gamma because of the most modern MOSFETs operate around this place corner or in the region where the surface roughness scattering dominates. So you can transform this relation to this form for immediate use.

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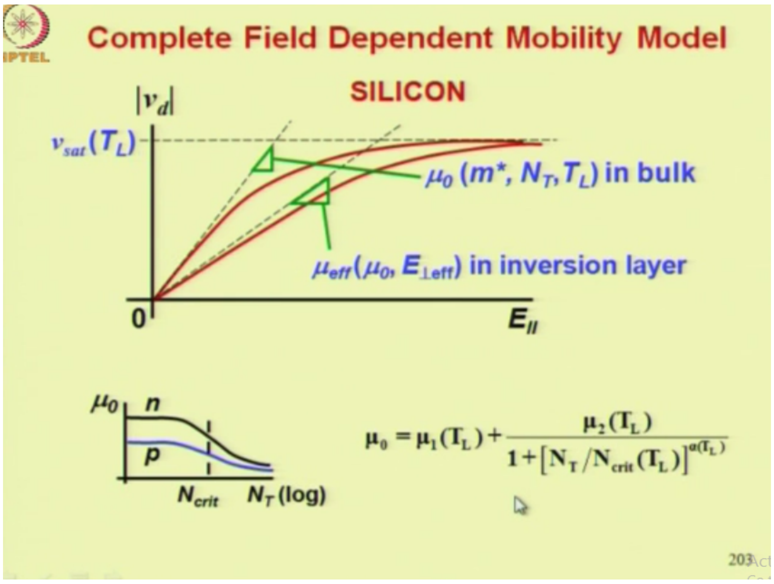

Complete Field Dependent Mobility Model

SILICON

$$\mu_0 = \mu_1(T_L) + \frac{\mu_2(T_L)}{1+\left[N_T/N_{crit}(T_L)\right]^{\alpha(T_L)}}$$

The bulk mobility, it depends on the effective mass of the carriers, the total doping concentration some of donor accept or defects and everything and lattice temperature. The behavior of the bulk mobility as a function of doping for any given temperature follows this kind of a curve. So, you have a high value for low doping concentration and ultimately it is saturates to a low value for very high doping concentration.

N critical is the doping concentration at which the mobility is half of the initial value. So, using the parameters N critical you can write an expression for mobility in this form, where the terms mu1 mu2 N critical and alpha are all functions of lattice temperature. So, you can see that for large values of Nt it approaches mu 1 and for Nt = 0 it has the value mu1+mu2.

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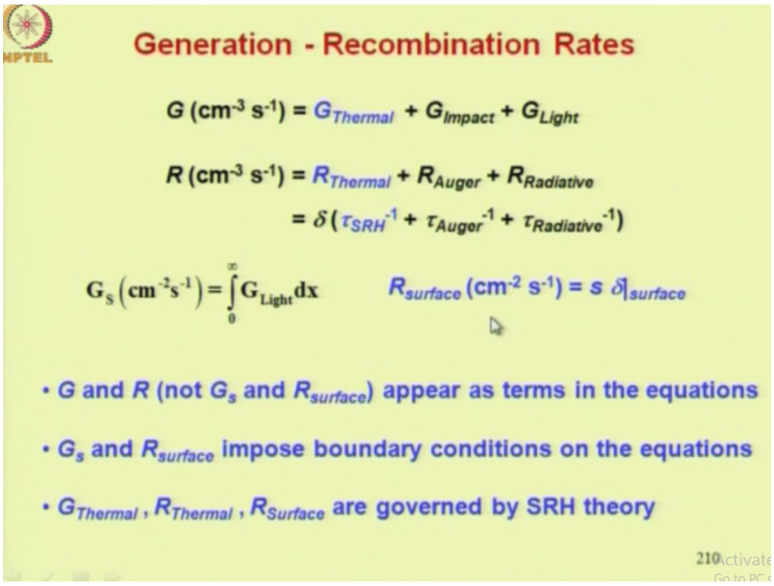

Generation - Recombination Rates

$$G\,(\mathrm{cm^{-3}\,s^{-1}}) = G_{Thermal} + G_{Impact} + G_{Light}$$

$$R\,(\mathrm{cm^{-3}\,s^{-1}}) = R_{Thermal} + R_{Auger} + R_{Radiative}$$
$$= \delta\,(\tau_{SRH}{}^{-1} + \tau_{Auger}{}^{-1} + \tau_{Radiative}{}^{-1})$$

$$G_S\left(\mathrm{cm^{-2}s^{-1}}\right) = \int_0^\infty G_{Light}\,dx \qquad R_{surface}\,(\mathrm{cm^{-2}\,s^{-1}}) = s\,\delta|_{surface}$$

- G and R (not G_s and $R_{surface}$) appear as terms in the equations
- G_s and $R_{surface}$ impose boundary conditions on the equations
- $G_{Thermal}$, $R_{Thermal}$, $R_{Surface}$ are governed by SRH theory

Now, coming to the generation recombination rates, the volume generation rate is that due to Thermal generation process + Impact ionization generation process plus generation because of light. The corresponding inverse processes for recombination, volume recombination are Thermal recombination, the Auger recombination which is inverse of Impact ionization and Radiative recombination which is the inverse of generation because of light.

Now you can express each of this recombination rates in an equation in which you have the rates as proportional to the excess carrier concentration and the proportionality constant is reciprocal of life times. So, for each of the processes you have a life time, the life time for thermal recombination process is the life time predicted by the Shockley–Read–Hall theory. Similarly, you have life times for Auger and Radiative processes.

And you can use this formula to predict the overall recombination rate. You also have processes for surface generation and surface recombination. So here this generation rate has the units of per centimeter square per second similarly for the surface recombination. The surface generation rate is obtained by integrating the volume generation rate.

Because of light over distance, assuming that the volume generation happens over a very thin layer near the surface. The surface recombination rate can be written as proportional to the surface excess carrier concentration at the surface following the approach for volume recombination. So, the constant of proportionality has a dimensions of velocity and this is called surface recombination velocity.

So, the G and R, the volume generation and recombination rates and not the surface generation recombination rates appear as terms in the equations. So, what is the utility of Gs and Rsurface? Gs and Rsurface impose boundary conditions on the equations. So, students often have this misconception they try to include G, S and R surface in the terms for the continuity equations where we have put G and R, right but that is not correct.

The G and R terms which appear in the continuity equation are volume generation and recombination rates. Finally, importantly the thermal generation, thermal recombination and surface recombination are all governed by the Shockley–Read–Hall theory. That is why these 3 recombination rates are all shown in blue and we shall consider their formulae together to begin with.

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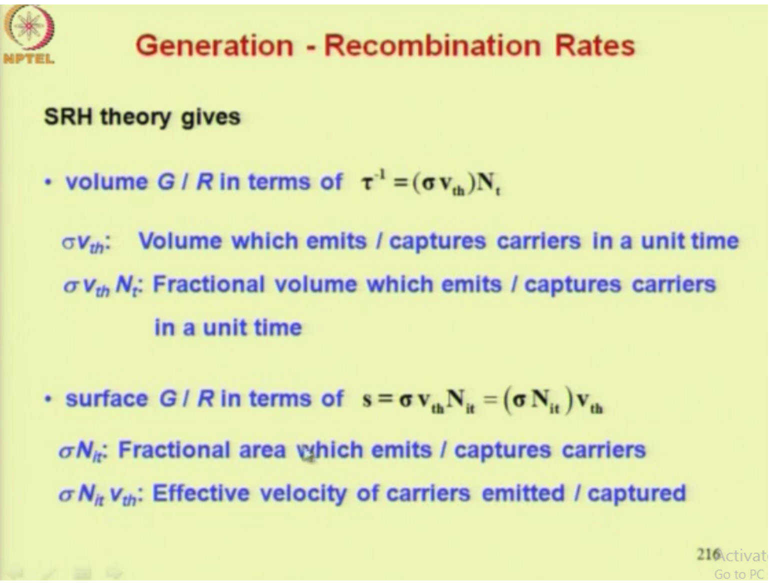

Generation - Recombination Rates

SRH theory gives

- volume *G* / *R* in terms of $\tau^{-1} = (\sigma v_{th}) N_t$

σv_{th}: Volume which emits / captures carriers in a unit time

$\sigma v_{th} N_t$: Fractional volume which emits / captures carriers in a unit time

- surface *G* / *R* in terms of $s = \sigma v_{th} N_{it} = (\sigma N_{it}) v_{th}$

σN_{it}: Fractional area which emits / captures carriers

$\sigma N_{it} v_{th}$: Effective velocity of carriers emitted / captured

A few points about the Shockley–Read–Hall theory. Now this theory gives the volume generation recombination in terms of reciprocal of time constant which is equal to what is called the capture cross section of the trap which is trapping electrons and holes in sequence and then crossing the recombination multiplied by thermal velocity, which is the velocity with which the carrier approaches the trap and the trap concentration per unit volume.

So Nt, here is the trap concentration per unit volume. Analogously, the surface generation or recombination rate are expressed in terms of a velocity s which is equal to the capture cross section or emission cross section of the trap multiplied by the thermal velocity multiplied by Nit is the trap concentration per unit area. Please note Nt here is the trap concentrating per unit volume whereas here it is per unit area.

Now for physical interpretation it proves useful to slightly reorganize this formula by putting sigma and Nit together and then multiplying this by V thermal. So, when you write it in this form it shown analogy to the volume recombination where we have put sigma followed by thermal velocity followed by trap concentration.

Look at the physical interpretation. The product sigma into v thermal is the volume which emits or captures carriers in a unit time. So, sigma is the capture cross section and thermal velocity. You can take it as a length travelled by a carrier in a unit time. So, you have a cylindrical volume there. This volume is swept in unit time because the length is the length that the carrier travels in that unit time.

So, what it means is all carriers within this volume will cross the trap in that time. Now, when you multiply this sigma v thermal/Nt it denotes the fractional volume which emits or captures carriers in a unit time. So, you take the per unit volume and you have several traps and you have this cylindrical volume for each trap. So, some of all the cylindrical volumes divided by the total volume is the fractional volume that is what this shows.

So, it is only this fractional volume which captures or emits its traps. Analogously, sigma into Nit, this product represents a fractional area which emits or captures carriers. So, you have a surface area for each of the traps imagine circle, circular area now there are Nit traps per unit area. So you have such Nit circles. So you divide the circular area by the unit area in which this Nit traps are there that gives you the fractional area.

So, this is the area which emits or captures carriers. Now sigma Nit into v thermal this is the effective velocity of carriers emitted or captured. So, what is happening is only carriers which are in this capture cross section are getting captured any other carriers which fall on the other area are not getting captured. So, you can define an effective velocity all the carriers which are captured how thermal velocity.

And all the carriers which are not captured which are falling in the areas between the circles, they are not captured so we can say their velocity, capture velocity is 0 and then you can find out an effective velocity. That is what this product is and that is the effective surface recombination velocity.

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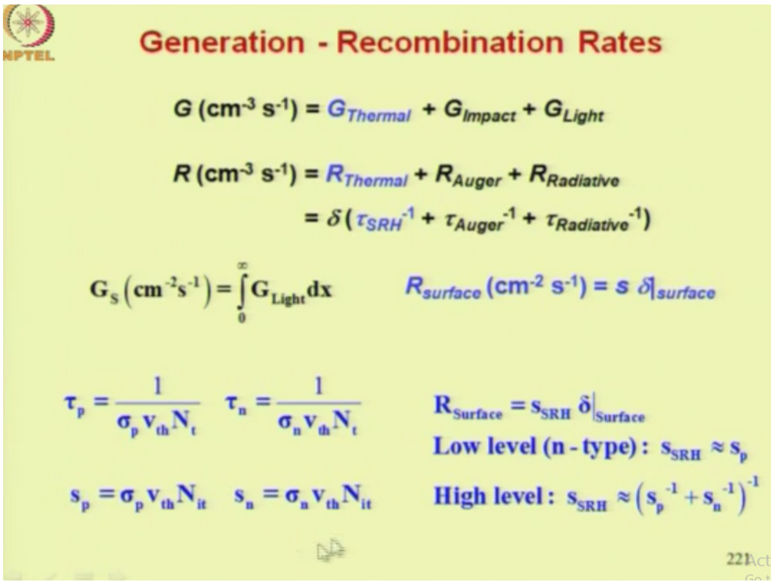

Generation - Recombination Rates

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$$R\,(\mathrm{cm^{-3}\,s^{-1}}) = R_{Thermal} + R_{Auger} + R_{Radiative}$$
$$= \delta\,(\tau_{SRH}^{-1} + \tau_{Auger}^{-1} + \tau_{Radiative}^{-1})$$

$$G_S\left(\mathrm{cm^{-2}s^{-1}}\right) = \int_0^\infty G_{Light}\,dx \qquad R_{surface}\,(\mathrm{cm^{-2}\,s^{-1}}) = s\,\delta|_{surface}$$

$$\tau_p = \frac{1}{\sigma_p v_{th} N_t} \qquad \tau_n = \frac{1}{\sigma_n v_{th} N_t}$$

$$s_p = \sigma_p v_{th} N_{it} \qquad s_n = \sigma_n v_{th} N_{it}$$

$R_{Surface} = s_{SRH}\,\delta|_{Surface}$

Low level (n - type): $s_{SRH} \approx s_p$

High level: $s_{SRH} \approx \left(s_p^{-1} + s_n^{-1}\right)^{-1}$

Now, what are the expressions in terms of these? So, the thermal generation rate works out to the form ni/generation time, where generation time is sum total of the time constant because of holes and the time constant because of electrons. So, we just now interpreted the reciprocal of the time constants, okay? Now this reciprocal of the time constants are different for holes and electrons because capture cross section of electrons and holes are different.

So, this is the formula for tau p and tau n. Same tau p and tau n are used in thermal recombination formula for expressing the Shockley–Read–Hall life time. Under low level conditions in an n-type sample for example the tau SRH is tau p. In a p-type sample it would be tau n. Under high level, it is sum total of tau p and tau n. So high life time is higher than low level life time.

Surface recombination is surface recombination velocity multiplied by the excess carrier concentration. Under SRH theory under low level for an n-type sample, the surface recombination velocity is the constant s suffix p. For high level the velocity is given by reciprocal of the sum of the reciprocals Sp and Sn due to holes and electrons. Now the formula for tau p and sp are analogues. So that is what is shown here.

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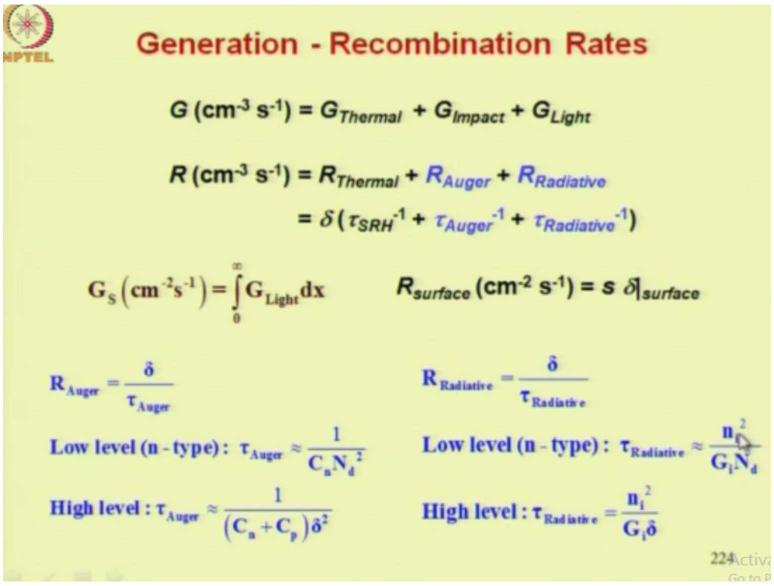

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$$= \delta\left(\tau_{SRH}^{-1} + \tau_{Auger}^{-1} + \tau_{Radiative}^{-1}\right)$$

$$G_S\left(\mathrm{cm^{-2}s^{-1}}\right) = \int_0^{\infty} G_{Light}\,dx \qquad R_{surface}\,(\mathrm{cm^{-2}\,s^{-1}}) = s\,\delta|_{surface}$$

$$R_{Auger} = \frac{\delta}{\tau_{Auger}} \qquad R_{Radiative} = \frac{\delta}{\tau_{Radiative}}$$

Low level (n - type) : $\tau_{Auger} \approx \dfrac{1}{C_n N_d^2}$ Low level (n - type) : $\tau_{Radiative} \approx \dfrac{n_i^2}{G_i N_d}$

High level : $\tau_{Auger} \approx \dfrac{1}{(C_n + C_p)\delta^2}$ High level : $\tau_{Radiative} = \dfrac{n_i^2}{G_i \delta}$

Let us look at the other recombination processes, namely Auger and Radiative recombination. Now these 2 also can be grouped together like we grouped G thermal, R thermal and R surface because Auger and Radiative recombination’s are both ban to ban recombination’s, okay. They are direct recombination’s no other particles is involved other than holes and electrons.

For Auger recombination the lifetime for n-type silicon, for n-type semiconductor is inversely proportional to the square of the doping. So, at heavy doping this life time becomes small and the Auger life time or Auger recombination process becomes important. For high level however, the Auger life time is not a constant it depends on the excess carrier concentration.

And particularly, it falls rapidly as inverse of the square of the excess carrier concentration and therefore you cannot write the Auger recombination rate as proportional to excess carrier concentration under high level. It is proportional to cube of the excess carrier concentration. The Radiative life time under low level conditions is similarly inversely related to the doping level.

So, whereas here it is inversely related to the square of the doping here it is just the doping. And under high level conditions it is inversely related to the excess carrier concentration. The Gi here is the intrinsic generation rate that is generation rate under thermal equilibrium and ni is the intrinsic concentration.

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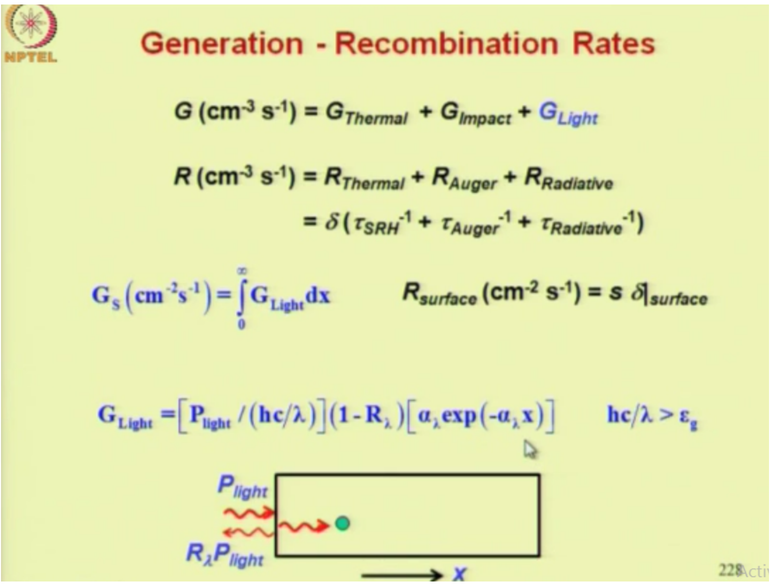

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$$= \delta\,(\tau_{SRH}^{-1} + \tau_{Auger}^{-1} + \tau_{Radiative}^{-1})$$

$$G_S\left(\mathrm{cm^{-2}s^{-1}}\right) = \int_0^\infty G_{Light}\,dx \qquad R_{surface}\,(\mathrm{cm^{-2}\,s^{-1}}) = s\,\delta|_{surface}$$

$$G_{Light} = \left[P_{light}/(hc/\lambda)\right](1-R_\lambda)\left[\alpha_\lambda \exp(-\alpha_\lambda x)\right] \qquad hc/\lambda > \varepsilon_g$$

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The impact ionization, the formula is given by this relation. Here alpha n is called the ionization coefficient which talks about the number of electron hole pairs generated by a carrier when it moves through a unit distance. The formula for this ionization coefficient is here it varies rapidly it increases rapidly with increase in electric field that is what this exponential relation shows.

The generation because of light, so volume generation and surface generation. So volume generation first, so light is incident on a sample part of it is reflected and remaining is absorbed. That is what is reflected in this formula. The P light is the incident power in watts per centimeter square when you divide it by the energy of the photon which has to be more than the energy gap to generate electron hole pairs,

this ratio gives you the photon flux per unit area per unit time. Part of this incident flux is absorbed so 1 - the reflection coefficient is the part that is absorbed and then whatever is absorbed as the photons move they generate electron hole pairs, so this alpha is like the ionization coefficient of the impact ionization instead of electron hole, electrons or holes it is the photons which are moving and ionizing the atoms electrons and holes and the absorption falls exponentially with x as more and more photons are absorbed.

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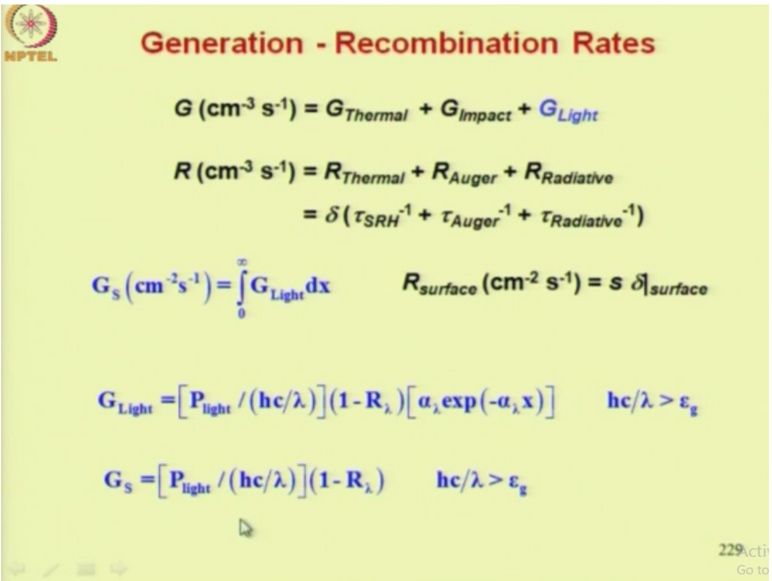

Generation - Recombination Rates

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$G_S\left(\text{cm}^{-2}\text{s}^{-1}\right) = \int_0^\infty G_{Light}\,dx$ $\quad R_{surface}\,(\text{cm}^{-2}\,\text{s}^{-1}) = s\,\delta|_{surface}$

$G_{Light} = \left[P_{light}/(hc/\lambda)\right](1-R_\lambda)\left[\alpha_\lambda \exp(-\alpha_\lambda x)\right] \quad hc/\lambda > \varepsilon_g$

$G_S = \left[P_{light}/(hc/\lambda)\right](1-R_\lambda) \quad hc/\lambda > \varepsilon_g$

You can integrate this formula assuming that the absorption happens in a very short distance. This can happen for some values of lambda because the alpha is a function of lambda. So, when you integrate as a function of x this term in alpha drops out leaving you with just this formula.

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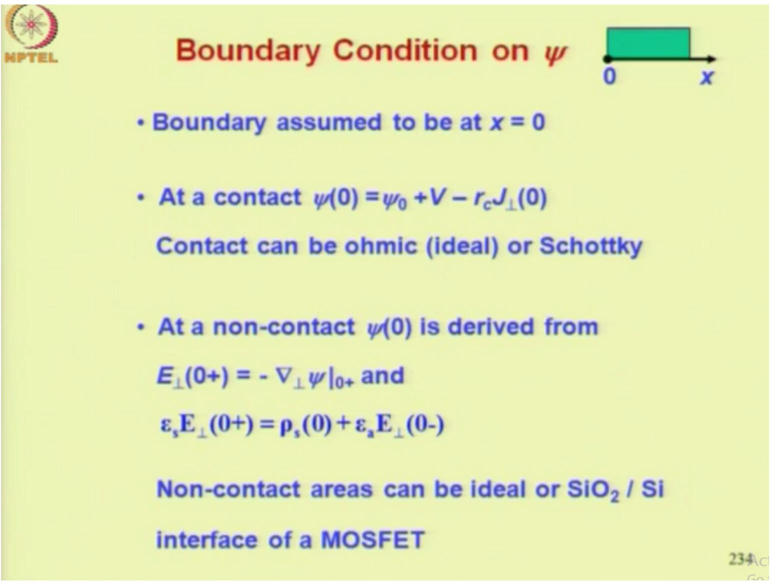

Boundary Condition on ψ

- Boundary assumed to be at $x = 0$
- At a contact $\psi(0) = \psi_0 + V - r_c J_\perp(0)$

  Contact can be ohmic (ideal) or Schottky
- At a non-contact $\psi(0)$ is derived from

  $E_\perp(0+) = -\nabla_\perp \psi|_{0+}$ and

  $\varepsilon_s E_\perp(0+) = \rho_s(0) + \varepsilon_a E_\perp(0-)$

  Non-contact areas can be ideal or SiO_2 / Si interface of a MOSFET

Finally, we discussed the boundary conditions. First, let us start with the boundary conditions on psi. So, you will recall we had said that we need minimum of 3 boundary conditions. 1 on psi, 1 on n and 1 of p because the 6 equations of the Drift deficient model can be reduced to 3 coupled equations in n, p and psi. Let us assume that the boundary is that x=0.

So this is this vertical boundary that we are talking about a 1-dimensional case, this is the semiconductor. Then for a contact that x = 0, the potential is sum of the built in potential plus the applied voltage at that contact minus the contact resistance into the normal current density. The contact can be ohmic or Schottky the same formula would apply.

If the boundary is a non-contact boundary, then you can derive size 0 or its gradient del psi from this relation. The perpendicular electric field in the semi-conductor 0+ here indicates the semiconductor side on the right of this origin, okay 0- is the ambient. So, perpendicular electric field is gradient of psi.

Perpendicular gradient of psi and the relation between the perpendicular electric field in semiconductor and the perpendicular electric field in ambient 0- is given by this relation, where rho s is a surface charge density at the boundary here. So, this is nothing but statement of Gauss's law. Non-contact areas can be ideal which means ideal contacts are those, non-contacts are those where there is no surface charge.

And there is no field going out into the ambient. So therefore this field, group field going out into the ambient that is this field. So, the field here in the semiconductor normal to the surface is 0 or silicon, silicon dioxide interface of a MOSFET in that case you will have surface charges and you are also normal component of the electric field and semiconductor will not be 0. However, these relations will apply for both these cases.

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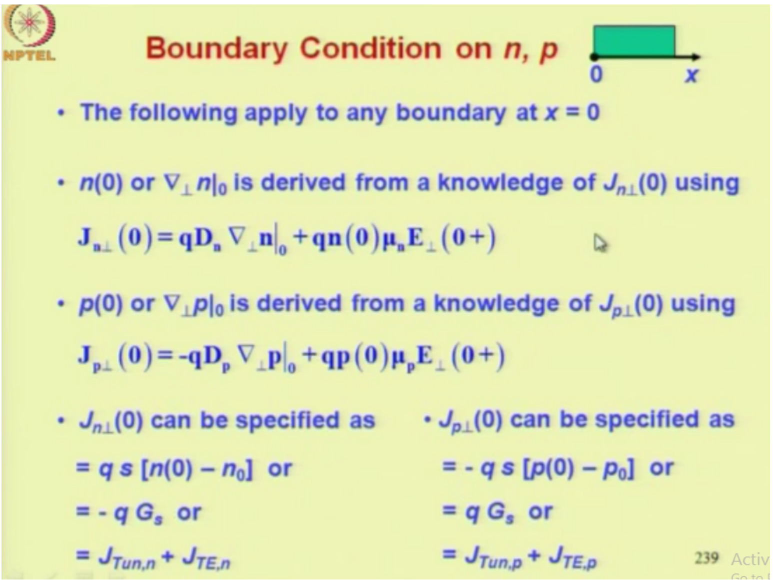

Boundary Condition on n, p

- The following apply to any boundary at $x = 0$
- $n(0)$ or $\nabla_\perp n|_0$ is derived from a knowledge of $J_{n\perp}(0)$ using

$$J_{n\perp}(0) = qD_n \nabla_\perp n|_0 + qn(0)\mu_n E_\perp(0+)$$

- $p(0)$ or $\nabla_\perp p|_0$ is derived from a knowledge of $J_{p\perp}(0)$ using

$$J_{p\perp}(0) = -qD_p \nabla_\perp p|_0 + qp(0)\mu_p E_\perp(0+)$$

- $J_{n\perp}(0)$ can be specified as
  $= q\, s\, [n(0) - n_0]$ or
  $= -q\, G_s$ or
  $= J_{Tun,n} + J_{TE,n}$
- $J_{p\perp}(0)$ can be specified as
  $= -q\, s\, [p(0) - p_0]$ or
  $= q\, G_s$ or
  $= J_{Tun,p} + J_{TE,p}$

Now let us look at the boundary conditions on n and p. Assuming the boundary to be at x = 0. You can get the carrier concentration at the boundary or the gradient of the concentration. Let us take electrons. You can get this from a knowledge of the perpendicular component of the electron current at the boundary. We will see shortly how you can specify this current but the formula for this current is the combination of deficient and drift currents.

So, this is where your n of 0 and gradient of n of 0 are coming. So, if this is specified you can use this formula to get either n of 0 or gradient of n. Now you can specify the perpendicular component of the current density either in terms of surface recombination velocity or in terms of surface generation rate or in terms of tunneling and thermionic emission currents. A parallel approach would be used for holes,

so you can get the whole concentration or its gradient at the boundary from this relation. The difference between this relation and the relation for electrons is that there is a negative sign here for diffusion current and similarly you can specify the normal component of the whole current using surface recombination velocity generation rate at the surface or tunneling and thermionic emission currents.

The difference between the relations for holes and the relations for electrons for their perpendicular current density is that you have a negative sign entering here in place of the positive sign for Jn. And on the other hand for the Gs relation you have a positive sign here in place of the negative sign for electrons. The reason for this is if there is a surface recombination here at this surface the both electrons and holes will move towards the left.

This will constitute an electron current to the right but a whole current to the left that is why the signs for the 2 are different. Similar comments applied for surface generation.

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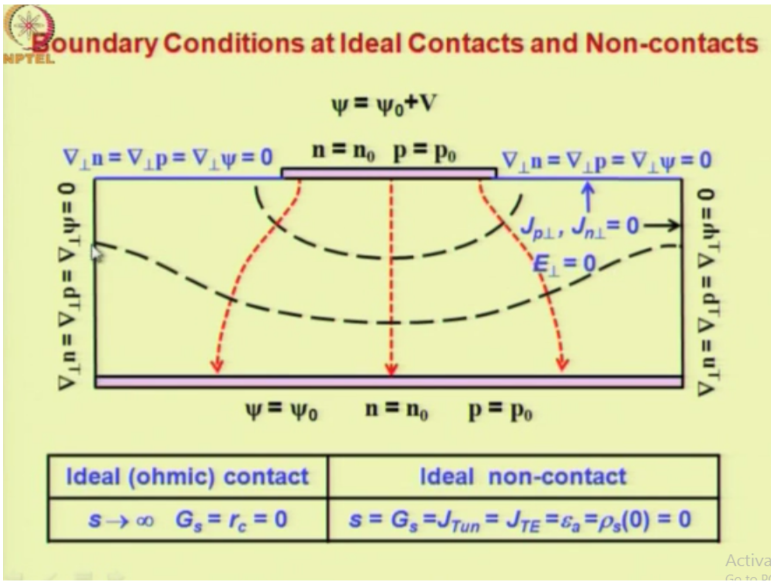

Boundary Conditions at Ideal Contacts and Non-contacts

Ideal (ohmic) contact	Ideal non-contact
$s \rightarrow \infty$ $G_s = r_c = 0$	$s = G_s = J_{Tun} = J_{TE} = \varepsilon_a = \rho_s(0) = 0$

Now let us look at the boundary conditions at ideal Contacts and Non-contacts based on what we have discussed about the boundary conditions regarding psi, n and p. Let us put all these 3 boundary conditions together for these 2 cases. So, for an ideal contact also called the ohmic contact, this will be characterized by surface recombination velocity very high pending to infinity and surface generation rate and contact resistance = 0.

The contact resistance has the dimensions of ohm centimeter square, please note that. So, for such a situation at the contact your potential is given by the applied voltage + the built in potential whereas the carrier concentrations are equal to the equilibrium values. At this contact the potential is simply the built in potential. So, this contact has been chosen as reference and a voltage V has been applied to the contact on the top.

Now ideal non-contact, this is characterized by all these quantities being 0. Now as a consequence what happens is that the normal component of electron and whole currents are 0 and the normal component of electric field also is 0 and therefore, the gradients of electron hole and potential the normal components of these gradients are all 0, okay. So, that is what is the ideal non-contact boundary.

Now these boundary conditions are very important in visualizing the flow of current. For example, the flow lines shown here current flow lines use the boundary condition. You see that they are emanating vertical to the contacts. This is because the electric field at the contact is vertical. No electric field is lateral because it is equi-potential surface. Similarly, if you take the equi-potential lines they emanate from the non-contact areas in perpendicular directions because this is the consequence of this boundary conditions here. You can see that this applies even to the equi-potential that is emanating from the vertical surface. So, here this equi-potential line is horizontal at this point.

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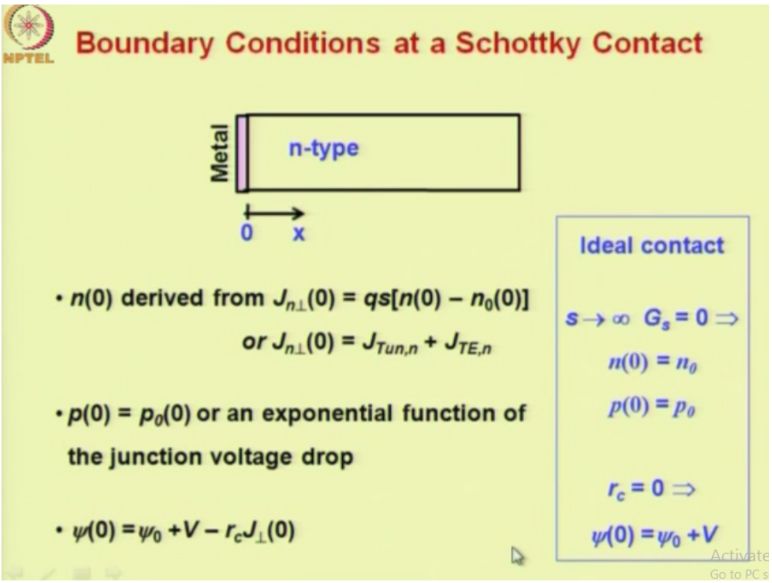

Boundary Conditions at a Schottky Contact

- $n(0)$ derived from $J_{n\perp}(0) = qs[n(0) - n_0(0)]$
  or $J_{n\perp}(0) = J_{Tun,n} + J_{TE,n}$
- $p(0) = p_0(0)$ or an exponential function of the junction voltage drop
- $\psi(0) = \psi_0 + V - r_c J_\perp(0)$

Ideal contact

$s \to \infty$ $G_s = 0 \Rightarrow$
$n(0) = n_0$
$p(0) = p_0$

$r_c = 0 \Rightarrow$
$\psi(0) = \psi_0 + V$

Let us look at the boundary conditions at a Schottky contact, will assume that the metals semiconductor contact is at x=0. We will also assume that the semiconductor is n-type. The electron concentration at the boundary is derived from a specification of the normal component of the current which could be in terms of a surface recombination velocity which is called in the context of a Schottky contact as thermionic emission diffusion recombination velocity, right.

So, that is the name given in the context of Schottky contact. You can have a current because of combination of thermionic emission and diffusion and net effect of these can be absorbed in terms of the parameter s. Now note here that the equilibrium concentration of carriers has to be taken at the boundary. At a Schottky contact here there will be a space charge region and therefore you have to be careful while choosing the equilibrium value of the concentration in this formula.

n suffix 0 normally would indicate the equilibrium value of the concentration in the bulk, neutral bulk region whereas you have to take the value at the contact and that would be different from the value under in bulk so that is why this bracket 0 is indicated here. So, this equilibrium value of the electron concentration at the boundary, including the variations introduced by the space charge region.

You can also alternatively specify the normal component of the current at a Schottky contact boundary in terms of the currents due to thermionic emission or the current due to tunneling. Now contrast this situation with an ideal contact, there the surface recombination velocity was infinity and there was no surface generation. So, as a result your electron and hole concentrations were simply equal to the equilibrium value whereas here the s is non-0 and non-infinity.

The hole concentration in a metal n-type Schottky contact at the boundary would be the equilibrium hole concentration at the boundary or it can be exponential function of the junction voltage drop so that depends on whether or not you want to incorporate the hole diffusion current along with the other current because of majority carriers in the Schottky diode. You know that the Schottky diode is a majority carrier device.

So minority carrier current is not that important, okay. Now that is why one can use any one of these 2 boundary conditions. For the contact, for the potential at the boundary this is the formula which we have already specified earlier just contrasted with this formula for ideal contact. Here the contact resistance is 0 and the potential at the boundary is simply some of the built in potential and the applied voltage.

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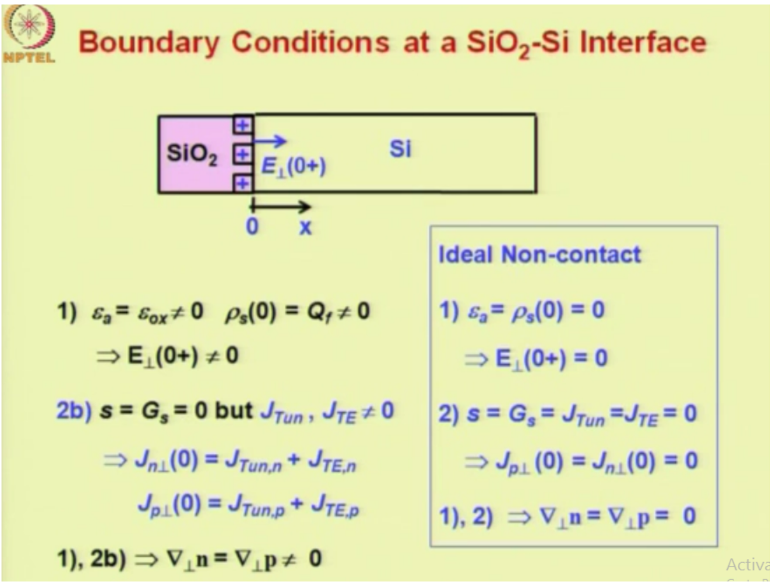

Boundary Conditions at a SiO_2-Si Interface

1) $\varepsilon_a = \varepsilon_{ox} \neq 0 \quad \rho_s(0) = Q_f \neq 0$

$\Rightarrow E_\perp(0+) \neq 0$

2b) $s = G_s = 0$ but $J_{Tun}, J_{TE} \neq 0$

$\Rightarrow J_{n\perp}(0) = J_{Tun,n} + J_{TE,n}$

$J_{p\perp}(0) = J_{Tun,p} + J_{TE,p}$

1), 2b) $\Rightarrow \nabla_\perp n = \nabla_\perp p \neq 0$

Ideal Non-contact

1) $\varepsilon_a = \rho_s(0) = 0$

$\Rightarrow E_\perp(0+) = 0$

2) $s = G_s = J_{Tun} = J_{TE} = 0$

$\Rightarrow J_{p\perp}(0) = J_{n\perp}(0) = 0$

1), 2) $\Rightarrow \nabla_\perp n = \nabla_\perp p = 0$

Finally, let us look at boundary conditions at silicon dioxide, silicon interface. So, the main difference between this interface and an ideal non-contact interface is that there is a perpendicular electric field either because of these charges or because of the presence of a metal or polysilicon electrode at the end of the silicon dioxide layer here.

So, the first important point about this interface is that, a perpendicular component of the electric filed in semiconductor that is at 0+ here at the boundary is non-0 because the surface charge is non-0 there is a fixed charge and the ambient dielectric constant, in this case the ambient is the silicon dioxide. So, it is the oxide permittivity this is non-0. So, both these contribute to a non-0 perpendicular electric field. At an ideal non-contact this field is 0.

Now there are 2 possibilities for specifying the remaining conditions at the silicon dioxide interface. The one possibility is that you assume this interface to be ideal in the sense that you assume that there is no surface recombination at this interface. So, s is no generation and no tunneling or thermionic emission, no leakage current in this oxide, okay. So, said all these = 0.

This would amount to setting the perpendicular compliant of hole and electron currents electron and hole currents to be 0. Now, this is very much like the ideal non-contact, okay. So, this part of the definition in terms of the current densities is similar to ideal non-contact. However, the perpendicular electric field is non-0, so that difference is important that remains.

Now combining these 2 conditions you realize that the normal component of hole and electron concentrations at this boundary would be non-0, whereas at an ideal contact these quantities are 0. Another possibility that you assume there is no surface recombination or generation but there is a leakage current because of tunneling or thermionic emission. So, leakage current is between the gate of a MOSFET that is present here

and the silicon when the silicon dioxide layer is thin. Now that is the case then you must specify the normal component of the electron and hole currents to be equal to the thermionic emission and tunneling currents, okay either only one of the 2 forms of either electron or hole tunneling currents may be present or both tunneling currents may also be present depends on the bias condition and so on. This we will discuss in the context of MOSFETs

Now, here again you will not have a 0 value of normal components of electron and hole concentrations, in contrast to the situation in ideal non-contact region. So, let us see what have we achieved in this module. So, we shall repeat the learning outcomes that we had set out at the beginning of this module.

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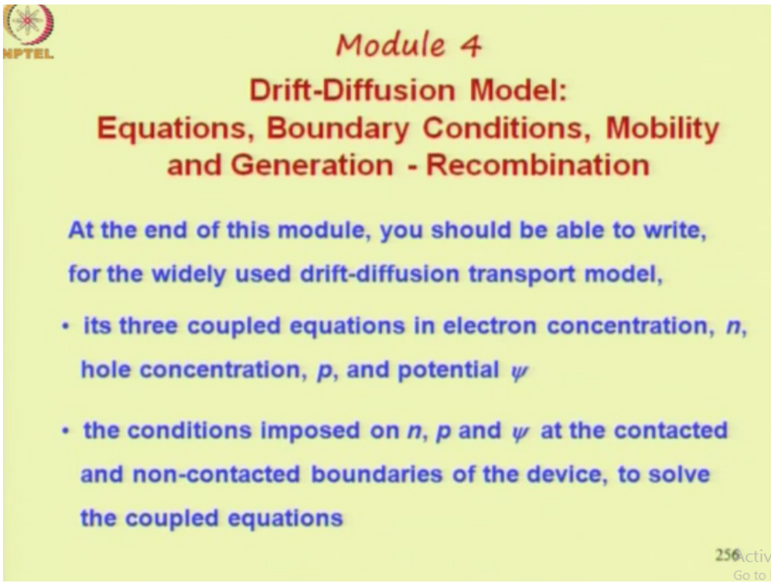

Module 4

Drift-Diffusion Model:
Equations, Boundary Conditions, Mobility and Generation - Recombination

At the end of this module, you should be able to write, for the widely used drift-diffusion transport model,

- its three coupled equations in electron concentration, n, hole concentration, p, and potential ψ
- the conditions imposed on n, p and ψ at the contacted and non-contacted boundaries of the device, to solve the coupled equations

So, at the end of this module, now you should be able to write for the widely used Drift-diffusion transport model. Ii is 3 coupled equations in electron contrition, hole concentration and potential. We should be able to write the conditions imposed on n, p and psi at the contacted and non-contacted boundaries of the device to solve the coupled equations.

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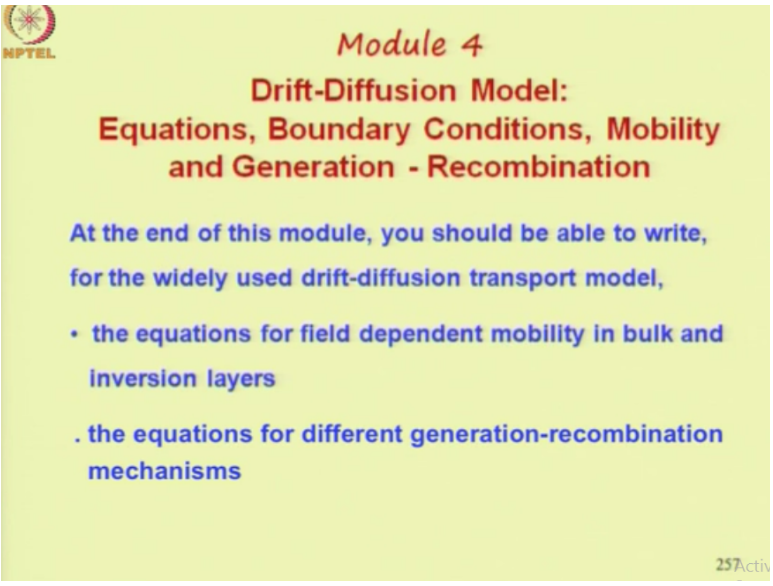

Module 4

Drift-Diffusion Model:
Equations, Boundary Conditions, Mobility and Generation - Recombination

At the end of this module, you should be able to write, for the widely used drift-diffusion transport model,

- the equations for field dependent mobility in bulk and inversion layers
- the equations for different generation-recombination mechanisms

You should be able to write the equations for field dependent mobility in bulk and inversion layers and finally you also must also be able to write the equations for the different generation-recombination mechanism. With that we have come to the end of this module. In the next module we will apply the Drift-diffusion transport model to analyze various situations more specifically we will consider characteristic lines and times which are useful for analysis of devices.