Analog Integrated Circuit Design
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Lecture No - 58
Continuous Time Active Filters

Hello and welcome to lecture 58 of analog integrated circuit design, we were discussing continuous time filters and we saw how to realize second order prototype in g m c and active r c architecture now what will do is extend this higher order filters and then analyze this filters in a little more detail. So, that we end up with practical implementation.

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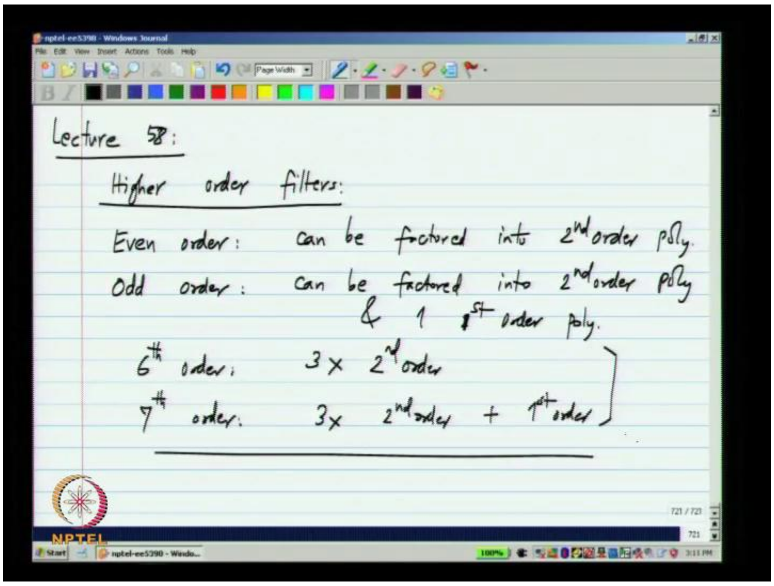

Now, if you have a higher order polynomial, if it’s even order polynomial, then it can be factored into second order factors and, if it is in order polynomial, it can be factored into a number of second order factors and 1 1st order polynomial that is if you have a higher order filter it can be realized is cascade of a number of second order filters, if it is of even order and, if it is a odd order a number of second order filters plus 1 1st order filter.

so that is one convenient way of realizing higher order filter we already know how to make second order filters and we can certainly make higher order filters for instance, if you have a 6th the order, it is 3 times second order filters and if you have a 7th order filter that can be realized as three second order filters plus 1st order. So, now, we already know how to make 2nd-order filters we need to figure out how to make 1st order filters that is very easy.

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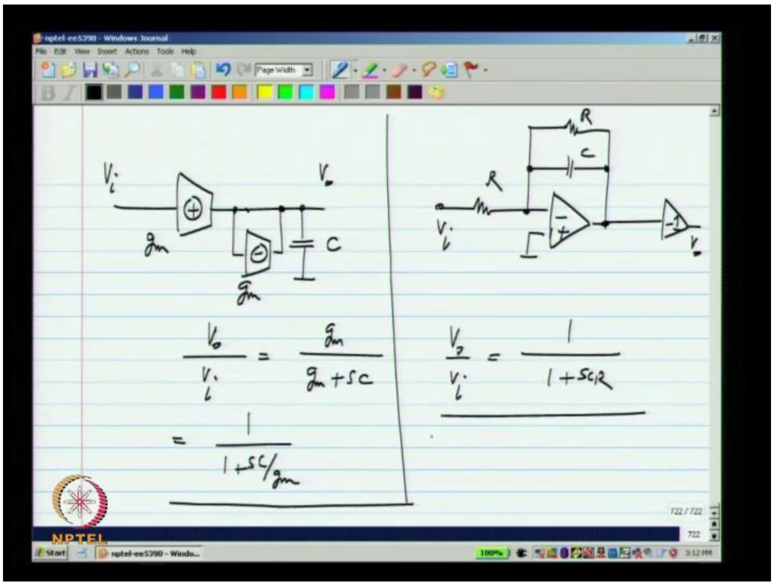

I will give the solution right away, if it is g m c it is have g m and g m there we know that v naught by v i will be g m by g m plus s c or 1 by 1 plus s c by g m similarly, if the active r c as structure v naught b v i would be 1 by 1 plus s c r. So, this is how to make 1st-order filters and this has to be combined with the 2nd-order filters that

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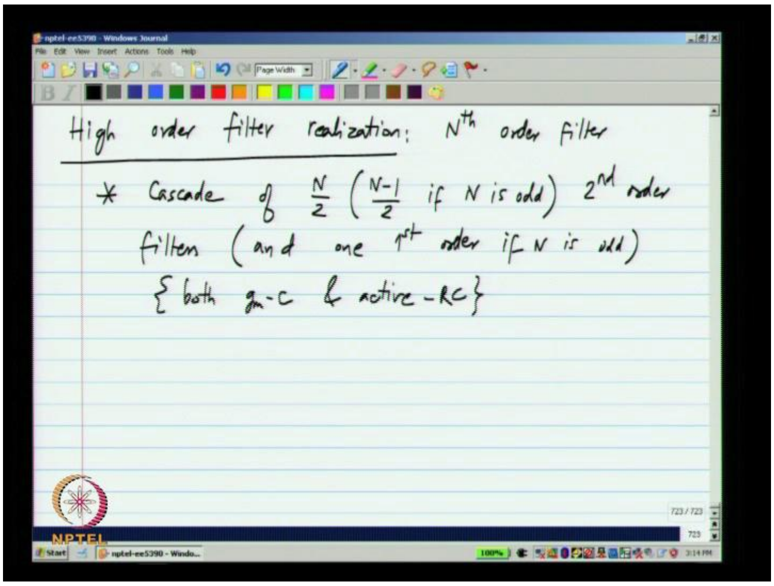

we already discussed and come up with the high order filter realization. For nth order filter we cascade n by two or n minus 1 by 2, if n is odd 2nd-order filters and 1 1st-order if n is odd and this is independent of other the realization is active r c or g m c. Now when you cascade it that is number of ways of cascading that is if you factoring into 2nd-order factors you can identify the natural frequency, and quality factor of each of the 2nd-order section now the order in which you cascade this is immaterial in the ideal case. So, any which way multiplied you will get the same polynomial, but in practice a certain types gives more noise and more distortion and. So, on so there is preferred order for cascading of these filters which tries to maximize the dynamic range now there are details optimization which try to do this, but what I am going to give is only some heuristic guideline.

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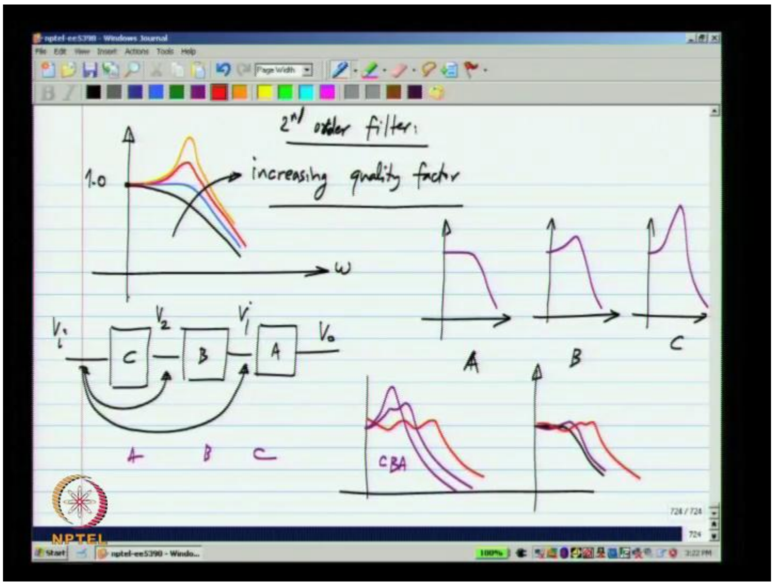

If you examine the magnitude responses of a second order filter what happens is that for low quality factors the response tends be like this and for a quality factor of one over square root of two it does that and for higher quality factors. It tends to do that and let normalizes the d c gain to one and this would be increasing quality factor now what is this mean for this lets say the orange case, if you apply a signal at this frequency the output voltage will be of much amplitude than the input voltage. Now what is this mean in term that it means that it reduces the swing limit at that particular frequency now we’re talking about filters, which has circuit which are sensitive to frequency it as different transfer function from the input to output as well as from the input to different points inside the circuit as you vary the frequency.

So what happens is the input swing limit is not constant overall frequency, but will depend on the actual frequency of input, because we can imagine the simple case you realize the active r c structure using the ideal op-amp which as some swing limit, which is equal to the power supply and that is the same for every op-amp now let us say from the input voltages to the output of a every op-amp the peaks of the transfer function are different. So, in one case it may the peak may be at zero d b itself and in other case it could be ten d b and so on. Now, what happens is let us say you apply in input and you go on changing its frequency and at some particular frequency that amplitude of the output voltage at a particular op-amp output peaks.

Now you go on increasing the amplitude at some point that peak will hit saturation, so in that case the filter is not usable any more. So, if any of the op-amp reaches saturation or any of the trans-conductor g m c filter reaches there swing limit clearly the system non-linear and you cannot use this filters any more. So, if you have taking at a particular frequency a certain output node in case of a trans-conductor or a certain op-amp output in case of active r c filter will reach the maximum first and that will limit the input swing what would like to do far maximizing the dynamic range is to maximize the input swing that can be applied. So, the trick is that let us say we have three sections we have we are talking about 6th order filter let us say and we have three 2nd-order sections a b and c let us say they has a low quality factor b has the higher quality factor and c as the anyone higher quality factor. So, if you cascade these as let us say c b a and examine the transfer function from v I v 2 v 1 and v 0.

What happens is that that transfer function from the input to v 2 will have a very strong peak remember your cascade them any order you will get the same transfer function perhaps that looks like this one what from v i to v 2 you will have a strong peak perhaps like that and v i to v 1 it will be the product of this 1 and that 1, which may have something at that sort and v i to v o is the cascade of all 3, which will be the red one. So, these corresponds to order c b a, and if you reverse it let us say you do a b and c what happens is when i will plotted down this particular axis the way i drawn it a does not have peaking at all. So, from v i to v 2 it looks like that and v i to v 1 will be the product of a and b, which may have a mile peaking and a b c will be the cascade of all 3 which is the desire filter.

Now this responses are not to scale, but the point I am trying to bring out here is that, if you put the lowest quality factor filter 1st then they will be less speaking in the transfer function from the input to different points in the circuit let us, because the first is does not have strong peak and the product to all of them is the same regardless of the order. So, if you put the lowest q value first and the lowest bandwidth one first what happens is even if the later stages is have peak there already attenuated by the first stage and you will not have as a large a peak from the input to different points now because the peaks in the transfer function from the input to different points are different is swing can be limited by swing limited of any of the stages you maximize the input signal that can be applied to the filter what you should try and do is equalized the magnitude responses from the input to all of the outputs.

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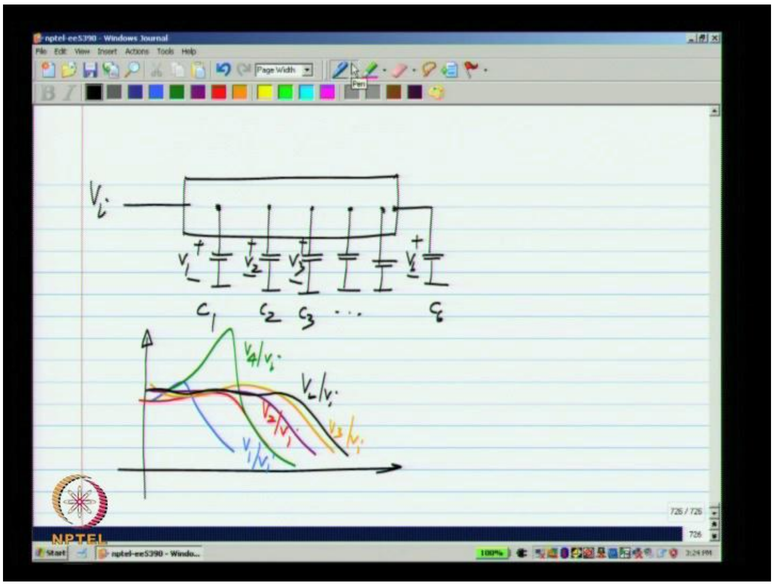

The preferred order is to put the 1st order filter first if you have it and second order sections with the lowest quality factor filter first rather an increasing order of quality factors. So, that is the preferred order this is to improve the dynamic range of the filter. Now, another aspect of this is if you examine in the filter let us say is 6th order filter and it has 6 capacitors in trans-conductance c filter or a g m c filter this 6 will be output of certain trans-conductance and you applied v i and across each of this you will have certain voltages and you can compute the transfer function from the input to each of this capacitance each of this nodes. So, now, let us say this is just some arbitrary example this is v one by v i and that is v two by v i and. So, on and that is v 3 by v i and we could have v 4 by v i like that and v 5 by v i this way let us say and finally, we have v 6 by v i these are transfer functions from the input to different nodes in the circuit as a function of frequency now if you look at this.

So, let us say you apply a given input signal and you sweep it across the frequencies the amplitude of signal at different points in the circuit will be different i meant in this particular example i will show that at a particular frequency v 4 will have much higher amplitude than any other node any other voltage v 1 to v 6. So, what happens is let us say and then you apply signal at this frequency and go on increasing is amplitude clearly the voltage across v 4 will be the largest and, because of the swing limits of active circuit connected to this node they will also reach the swing limit the fastest.

So, the swing limits are limit by this particular voltage v 4 whereas, all the others are operating well within the swing limits now this is the inefficient situation and for maximizing the dynamic range it would be better, if the maximum of all the transfer function from the input to different points in the circuit at the same peak value. Now this is the frequency dependent circuit. So, they will not reach the peak at the same frequency, but it will be more optimum in the sense that you will not have case where as single output node is as very large swing. So, it limits the swing limit whereas, the other are operating well within the swing limit and therefore, inefficiently.

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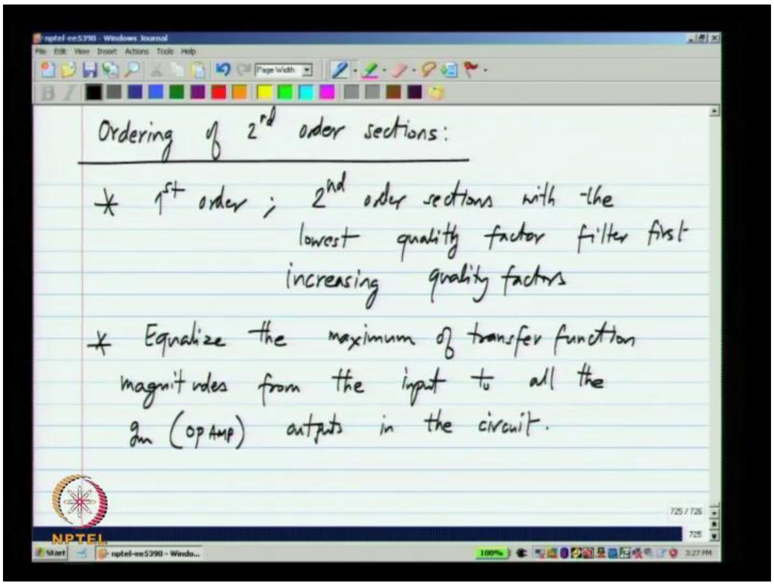

What this means is the following that transfer function from the input to output will be fixed that is given by the filter that you want to implement now depending on which way you do the altering and. So, on you will have different transfer function between the input and different points in the circuit. Now the peak values of all those transfer function should be identical. So, that the swing limit not limited by any one particular output, but by all of them as we sweep across frequency now you just the start of with the prototype and designed the filter you will find that this will not be the case the sweep will be very different at different outputs. So, what you have to be able to do is manacle the circuit. So, that the peak value at particular output whether it is a g m is output or op-amp output is changed, but the overall transfer function is not change now in turns out its very easy to and I will show how for the case g m c filter and exactly the same can be done for active r c filter.

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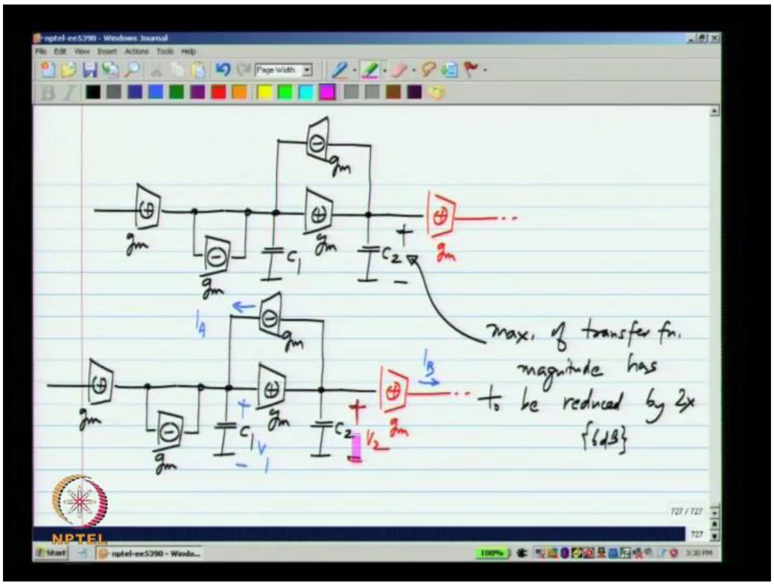

What I will do is I will just show is single 2nd-order section which assume is part of a higher order filter let me assume all the trans-conductor value are identical just to start off with and let us say that the voltage across this c two I will also show the following section so, but I have that also as a value g m and so, on let us say what I want to do is here maximum of the transfer function magnitude has to be reduced by factor of 2, which is 6 d b let us say it is 6 d b too much compared to what would you like to have. So, you would like to reduced by 2 x now none of the other transfer function change that is from the input to any other point are certainly the output now way to ensure that is that basically we recognized the only the voltage here has to be change the current and voltages everywhere else remain exactly the same right. So, than let me just copy this over. So, let me call this v 1 and this 1 sum i a arbitrary and this is some i b and. So, on and i will call this here v 2 now compare to the original circuit only v 2 must change.

Now, the other thing like v 1 i a etcetera etcetera must be and change now how will be able to scale the value of v 2 v 2 is nothing, but the voltage across the capacitors, which is the current flow into capacitors divided by the admittance of the capacitor. Now the current flow into this capacitor will not change right that is because the voltages at every other node as remain the same. So, in this particular case that is not going to change. So, we must do to scale the voltage here by a factor of 2 is scale down the current by a factor of 2 in this capacitor. Now, because the voltages scale down by factor of 2 there are trans-conductor. So, the inputs are connected to this nodes under current will reduce by factor of 2 and that should not happen because we should not change any other transfer function. So, all those trans-conductor values must be double.

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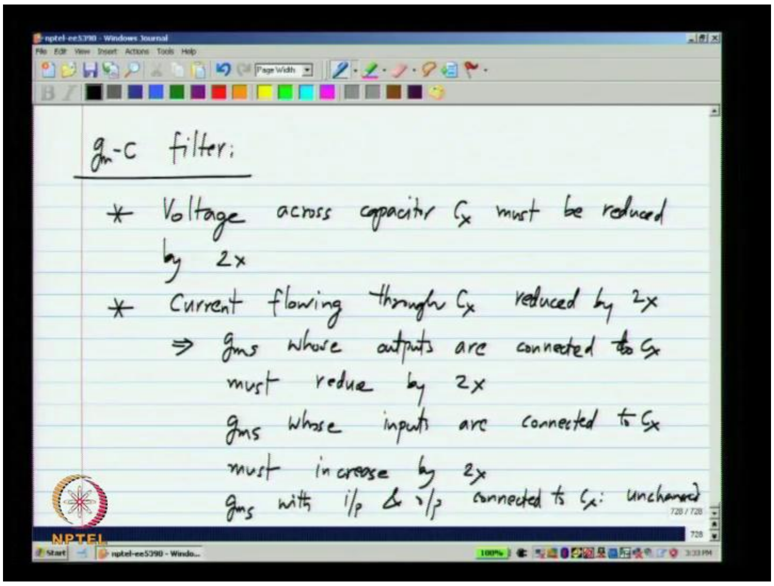

So, let us say there is a capacitor c x and it has to be reduce by factor of 2. So, what can be done current flowing through c x reduced by a factor of 2. So, which means that the g m stages that trans-conductors whose outputs are connected to c x must reduced by 2 x and we would not change anything else. So, g m s whose inputs are connected to c x must increase by two x and if you have g m whose input and output and both connected to c x value should not change this is one way of scaling it.

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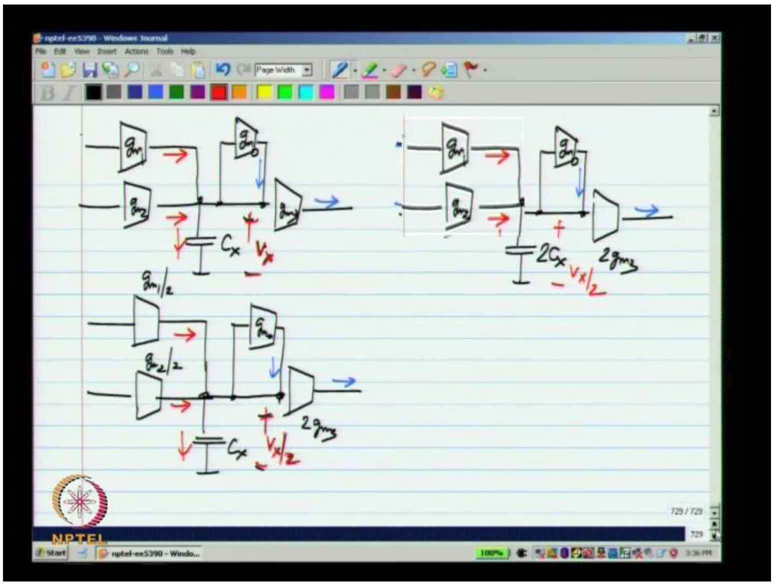

That is we have c x g m one g m one two lets g m three going like that and I have v x over there lets I have something like this where g m is connected up on itself and I call this g m 0 for change these 2 g m 1 by 2 g m 2 by 2 and 2. Times g m three and this remains at g m 0 what happens is if originally we have v x over there now they will be v x by two and all the current will be unchanged this one will show in blue this thing will be unchanged they will be the same in two circuits the only thing that would have changed is the voltage across this and also the current here which in term means the current in the capacitors. So, this is one way of reducing the node voltage by two this is known as the node scaling and on alternative way, which is quite obvious is also to change the capacitor value.

So, instead of changing the g m s what we do is to change the capacitor value alternatively let us say I have make this 2 times c x. So, this case current in the capacitor not changed, but the voltage across it as changed and again to keep the output constant we must make this 2 times g m 3. So the voltage here change is 2 x by 2 the output current does not change. So, either of these is possible.

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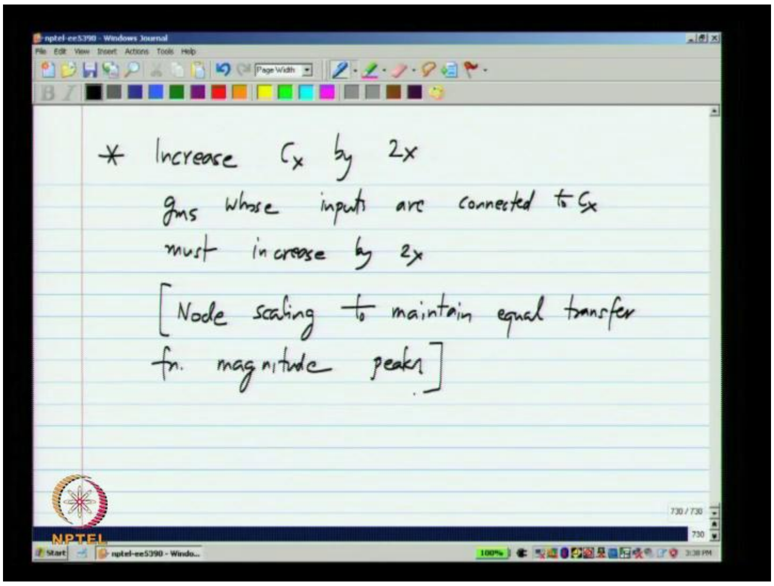

Let us say another alternative is to increase c x by factor of 2 and the remaining stuff this part is the same as before. So, these are the two possibilities by which you can do node scaling and you do this node scaling you initially similar your circuit to determine the transfer function magnitude at each note under you determine how much is node must be changed by and you scale all of g m s are capacitor, whichever you choose to do and you will be able to to make all the of peak values equal. Now instead of using cascades of biqnads, we can also realize the high order filter directly by a stimulating the question of the ladder filter, I will show it for a third order filter example with g m c filters, but exactly the same thing can be done with active r c filter as well this shows the single terminator 3rd order filter.

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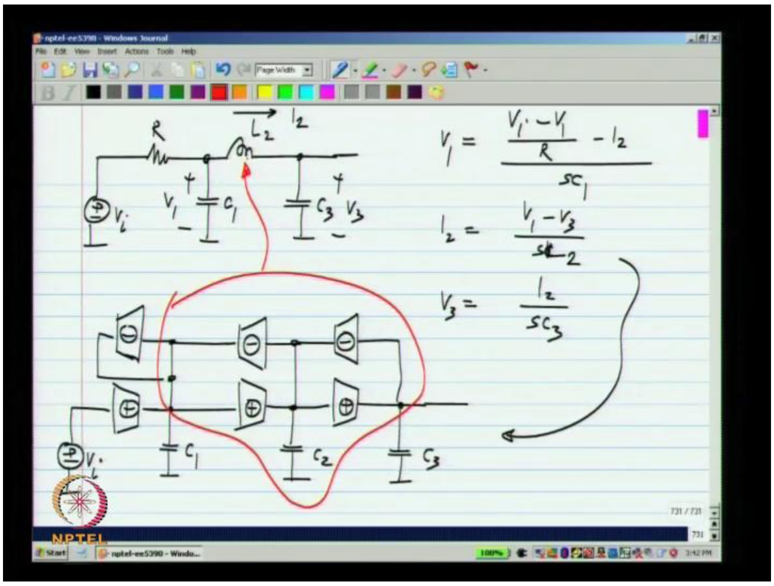

As an exercise you can write down the equations in terms of this street variable now we know that v 1 is v i minus v 1 divided by r minus i 2 that is a current flowing through the capacitor c 1 divided by c one and similarly i 2 will be v 1 minus v 3 by s l 2 and v 3 would be i 2 by s c 3 and this i 2 will be scaled to be a voltage and will get some relationships which as before will be some voltage will be equal to linear combination of voltages divided by s times something.

Basically some integration operations and this can be implemented it turns out we can verify the of yourself using a circuit of this sort this is a result of implementing these equation directly fall I did was the same as before I did not show all of this step, but the equation relating the state variables of the circuit one for the each state variable was written out as the state variable equals some voltages which are integrated and in the integration is done using g m c integrators. So, by expanding this you can realize the filter of any order and this is another way of doing it instead of cascading by codes sometime this filter are preferred over cascading biqnads.

Now in this particular realization earlier I had shown that an inductor itself and inductor l 2 like this can be realize by gyrator, which you can clearly identify over here gyrator a capacitor c 2 is the inductor l 2 you can see that are insight for g m c realization there is no different between try into realize the inductor using this trans-conductors and capacitors are doing this operations simulation of l c ladder an active r c it will look some are different, and you can take of an exercise to translate this filter into acting r c realization that is…

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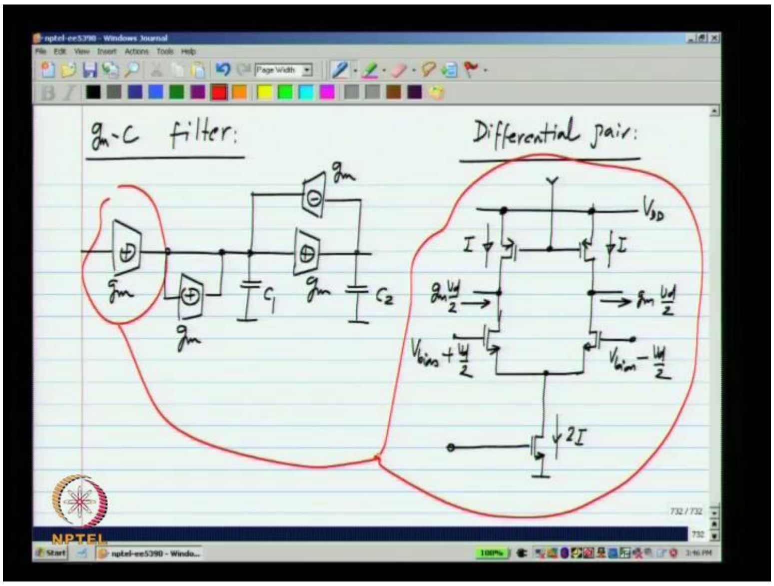

Next thing to see is an example of how to implement the trans-conductors, if you make a g m c filter g m c filter again showing your second order example. I have taken some particular example, now we would like to realize fully differential g m which taken voltage and drive out a voltage current and the simplest way to do that is using differential pair. So, let us say the tail current is to i and this currents are i and i.

And if I apply the differential voltage we know that is a current g m times v by 2 tense to flow is that differential and this would replace each of these self's this for instance could be implemented by this whole thing, but in a fully differential form now we already seen for other discussion fully differential op-amps that the common mode feedback has to be stabilized using separate loop, because the functional feedback loops will stabilize only the differential values, but the common mode feedback values have be stabilized.

So, that the circuit wires are properly in this particular case you see that we have to current source i and i injective from the top, but they have to be exactly equal to i. So, that what is that pushed out is only the differential current if you realize them independent current source the output will either drive it is drive itself to v d d are towards ground because of mismatches the only way to make them exactly equal to i is by driving there gates with a common mode feedback circuit now if you observed let us say this note which correspond to pair of differential notes in the circuit he has three trans conductor whose outputs are connected to it this one this one and that one and each of them will have a pair of current sources like this and there gates have to be driven by common mode feedback the common mode feedback should be such that these node voltages as stabilized certain common mode. Now is all three trans-conductor are driving the same output node. So, the measurement of common mode will made on this node are which correspond to a pair of is differential nodes over here and from that information we drive the common mode feedback all of the trans-conductor.

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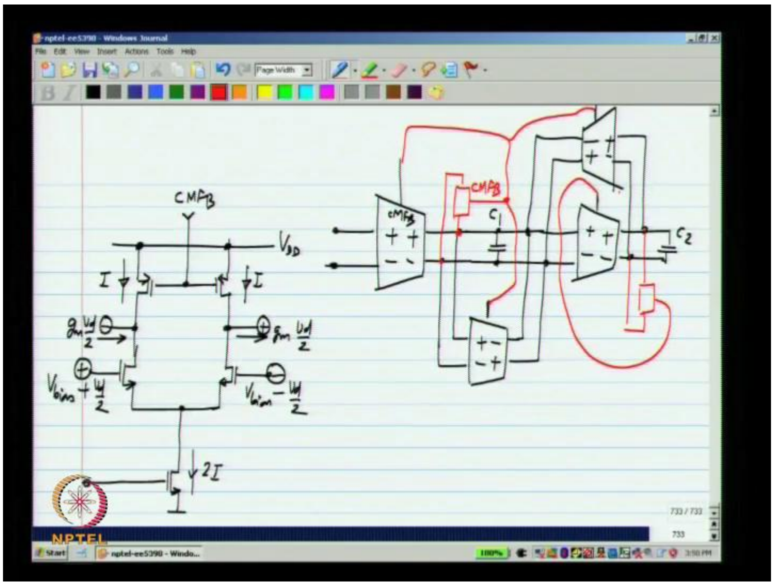

So, what I mean by that is let me copy over this cell, I will show it like this there are this is the plus input that is the minus input that is the plus output minus output for this one and I will have something over here and something over here the voltage three trans-conductor are driving the same pair of output nodes. So, what we do is will show the common mode feedback circuit in ah schematic way and you can you similar common mode feedback circuit as you did in the fully differential of op-amp and the output of this will derive all of them together that will stabilize the common mode of this pair of nodes and similarly here we have only a single trans-conductor and that will be driven by common mode feedback which measures the common mode of these 2. So, if you have a bunch of trans-conductors. So, the outputs are connected to the same pair of nodes all of them will have the same common mode feedback derive and this makes sense, because you have only one common mode feedback measurement and other way to think about it is that these are trans-conductors which is some current sources on top and all this current sources are connected up in parallel that is all that. So, if you have a fully differential g m c filter you will need a common mode feedback for each pair of nodes that is for e state variable.

In case of an active r c filter is assume that the op-amp have been designed along with the common mode feedback circuits. So, what we have been discussing is a very brief view of continuous time filters there is lot of literature on filters with lots of different topologies and. So, on what I will give here is only a very, very quick overview and you can consent the literature for further detail we have studied the differential trans-conductor in some detail while studying op-amp.

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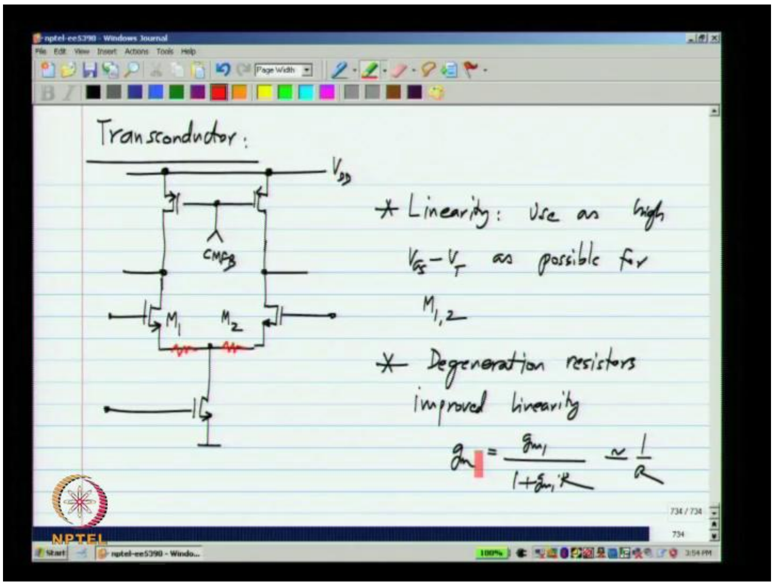

So, let us say this is what we have now we would like trans-conductor to be linear as linear as possible and we know that that is related to the v g s minus v t, the gate overdrive of transistor m 1 and m 2. So, if you want a i linearity use as high v g s minus v t as possible for m 1 and m 2 when you designed the trans-conductor you make sure that you choose the sizes of m 1 and m 2 and the current. So, that there overdrive is larger as possible even the constraints of your power supply now another possibility is to use degeneration resistors and again this is a topic that has been widely studied in literature that is a number of ah topologies proposed for making trans-conductors more linear, but we know that if you have this this acts like negative feedback and you will end up with greater linearity, if you have degeneration resistors than the linearity will be improved by the g m will be reduced actual g m is the gm of the transistor m 1 divided by 1 plus gm 1 times r and approximately 1 by r if gm.

One time r is very large am in the low voltage supplies that we have today it is very difficult to make gm 1 r very large, but you try to make it as large as possible now this is 1 area where the active r c topology wins over g m c topologies the active r c topology is a circuit involving op-amp in feedback and is inherently is more linear compared to the trans-conductor which are operated in op-amp.

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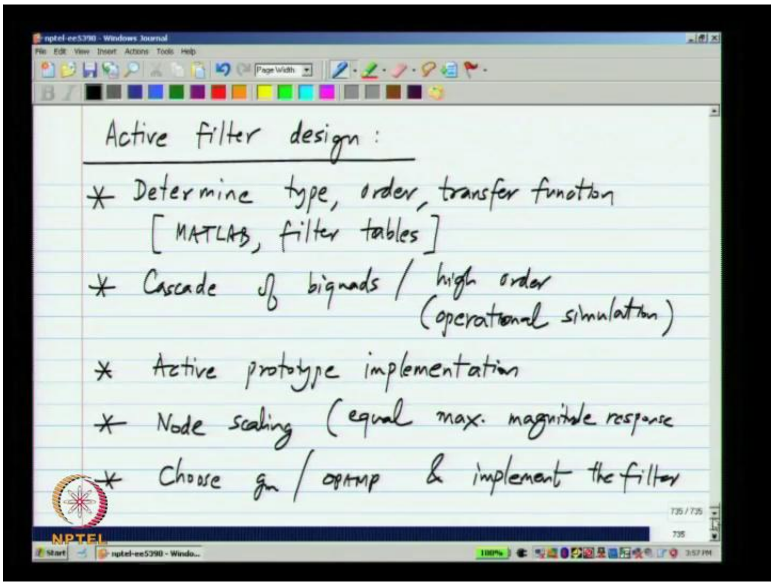

So, as a very quick summary active design filter involves the following steps, determine the type order transfer function and may be choose the prototype et cetera and this is usually done using a tool like mat lab or you could also use filter tables there are published tables of filter types and so, on and then you choose either cascade of biqnads or high order realization that this operational stimulation of a high order filter.

then you implement it implement the active prototype this implementation of active proto-type in some terms of g m c s then you do node scaling for equal maximum magnitude response then implement this with either g m's or op-amp you choose your gm or op-amp and implement the filter then of course, you stimulate the noise distortion et cetera if the noise is too high or too low for your requirements you can do impedance scaling which have discussed in detail earlier to change the noise values without changing any of the transfer function.

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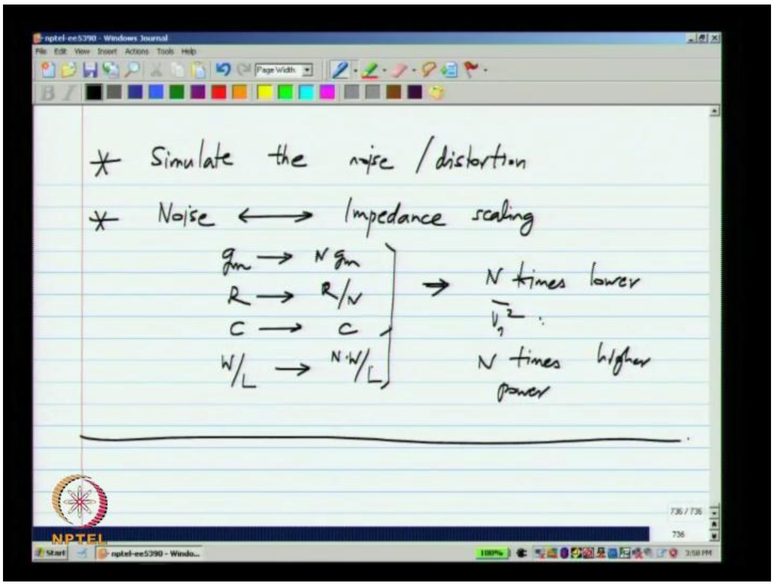

This means that all gm's will become n times gm all resistors would be r by n all capacitor is will remind exactly as they are and in case of mosfet of course, with w will become n times w all of this will imply n times lower voltage noise variance and also n times higher power. So, that is a very very quick summary of how to do active filter design we have only touched upon it very superficially, but it should give you a starting point and enable you to go through the literature and look for further detailed information and implement filters. Thank you, I will see you in the next lectur