Analog Integrated Circuit Design
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Lecture - 45
Phase Locked Loop as Frequency Multiplier

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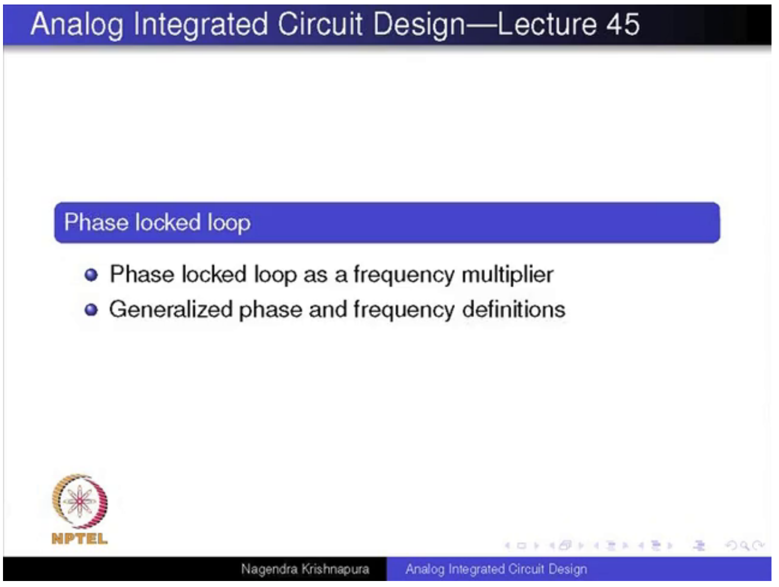

Hello, and welcome to lecture 45 of analog integrated circuit design, so far we have discussed negative feedback amplifier in great detail many kinds of feedback amplifier including fully differential amplifiers how to design them, how to similar them and swan. Now, we will move on to another kind of negative feedback block which works in a way similar to the negative feedback amplifier, but it is very different in function it is a device to multiplying frequency.

Just like an amplifier in input voltage this frequency multiplier or the phase locked loop will take a given frequency and give a multiple of that frequency at its output and this is an extremely important and useful block. It is used in all kinds of circuit including digital circuits basically it generates periodic waveforms with possibly varying frequency. Now, even digital circuit need periodic clock, so it is used over there it is used for generating carriers for a radio where you need number of carriers.

So, that you can generate the modulation over them, so in general wherever you need a frequency references you will need a source of periodic signals and this solves us the source. Now, what we will do is to synthesize this block in a manner similar to how we got the negative feedback amplifier and we will analyze it, and evaluate the short comings if there are any and then overcome them, and that will finally lead us to a very popular topology of the phase locked loop.

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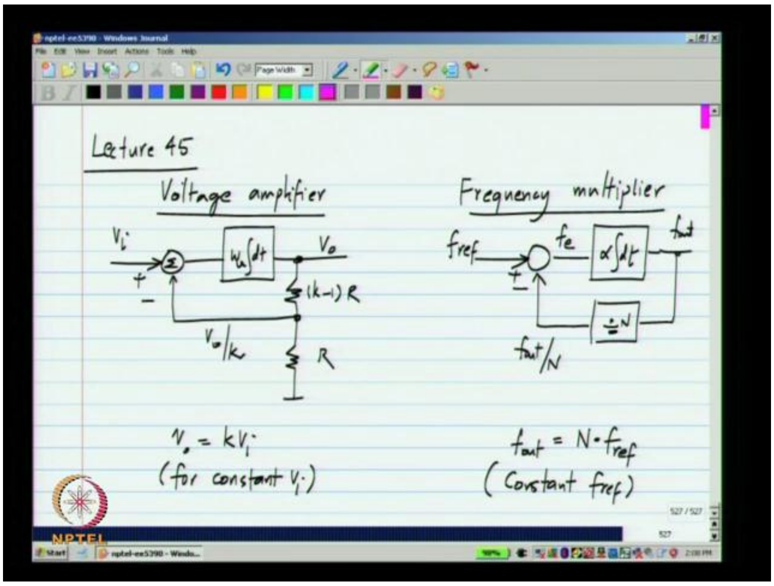

Now, let us take our prototype voltage amplifier what did we do with the input voltage compared it with a fraction of the output voltage. If this is v naught this will be some v naught by k and integrated the difference to drive the output this implements v naught equals k v i for constant inputs v i. Eventually, when the circuit reach steady state v naught by k will be equal to v i, the input to the integrator will be 0 and v naught will be k times v i. Now, let us say we need to make frequency multipliers whose output frequency will be some N times the input frequency I will call the input frequency is reference frequency because that is the common terminology that is used.

Of course, we will assume that the input frequency itself is constant, we can do it in an analog is manner let us say we know what the input frequency is. We compared that to a fraction of the output frequency that is let us say this is the output frequency and, here we get a fraction of the, a foot by N and what we should do is based on the frequency difference f e. We should integrate the difference and drive the output, so this is the frequency multiplier and it is very easy to see that when f ref is a constant steady state is reached when f e is 0, that is the input to the integrator is 0 and f out by N equals f ref.

In other words f out will be equal to N times f ref there are some details that we have to fill in, first of all in a voltage amplifier is very easy. Our electrical signals are in the forms of form of voltages or sometimes currents and we either compare the voltages or drive the output with the integral of the voltage difference or we take the difference between currents and drive. The output with the integral of the current difference those things are possible, but here again we have electrical signals which are voltages or currents and their periodic with a certain frequency.

But, what we need to compared according to the diagram that we have come up with there is a comparison of frequencies not comparison of voltages. So, the voltage will have a certain frequency and we need to be able to compare the frequencies will see how to do that shortly, but before that essentially will be dealing with periodic signals whose period can be change slowly and so on. For instance, the output frequency could be changing, now strictly speaking such a waveform is not periodic at all, but we need to be able to handle that signal. So, first we will look at definition of frequency and phase in a more general way than for constant frequency sinusoids.

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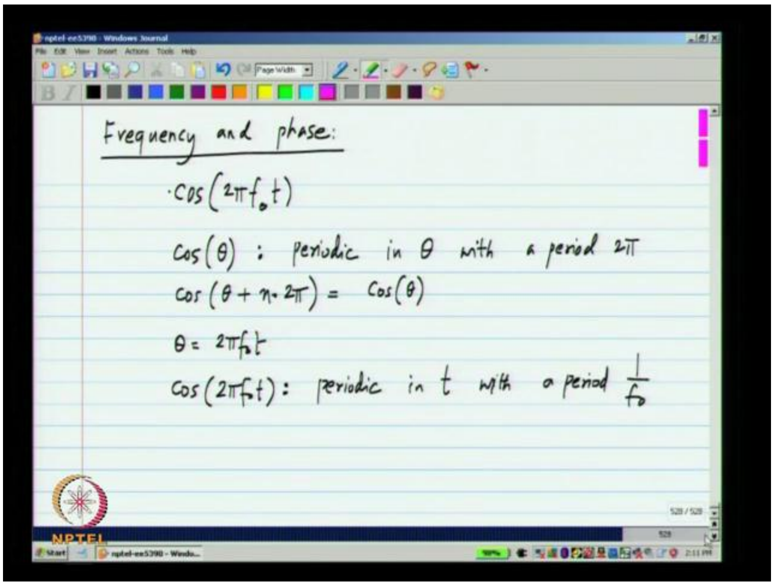

Now, let us start with the periodic signal that everybody is familiar with that we just take cos 2 pi f naught t, we know that the periodic signal and this could be some voltage with a peak value v p. Now, will break it down into the basic periodic function that is cos theta, let me remove that v p from here on which cos theta, this is a function, that is periodic in theta with a period 2 pi.

That is the cos theta plus N times 2 pi for any integer will be the same as cos theta, so this is something we all know, now if we substitute theta equals 2 pi f naught t the function that we get cos 2 pi f naught t will be periodic in t as well with a period one over f naught. So, this is a signal that is periodic in t, so what we will do is we will consider this form of the signal where which is by definition periodic in theta. Then define theta as a function of time this looks like a round way of doing things, but it is very useful as we will see.

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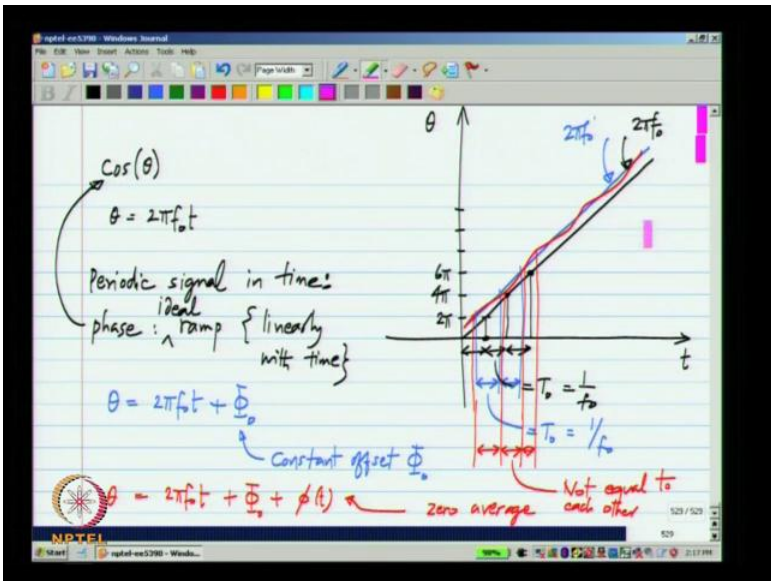

So, let me now plot this state of a t when theta equals 2 pi f naught t, obviously this describes ramp whose slope is given by 2 pi f naught. Now, the function cos theta is periodic with the period 2 pi, so on the y axis I will mark integer multiple of 2 pi and so on. Then I will look at the times at which the ramp process these thresholds crosses 2 pi, 4 pi, 6 pi etcetera because this ramp of course is a straight line these intervals will be equal to each other. There will be equal to t naught which is the reciprocal of f naught, so this is pretty clear, so essentially we can define periodic signal in time as signal that is periodic in time we say that it has a phase that is a ramp that is an ideal ramp.

The phase changes linearly with time and this phase is the phase of a periodic functions such as cos theta, so the resulting function will be periodic in time. Now, we can see what happens if we deviate from this, first of all let me take a simple modification I will add a constant of set phi naught. So, what happens in that case I will simply get a ramp that is shifted by phi naught the slope is still the same as before it is 2 pi f naught, here I am drawing the new phase theta with refer to time.

Then I will look at the points at which the times at which the new ramp crosses 2 pi, let us clear that these intervals are also equal to each other and also equal to t naught which is one over f naught. So, a constant phase of set does not take away the periodicity of the signal and this is pretty obvious as well because we know that instead of cos 2 pi f naught t, if we have sine 2 pi f naught t. That is like adding a phase of minus 90 degrees and that is periodic as well next let us take at different case where I have this of set as well as some part that varies with time and I will assume for now that this as a 0 average.

That is any average value is captured in this constant of set and this is phi of t itself as a 0 average in the what happens of course it depends on phi of t. But, let me plot it for some arbitrary case that I will consider, so let us say deviates like that, so you have the ramp plus the of set plus also time varying quantity. Now, if you, now look at the time instant at which the new phase crosses integer multiples of 2 pi, so let me pull out these things than, here it is clear now that these intervals are not necessarily equal to each other. So, depending on their type of variation and phi of t, you will have a certain variation in the period of the signal.

So, basically this representation is very useful to describe signals which are deviating from ideal periodicity ideally the phrase of the signal should be a ramp. That is if you have the phase to be an ideal ramp a straight line with time than the resulting signal will be periodic in time. Now, you have the phase of a periodic function, but the phase is not exactly a ramp let us say it is a ramp plus some time varying quantity then the successive periods of this function will not be equal to each other it is a periodic.

It turns out that anything that you generate will not be exactly periodic there will be all kinds of deviation from ideal periodicity be this could be due to systematic variations. It could be due to random variations such as we encounter of set and noise in voltage and current domes we can have this kind of deviation in the phase domain. This becomes extremely important in the generation of periodic signals. And also in the analysis of systems with generates these signals so that is why we discuss this.

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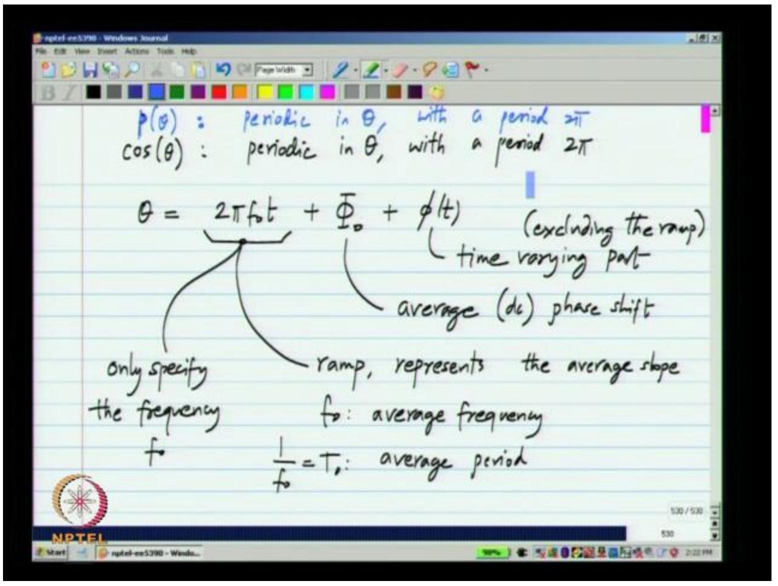

So, bottom line is that start with the periodic function is periodic in theta with a period 2 pi and to describe a signal that is periodic or almost periodic in time, what we do is we represent the theta as consisting of 3 parts. In general this is the ramp and it represents the average slope that is these two parts will have a slope of 0 and this part represents the average slope. So, this f naught is really the average frequency because the signal is not exactly periodic and the average period is t naught and there will be deviation from the anti average deviation is captured in this phi naught.

Finally, the time varying part of the phase excluding the ramp, the ramp itself is time varying, but we know that a periodic signal as a phase which is a ramp. So, that is excluded from this excluding that all of the time varying parts are included in this phi of t, so to specify a signal that is periodic in time with some deviations. All we have to specify or these numbers in fact for the first number will only have to specify the frequency f naught. So, you have 3 parts with one in the frequency f naught, the other one is the average phase shift phi naught and the rest is the time varying phase shift phi of p.

So, of these f naught and phi naught of trivial and many times you end of working only with this time varying part with is phi of t and by knowing the nature are phi of t. We can also reduce the deviations from periodic city again the reason, we do use this representation is we are on the interested in if the signal is periodic or not. Now, I started with cos of theta, but it does not have to be a cosine function it can be any other function, many times you could have a square wave or some other wave shape.

The shape is not what is important, what is important in the, in these systems we generate frequencies is whether the signal is periodic or not, so instead of cos theta, we would have any other function p of theta which is also periodic in theta with a period 2 pi. So, then only the wave shape will be change, but the periodic city will be the same as long as this theta values we plug in here are the same just to give you an example.

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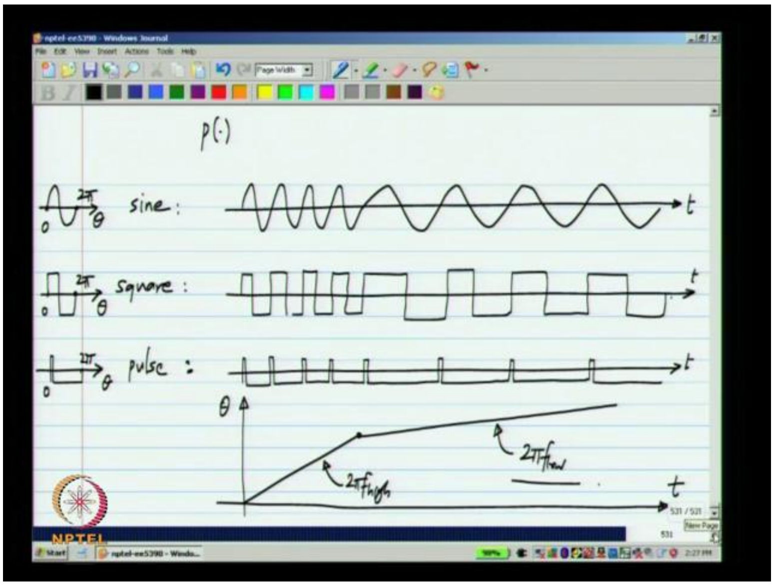

Let me draw to a forms, here in this particular case I have drawn 4 periods of the sine wave with high frequency and 4 more periods with a lower frequency. This is a sinusoid and I could have a square wave which behaves exactly in the same wave that is the frequency shift to a lower value after 4 sin. It would have a pulse line whose frequency again where is in a similar fashion it shift from a higher frequency to a lower frequency and all these cases it is only the basic shape that as changed. So, this is by the way plotted versus time and all these cases the basic shape with the respect to theta is what is different.

So, the first one is the sinusoidal shape, the second one as the square shape and third one as some narrow pulse shape and in this with respect to theta all of them have a period of 2 pi what is different is this function p that is periodic. Now, there may be applications in fact there are applications where the shape of the waveform is important, but it is not important for the generating these periodic signals. We know that we can turn a square wave into a sine wave by putting in through a filter or turn a sine wave into a square wave by putting it through again amplifier which as some limiting.

So, we will not, now worry about the waveforms and most of the discussion in the generation of periodic signals that is for the phase locked loop will be in the phase domain. This would have a commonly seen if you refer to any textbooks the main reason is that we are only interested in whether the signals are periodic or not and not really what the shape of the signal is.

So, in all these cases if you plot the plot this theta versus time what happens is up to this point it as a higher slope and after that it as a lower slope what I mean to draw here is a constant frequency and a lower constant frequency. So, that means that phase is ramping up to this point and again after that it also ramps up, but with a much lower slope so this is the general definition of phase of a signal.

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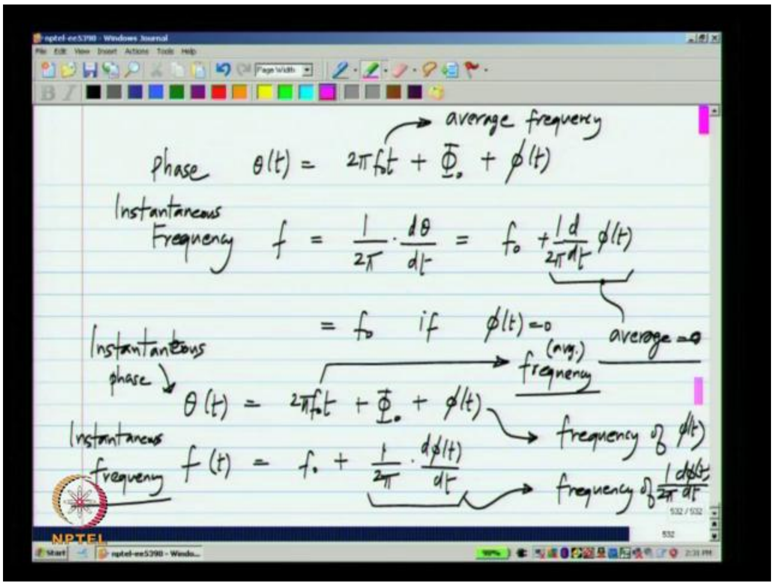

There is also, from this we can define the generalized frequency this by the way I emphasize is the average frequency and the frequency that I am going to calculate is known as the instantaneous frequency f. This is given by 1 over 2 pi time derivative of theta which we can see f naught plus time derivative of phi of p, now if the time varying part is 0, if phi of t is 0 the instantaneous frequency equals f naught. Now, this makes sense what that is saying is if the signal is exactly periodic possibly with some extra constant phase shift average frequency will be the same as the instantaneous frequency.

If the phase as time varying component in it, then the instantaneous frequency will be defined from the average frequency and they are frequency will be equal to f naught. So, clearly this also will have an average value of 0, so earlier I said that anything that is ramping up is captured, here the constant of set is captured here and the time varying part with 0 average is captured here. Similarly, the average frequency is captured in the form and the time variations on top of that are captured in that with an average of 0.

So, it is an important to get familiar and comfortable with this you would have come across this in communication systems if you have taken such courses when you study modulation and so on. Notion of generalized frequency is very important in frequency modulation and phase modulation and it is also important for as for the study of phase locked loops. One of the confusing things in this may be the use of the term frequency when you from anything or instants let me write out the expression for the phase again and this can be called frequency and what is meant.

Here, it is of course the average frequency and the instantaneous frequency is could also be called as frequency, so this is another thing and also because we have phi of t that is time varying. It can be varying at a certain frequency, so will also use the term frequency in connection with that, so that is use like that that is this phi of t itself could be a sinusoid. At some frequency that frequency is absolutely nothing to do with this f naught it could be coincidently related to f naught. But, it in general as nothing to do with it and it as it is shown frequency, similarly we will not deal with instantaneous frequency too much we will deal with this instantaneous phase.

But, it is possible to do all the analysis with the instantaneous frequency and then this time varying part of the frequency can have its own frequency again there is nothing to be confused about. There is just different signals that we are talking about as long as you understand that f naught means the average frequency this f of t means instantaneous frequency, and this time varying part can have their own frequency everything should be fine.

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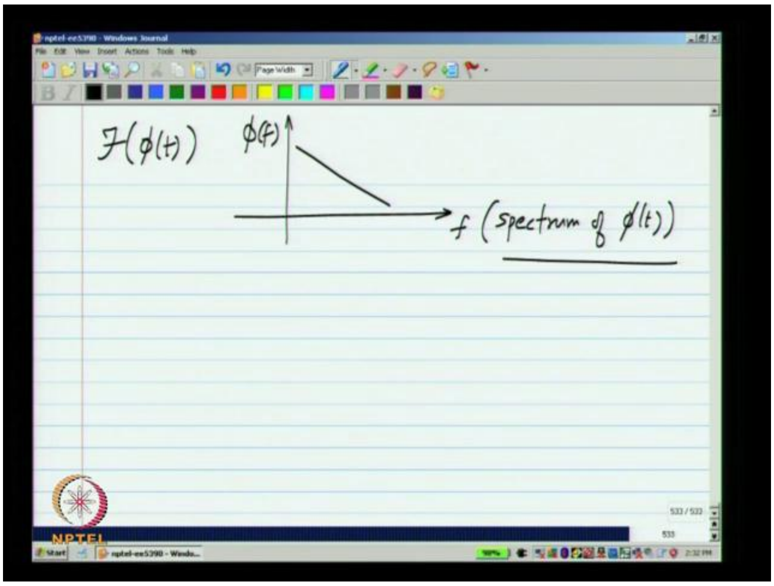

It is also quite common to sometimes take the Fourier transform of phi of t and plot it obviously on some frequency axis and it could have any kind of shape that is basically phi of f. So, this frequency axis which denotes the frequency of this phi of f that is this phi of f denotes the content of a phi of t at different frequencies, so there is it another frequency here which is basically the x axis of the spectrum of phi of t.

So, the same term frequency is use from anything just go through the definitions and be clear about exactly what frequency were talking about in what context usually it will be quite clear. It only takes a little of getting used to initially with that introduction to general notions of phase and frequency we can go head and tried to realize the frequency multiplier that I earlier described.

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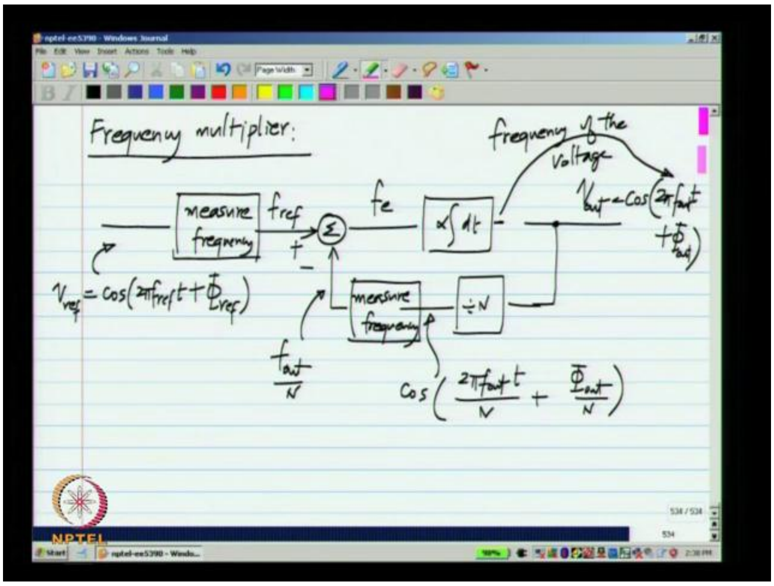

So, what do we need, here I said that I have the frequency of the input signal and frequency of the feedback signal which is 1 over N time the frequency of the output signal we need to have this integrator which will control the output frequency f out. We need to have this divider which will take the output frequency and divided by a factor of N as I mentioned earlier the input will not be a frequency. But, obviously it will be a signal of a certain frequency let me call it b ref, and I will denote the input by a sinusoids of a frequency f ref. As I emphasized earlier the waveform does not have to be a sinusoids, but it can be any periodic signal p with a period of 2 pi.

So, that is what is the applied here and we measure the frequency of that if we do that will obviously get f ref and that has to be compared to the output will be a signal at the output frequency, let me say cos 2 pi, phi of t plus some phase of set phi out. So, in general the phase of set can be different, here also I will show it as some phi ref, now what the frequency divider will do is to give a signal whose frequency is 1 over N times the output frequency.

It turns out this phase of set will also be 1 over N times the phase of the output will see quickly will see soon how this comes about and we have to measure the frequency of this one and compare the two, so at this point will be getting f out by N. So, what I am trying to describe here is that will have a signals that is voltages are currents with the certain frequency I will usually take the example of voltages. The input will be voltage with a frequency f ref, the output will be a voltage again with the frequency f out. The frequency divider will take the output voltage and generate another voltage whose frequency is f out by N.

Now, the two things we wanted to compare to compute the error the two things we wanted to take the difference of r the frequency f ref and the frequency f out by N. So, from the voltage signals v ref and a the feedback signal at the output of the frequency divider we need to measure the frequency of those signals and get f ref and f out by N. These two quantities we compare that is we take the difference and integrate the difference control the output frequency, so here I should be careful with, how I specify this I will show this box disconnected.

What it meant here is that the integrated frequency error is not the voltage it is the frequency of that voltage again this simply follows an exact analogy with the voltage amplifier. The only confusing parts can be that we are dealing with voltages which have certain frequency, but we have to do our operations on the frequencies of the signals not on the voltages themselves that is why I am doing this. So, we need to able to control the output frequency that is we need a block whose frequency can be electrically control if it can be done. Then we can drive it with the integral of the frequency difference that is driving it with this signal and control the output frequency.

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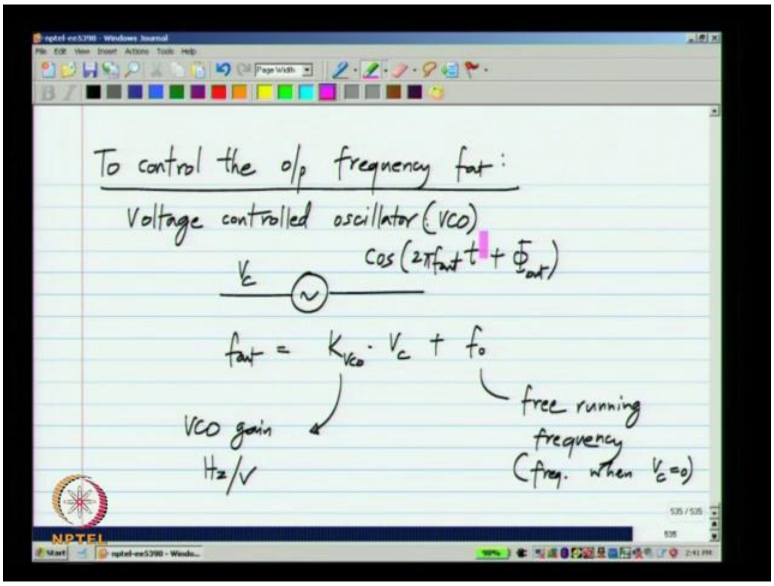

Now, it turns out that to control the output frequency f out there is a block that is available which is known as the voltage controlled oscillator. Now, you know that oscillators are autonomous blocks with generate a periodic signals, now what we are trying to do here is to control. The frequency of periodic signals using negative feedback, now there are also certain kinds of oscillators whose output frequency will depend on some electrical variables such as voltage or current.

So, you could call them voltage controlled oscillators and current controlled oscillators i will call them voltage controlled oscillators are V C Os, so what it does is we receive an input voltage V c and give a periodic signal. So, that is as usual I will denote with a cosine at a frequency f out and there will be some arbitrary phase shift phi out, now how is this f out related to V c in general it will be something non linear. But, we can very usefully model it as f out being some constant of proportionality times V c plus some f naught that is some of set frequency, and this is known as the free running frequency.

This is nothing but the frequency at which the oscillator will oscillate if V c is 0 that is pretty obvious from this expression V c is 0, f out will be equal to f naught. This f V C O is the gain of the V C O is voltage controlled oscillator is usually obviate at V C O, and this is V C O again is measured in volts per volts. That is for unit change in the controlled voltage of much does the output change output frequency change. So, that is given by this, so this voltage controlled oscillator can be used as a block that electrically controlled the output frequency.

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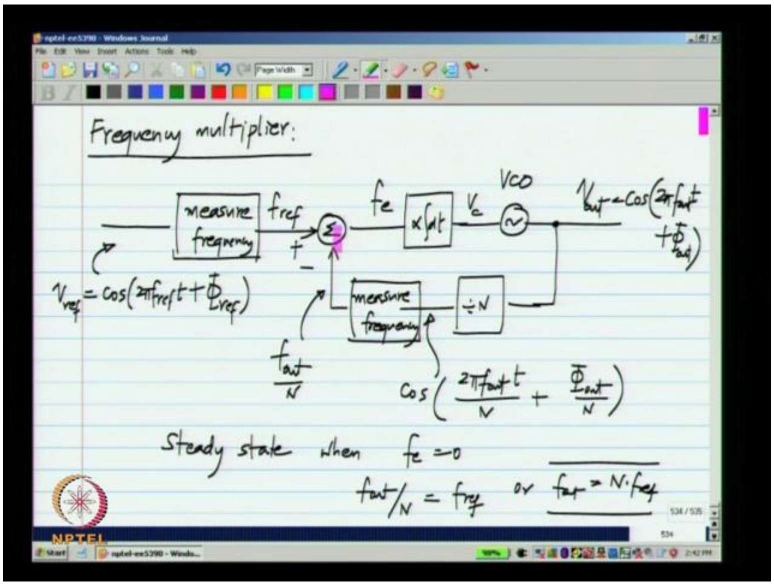

So, what we do, now we can generate the output using a voltage controlled oscillator and the control voltage oscillators must be related to the integral of the frequency different. Once again, if you work through this what happens is the system will read the steady state when the input to the integrity 0, then V c will be constant where V c is a constant the output frequency will be some constant. So, we will have constant frequency everywhere and steady state is reached under the condition f v equals 0 that means that f out by N equals or f out equal N times f.

So, all we have done is split the forwarded part into two parts, one which is tool for controlling the output frequency electrically. The other is when which is to avoid this control is the integral of the error between the input and feedback frequencies, so this is the model of the V C O, now one important thing here is that.

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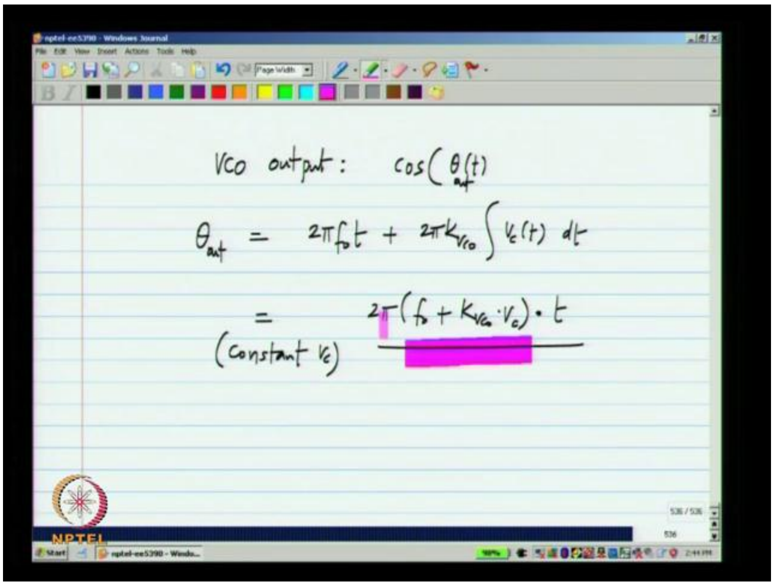

Again, if we see output can be written as cos or any other periodic function which is period 2 pi theta t, let me call theta V C O or theta out of t were theta out is given by 2 pi f naught t plus 2 pi K V C O integral V c of t. This will be if V c happens be a constant, 2 pi f naught plus K V C O times V c times t, what I am trying to emphasize here is that if V c if time varying you cannot use the second relationship. It is not f naught plus K V C O, V C O of t times t it is integral of V c of t d t, now is V c happens to be a constant. Then simply the frequency shifted by K V C O time V c and if V c is 0 then we will have the output frequency to be the free running frequency f naught.

So, this is the phase model of the V C O which will be using continuously from here onwards after a while after we complete the design of this block, we will model everything in terms of the faces of signals. This is the model we have to use for the V C O, now this block diagram is fine except we do not get, now how to do this measurement of equity and so on. We will quickly see out to do that this V C O, it can be made you may have heard of the world V C O. Later in this course, we will see some examples how to make the voltage controlled oscillators, so will assume that this can be made and how do we make this frequency divider.

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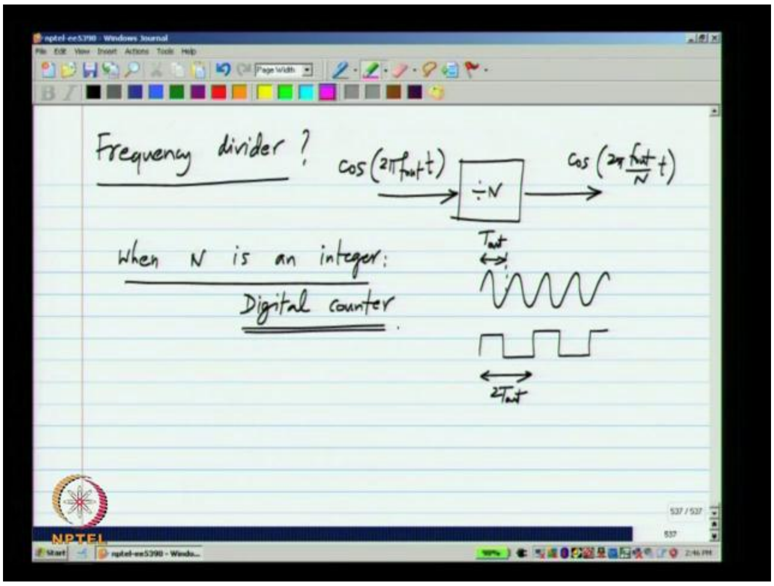

That is if you feed in a voltage whose frequency is left out then the output must have a frequency of f out by N, now there may be many ways of doing this. But, the most convenient when N is an integer is the use a digital counter, a digital counter you know the divide by 2 counter. If the input is like that, the output will be 1 for 1 cycle and 0 for the next cycle, 1 for 1 cycle and 0 for the next cycle and so on. Clearly you see that the output as of the frequency or the twice the time period of the frequency of the input waveform now we will only be considering the case when this N is an integer.

So, this frequency divider will be realized using digital counters, now the one other thing is here I showed cos that is sinusoidal wave forms at the input and output of a divider. You may be wondering how the digital cycle, it will operate with such a wave forms as I said I will keep showing functions as cosine, just to show that I have to show something. But, it can be any other waveform, any other wave shape which is this same period of 2 pi, so obviously if we have a digital counter most of the time. It operates with waveforms that are more like a square wave than like a sine wave, but without loss of generality I can write cosine everywhere.

So, now we know how to make the frequency divider as well what is left is the rest of these blocks, let us consider this part out of it what is happening. Here, we measured the frequency of the signal we measured the frequency of the other signal when the difference between frequencies and integrate the difference will get the control voltage.

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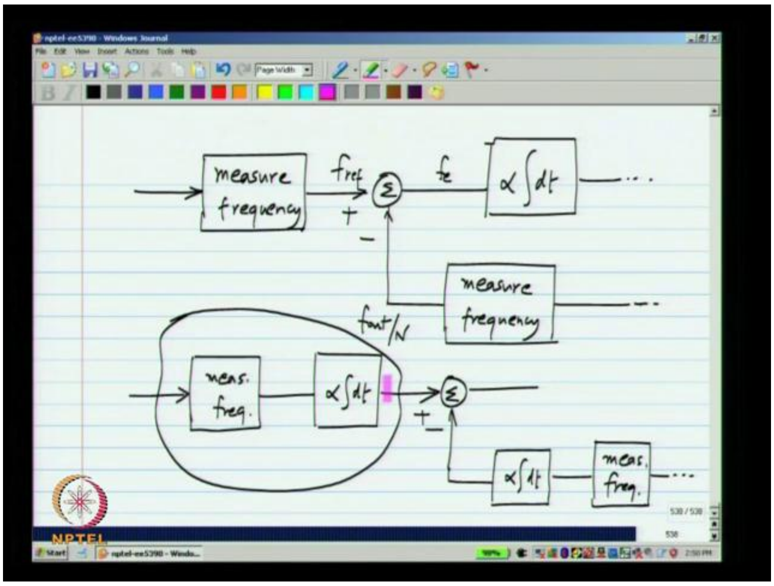

We integrate the difference, now I can equivalent write this as measurement frequency and I can pull this integral into both of these branches this branch and that one a little the integral and both of those paths. Now, what we are doing is taking the input signal measuring the frequency integrating it taking the feedback signal from the frequency dividers measuring its frequency and integrating it and taking the difference. What is this entire block doing, it is taking the frequency and integrating it which is the measuring the phase of the input because what is phase after all.

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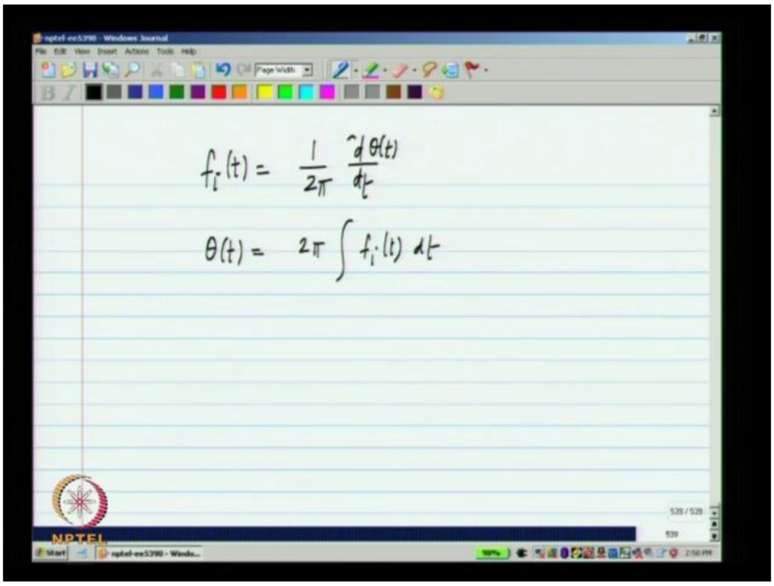

I said the instantaneous frequency is 1 over 2 pi times the time derivative of phase, if I invert this I will get phase to be 2 pi times integral of the instantaneous frequency.

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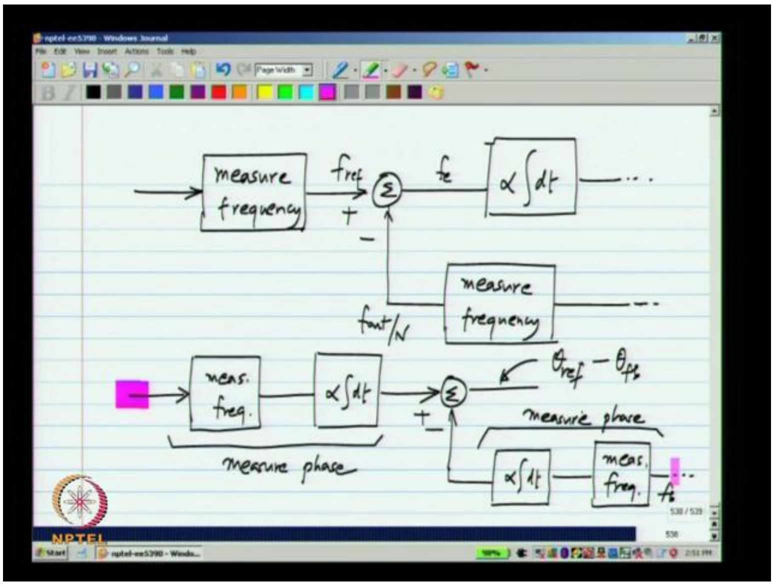

So, all this is doing is to measure the phase of the input signal this is measuring the phase of the feedback signal output of the counter. So, the quantity that we get here is nothing but the difference between the phase of the reference signal and the phase of the feedback signal I will denote the signal by the subscript of that. So, what really need to do is we do not have to measure the frequency and so on we do not yet know how to do that. But, what you need to do is to simply measure the phase difference between these two waveforms, that is measure the phase difference between the reference signal or the input signal and the output of the frequency dividers, so what do we get in that case.

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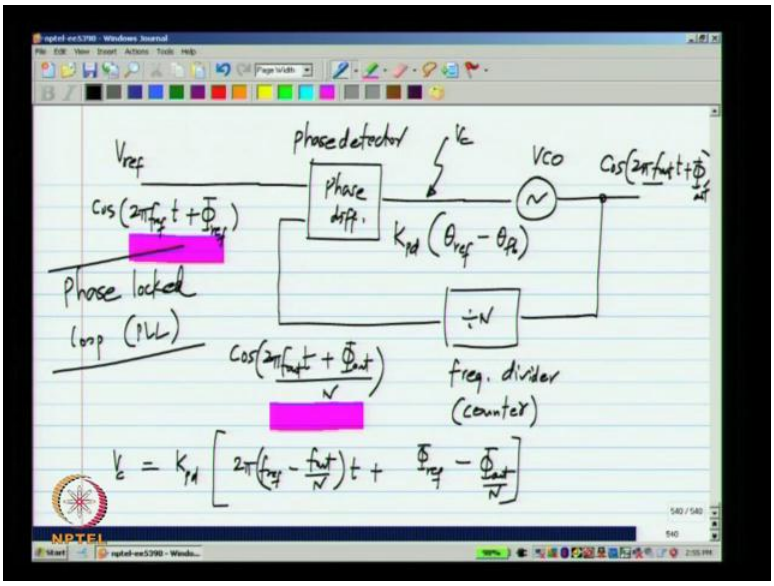

We need some block that measures the phase difference between these two and this is nothing but a controlled voltage V c of the voltage control oscillator because we have combined this entire thing. Measuring frequency and integrating the difference, which was giving as the control voltage V c into this single block which measures the phase difference and is it is measuring the phase difference between the outputs of the frequency divider. But, the output we would have was to phi f out p plus pi out and, here we would have cos 2 pi f out p plus phi out the whole thing divided by N.

So, this the frequency divider for the counter and here this we see related to the phase difference between these two signal which is basically a constant of the phase deductor K P d times the phase difference. Here, theta ref is this much and theta f e is that much and for the system to be this steady state, this ramp part has to be 0 it is f ref has to be equal to f out by N. This we had come up with using a different kind of reasoning in fact this is the objective of design to make f out equal to N times f ref, so this part will be 0 and this part will be something that is related to V c and some parameters of the circuit.

So, this entire block with this phase deductor the block that take the phase difference is known as the phase deductor a voltage controlled oscillator and a frequency divider. This is known as a phase locked loop or P L L, very widely used block and by changing N, you can also change the output frequency from the same input. In fact, that is mod in which the phase lock is used very often it as many other uses one of the main uses is for what is known as frequency synthesizers.

By changing the value of N, the output frequency will be N times f ruff N times reference to frequency, so by changing the value of N, the output frequency can be changed. You know that you can make digital counter with programmable division modules, so by programming the modules you can change the output frequency. So, that is the basic phase lock loop in the next few class will lock at exactly how it behaves, how to realize the a different blocks of for this phase lock loop and so on, thank you I will see you in the next lecture.

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