Analog Integrated Circuit Design
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Lecture No - 4
Loop Gain and Unity Loop Gain Frequency: Opamp

In the previous lecture, we have seen, we have analyzed the negative feedback amplifier in the frequency domain and we have obtained the magnitude and phase plots of the transfer function. The purpose of this goes to evaluate what happens to the output of the negative feedback amplifier when you drive the amplifier with the sinusoidal. So, this is the magnitude response and this is the phase response. The magnitude is close to the idle value of k for frequency as well below this pole value omega u divided by k, and well beyond omega u by k, it goes of it 20 db per decade.

Now, at the frequency omega u by k, you can see that when omega equals omega u by k, you get k divided by square root of 2. So, they divide it by square root of 2, basically 3e db below k. So, at omega u by k, it will be k divided by square root of 2 or 3 db below k. So, the actual magnitude response will do something like this, and this omega u by k is known as the 3 db bandwidth of the system. So, for a first order system, the bandwidth equals the pole frequency and the magnitude response of this amplifier at the pole frequency equals k divided by square root of 2. The k is the dc gain or the gain at low frequencies.

Now, when you are drawing bow tie plots, we usually do not make such distinctions between 0 db and 3 db attenuation and we showed by straight lines which are asymptotic toward. So, a bow tie plot, you do not necessarily have to show this 3 db, but you have to recognize that at the pole frequency, there is the 3 db reduction in the gain compared at the low frequencies. As for phase at low frequencies, the phase lag is small and then, it increases to 5 by 4 at omega u by k and it increases to minus 5 by 2. The phase lag increases to 5 by 2 at very high frequencies. So, we will see what happens to the sinusoidal when you pass it through this type of amplifier.

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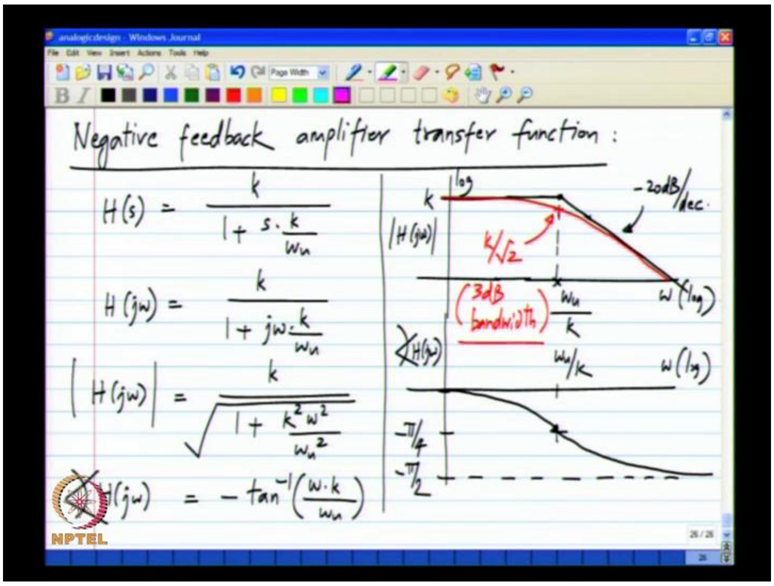

So, we know that if the transfer function of the system is H of x and you apply an input, let us see v p cos omega t to such a system, so the output will be also be sinusoidal, and we know what sinusoidal is going to be with a magnitude H of j omega or times v p. The amplitude of sinusoidal gets multiplied by the magnitude of the transfer function at the frequency of the sinusoidal, and you get an added phase which is the phase of the transfer function at the frequency of the sinusoidal. We will examine this for our particular case.

So, if we apply an input of the V p cos omega t to our amplifier, the output will be k V p by square root of 1 plus omega square k square omega u square. This is V p multiplied by the magnitude of the transfer function and cosine of omega t minus because the angle is always negative for this particular transfer function than inverse omega k by omega u. Of course, these are the other complicated expression, but we have to make sense of it as usual by making suitable approximations.

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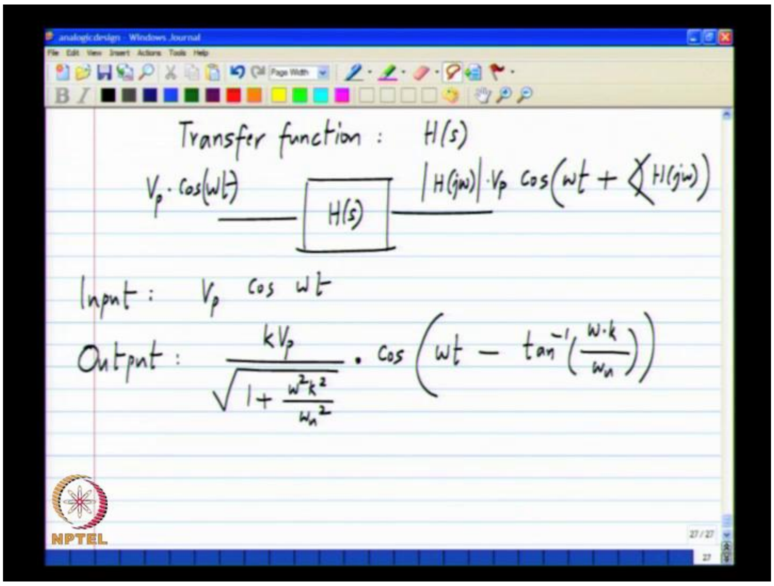

These are inputs and outputs, and first we will look at a frequency range which is much smaller than omega u by k, that is we will be looking at this range of frequencies. So, in this range of frequencies, the output can be approximated by k V p because this term, the second term under the square root in the denominator is much smaller than 1 and the angle can be approximated by that for very small values of the argument tan inverse x approximately equals x. So, instead of tan inverse, we simply take the argument of the tan inverse which can be rewritten as k V p cos omega t minus k by omega u.

Basically, you get an output sinusoidal whose amplitude is k V p, and you get a delay of k by omega u. Here, you see that simply instead of t, the input as omega t, the output is omega times t minus k by omega u. So, this is simply like delaying the sinusoidal by time equal to k by omega u. You also recall that this is the time constant of the system k by omega u. So, from this we see that for low frequencies, where low-frequencies are defined by frequencies much smaller than omega u by k, the magnitude is almost exact the amplitude of the sinusoidal is k V p, that is you still get a gain of 10 and there is a delay of k by omega u.

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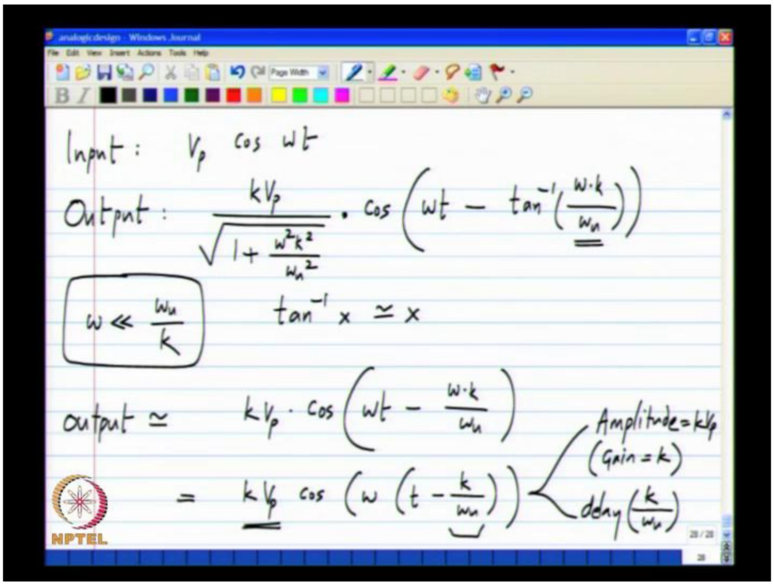

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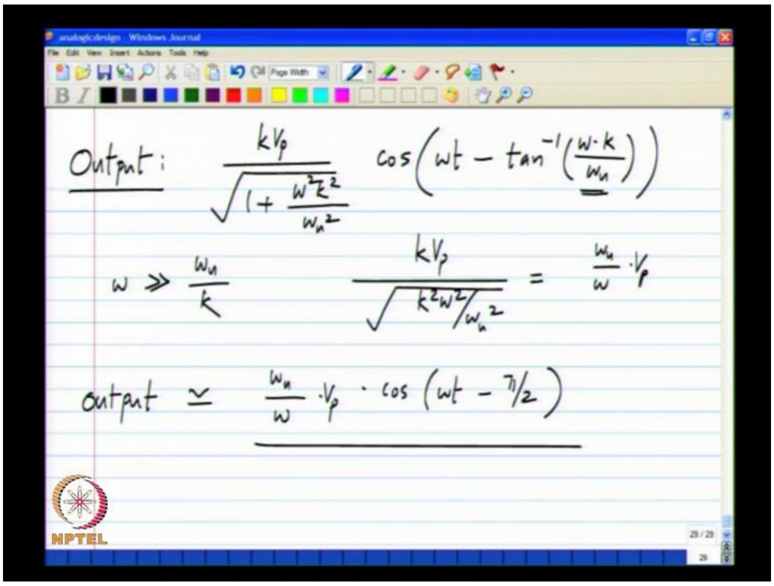

So, it appears that the amplifier is behaving more or less ideally, right. So, again this is the behavior of the amplifier, approximate behavior of the amplifier for frequencies much smaller than the pole frequency or frequencies, much smaller than the 3 db bandwidth of the system. For these frequencies, the output can be approximated by neglecting the omega square k square by omega u square term as well as approximate tan inverse by the ad agreement of the tan inverse. When you do that, you see that simplicity again of k and you also see a delay of k by omega u. So, the sinusoidal is naught modified in. Anyway, it is simply delayed. So, if you delay any signal, it will retain its shape. So, what you get for very low frequencies is almost ideal behavior.

Now, we can redo the approximation for high frequencies. The output I am rewriting here now for very high frequencies, that is omega is much greater than omega u by k. What happens is that, one is negligible compared to this term under square root. We get the amplitude of the sinusoidal to be k V p by k square omega square by omega u square which is the same as omega u by omega times V p, and also you get the argument of the tan inverse to be a very large number because we are saying that omega is much more than omega u by k. So, this argument is very large and for a very large of argument, we can simply approximate the tan inverse by 90 degrees. So, the output for very high frequencies approximately is omega u by omega times V p cos omega t minus 5 by 2.

So, you see that first of all the attitude of the sinusoidal as now has nothing to do with k. The gain has become pretty much independent of k. It is dependent only on the parameter of the integrator and secondly, the sinusoidal experiences phase lag of 90 degrees. It is not following the sinusoidal with the fixed delay, but it has a fixed phase angle of 90 degrees which means that it is you can say that it is not following the sinusoidal at all.

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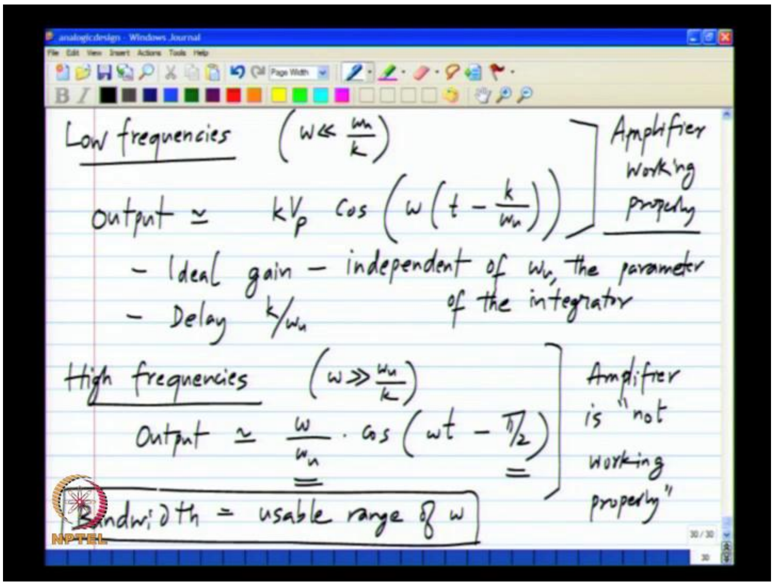

So, this omega equals omega u by k defines sort of the boundary between the ideal behavior of the amplifier and the non-ideal behavior of the amplifier. For frequencies much smaller than this omega u by k, you get almost the ideal gain. The signal is simply delayed and the frequencies are much more than this omega u by k. You get gain which is unrelated to the gain k. You get some arbiter again that depends on the parameter of the integrator and also, you get a phase lag of 90 degrees to summarize at low frequencies.

So, this w naught e is the gain value ideal and there is a delay of k by omega u, and also value of the gain is independent of the parameter of the integrator. See it depends only on k which is the gain that you want to have in the amplifier. It is not dependent on what kind of integrator you use as long as you stay away from sufficiently belong omega u divided by k and for high frequencies. So, now, gain is not dependent on k. So, the output is given by, so the gain is omega by omega u and the output is out of phase with the input. So, we can effectively see that the amplifier is working here because you get the correct gain. You get a delay, but delay is usually of no consequences in most situations. So, you do not worry about it.

So, amplifier is working properly in this situation and then, in this case we can pretty much say that the amplifier is not working properly. So, the frequency omega u by k divides regions, where the amplifier is working properly and amplifier is not working properly which is why omega u by k is called the bandwidth of the system. So, it is put a limit on the frequencies that you can apply to the system and still expect proper behavior. In this case, the proper behavior is the behavior of an amplifier. You would like the amplifier to have a gain k. It has gain k only for our frequencies well below omega u divided by k.

So, this amplifier is usable only for sinusoidal source. Frequencies are below omega u by k that is why it is called the bandwidth. So, bandwidth simply means the usable range of frequencies and for this particular case, it is equal to omega u divided by k. Now, just like we analyzed the time domain response and we saw that if you use an integrator with the higher value omega, that is if you have a faster integrator, what happens is that it will re-steady state faster.

Similarly, here if you have a faster integrator that is if you have an integrator with a higher value omega u, what happens is the amplifier will be usable for a wider range of frequency. So, the bandwidth of the system is omega u divided by k and the amplifier will be usable for a wider range of frequencies. If you have a higher value of omega u, conversely this is usually obtained from the specification. You are usually given the range of frequencies for which it has to operate properly like an amplifier which means that you have to choose an appropriate value of omega u, so that it behaves like an amplifier.

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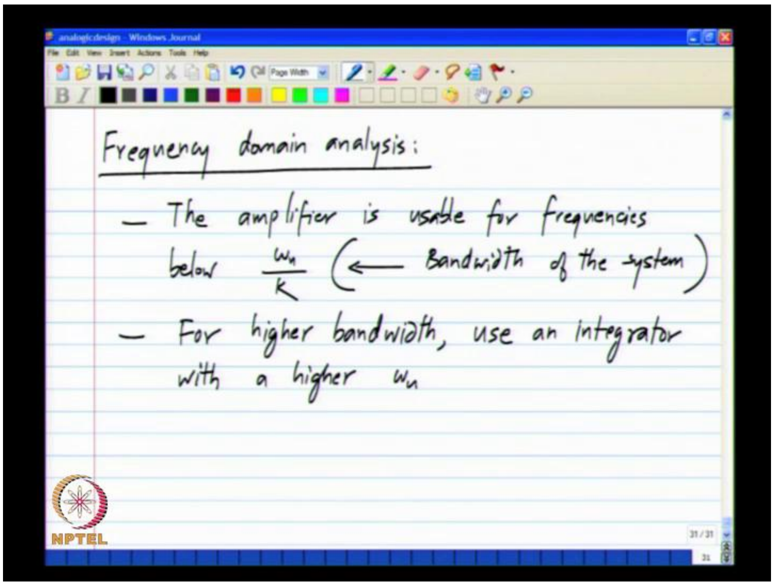

So, the result of the frequency domain analysis is that the amplifier is usable for frequencies below omega u by k, which is why this is the bandwidth of the system and if you want to make an amplifier with the higher bandwidth, you have to use an integrator with a higher omega u. So, from this we see that first of all, we can make an idle feedback amplifier, but it has some limitations depends on the kind of integrator that you choose to have.

Now, the integrator described by single parameter omega u, it describes the speed of the respond in the time domain or the bandwidth in the frequency domain and depending on your requirement, you have to choose the appropriate value of omega u. Now, as we go little more and more into the circuits, you will see that achieving higher and higher values of omega u is more and more challenging and that is where the difficulties in circuit design is coming.

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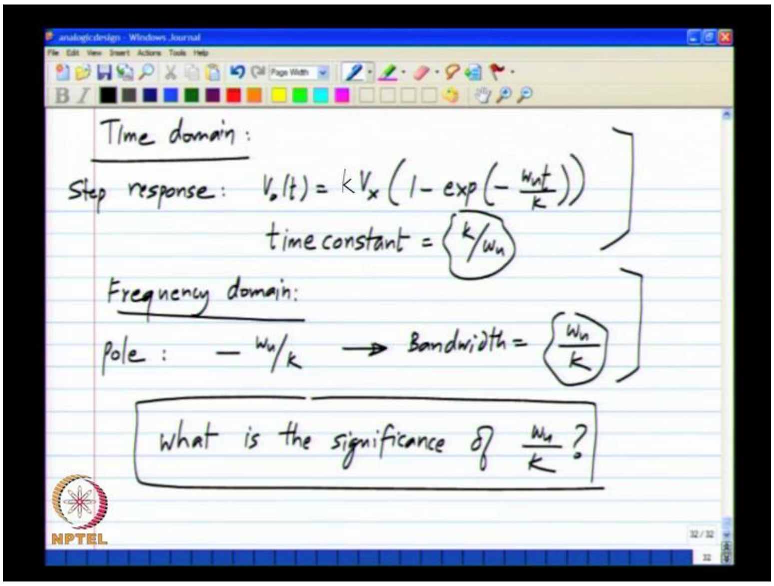

Now, let us take a look back at the time domain and frequency domain analysis of our negative feedback amplifier. In the time domain, we saw that the step response that is if you have an input step going from 0 to v x, the output as a function of time will be given by the time constant of this response is k by omega u, and from the frequency domain analysis, we see that the pole is at minus omega u by k which results in a bandwidth to be omega u divided by k. Now, we have analyzed each of these aspects and we know exactly how it comes about. Certainly we know that if you have increased omega u, the time constant reduces and the bandwidth increases and this we also know why. Because higher omega you mean faster integrator and the response to a step will be faster if you user faster integrator.

Now, we still have to look at exactly why this quantity k by omega u comes up in many different places. We see that it comes up in the time domain and in the frequency domain. What is the significance of this, the frequency omega u by k? We already know that it is the pole system, but we would like to dig deeper into it and see what it means. So, what is the significance of omega u by k? Why does that appear in all these expression? This is just a coincidence or is there something more to it? So, that we can look at by examining the system in the frequency domain and going back to some of the concepts of control systems negative feedback control systems.

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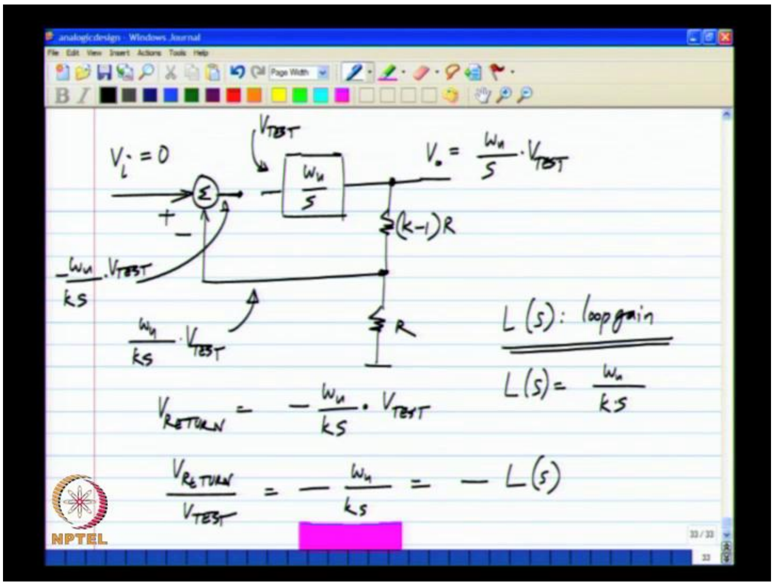

I will redraw my amplifier here. Now, if you look at the input to the integrator, the input to the integrator is given by V i minus V naught by k. The input to the integrator is given by V i minus V naught by k. So, it has a part that is related to the input to the amplifier which is applied externally. It also has a part that is related to quantity inside the amplifier which is the output of the integrator. So, the negative feedback part is the second part here, which is basically the output of the integrator being feedback to its input. So, one of the crucial quantities in the analysis of a negative feedback system is what is known as loop gain. So, it refers to reviewing that the certain quantity into the system.

So, let me call it V test. What part of it comes back? So, here we are not interested in the input to the system. So, we will reduce V i to be 0 and then, I will apply a test input to the integrator. So, what happens is, I will get a quantity here which is equal to omega u by s times V test and here, I get omega u by k s times V test k and at this point, I will get the same this, but with the negative sign. So, at this point I get minus omega u by k s times V test. Now, we are talking about negative feedback system that by definition means that if you apply something into the loop of the negative feedback system, something comes back to it and here, we have evaluated exactly how much comes back.

I have broken the loop at the input to the integrator and then, I have applied a voltage V test. It goes around the loop like this and finally, I get back an output which is minus omega u by k s times V test. I will call this V return whatever voltage returns to the same point or I can define the loop gain, the ratio V return by V test s minus omega u by k s, and I will define this 3 minus L, where the loop gain keep in mind that the quantity that returns at the point at which you have broken the loop is minus L s and L s is defined to be the loop gain. Essentially this says that if you apply some signal here, how much of a comeback?

So, something has to come back because we are talking about a negative feedback loop. If there is feedback, something has to come back. Exactly how much comes back is quantified by the loop gain. Now, let us evaluate looking for this. In our case, the loop gain is omega u divided by k s and for our case; the loop gain is L of s is omega u by k times s.

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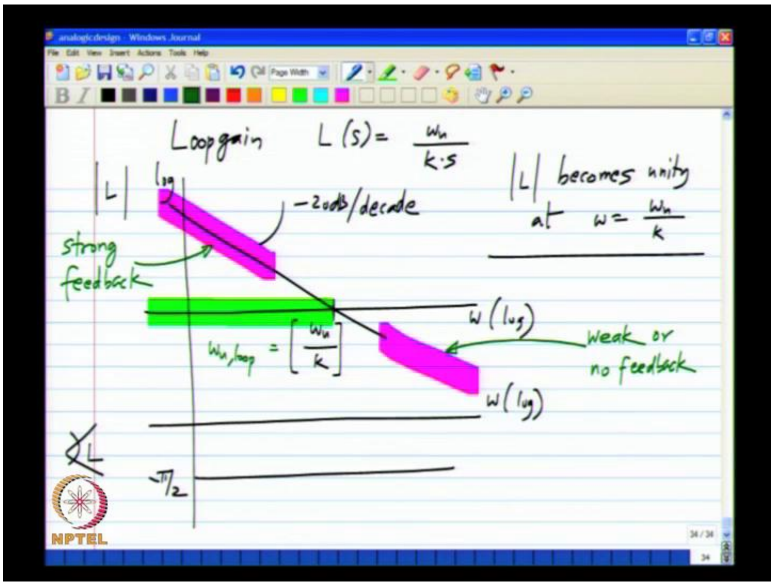

We can draw the broad a plot for this function as well. This is very easy because this function also denotes integration and the magnitude plot of L. Again on lag scale is a straight line with a minus 20 degree for decades of the frequency at which the magnitude of L becomes unity. It is given by omega u divided by k and we can also plot the angle of L. This is not very interesting. It is equal to minus 5 by 2 for all frequencies. So, magnitude of L becomes unity at omega equals omega u divided by k.

So, again you see this quantity omega u divided by k. So, what k? So, the unity gain frequency of the loop gain happens to be omega u by k. What physical significance does this have? Actually, this has trim bender physical significance. Like I said earlier, we are talking about negative feedback loops. That means that something has to be fed back in the loop. You apply something here and then, something comes back in the loop. Now, what comes back is described by the loop gain. So, now to have negative feedback loop, there is not enough to have a connection back from the output to the input. The significant amount of feedback signal has to be there.

So, what does loop gain signifies is the quantity of the feedback. So, you see that below frequency omega u by k, the value of L is large and there is the significant amount of feedback, and if you go well beyond omega u by k, the magnitude of L is much smaller than 1. So, naught much signal is being fed back although you have a connection from the output the input the magnitude of the loop gain is very small means that there is practically no feedback here.

So, in these range of frequencies, you have a high feedback or a strong feedback. In this range of frequencies, you have weak or no feedback. So, this omega u by k separate regions where you have strong feedback and a weak feedback and that makes all the difference. So, you can clearly see that you have strong feedback. The amplified behaves ideally because of troll. The way we derive the amplifier was by saying that using negative feedback will adjust the output to be of the ideal value, that is you look at the error between the desired value and actual value, and you drive the output in a direction that will minimize the error.

So, that is the negative feedback loop. Now, for there to become very small, the feedback has to be very strong. Now, the loop gain is the quantity that quantifies the amount of feedback. If you break the loop in someplace, you inject a signal there and then work comes back to the same point that is the loop gain. We exclude the negative sign by convention, but that is the loop gain. Now, you see that for our system the magnitude of the loop gain is much more than 1. When frequency is much less than omega u by k, that is why for frequency is much smaller than omega u by k. The amplifier behaves almost ideally.

So, we have already seen that from the analysis of the transfer functions, the amplifier is usable for frequencies. Well, below omega u by k, the gain is ideal. Below this value almost ideal and there is simply a delay whereas, for frequencies much more than omega u by k, there is no feedback. You may have a connection from V naught through the resistive divider back to the input, but there is no signal coming back at those frequencies. So, that means, the reason we have no feedback and it is not behaving like a negative feedback amplifier at all in this higher range of frequencies. So, the usable range of frequencies is given by this much, and this omega u by k I will denote it as omega u loop. The unity loop gain frequency that is omega u by k and below omega u loop where that feedback is strong, we have almost ideal behavior. Above omega u loop, the feedback is weak. You have one ideal behavior.

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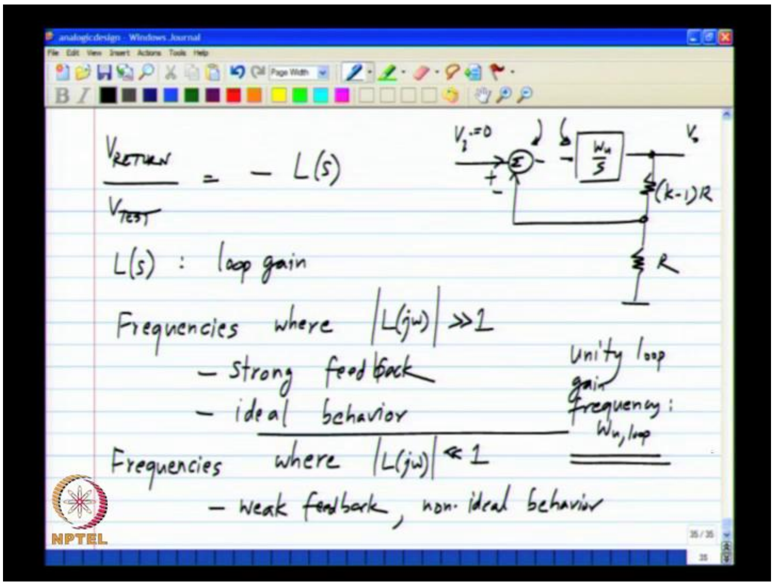

You consider our negative feedback amplifier and then, in this case we are interested in what is happening inside the loop. So, as we said inside the loop 0 and we break the loop somewhere. It actually does not matter where you break it as long as it is in the loop that you have breaking, and you apply something here. See what comes back here. The ratio of the 2 V return by V test is given by some minus L of s. We take the minus sign because we already know that we are implementing a negative feedback loop and d, L of s is term for the loop gain. The range of frequencies where the magnitude of L is much more than 1, that is strong feedback and the amplifier behaves almost ideally that is as you expect using the principles of negative feedback and the frequencies, L j omega is much less than 1. You have weak feedback and non-ideal behavior.

So, what separates these two regions is the unity loop gain frequency. The unity loop frequency is where the magnitude of the loop gain magnitude of L of j omega becomes equal to 1. So, this omega u loop is the unity loop gain frequency defines the bandwidth of the system in a negative feedback system. So, to be able to have a large bandwidth to have a large range of frequencies, where the negative feedback system behaves ideally, you need to have a unity loop gain frequency that is very high. This is again a challenge in a design.

Well, what I am trying to do here is to give an intuitive feel for why we get this omega u by k everywhere is not arbitrary. It is not a coincidence that omega u by k happens to be the unity loop gain frequency for this particular system. So, that is the bandwidth of the system and this will be true of every negative feedback system. There may be small changes from that, but the unity loop gain frequency is what is at the bandwidth of the system.

So, you expect certain behavior assuming negative feedback and that behavior will be there only of the negative feedback is strong. As I emphasized several times is not enough to have a wire connecting from the output to the input. You have to have a significant feedback signal coming back on that wire. So, the regions of frequency where a significant signal is coming back on that wire, that is where negative feedback is strong and also where negative feedback is weak is divided by the unity loop gain frequency.

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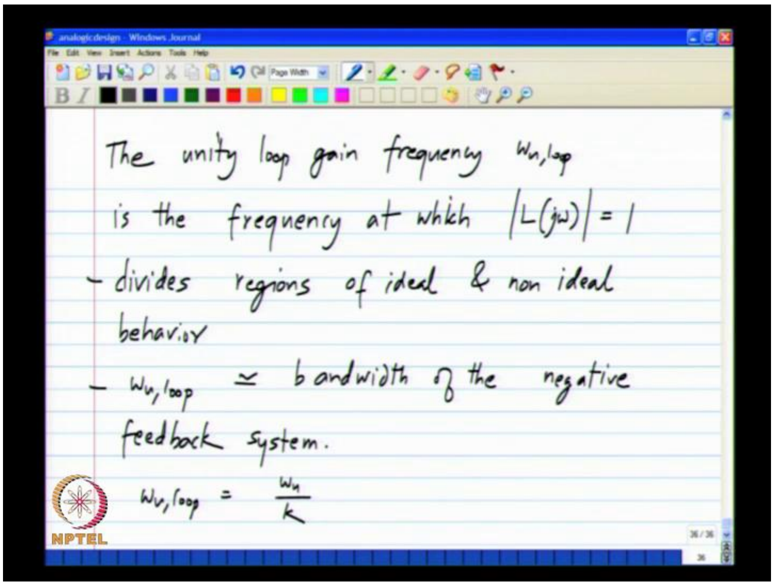

The unity loop gain frequency omega u loop, this is the frequency at which magnitude of L of j omega becomes unity, and this divides regions of ideal and non-ideal behavior and this is simply because it divides regions where there is a strong feedback and where there is no feedback or weak feedback. That is why omega u loop typically is the bandwidth of the negative feedback system, and for the system that we have considered. So far it is equal to omega u by k, we already know that omega u by k is the bandwidth of the system and k by omega u is the time constant of the time domain step response.

So, we will come back to this later when we do the stability analysis. The frequency at which the loop gain becomes unity is the crucial point for the analysis of negative feedback systems. So, we will come back to it, but here you get a feel for what it means is, it divides the regions of strong feedback and weak feedback. That is why it said the bandwidth of the system.

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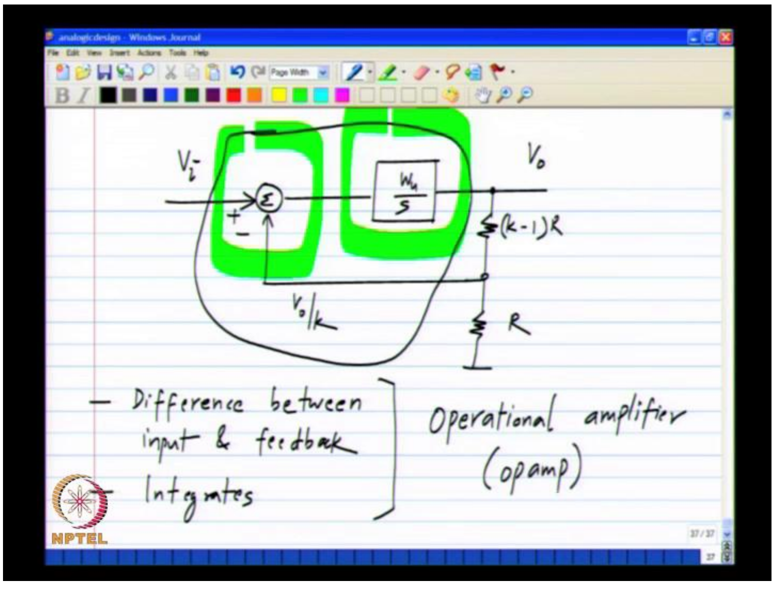

So, I have drawn this circuit repeatedly and this is our negative feedback amplifier, and to make a negative feedback amplifier, we have to have first of all we wait to take the difference between the input and the feedback quantity. In this particular case, it is between V i and V naught by k and we have to have a way to integrate the difference. Now, this negative feedback amplifier scheme is so useful that people have put in a lot of effort to put both of these into a single block, and is very useful block.

So, a block which does sensing or difference between input and feedback, and also integrates the difference, such a block is known as an operational amplifier or an opamp. Opamp is an extremely useful block which is probably venues the billions of times and it as a facility to take the difference between the input and the feedback quantity, it also integrates the difference.

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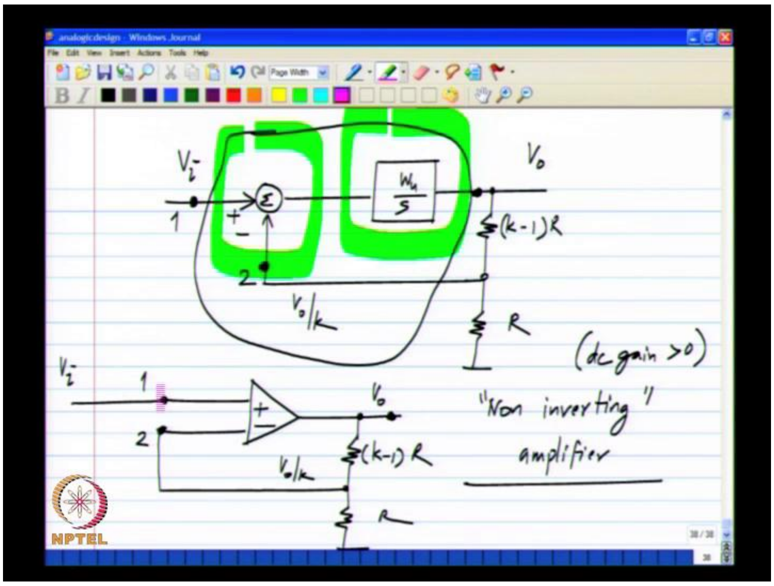

So, the symbol for the opamp does give by where this refers to this particular input and this refers to this particular input. So, by connecting into these two points, you can take the difference between the input and the feedback quantity and the output is nothing, but the output of the integrator. So, using this opamp, we can make the negative feedback amplifier and it is very easy as simple as what I have on the upper schematic to the lower one. So, I have a resistive divider k minus 1 R and R. So, this gives meaning if this is V naught. This gives me V naught divided by k, then I have applied to the appropriate input of the opamp and I apply the input voltage here.

So, this is my basic negative feedback amplifier using opamps and it is nothing, but the realization of the negative feedback amplifier that we have been discussing so far and this is known as the classical non-inverting amplifier. Non-inverting simply means that the dc gain is positive because in this case, we know that the dc gain is k, right. The dc of this particular system is k. So, you think of the opamp as an integrator and what happens is the integrator will reach steady state only when the difference between 1 and 2 is 0.

So, we will still assume that the input V i is a constant. The steady state will be reached only when the difference between 1 and 2 is 0 and output is also constant. So, when that happens, the output will be exactly k times the input voltage and we get our ideal amplifier. So, the opamp is nothing, but block that can take the difference between two signals and integrate it to drive the output, and the opamp is disconnected by a single parameter which is the parameter of the integrator.

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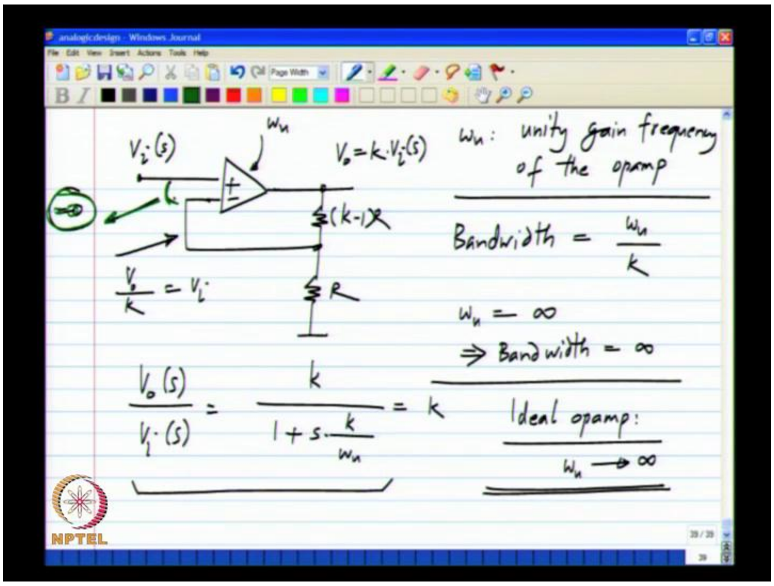

So, the opamp, certain omega u is the unity gain frequency of the opamp and this is the most important parameter in the opamp. If you look at any opamp data sheet, you will see the unity gain frequency listed. Typically, it is listed in hertz we have been using which means that our omega u is in radiance per second, but whatever it is, once the unity gain frequency of the opamp is known, you can calculate either the time constant of the bandwidth of any circuit that you make using the opamp. So, we also know that V naught by V i will be k by 1 plus s times k by omega u for this particular amplifier structure.

Now, there is also a concept of an ideal opamp. If you go on increasing the unity gain frequency of the opamp, if you go on increasing the value of omega u, what happens is the bandwidth of the system goes on increasing. Bandwidth of the system is given by omega u divided by k and if you keep on increasing the value of omega u, the bandwidth goes on increasing. Now, this is set omega u to the infinity. Then, the bandwidth will be infinity. So, that means that for all frequencies, the gain of the system will be k. The transfer function will not have any s term in it. The transfer function will simply be equal to k.

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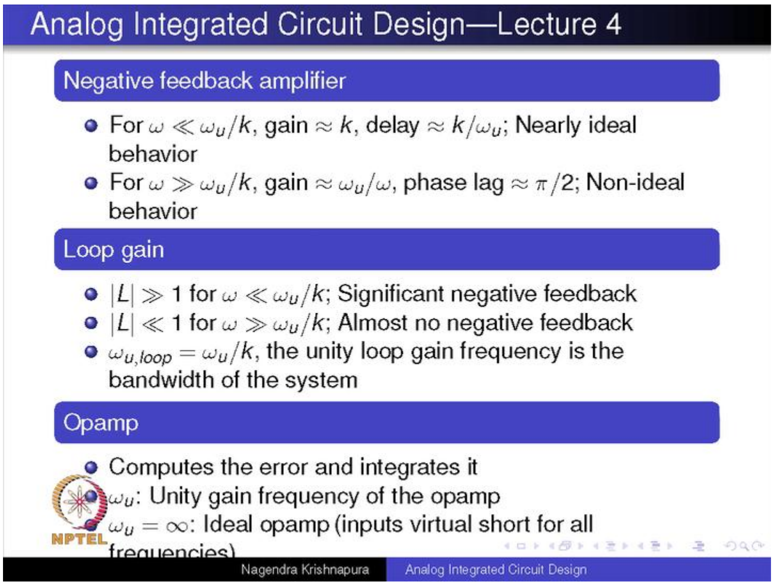

When the opamp is an integrator with the unity gain frequency omega u, it behaves ideally for dc. That means the dc gain is k. So, we know that if you apply constant V i, finally a steady state is reached where the plus and minus terminals. So, the opamp for the same voltage and output equals k times V i. So, it always shows ideal behavior for dc inputs. Now, if you set omega u to be equal to infinity, the bandwidth becomes infinity and it shows ideal behavior for all frequencies. So, this is what is called an ideal opamp. Ideal opamp simply means it is an integrator with omega u tending to infinity.

So, what happens in this case is for any frequency V naught will be k times V i. The negative terminal of the opamp will also be equal to V naught by k equals V i and the difference between these two terminals that will be equal to 0. So, that is the definition of an ideal opamp. So, many times the ideal opamp is presented first and then, you are shown the actual opamp. We have gone the other way. We have gone from first principle to try and drive negative feedback amplifier. For that we see that we need a block that sense of the difference between two quantities, and something that integrates the difference and we can arrange it as the negative feedback amplifier. So, that is an amplifier in which the error is gradually driven toward 0.

So, such a block when you have an integrator behaves ideally when the input is constant, and you wait for the output is steady state, the error or the difference that is sense will eventually become 0. In steady state, the error will be 0. Now, if you make the speed of the integrator unlimited, then it does not matter how the input changes always the output will be able to track the input.

So, such a concept is known as an ideal opamp. It is a very useful block for analysis and sometime for coming up with circuits and so on. Of course, later we have to recognize that every real opamp as a finite value of omega u and then, put that analyzed, but nonetheless the ideal opamp with omega u equal to infinity is a useful concept. Basically, the behavior of that opamp is given by the fact that the difference voltage between the plus and minus terminals. So, the opamp equals 0.

So, using that you will be able to analyze any ideal opamp circuit and this is a good first step. So, let us see an opamp circuit that you have never seen before and you think that it is an opamp circuit that uses negative feedback, then you can assume that the opamps are ideal meaning the input terminals. So, the opamp for at the same voltage, that is the error in the negative feedback system is 0 and added analyzed after that you can put in all the details to the circuit and then, analyze the circuit.