Analog Integrated Circuit Design

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Lecture No - 2
Negative Feedback Amplifier

In the previous class, we were looking at how to make a negative feedback amplified, and to really try to do it was to take an analyze with an everyday system in which we use a negative feedback, and the system took was the way every one drive their vehicles and tries to get it to a desire speed that is your driving a motorcycle, and I want you want to reach 50 kilometers.

Now, what we will do is, you look at the actual speed. That means, you sense the actual speed by looking at the speedometer. When you compare the speedometer leading to the desired speed, that is effectively you take the difference between the desired speed and the actual speed and based on that, you take some action which will make the motorcycle go faster and slower and then, you will eventually be able to be reach 50 kilometers in an hour. Now, what you want to get here is the mathematical description of what you are doing, so that the negative feedback system words properly and we try to do that by looking at what happens if speedometers stuck.

Now, this is easier to analyze because in this case of speedometers stuck; that means, that you think the actual speed is 30 kilometers per hour. It is not changing and the desired speed is also constant. So, the error, the result of comparison that you are acting on is constant. So, you are basically looking at what you do for a constant input. So, it is easier to analyze and this will give the insert into what kind of system we need to have here, so that our negative feedback system works and it is very obvious what you do. Let us say the speedometer stuck at 30 kilometers in an hour.

So, basically you think the motorcycle is going at 30 no matter what you do. So, you will go on accelerating. So, the actual speed will go on increasing, but you do not realize because the speedometers is stuck, and what happens to the actual speed is, it goes something like this. It goes on increasing with time. So, this curve shows the behavior of actual speed with time when you think speedometers stuck.

Now, you get further inside into this was you assuming that the speedometers stuck at 49 kilometers an hour. You have delivery chose a value that is very close to the desired speed of 50 kilometers per hour. So, if you see this speedometer at 49 kilometers an hour, what you do is you still accelerate. So, you think you can reach 50 kilometers an hour, but in this case you do it much more jointly, right. You do not do this same thing that you get and then, the speed was 30 km per hour.

So, in this case the speed, well jointly according to the red curve here, it is still going, but it will go much slower, right. So, this is the behavior of the actual speed of motor cycle with time for difference values of error. So, simply looking at this tells you that the action that you need is, what the block is that gives you an increasing speed, increasing output when an input is constant and also, the rate of increased dependence on how much the input is. If the input is very small, the rate of increase is very small. If the input is very large, the rate of increase is very large. So, very obvious simply while looking at this curve, what you need here is an integrator. An integrator simply says that if there is a non-zero input, the output goes on increasing. If it is positive, if the input is positive, output will go on decreasing. The input is negative.

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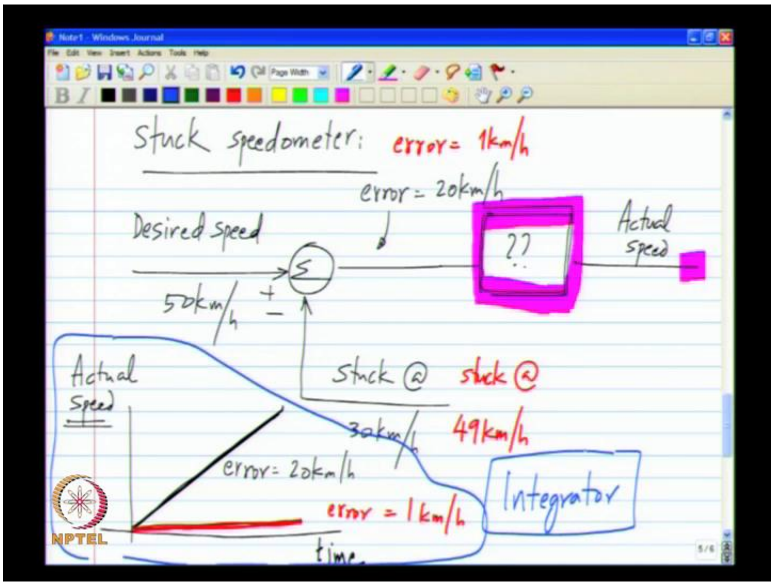

So, that is what integrator is. So, as you are driving, you are effectively integrating the difference between the desired speed and the actual speed. I took case where the desired speed was more than the actual speed, but if it is less, what you do is simply instead of accelerating, you go on breaking and till the speed reaches the desired speed. Of course, when you are driving, you do not calculate the integral and so on, but that is what is exactly happening, ok. Take another example. Let us say you are controlling the volume knob of your radio or your music player. What is that you do? You have actual volume which you pursue through your ear, and you have some desired level of volume, some comfortable level that your use to. So, you go on turning the volume knob. If the volume is too low, you go on increasing the volume until reaches the desired volume that is integration.

You are effectively integrating difference between the actual volume that you hear and the desired volume which is your idea of comfortable level. Similarly, if the volume is too high, you go on decreasing, you will go on rotating the volume knob to left and till the volume becomes comfortable. So, in this case also, the sensing mechanism is not working that is let us say, lose your hearing. Suddenly you will go on increasing the volume because you do not see that the actual volume is increasing because you cannot hear, and the volume will go on increasing with time. So, essentially the actual volume is an integration of the perceived difference between the desire and the actual volumes.

So, you need for feedback is first of all way of sensing the difference between the desired value and actual value and secondly, a way of integrator, the error integrating difference between the error and actual quantities and the output should be driven by the integral of the difference. So, what happens finally is let us go back to the complete system. So, I said this is the integrator which simply denote in this way. For now, output of integrator as actual speed of the motorcycle or any vehicle that you are driving and the difference between desired and actual is the error.

So, what happens is an integrator if the actual speed is different from the desired speed, it could be higher or lower. If the actual speed is lower, the error is positive. You will go on. The integrator will go on increasing the actual speed. The actual speed is higher than the desired speed, the error with the negative and the integrator will go on reducing the actual speed, either way. The only way the integrator will stop increasing or decreasing the output is if the input happens to be 0. To have a steady state that is to have a constant output from the integrator, the error has to be zero. If the error is 0, what it means is that the actual speed is equal to the desired speed.

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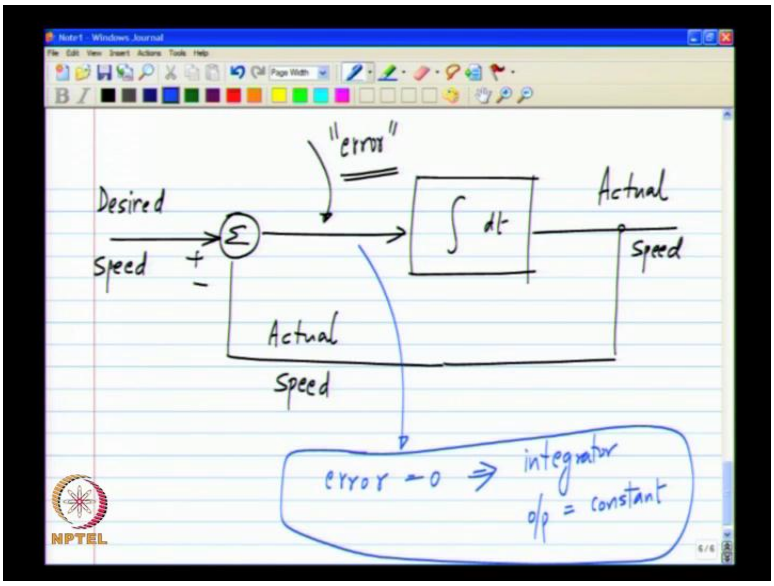

What happens in your vehicle is you will not stop until the actual speed becomes equal to the desired speed, right. If the actual speed is higher, you go on breaking. If the actual speed is lower, you go on accelerating and the early time you do not do that, neither break nor accelerate, further is when the actual speed is exactly equal to the desired speed. So, this system achieves the desired value by using negative feedback. This is called feedback because you are comparing constantly the actual output and the desired output.

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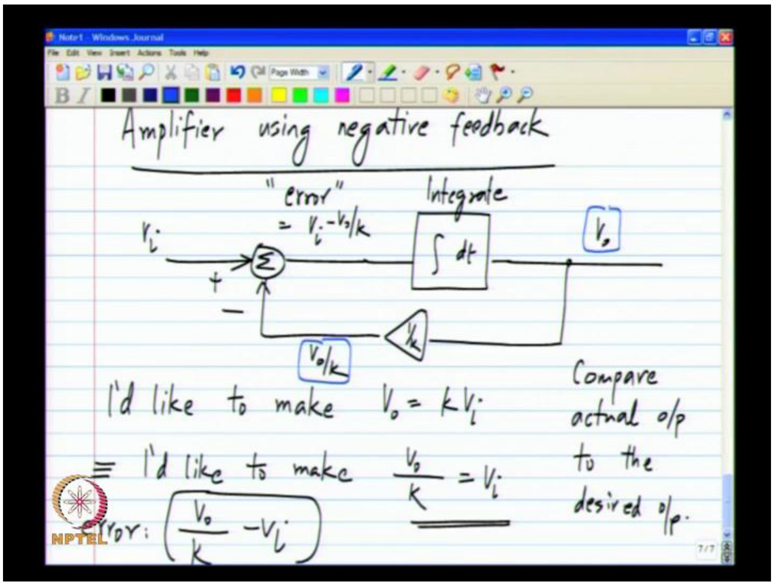

Then, the way to drive the output is by integrating the error, integrating the result of such a comparison and what we would like to do is to make a negative feedback amplified using exactly the same principle. So, I have the input V i and I have the output V o, and we have to make some k time V i. Wave k is the gain and as I mentioned earlier let us assume that we are using dc, that is this is constant and it is not changing with time. So, what we have to do? Based on principle, we just learnt that we have to compare actual output to the desired output. This is what we have to do.

Now, what is the actual output? The actual output is V o or V i and the desired output is k v i. So, I have to measure the error which is k times V i, desired output minus the actual output V o. This is the error. This is difference between the desired output and the actual output. Can we do this? Now, you say that obviously flow in this because if I had k time V i, already I do not be building the amplified. The whole point of the amplified is to take V i and build k times V i from it. So, I do not have k times V i. I have V i. I should generate output which is equal to k times V i. I do not have k times v i if I do not build the amplified in the first phase. So, what is that I should do? So, if you think about it, what the negative feedback system is trying to do is to make V naught equals k times V i.

It is the same as the sign. I am trying to make V naught by k equals V i. Let us say I like to make V naught to be k times V i, but I do not already have k time V i, right. That is the purpose of my building the amplified which is the same as saying I would like to make V naught by k equals V i, and this comparison I can make that is instead of taking the difference V naught minus k, V i will take my error to be V naught divided by k minus V i. So, this is the comparison I can make because I have the value of V i. So, with this we have slide extension to what we did earlier. I earlier said that you compare the actual output to the desired output. Now, it does not have to be the actual output that you desire. Output does not have to be equal to, sorry. So, the output does not have to be compared to the desired output. It can be a function of the output.

So, you can compare a function of the output to the input. So, that is what we are doing here. In this case, the function has simply V naught divided by k. So, this comparison I can make and I will integrate the error to drive the output. So, I take V naught, I multiply by 1 by k. So, here I get V naught by k i compared this to. So, here I get the error and then, integrator will come to the details of the integrator soon. So, if I do this, I have my negative feedback system.

How do we get my V naught by k from v naught? That is very easy. It is difficult thing is to have amplified voltage, but is the very easy thing to reduce the voltage. If I want V naught by k, from V naught I can simply use a resistive divider. So, let me redraw this using the actual circuit implementation to obtain V naught by k. From V naught if I use the voltage divider like this where one of the resistance k minus 1 r and other assistance is r, I will get V naught by k and the output of the resistive divider and I subtract it from V i, and I integrate the error I used than integrator. What will happen now? Let us say V i is 1 volt and my k equals 5. I will get V naught divided by 5 here.

So, let us say V naught by 5 is less than 1 volt. What will happen? The error here will be greater than 0. So, that means, that the output will go on increasing. So, in this case, the output will go on increasing and the only way the output remains constant is if this error is 0. Now, if error is 0, the output V naught is constant. If error becomes 0 and error being equal to 0 implies that V naught k equal to V i or V naught equals k. V i is exactly what we wanted. So, this is an amplified using negative feedback. So, this is the basic. We will amplify using negative feedback and the crucial part of it is than integrator the two important things. One is able to sense the difference that is compute the error and then, the second thing is able to integrate the error to drive the output.

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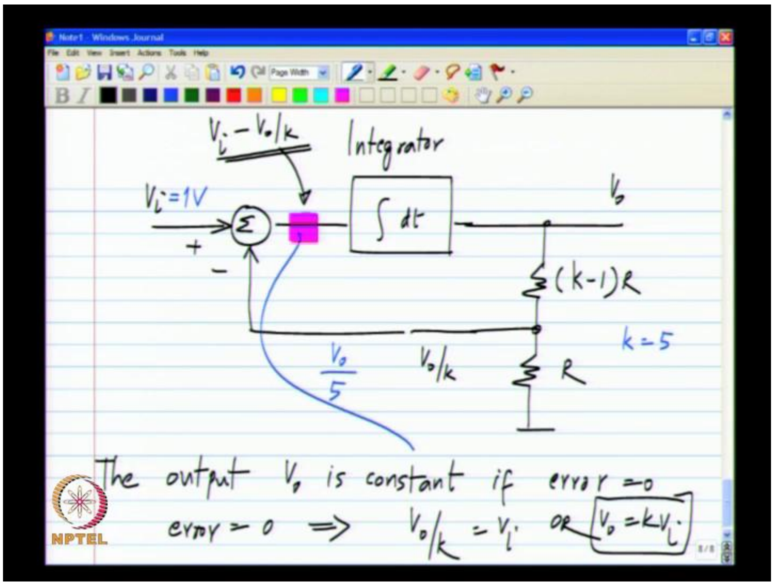

So, these are two essential component of any negative feedback system. We have taken amplified of gain k as an example, but the system is same. For any other kind of negative feedback, you control the output using an integrator. You make the error equal to 0.

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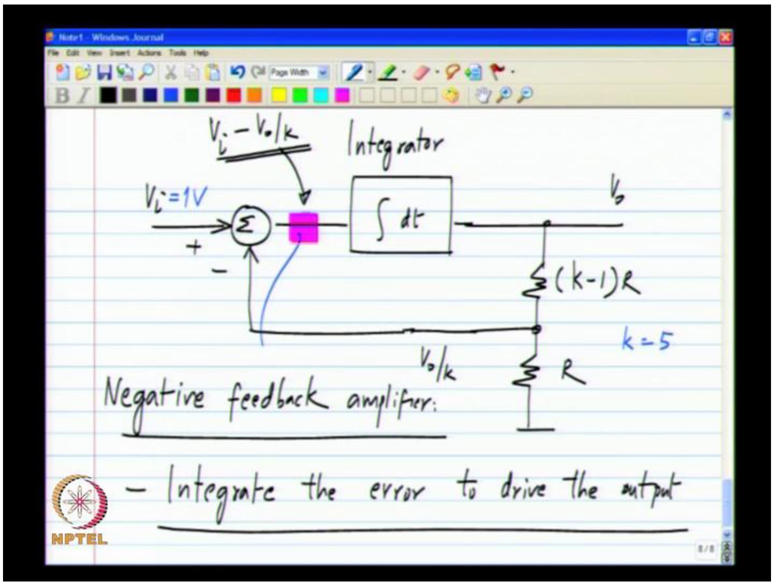

The basic principle is that you integrate the error to drive the output, and this is due to ability similar to what you do when you are adjusting the volume control norm for your comfort or when you are adjusting the speed of your automobile. You got the result speed. The principle is the same. You integrate the difference between the desired and actual values to drive the output for how that we have the basic negative feedback amplified. We will analyze it and see how it behaves under different conditions. This was the integrator and we have the voltage divider. Here I will denote this error by V e and now, when we need to make the definition more precise, just look at the integrator. So, if you look at the integrator, it has a voltage output under voltage input. So, for dimensional consistent, see there has to be a constant here which is dimensional of V frequency.

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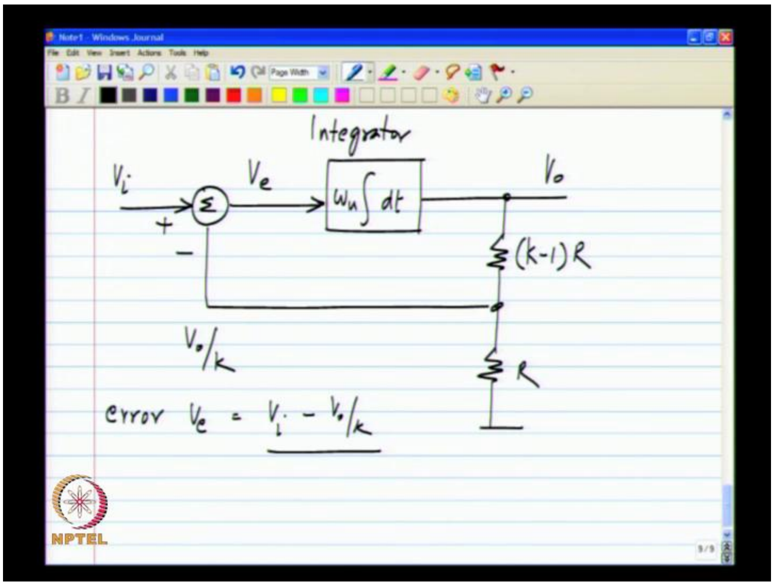

So, we will examine the integrator in detail. It will be clear what the role of this constant is, but before the integration, we need to have constantly which has dimensions of frequency. So, we will examine this. We will see what happens when the output is different from k times V i. You already know that only the output will be constant when the error V e is 0. It means that V naught by k equals V i not equals k.

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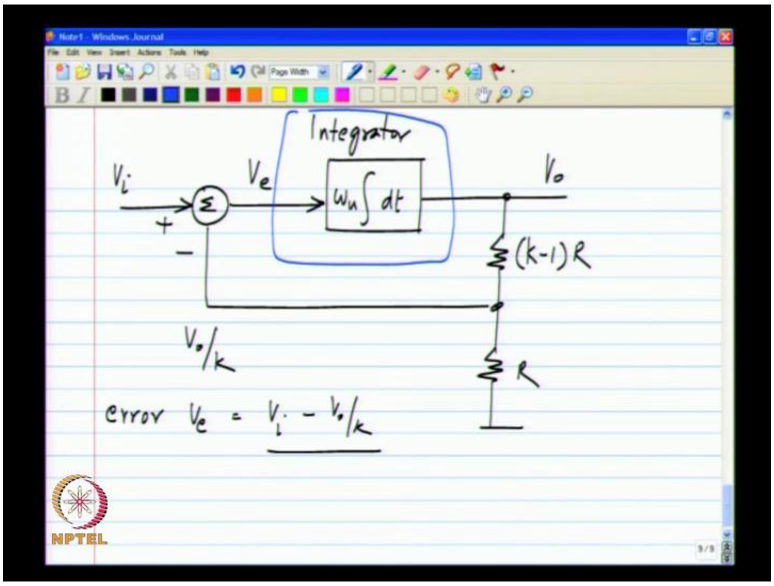

So, we already know that the steady state solution to be this system is V naught equals k V i that is the output equals k times the input voltage. So, we need to know what happened when before reaching steady states that, let us say the output is supposed to be 5 volts, but it is 4 volts. So, how does it get to 5 volts? All that analysis we will do, but before we go there, we will analyze the crucial part that is integrator. We will see how that behaves.

So, the integrator integrates error voltage V i V e and as I mentioned, there has to be proportionality constant omega u, so that the dimensions are consistent. So, the relationship is V naught is omega u integral V e V t, and omega u dimensions of frequency. So, just to have numerical example let me take omega u to be 1 giga radians per second, that is to say 10 to the 9 radians per second.

So, what happens in this case is let me plot the output versus time as if that V e is constant at 1 volt. Let me also assume that the integrator output via initially, start from 0. So, what will happen be the output? Now, it is 10 to the 9 radians per second integral 1 volt d t, essentially this 10 to the 9 t and because the initial condition is 0, this is all what we are going to get.

So, we plot the output at let us say 1 second. The output will increase Lagrange. So, at 1 neon second the output will be 1 volt, at 2 neon second the output will be 2 volts and three neon seconds, 3 volts and so on. So, if V e is the constant, the output will increase like a ramp will increase with the constant slope, and the slope is related to this constant omega u and the value of v for instance.

Now, that is v e become a volt, what happens at 1 neon second? The output will be of a volt. The 2 neon second will be 1 volt, at 3 neon second, it will be 1.5 volts. So, if the error is smaller, the output will rise slower. This we already know. So, that is the behavior of an integrator. If the input is larger, the output will rise faster. If the input is smaller, the output will rise slower. So, similarly if the input is negative and large, the output will fall very quickly. If the input is negative and small, then the output will decrease, but rather slowly.

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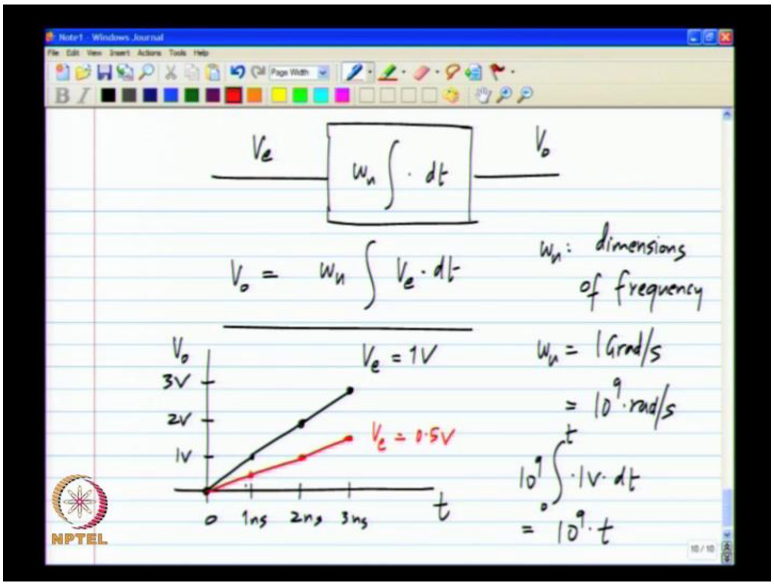

So, another comparison we can make is what happens if let us say, I take indifferent integrator? This omega u is constant and that describe the integrator. It basically tells you how fast the output will raise if you apply 1 volt input to the integrator. So, if omega uses 1 giga radius per second or 10 to the 9 radians per second, what happens is, the output will rise at 10 to the 9 volt per second if the input is 1 volt.

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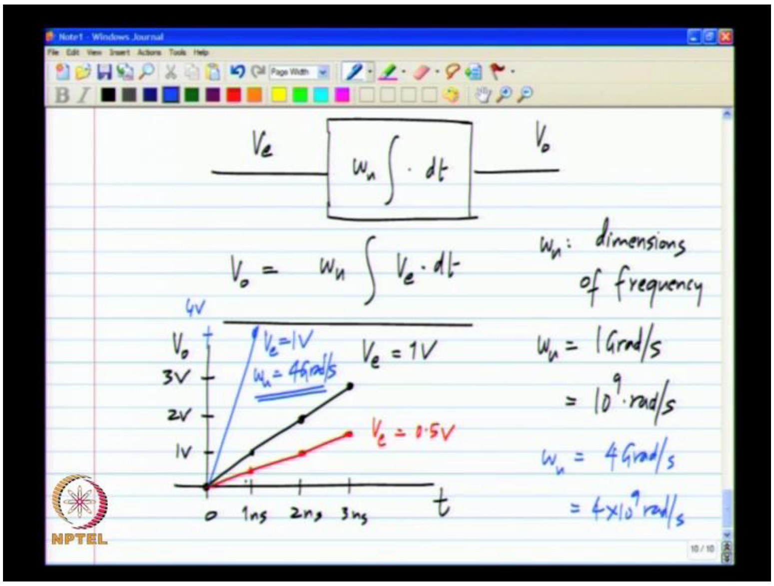

So, omega u is the proportionality constant if we use larger omega u. Let us take a different case. Omega u is for giga radians per second that is 4 times 10 to the 9 radians per second. What will happen in that case? Let us say V e, the error voltage is 1 volt and t equal to 1 neon second, the output will be 4 volt. This can be easily evaluated. So, it will rise very fast. This is for V equal to 1 volt and with our new integrator.

So, essentially a speed of integration is defined by omega u when you get integrator as single parameter omega u which is how fast integrates it. If omega use very large, then it integrates it quickly. If omega use very small, it integrates very slowly. Then, also the output, actually output of the integrator depends on constant omega u as well as what the input voltages. If the input voltage is very large, integration is very fast and if the input voltage is very small, the integration is slow.

So, this is what we need to know about the integrator for now is if this will integrate into our system and then, how it behaves. So, we know that V naught omega u integral V d t, so V i remains constant. So, now, we have to find exactly how V naught behaves with time. To do that we have to write the equation describing the system and I just substitute for V e, the difference between i and V naught by k. Now, this can be solved, but it is easier if you write this in differential form instead of an interval form.

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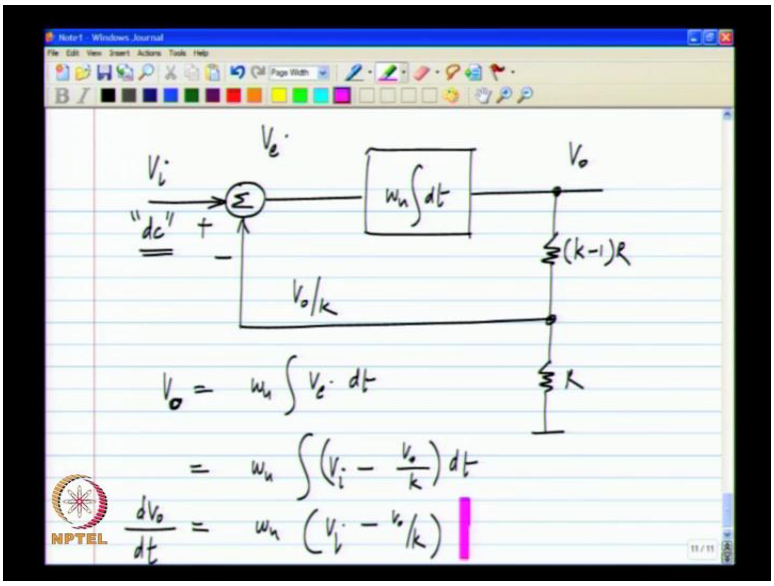

So, that is differentiate both sides and I will write the derivative of output equals omega u V i minus V naught by k. So, this is the differential equation in this part. Here, this is the differential equation governing the system. We have to solve this equation and we will be able to see the behavior of this system. That is how V naught behaves when it is not equal to the desired value.

How exactly we get to the desired value? This is the first order differential equation and this can be easily solved for a constant V i by re-arranging the terms. So, we integrate this. Solving this, what we will get is that rearranging the terms, you get this and integrating this, you will get this. You have to integrate from the initial condition to the final and here, from 0 to t.

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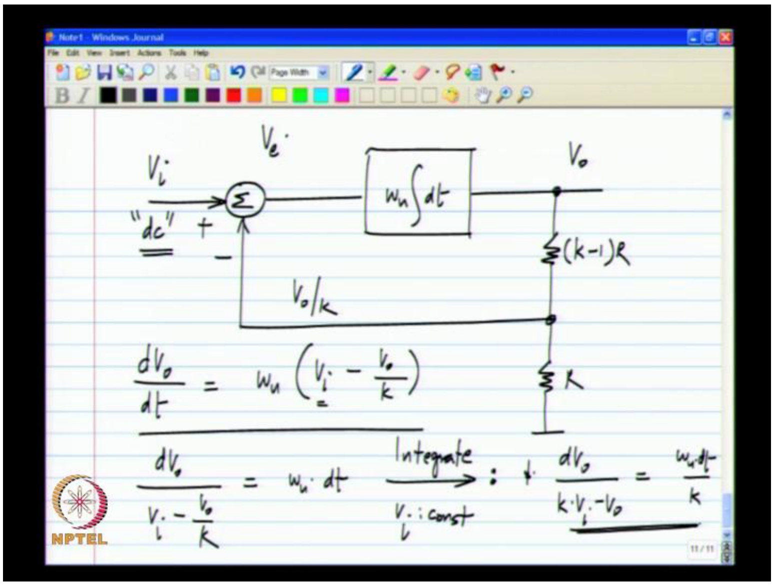

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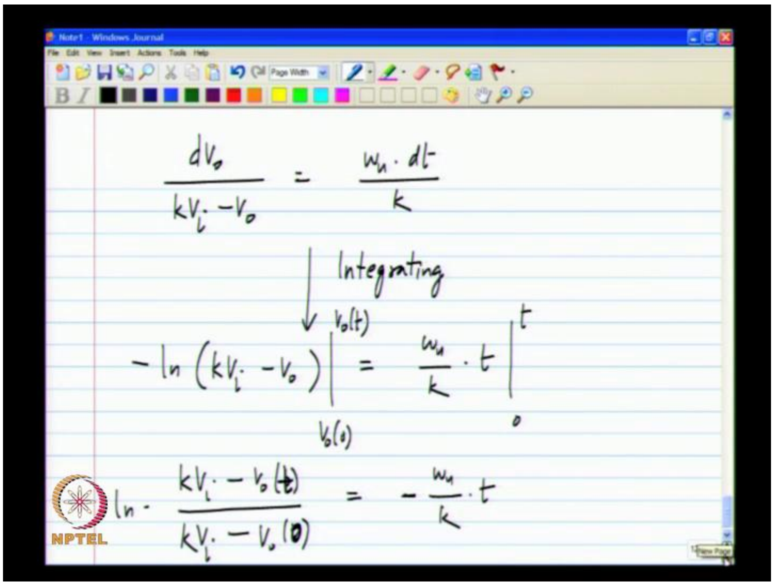

So, what we will end up getting is that the logarithm of k V i minus V naught at t equals to 0 divided by the log of k V i minus V naught at t will be equal to omega u by k times t or if I take the reciprocal on the left side as just I get this one.

So, if I rearrange terms of this equation, I will get V naught of t to be V naught of 0 exponential of minus omega u by k times t plus k times V i 1 minus exponential of minus omega u by k times t. So, this is the solution to the first order differential equation which governs negative feedback system. So, what it says? The initial condition of output is the case with an exponential and the desired output grows as an exponential. So, let us first take the same example, omega u 10 to the 9 radians per second. Let us say we take k equal to 5, and let me plot V naught versus t.

So, what is the curve looking like? First, let me take an initial condition to be 0. So, in this case, the first part of the solution is not there and we have only the second part, and we also have to specify the value V i. Let me say that V i is 1 volt. So, k time is V i will be 5 volts and if you look at this term for t equal to infinity for very large values of t, this particular term, the second term within in the bracket will become 0. So, the output becomes equal to k V i, but for very small values of t for t equals to 0, the exponential unity and output will be equal to 0. So, what it says is the output will start from 0 and go as exponential that is 1 minus exponential, and finally assumes that particularly reach 5 volts. So, that is what happened.

So, if k equals 5, then the output will reach 5 volt. When the input is 1 volt, but it is a start from 0 volt instead the initial condition of 2 volts. What will happen is, it starts from 2 volts and the contribution of the first exponential goes and decreasing with time, the contribution of second exponential goes on increasing with time. So, finally, we will still reach the same steady state. You can see that in either case, 1 t equal to infinity. This term becomes 0, this term becomes 0. The output equals k V i. The steady states output is always equal to k times V i. That is what happened.

So, let us say start from 2 volts. So, wherever you start from, it will always reach k times V i which is the behavior that we want, but that is the period for which, that is the transient, that is the output is strict 5 volts. It is changing from the initial condition to the desired value. This is equivalent of accelerating phase of your motorcycle that is when you are accelerating, the speed is the desired value. The speed is changing.

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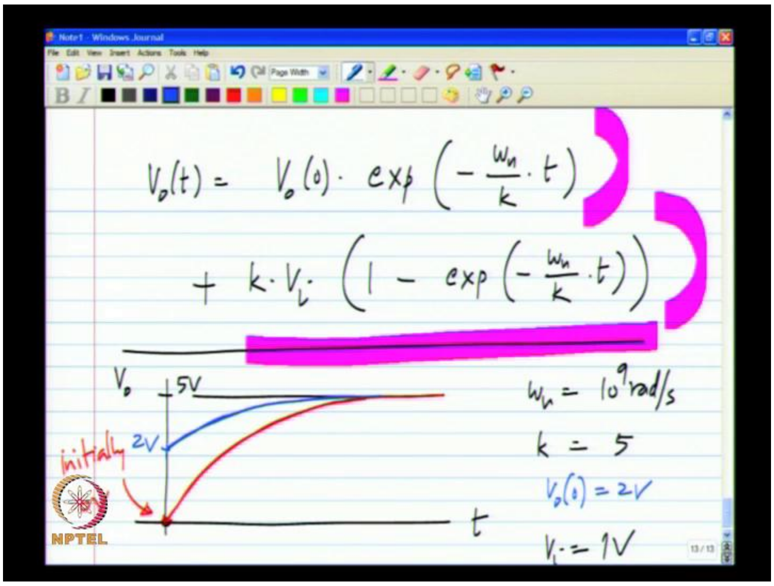

Now, exactly how much time it takes? We can also evaluate and we will do that shortly, but one thing that we can tell from this expression is that V naught t equals k V i when t equals infinity.

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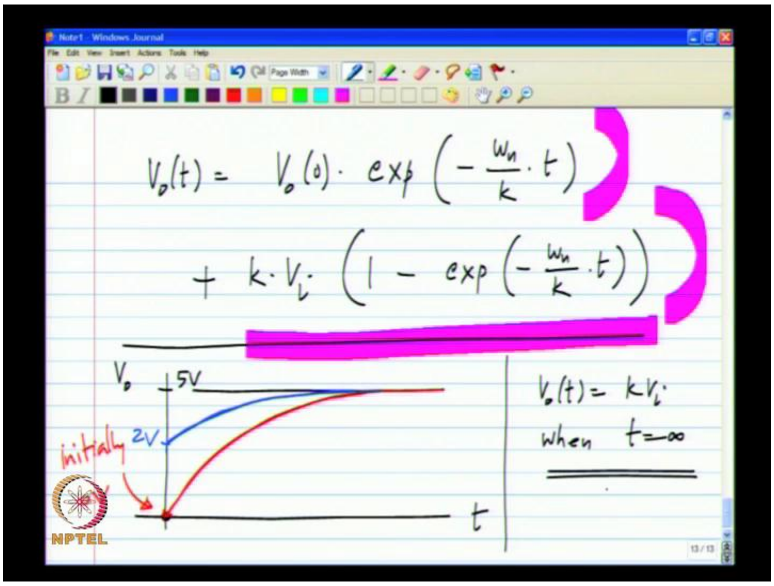

So, before we said that integrate output remain constant only if the error is 0. That is why we not equals k V i. Now, we see from the regression analysis that V naught is k V i t equals infinity. So, you can examine some more details of this. So, as I mentioned, I will take k equals to 5 and V i equals to 1 volt which means that k times V i is 5 volt. So, if start from 0, it reaches 5 volt. Eventually if you start from 2 volts, it will also reach 5 volts and here start from high value which you could for some reason start from 6 volts. You will again reach 5 volts regardless where you start from, you will always reach 5 volts, ok.

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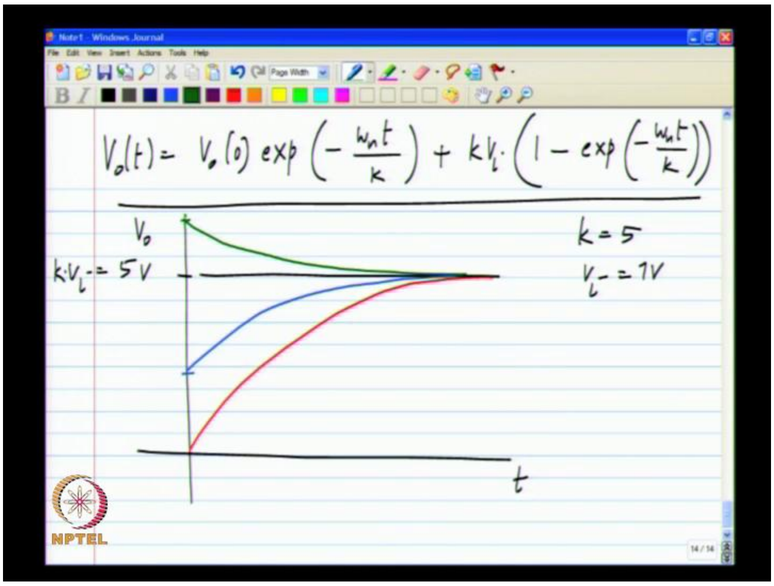

Exactly how much time does it take to reach 5 volts? First of all, it never reaches 5 volt exactly because this exponential never become exactly equal to 0. They will go towards 0, but they never become exactly equal to 0, but still we can make an estimate of when the output reaches very close to 5 volts. So, let us take case where the initial condition is 0. The initial condition is 0 only when the second part of the expression contributes to the output.

So, in this case, this is the output and I want this to be equal to let say 99 percent of the ideal value which is k V i. So, this exponential should be 1 percent because the exponential is simply the error from unity. So, the exponential should be 1 percent. So, what does this tell you? It tells you that t will be equal to 0, that is two times natural algorithm times k dived by omega u.

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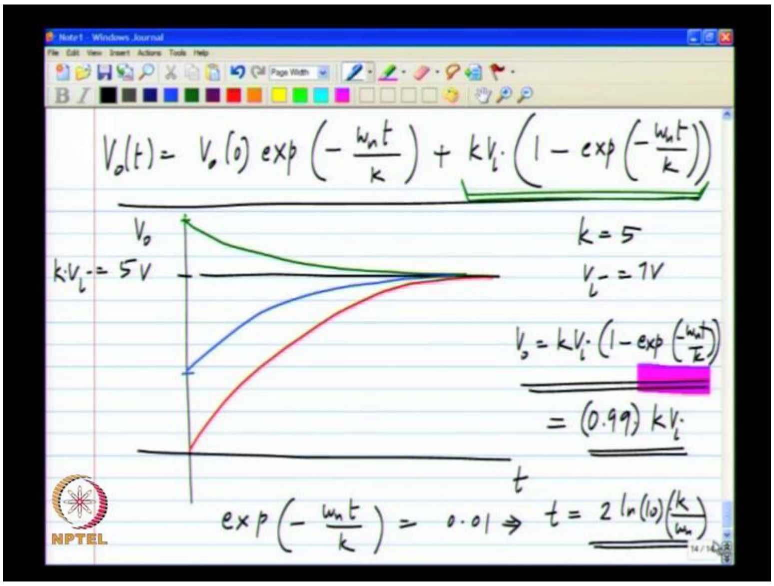

Note that omega u has dimensions of time because omega u has dimensional of frequency and this part two times long-term is about 4.6. So, within five times constant, the output will reach 99 percent of the ideal value. If you start from zero initial condition that is what the expression is same. So, this gives an estimate of how much time it will take for to reach close to ideal value.

Now, you can choose how close it could be? It could be 99 percent, but in every case you will be able to evaluate. How much time it takes to reach that particular value? Now, you can evaluate the numerical example for omega u of 10 to 9 radians per second and k of 5. So, in this case, this implies that. So, this k by omega u turns out to be 5 by 10 to the 9 radians per second which is equal to 5 neon second, and two long-term is 4.6. So, t equals 23 neon second output reaches 99 percent of the steady state ideal to value and here, I assume is 0 initial condition.

Now, let us say I use a faster integrator. What you mean by faster integrator? It has a higher value of omega u. Omega u let say is 5 times 10 to the 9 radians per second 5 giga radians per second. So, what will happen in this case is simply k by omega u will be 5 by 5 times 10 to the 9 radians per second which is equal to 1 neon per second. So, k by omega u is 1 neon second and this t instead of the 23 neon second will become be 4.6 neon second.

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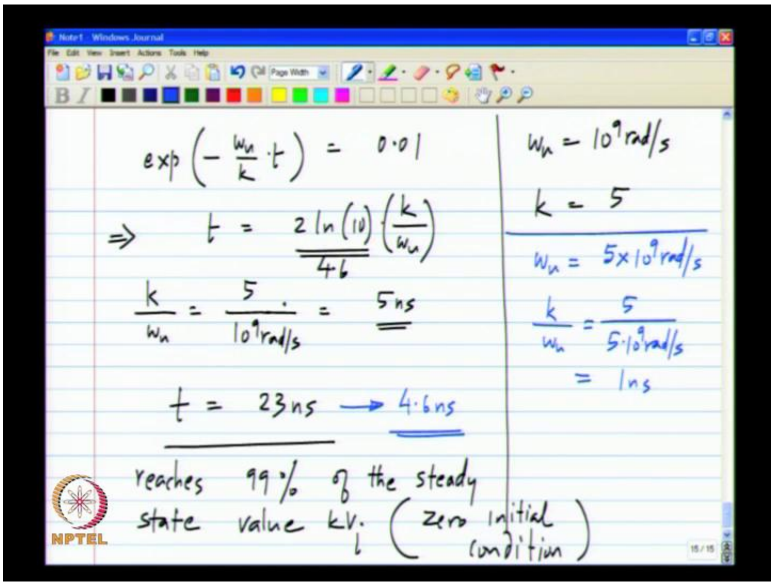

So, the effecter using a faster integrator in your circuit that is an integrator with higher omega u is simply that the output will reach steady state faster. Now, this will be an important design criteria. In many amplified will be told that of course you make an amplified gain 5, but you have to reach steady state in given amount of time. This relates to this speed of amplified. So, if you want to make a high speed amplified, we need to also have a high-speed integrator.

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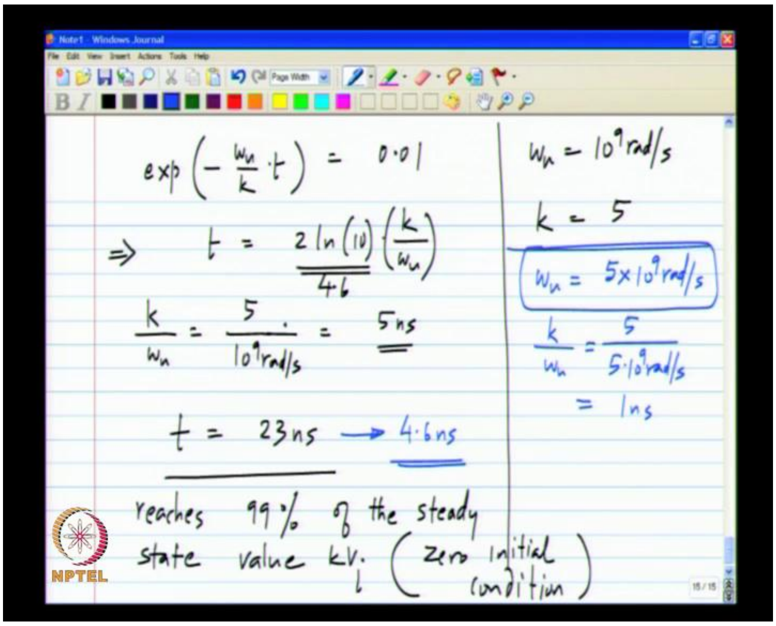

So, you already see that the parameters of the integrator will inflame the performance of the amplifier. So, if you can do with slow amplifier, you use the slow integrator. If you want a fast amplifier, you have to use past integrator and exactly how much is also you can calculate. For instance, if you want to make an amplifier of gain 5 and it has reached steady state within 5 neon seconds, then you know that you have to choose a unity gain frequency that is about 5 giga radians per second.

I hope all of this is clear. So, what we learn today is first of all, how to make a negative feedback amplifier and the way to do that is to compare the divided output to the input and integrate the difference to drive the output. From this, we know that the steady state will be reached only once the error is 0 or V i. V naught by k equals V i that is to say that V naught equals k time V i. We look at the characteristics of the integrator it as single parameter omega u which influences the speed of integration. It has the higher value of omega u, the faster the integration. The lower value of the omega u, slower the integration.

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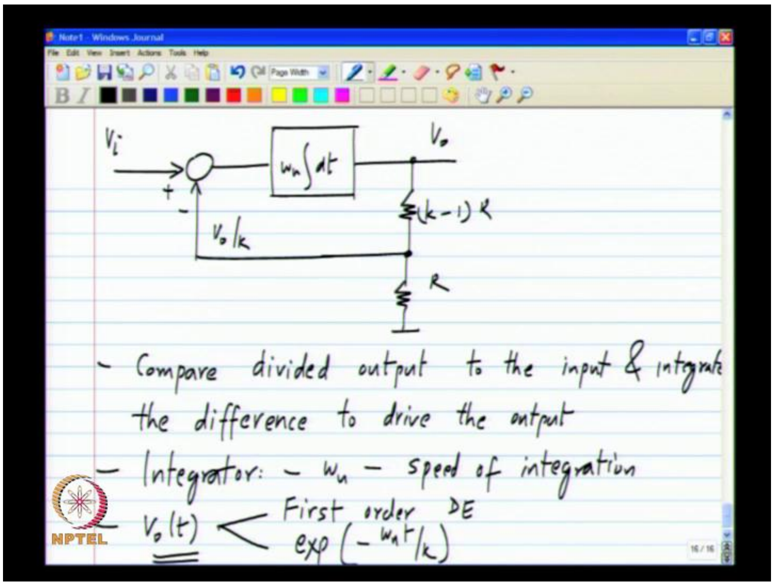

We also looked at how V naught changes with time. We know that in steady state solution, the output V naught reaches the ideal value of k times V i that how does v naught changes with time. That also we looked at and to do that you have to solve first all differently equation, and you will find that V naught of t has some exponential of the form minus omega u t by k, which represents the transient. There is the steady state value of k d by V i that in addition to there are transient which d k as exponential of minus omega u time t divided by k. So, based on this, you calculate how fast it reaches steady state.

So, when I say it reaches steady state, it never exactly reaches the steady state. It reaches certain percentage of the steady state. Now, depending on your application, you can decide how much that percentage is. It could be 99 percent, 99.9 percent or 90 percent, whatever it is evaluated some numerical examples for 99.9 percent. So, it says that if you start zero initial conditions, you reach 99.9 percent of steady state. You have to wait for 4.6 times k divided by omega u. It is k divided by omega u is like a time constant of the system, and you have to wait for 4.6 time constants to be able to reach 99 percent of the steady state.

So, once you do this, the output is close enough to steady state and then, you can say it has reached steady state. We also saw that if you want to reach steady state faster for whatever reason, you have to use faster integrator. There is no other way to do that. If you want to reach steady state faster, you have to use faster integrator and this gives you to design guideline. Then, you are designing amplifier and as also told that you have to reach these percent of steady state within this much time. Let us decide how fast integrator you have to design and then, you have had designed the integrator and you will be able to complete the amplifier.

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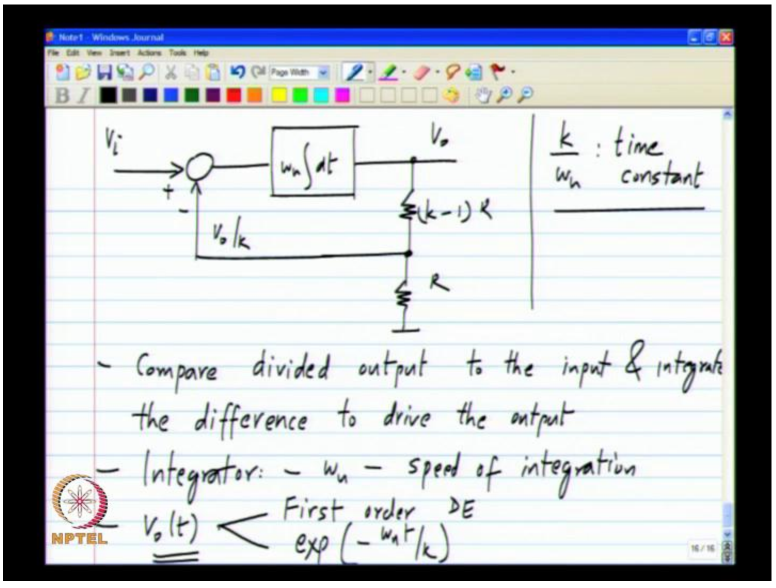

So, that is it for this lecture. So, we have looked at how the output changes when the input is the constant. In the following lecture, we will also analyze what happens when the input is changing.