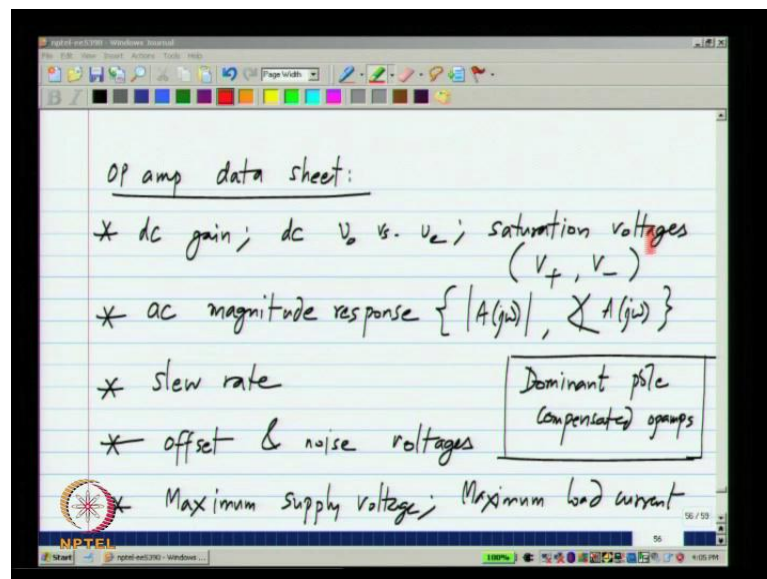


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**Lecture - 19**  
**Opamp Offset and CMRR, Trans impedance**  
**Amplifier Using an Opamp**

Hello and welcome to the lecture 19 of analog integrated circuits design. In the previous lecture, we were looking at a typical opamp data sheet what specifications are listed, and this we did based on knowledge of opamp. So far as a micro model using trans conductor or voltage control currents sources, what will be listed as typical specification of an opamp. A standard opamp are given here.

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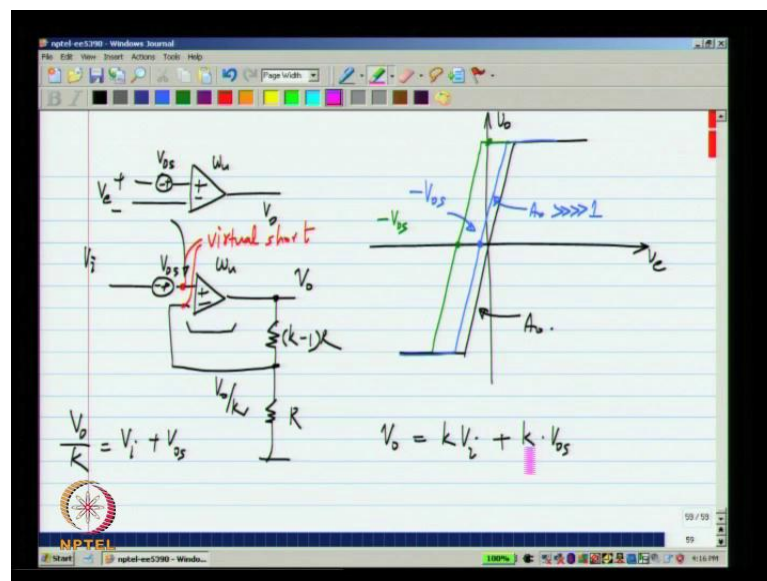


You have the dc gain and saturation voltages, ac magnitude responses which also gives you the phase margin  $A$  for unity gain. You can also calculate the phase margin for other values of gain by appropriately lowering in the magnitude plot. You also have the slew rate which relates to the maximum rate of change, and this comes about because internal to the opamp, there will be some current limitation which limits the rate of charging of the integrating capacitance or the compensator capacitor, and you have offset and a noise voltages in the circuits which lead to some error in calculating the difference between the

desired and the actual values consequently give you an error in the output of the feedback circuit.

We also have a maximum rating like maximum supply voltage of opamp can tolerate and maximum supply current the opamp and tolerate these specifications are to through even for opamp are used in ic for specific purposes, but in general this specification assets hold good even for the specifications are necessary. Even for opamp used in ic's for specific purposes, there are more elements for catalog of part opamp for which you will study the data sheet, and you will use the opamp. When you design an opamp for integrated circuits which you designing yourself, you can change some of the parameters out of these things will look at the effect offset, right. Now, later we will see how the offset comes about, but now we will see what offset does to our circuit.

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this opamp is purpose to be something that takes the difference between two values and integrated it with the certain multiplying factor omega u which is the unity gain of the frequencies of opamp. I mean in reality, there will be an error that is added which I call  $V_{os}$  and that is equivalent to adding voltage  $V_{os}$  scale. In other words, it is like having a voltage source in series with the input.

Now, this voltage is the random quantity. Its statistics are unknown, but voltage exact number is not known. That is why it is a problem. Now, what is this our circuits do? It depends on which circuits? We are talking about in general. You can see that what you

being is compared by the opamp is not the voltages  $V_1$  and  $V_2$ , but  $V_1$  minus  $V_2$  plus  $V_{os}$ .

There is an error that is added to the comparison. So, consequently when steady state is reached for constant inputs, the inputs here, the two inputs here will be at the same voltage. So, that means, the two inputs  $V_1$  and  $V_2$  will not be at the same voltage. Whatever we use to call the virtual short before the voltage between the two inputs, will now be separated by the offset voltage or  $V_{os}$ . So, this leads to error in the circuits. First of all dc characteristics of the opamp itself  $V_o$  versus  $V_e$ , ideally it should have been a straight line passing through the origin and having the slope of  $A_{naught}$ . Now, clearly you see that if  $V_e$  is 0, the output will not be 0, ok.

So, what happens now is that instead of being like this, the output will be like that. So, when the input  $v_e$  equals minus  $V_{os}$ , the input here will be 0 and the output will be also 0. So, this point will be equal to minus  $V_{os}$  the way we have defined it and that is the offset voltage. In fact, the effect of offset can be quite significant because the slope here, this  $A_{naught}$  is very large. It can be even to be the million.

So, it can be that due to the offset voltage. When the input is 0, the output is already saturated. So, this is the offset voltage in this case, and when the input is 0, the output is already saturated, right. This is not because the offset voltage itself is very large. This is because the gain can be extremely large. This curve is wrong with the modulus slope because I have to show the characteristics, but if I plot  $V_o$  and  $V_e$  on the same scale, the characteristics of the opamp looks something like this, ideal characteristics look something other short.

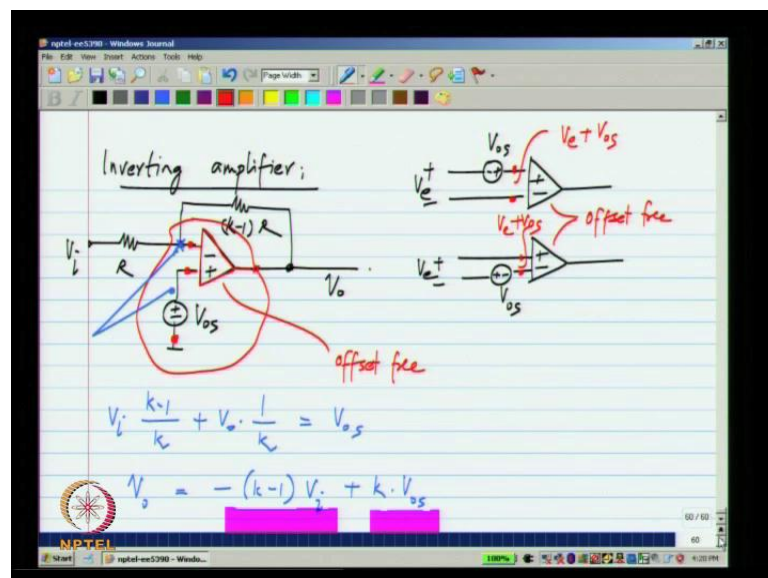
So, almost vertical you can see that a very small shift in this will take the opamp saturation when the input is 0. So, this is the important point because you cannot use an opamp in open loop, right. If you look at only this black curve given here, it looks like I can use the opamp as an amplifier like this without having any feedback, but because offset is very light, the opamp output will be saturated when the input is 0, and you do not know exactly what input you have to apply to operate in this region. You have to operate in the high slope of region of the opamp, so that you get all the benefits of negative feedback. So, the only way to fix this situation is to have dc negative feedback around the opamp, so that the opamp gets biased in the high slope region.

So, circuits which do not have dc negative feedback will have an output that is most lightly saturated. You cannot use that circuit. Now, what does this offset do to our amplifiers? Let us take our proto type amplifier. This is R, this is K minus 1 R and the opamp has an offset  $V_{os}$ . If I apply  $V_{ei}$  have to find out what  $V_{os}$  and for simplicity in this case, I will assume that the dc gain is infinite, that is this opamp act like an ideal integrator with the unity and frequency  $\omega_u$ . So, that means, when reach its steady state, these two voltages that is plus and minus inputs of the opamp show here are identical to each other.

So, what does it means? The voltage here is  $V_{naught}$  by K. So, the voltage here also will be  $V_{naught}$  by K and that will be equal to  $V_i$  plus  $V_{os}$ . So, this means that the output of the amplifier will be K times  $V_i$  plus K times  $V_{os}$ . So, this is the desired output and K times  $V_{os}$  is the offset in the output. So, this means that when the input is 0, the output is not equal to 0, but the output will equal to K times  $V_{os}$ .

So, the effect of the offset is general to create an offset in the output as well, and in this particular example, it turns out that the desired output is K times the input voltage and the offset that we get is also K times the offset voltage of the opamp. The gain from the offset to the output is the same as the gain from the input to the output. Now, this does not have to be the case depending on the circuitry bill. The gain  $g$  from input to the output and from the offset can be quite different.

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So, let us take the other amplifier that we are familiar with which is the inverting amplifier. This is the offset voltage and let say I call this  $R$  and again,  $K - 1$   $R$ . If I apply  $V_i$ , let see what happens to the output voltage. Now, the way I have drawn this opamp is the complete opamp. So, this is the negative terminal and that is the positive terminal, and the opamp that is drawn inside is free of offset, and the offset is represented by this voltage source  $V_{os}$ .

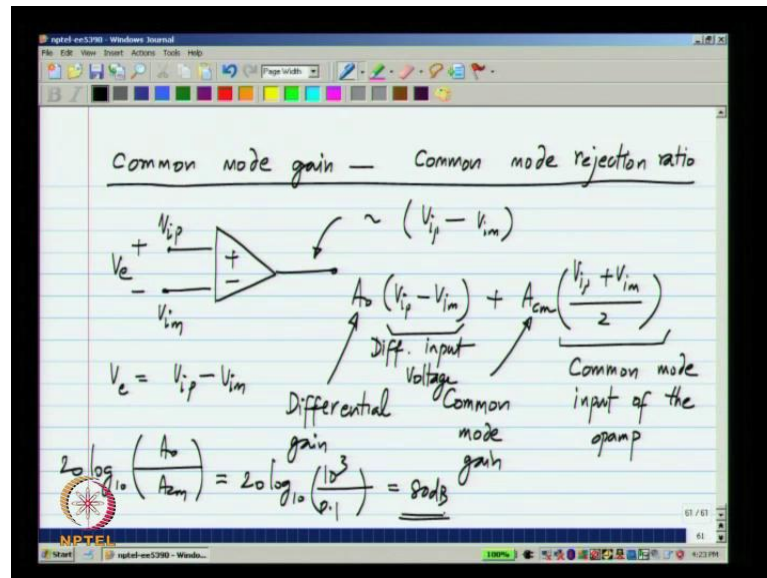
That is very common representation of offset. You represent the offset as a voltage in series with either the plus terminal or the minus terminal. What will the output voltage be? These two voltages will be equal to each other and we know by resistive division formula, the voltage at the minus input will be  $V_i$  times  $A - 1$  by  $K$  plus  $V_o$  times  $1$  by  $K$  and that will be equal to  $V_{os}$ .

So, now we can write  $V_o$  be minus  $K - 1$  time  $V_i$  plus  $K$  times  $V_{os}$ . This is the desired output minus  $K - 1$  times  $V_i$ . That is the gain of the inverting amplifier and you also see that here is an offset  $K$  times  $V_{os}$ , and here you see that the gain from the offset form output is  $K$  and gain from input, the output minus  $K - 1$ . So, in general the two do not have to be equal to each other. You have to analyze the circuits. We will see what the gain from offset and what the gain from input is, and you can also think of it as just the circuit with two inputs.

One of the inputs is the offset voltage, and one of the inputs is the actual input voltage that you want to apply and you will have a different gain from the two sources. Later we will see that any noise voltage of the opamp will be represented by voltage source similar to the offset in series with either plus input or the minus input. Now, so far I have been representing offset like this. It does not have to be. Only that way I could also represent in series with the negative input where this opamp is considered to be offset free.

These two representations are equivalent and that is very clear. When you consider the input voltage applied here  $v_e$ , and you see that the voltage between different, these two points will be  $V$  plus  $V_{os}$  and it is exactly the same here as well  $V$  plus  $V_{os}$ . So, that is about the offset of the opamp. Now, you can see that the offset creates an error in the output and it creates in the output as well, and you have to either choose an opamp which has a low inner offset of your applications or find some way of getting  $(\Delta)$  of the offset even though the offset itself has an offset.

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Now, there is one more specification or opamp which we did not consider earlier which is known as the common mode gain, which is usually specified by the common mode rejection ratio. Now, what are these things? Again, the opamp is supposed to consider only the difference voltage between two inputs. Let me call it  $V_{ip}$  and  $V_{im}$  and  $V_e$  equals  $V_{ip}$  minus  $V_{im}$ , and the output is supposed to respond to only  $V_{ip}$  minus  $V_{im}$ . It should look at only the difference now because of the limitation of circuits with which opamp is built. It turns out this is not exactly true, right. So, what happens is that it also responds to the average value of  $V_{ip}$  and  $V_{im}$ . For instance, if you consider the opamp and dc, the output is supposed to be a naught times  $V_{ip}$  minus  $V_{im}$ , where  $A_{naught}$  is the dc gain which in ideal condition is infinity.

In reality, it turns out  $A_{naught}$  times  $V_{ip}$  minus  $V_{im}$  plus some common mode gain  $A_{cm}$  times  $V_{ip}$  plus  $V_{im}$  divided by 2. This  $V_{ip}$  plus  $V_{im}$  divided by 2 is the average voltage of the input or common mode input of the opamp, and this  $A_{cm}$  is known as common mode gain.  $A_{naught}$  is called the differential gain because this  $V_{ip}$  minus  $V_{im}$  is the differential input voltage. Now, this  $A_{cm}$  should ideally be 0, so that the opamp respond only to the different input  $V_{ip}$  minus  $V_{im}$ , but in reality is not 0. It will be much smaller than  $A_{naught}$ , but it will be non-zero.

To figure out how small it is compared to  $A_{naught}$ , you can specify the value of  $A_{cm}$  itself, but usually what is done is this. Specify the value of the ratio  $A_{naught}$  by  $A_{cm}$

represented in dB 20 log to be base one. So, let say if A naught is A, I will just take some arbitrary example. Let say the value of A naught is 1000 and value of A cm is 0.1, then the common mode rejection ratio will be 80 dB.

So, you can see that the ratio A naught by A cm to represent how large the differential gain in comparison to the common mode gain. So, that is called the common mode rejection ratio and it represents in dB and this is also a part of the opamp data sheet. Now, what is the effect of the common mode gain? That we will see actually the two classic circuits that we have the inverting amplifier and the non-inverting amplifier give perfect examples of what can happen because of common mode gain.

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The slide shows a handwritten diagram of a non-inverting amplifier circuit. The input voltage is  $V_i$ , which is applied to the non-inverting input ( $V_p$ ) and also to a voltage divider consisting of two resistors,  $(k-1)R$  and  $R$ , connected to the inverting input ( $V_m$ ). The output voltage is  $V_o$ . The slide includes the following text and equations:

- Effect of non-zero common mode gain  $A_{cm}$**
- Non inverting amp:** Differential gain is very large
- $V_p = V_i, V_m = V_i; V_o = kV_i$
- $V_o = kV_i + A_{cm} \cdot V_i$
- The term  $A_{cm} \cdot V_i$  is noted as "Ideal" and "Non-linearly related to  $V_i$ ".
- Equation for output:  $V_o = A_d (V_p - V_m) + A_{cm} \left( \frac{V_p + V_m}{2} \right)$  (with a pink highlight under the fraction)
- Equation for  $V_m$ :  $V_m = \frac{V_o}{k}; V_p = V_i$
- Instruction: "Calculate  $V_o$ "

As I said A cm should ideally be 0. In reality, this is much smaller than A naught, the difference gain, but non-zero let us take the non-inverting amplifier. Now, I cannot simply use the different voltage here. I would use these two to represent the output of the opamp to calculate output of the opamp. So, V naught is given by A naught V ip minus V im plus A cm V ip plus V im divided by 2, and we have the relationship due to the feedback network which says that this V im equals V naught by k and we also know that V ip simply equals to voltage V i. Now, from this we can calculate V naught exactly using algebra, right. Please take this as an exercise and calculate V naught using these relationships. It is a bunch of linear equations. It is really simple, but I am not going to do it here. I will just explain the situation more intuitively.

So, what I will do is first of all, let me assume that the differential gain is very large and initially, I will consider a case where the common mode gain is 0, right. If the differential gain is very large, let say reaching infinity and the common mode gain is 0. This is the situation that we had earlier ideal of opamp. So, what we will have is,  $V_{im}$  will also be equal to  $V_i$  because of the virtual short and output will be just  $K$  times  $V_i$ . This is the ideal case. Now, what I will do is, I will assume now that the common mode gain  $A_{cm}$  has a small non-zero value.

What will that do? That will change the output. I will assume that the output has not changed significantly. So, if the output is not changed significantly, then  $V_{im}$  has not changed significantly. Output is changed, but not by very large quantity. That is my assumption. So, what does it means?  $V_{ip}$  and  $V_{im}$  are more or less same as before and the average value  $V_{ip} + V_{im}$  by 2 will also be exactly the same as before and it is just equal to  $V_i$ . So, in this circuit, the common mode input of the opamp equals the input voltage. That is very clear that if the input is by some amount the negative input follows that. So, the average value of the two is exactly same as the input.

Now, because of the common mode gain, I will assume that the output has some changed output experiences and that is equal to the common mode gain times, the average value which is the same as  $V_{ip}$ . What happens because of this, the output voltage will be the ideal value of  $KV_i$  plus the common mode gain times, the common mode voltage which happens to be  $V_i$  in this particular circle.

So, what is the problem? First of all, it looks like the gain is not exactly what we want. The gain is  $K$  plus  $A_{cm}$ , instead of just  $K$ . That is the problem in itself. The value of  $A_{cm}$  is uncertain for this causes small error, but the bigger problem is that although I represented the transfer from the common mode voltages to the output as a linear function, I said that the output is simply the common mode gain times. The common mode voltage, it is frequently not that it is some non-linear function of the common mode voltage.

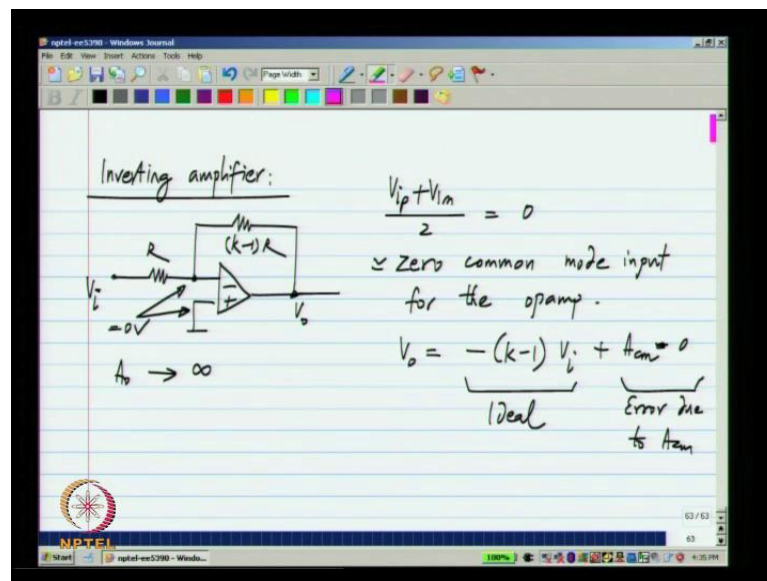
So, the output will be some  $A_{naught}$  time differential voltage plus some non-linear of the common mode voltage which I have approximated to be  $A_{cm}$  times  $V_{ip} + V_{im}$  by 2. Now, because it is non-linear, the error gets here is also non-linearly related to  $V_i$ . So, that means, the output is non-linear is related to  $V_i$  and it creates disquisition and so



on. So, this can be a problem. Now, you see that in the case, the common mode input to the opamp is exactly the same as the input signal that you apply, and this will contrast now with the inverting amplifier in which case the common mode voltage will be transferred to be much smaller. Now, I did not go through calculations.

What I did was I just assume that the output voltage changes by a small amount, so that the effect of that on the input common mode voltage is not by very large. I calculated the common mode voltage assuming there is no common mode gain and then, I used the same thing. In later calculations, you can do the exact calculations and say how much the error will be. The error will be not large. It is a sum that you can use if you understand the relatives quantities of different things. Now, let me take an inverting amplifier like before. You can put down all the expressions including the relationship between the input voltage of the opamp and the output including the common mode gain and solve for the output.

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I will not do that. I will use the same shortcut I will use. Before I will calculate the inputs of the opamp assuming there is no common mode gain and use to common mode voltage to calculate the error. Due to common mode gain, it is very easy. First of all, again I will assume in naught training to infinity. A naught being very large, what happens then the voltage here, this is the same as voltage here and that is equal to 0. The V naught is very large. These two voltages are close to each other. Even A naught is finite. This voltage

will be a small fraction of the output. So, you can now see that the average value of these two, the common mode voltage is very close to 0. So, the inverting amplifier is almost 0 common mode input for the opamp.

Now, because of the differential gain, the output of the opamp has to be, this is the ideal output. Now, we also have plus  $A_{cm}$  times the common mode voltage which is 0. It is almost 0 even if it is not exactly 0. It is quite small compared to  $V_i$  and  $V_o$ . So, this is the error due to  $A_{cm}$  as before it is not just  $A_{cm}$  times the common mode voltage, it is some non-linear function of the common mode voltage. It does not matter because common mode voltage itself is so small that the error due to that is small. So, this brings out one of the key differences between inverting and non-inverting amplifiers. Let us say you wanted amplifier of gain 10. You could make it either as inverting amplifier or a non-inverting amplifier.

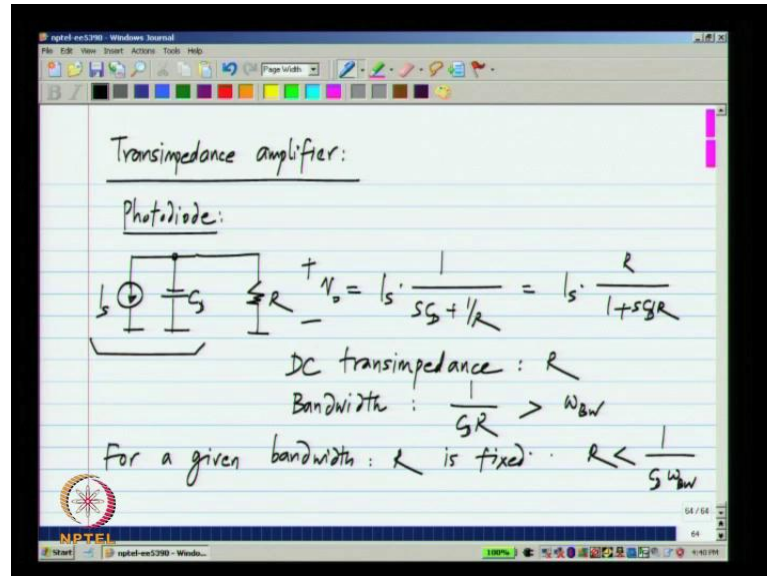
Now, how do you choose one versus the other. You will look at different performance matrix. For instance, the non-inverting amplifier has an input resistance. The amplifier has input resistance equal to  $R$ , the resistance that we use here, but there is also another significant difference based on the common mode performance of the opamp. You expect that because the common mode voltage is very large in a non-inverting amplifier.

The effect of non-linearity will come out much more strongly. We saw that the non-inverting amplifier has a very large common mode voltage. So, any effect of common gain and consequent non-linearity will be much larger in a non-inverting amplifier. Then, in inverting amplifier, the common mode voltage is 0. So, its effect will be very small. So, sometimes in destructions sensitive applications, you tend to use inverting topology instead of non-inverting topology. So, that is about the standard opamp and the opamp data sheet and so on.

So, in the rest of this lecture, we will discuss one more interesting circuits and trans impedance amplifier which is used frequently with photodiode. Now, this is somewhat disconnected from the rest of the lecture, but I will show that you get a better understanding of loop gain and stability criteria and so on. So, we have already articulated the stability criteria in many ways. First of all, you can look at the loop gain and use the nacre plot stability and in most practical cases, you did not have to draw the

nacre plot. You can simply look at what I plotted as and stability and see that only around the unity gain frequency, it has to have a 20 degree per decades loop.

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So, the circuit that I am going to discuss now also illustrates this in a slightly different way as you probably know a photodiode response to a light starting on it by producing the current. Usually you have a reverse diode and then, you have some light shining on it. There is some current that tends to flow through the diode. Normally, reverse biased diodes carries no current, but in a photodiode, when light falls on it, they will be current.

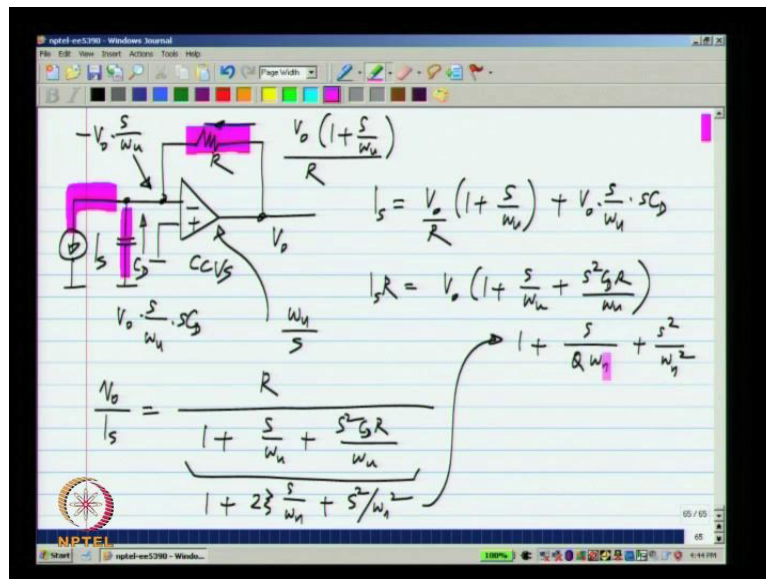
Now, this is represented by the current source. I will just call it  $I_s$  and the photodiode has reverse biased capacitance which can be significantly large and I will call this  $C_D$ . So, we will use this as the model for the photodiode and this is used very frequently in communicate systems where you have a light coming through in optical fiber and then, at times on a photodiode it gets converted to current and that current has become converted to the voltage that is detected later.

So, what we do? There are many ways of converting a current to a voltage. We already discussed this in different context earlier. What are the possibilities? First of all, I could simply connect the resistance across the photodiode. Now, the transfer function between this  $I_s$  and this voltage is given by  $I_s$  divided by  $1$  by  $R$  plus  $sC_D$  or in other words, the output voltage is  $I_s R$  by  $1$  plus  $sC_D$  times  $R$ . So, at DC, the trans impedance is  $R$  and the bandwidth of this is the first order circuit. So, I am assuming familiar with the

bandwidth of the first order system, it is 1 over CD times R and please remember that CD is  $(\omega)$  that you want to give. You cannot change that. You certainly cannot reduce it. You can add to it.

Now, you can see if you add any more capacitance to it, the bandwidth only reduce this, but you cannot reduce it. So, what does it mean for a given bandwidth R is fixed. So, if you want bandwidth is greater than let say, some A omega BW, this means that R is smaller than 1 over CD times omega BW. Now, if you want a large bandwidth, you can only have a small gain. In this case, the gain from the input currents, the output voltage is the DC trans impedance and that is R, and if you want wide bandwidth, you can only have a small gain and that is constant. There is nothing you can do in this circuit to increase the gain. If you increase the gain, the bandwidth will reduce. So, instead of this kind of circuits, let us strained use the current control voltage source and see what happens.

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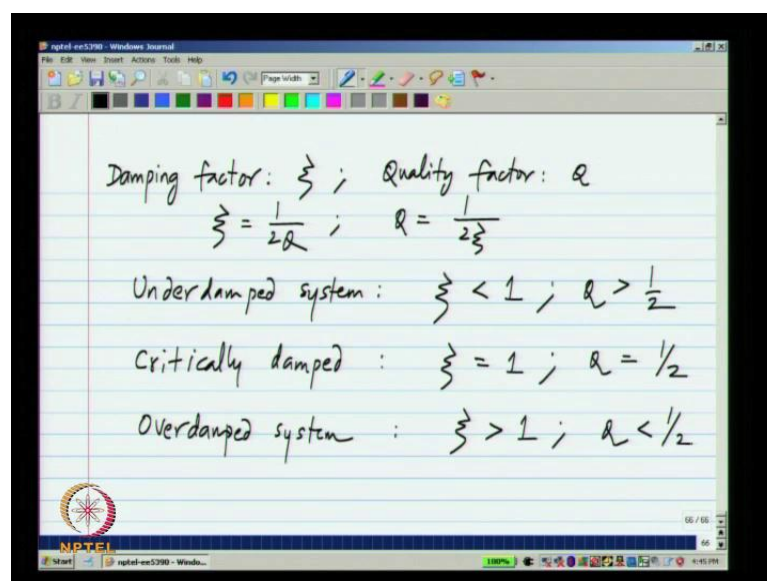
I know what the current control voltage looks like, this is the current control voltage source and I connect my photodiode to this, and the capacitance that will always with be there with the photodiodes will get connected to that one. I will assume that the opamp has certain unity gain and frequency omega U and I will model it as an integrator. So, the transfer function of the opamp itself is mega U by S, right.

So, I will assume that the DC gain is infinite that is for this circuit, it does not turn out. You have to consider the effect of the DC gain only if you are looking specifically at the accuracy of the amplifier around DC. So, if you are looking at stability and other things which are going to look at, you can usually model it as a generic rater and better way of it. So, what will be the voltage here? If this has a transfer function of  $\omega U$  by  $S$ , the voltage here is nothing, but minus  $V$  naught  $S$  by  $\omega U$ .

So, the current flowing here is  $V$  naught  $1$  plus  $S$  by  $\omega U$ . The current flowing through the capacitor upwards is  $V$  naught  $S$  by  $\omega U$  times the admittance of capacitor which is  $SCD$  and the sum of these two, this and that has to equal  $I$   $S$ . Sorry, I made a mistake here. The current flowing through the register  $V$  naught times  $1$  plus  $S$  by  $\omega U$  divided by  $R$  of course and we have plus  $V$  naught  $S$  by  $\omega U$   $SCD$ . So, I will take  $R$  this side and I will have  $(\ )$ . So, now, the transfer function  $V$  naught by  $S$  to be transfer  $R$  which is the DC gain or DC trans impedance of this divided by  $1$  plus  $S$  by  $\omega U$  plus  $S$  square  $CDR$  by  $\omega U$ .

This is the second order system. We know that those standard for  $S$ . So, this is the damping factor. Alternatively, we can represent as a quality factor. It will be  $1$  plus  $S$  by  $Q$   $\omega N$  plus  $S$  square by  $\omega N$  square the damping factor, and quality factor alternative ways of representing the same second order function whether it is critically damp, over damp or under damp.

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The damping factor usually denoted by zeta and quality factor is denoted by Q. The two are related by that relationship and under damp system will have zeta less than 1 or quality factor greater than half. A critically damped system will have zeta equals 1 or quality factor equals half and finally, over damped system will have zeta greater than 1 or quality less than half. So, you can use whichever is compatible with.

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The image shows a handwritten circuit diagram of an inverting op-amp with a feedback network consisting of a resistor  $R$  and a capacitor  $C$  in parallel. The input is a voltage source  $V_i$  through a resistor  $R$ . The output is  $V_o$ . The equations derived are:

$$I_s = \frac{V_o}{R} \left(1 + \frac{s}{\omega_n}\right) + V_o \cdot \frac{s}{\omega_n} \cdot sC$$

$$I_s R = V_o \left(1 + \frac{s}{\omega_n} + \frac{s^2 CR}{\omega_n}\right)$$

$$\frac{V_o}{I_s} = \frac{R}{1 + \frac{s}{\omega_n} + \frac{s^2 CR}{\omega_n}}$$

$$\omega_n = \sqrt{\frac{\omega_n}{CR}}$$

$$2\zeta = \frac{1}{\omega_n} \Rightarrow \zeta = \frac{1}{2} \frac{\omega_n}{\omega_n} = \frac{1}{2} \sqrt{\frac{1}{\omega_n CR}}$$

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The image shows handwritten equations for calculating the damping factor  $\zeta$  and natural frequency  $\omega_n$ :

$$\frac{V_o}{I_s} = \frac{R}{1 + \frac{s}{\omega_n} + \frac{s^2 CR}{\omega_n}} \quad \text{dc gain: } \frac{R}{\omega_n}$$

$$\zeta = \frac{1}{\sqrt{2}} \quad ; \quad \frac{1}{2} \sqrt{\frac{1}{\omega_n CR}} = \frac{1}{\sqrt{2}} \quad \zeta = \frac{1}{2} \sqrt{\frac{1}{\omega_n CR}}$$

$$\omega_n CR = \frac{1}{2} \quad ; \quad \omega_n = \frac{1}{2CR}$$

$$\omega_n = \sqrt{\frac{1}{2CR} \cdot \frac{1}{CR}} = \frac{1}{\sqrt{2} CR}$$

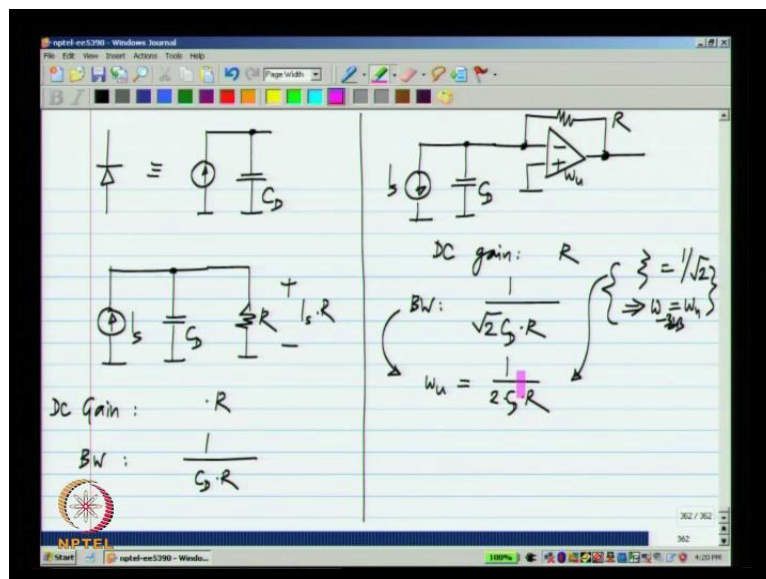
This case I will use zeta. Let us calculate quality factor and natural frequency. The natural frequency is  $\omega_n$  by  $CR$  times  $R$  and we know that  $2\zeta$  by  $\omega_n$  is 1

over  $\omega U$  from this comparing this is to standard form. So, zeta is given by half square root of  $1 + \omega UCD$  and  $R$ . In other words, let us repute the results had  $V$  naught by  $SR$  by  $1 + S$  by  $\omega U$  plus  $S$  square  $CDR$  by  $\omega U$ , and this transfer function as DC gain of  $R$ , natural frequency  $\omega N$  of square root of  $\omega U$  by  $CDR$  and damping factor of zeta of half square root of  $\omega UCDR$ . Now, we would like to constrain the damping factor. It may not be 1, but certainly it cannot be very small like 1 and so on. Usually use damping factor of same were between 1 and half for simplicity. Let us assume that zeta happens to be  $1$  over square root of  $2$ .

So, this means that half square root of  $1$  over  $\omega UCDR$  is  $1$  over square root of  $2$ . So,  $\omega UCD$  times  $R$  happens to be half. Here, that is what comes out from that. Now, natural frequency is given by square root of  $1$  over  $2 CDR$  times  $1$  over  $CDR$  which is basically  $1$  over square root of  $2 CD$  times  $R$ . Now, it turns over that when the damping factor is  $1$  over square root of  $2$ , the bandwidth of the circuit is exactly equal to the natural frequency and that is  $1$  over the square root of  $2 CD$  times  $R$ .

Now, what is that saying? So, first of all, please be aware of the constant. We want to make the damping factor equals  $1$  over square root of  $2$ . So, that means that the value  $\omega U$  should be adjusted to be  $1$  over  $2$  times  $CDR$  and the bandwidth of the circuit will also be  $1$  over square root of  $2$  times  $CD$  times  $R$ .

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These are the expressions that we get for the trans impedance amplifier that we are made, that is current control voltage source for converting the current of the photo diode voltage. You compare this previous case originally. I had a photo diode which I am model as a current source with sum capacitance  $C_D$ .

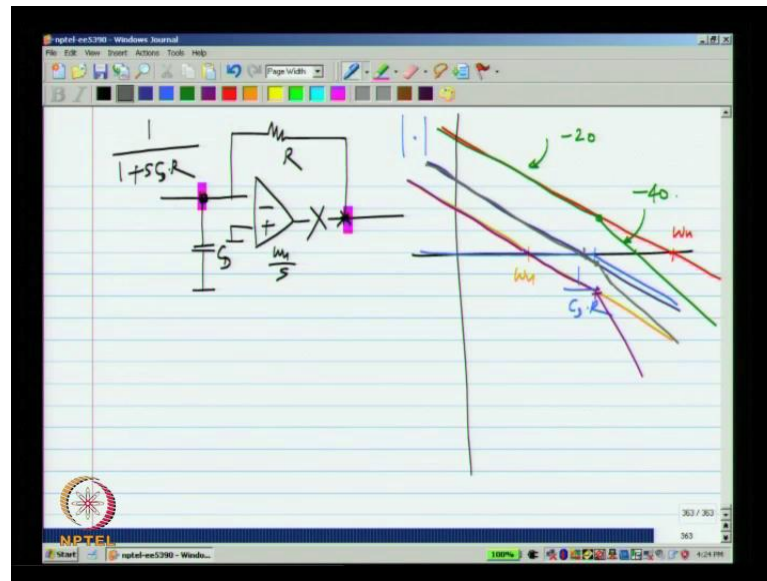
The first circuit I had was to simply connect a resistance across the photo diode to get the voltage if this is  $I_{SCD}$  and I have a resistance  $R$ . So, the output voltage is times  $R$ , the gain of the circuit is  $R$  and the bandwidth of the circuit is  $1$  over  $C_D$  times  $R$ . Now, I have a new circuit which I think is improvement inside of having resistance directly across the photo diode. I will use the current control voltage source  $I_S$ . In this case also, the DC gain is  $R$  and the bandwidth, it depends on damping factor and so on, but here what I have calculated is for a damping factor  $\zeta$  of  $1$  over square root  $2$ , it turns out for this particular value of damping factor that bandwidth of the circuit equals the natural frequency of the circuit. So, it will be  $1$  over square root  $2$   $C_D R$ .

The damping factor is  $1$  over square root of  $2$  which means that  $3$  DB bandwidth happens to be equal to the natural frequency of the second order system. Now, is this really improvement? First of all, it looks like the bandwidth has to be reduced by the way to get particular bandwidth. This particular damping factor we need to use an opamp unity gain frequency is  $1$  over  $2$  time  $C_D$  times  $R$  inverse. It is not a good idea to use a higher unity gain frequency for the opamp. We will see that. It will be a result in a smaller damping factor or a lot of ringing. So, what it means is we can use this circuit and it converts a current to voltage, but for it not to have a ringing, we are let us say we choose data equals to  $1$  over square root of  $2$ . We have to choose a particular value of  $\omega_U$ .

Now, it is assumed here that the diode capacitance  $C_D$  and the DC gain are given. Now, the problem with the first circuit was that once these two are given, the bandwidth is fixed. So, if you specified the large gain, you will have to suffer a low bandwidth. Now, it seems the same is going to happen even when we have the opamp. When we have a opamp, first of all we can have a high natural frequency by using high unity gain frequency for the opamp by choosing an opamp with a very high unity gain frequency when it turns out that also reduces the damping factor to an  $(\zeta)$ . So, another way to look at this is to examine in the loop gain.



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Now, for a loop gain calculations, I do not need the source and I can break the loop here. I will assume that this is an integrator of the form  $\omega U$  by  $S$  and the transfer function here to there from this point to that point is  $1$  over  $1 + SCD$  times  $R$ . Let me plot the two magnitudes separately. This is  $\omega U$  by  $S$ . Rather let me plot this one by  $1$  over  $SCDR$ . Magnitude response looks something like that where this is  $1$  over  $CD$  times  $R$ .

So, this is bode magnitude plot. Now, how do you choose  $\omega U$ ? Let us say, we choose  $\omega U$  over here sum very large value of  $\omega U$ . What happens then is the response from the opamp the magnitude opamp will be like that and combine loop gain response which is the product of this to  $\omega U$  by  $S$  times. That whole thing will be something like this. Now, after this point, it falls of it 40 degree per DB.

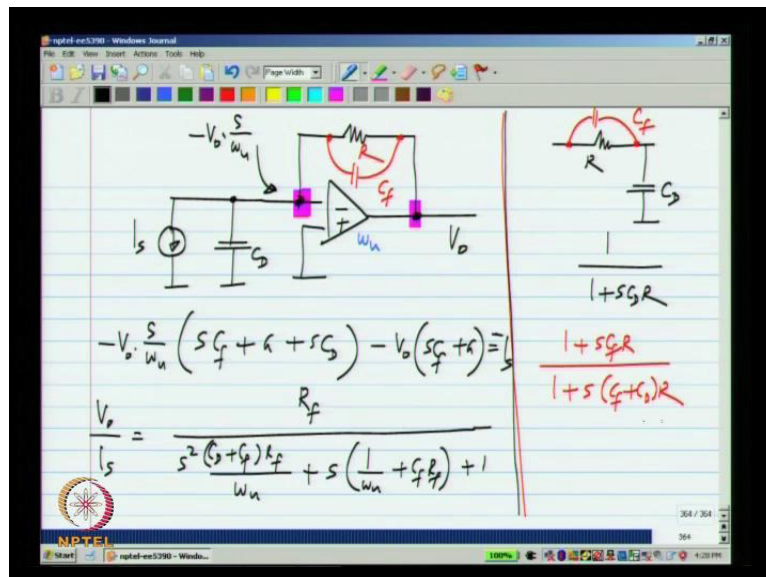
This is minus 20, minus 40. Now, this is clearly a problem. Our stability criteria show that when loop gain, cross unity loop have to be around minus 20 degree per DB, that is it should have an integrator like behavior or a first order behavior. When it crosses unity gain frequency, they have annualized this in so many ways. We said they should behave like integrator or the nacre plot fall should cross the circle around the negative access action and so on, and this is clearly not doing that and this is a problem because we have chosen very high valve of  $\omega U$ .

Let us try a smaller value of  $\omega U$ . So, let us say we had  $\omega U$  somewhere over here. Then, everything seems here. Then, the loop gain will be something like this. It will

have first order behavior up to sum value way below the unity loop gain frequency and it will do that this is perfectly fine. Now, this is the value required for damping factor of 1 over square root of 2 and that corresponds to somewhere over here. In this case, what happens is the loop gain will have 20 degree per DB slope.

This point just behind the unity loop gain frequency and from it will flow up with 40 degree per DB. So, that is the limit and this is why we cannot have a very high omega U. So, even if you are able to build an opamp with a very high unity frequency, it is useless, even the value CD and R. We simply cannot choose the higher value of unity gain frequency and prevent severe ringing.

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So, what can be done about this is there is the clean solution to this as well which can be also thought of many ways as usual. We know that ringing appears when we have too much delay in the feedback loop. That is a one way to think about it or when we have low frequency parcels pool. So, on one way to counter the effect of the pool is by adding a 0. Now, from the output of opamp input, we have a first order system R and CD which has the transfer function 1 by 1 plus SCDR to this. We can have a 0 by adding a capacitor across the resister. This will have a transfer function 1 plus SCDR divided by 1 plus SC F plus CD times R.

So, this can be through of many ways we can think of it in the frequency domain. This 0 will prove a space lead alternately you can think of. It does the resister take long time to

charge the capacitor. So, you provide a lot of charge by connecting capacitor across it. So, apply a step input. The output also a step because the C F provides the capacitor, C F provides lot of charge or alternatively you can think of fast path that is by passing a slow path R. So, either way you can, I mean the result is that in the frequency domain, you have 0 that provides (()) lead or in the time domain, you have advance compare to what you have earlier.

Advance meaning a negative delay having delay. Of course, the delay of this circuit is not negative. It is just less than what it was before and again, I will use an opamp with certain unity gain frequency omega U. Now, what happens if I write down the node equations here is this V naught what has to be minus V naught S by omega U. So, I know that minus V naught omega S by U time SC F plus G plus SCDG is the reciprocal resistance R minus V naught C F plus G has to be equal I S minus, I S in this case. So, if I solve for this, I will get V naught by I S to be R F divided by S square C D by C f R f by omega U plus S 1 over omega U plus C f R f plus 1.

If you compare this, what you have earlier is exactly the same. If you reduce C F2, the 0 that is what we will get. Now, what is good about this is let us again try to get a damping factor of 1 over square root of 2 and see where it leads. Remember our problem was that regardless of what opamp will use, we are not able to get a higher bandwidth, given the photo diode capacitance CD and the required DC gain or the DC transe impedance R.

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The image shows a digital whiteboard with handwritten mathematical equations. The equations are as follows:

$$\omega_n = \sqrt{\frac{\omega_u}{(G+G_f)R}}$$

$$\zeta = \frac{1}{\sqrt{2}}$$

$$\frac{2 \cdot \zeta}{\omega_n} = \frac{1}{\omega_u} + G_f \cdot R$$

$$\sqrt{2} \cdot \sqrt{\frac{(G+G_f)R}{\omega_u}} = \frac{1}{\omega_u} + G_f R$$

$$2 \cdot \frac{(G+G_f)R}{\omega_u} = \frac{1}{\omega_u^2} + \frac{2 \cdot G_f R}{\omega_u} + (G_f R)^2$$

Additional annotations include a red box around  $\omega_{MB} = \omega_n$  and a red bracketed equation:  $(G_f R)^2 = \frac{2 \cdot G_f R \omega_u - 1}{\omega_u^2}$ .

So, once the CD and R given, our bandwidth seems to be fixed in this case natural frequency  $\omega_n$  equals square root of  $\omega_u$  divided by CD plus C f times R and 2 times the damping factor divided by  $\omega_n$  will be 1 over  $\omega_u$  plus its C f times R. Now, I will show the damping factor 1 over square root of 2. It is a convenient value and for this particular value of damping factor, the 3dB bandwidth happens to be equal to  $\omega_n$ , but the conclusion equals to same if you do the calculations with any other reasonable value of damping factor from half to 1 for this damping factor being half, I will have square root 2 times square root of CD plus C f R divided by  $\omega_u$  2 equal to 1 over  $\omega_u$  C f times R squaring both sides. I have that equal to 1 over  $\omega_u$  square plus 2 times C f R by  $\omega_u$  plus C f R square.

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The image shows a digital whiteboard with handwritten mathematical derivations. The top left shows  $\omega_{-3dB} = \omega_n$ . To the right, the equation  $(C_f R)^2 = \frac{2C_D R \omega_n - 1}{\omega_n^2}$  is written. Below these, the derivation for  $\omega_n$  is shown:  $\omega_n = \sqrt{\frac{\omega_u}{C_f R + C_f R}}$ , which is then rearranged to  $\omega_n = \sqrt{\frac{\omega_u}{C_f R + \frac{2C_D R \omega_n - 1}{\omega_n^2}}}$ . This is further simplified to  $\omega_n = \sqrt{\frac{\omega_u}{\omega_n C_f R + \frac{2C_D R \omega_n - 1}{\omega_n}}}$ . The final result is  $\omega_{-3dB} = \omega_n = \sqrt{\frac{\omega_u}{\omega_n C_f R + \frac{2C_D R \omega_n - 1}{\omega_n}}}$ , with pink annotations highlighting the terms  $\omega_n$  and  $\frac{2C_D R \omega_n - 1}{\omega_n}$ .

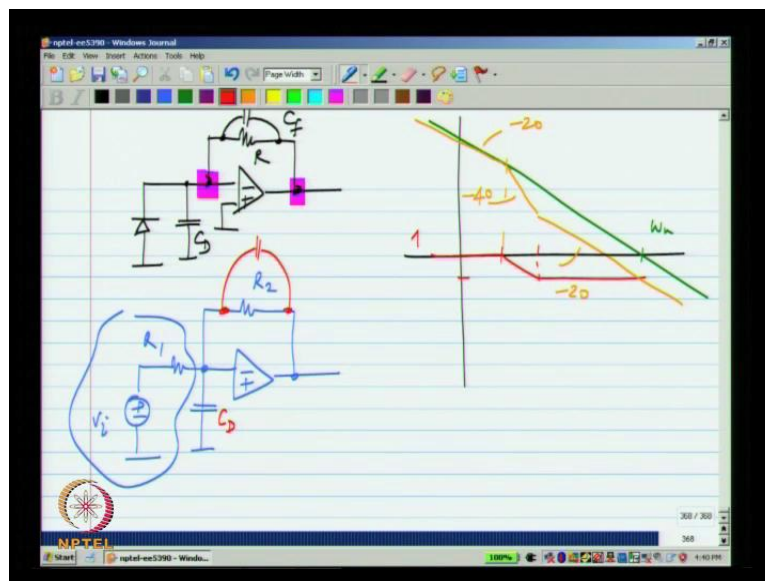
So, this part times of with that part and we have C f R square equal to 2 times CDR  $\omega_u$  minus 1 divided by  $\omega_u$  square, ok. Now, given  $\omega_u$ , you can chose C f for this damping factor. It basically use a relationship between in different quantities, but what we are finally interested in is the value of  $\omega_u$ , which is equal to square root of  $\omega_u$  divided by CD times R plus C f times R. Now, I have the equation for the C f times R. I know that C f times R square equals 2 CD times R times  $\omega_u$  minus 1 divided by  $\omega_u$  square.

So, if I substituted by here and I will get square root of  $\omega_u$  divided by CD times R plus square roots of these expression which is 2 CDR  $\omega_u$  minus 1 divided by

$\omega_u$  which finally simplifies to  $\omega_u$  divided by  $\sqrt{2 \text{CDR} \omega_u - 1}$  and square root of whole. The expression looks to be complicated, but the point I want to make here is that 3D bandwidth depends on  $\omega_u$ .

Now, both the numerator and denominator depends on  $\omega_u$ , but the numerator is directly proportional to  $\omega_u$  whereas, the denominator as is under some square roots. So, the point here is that is possible to use the faster opamp meaning an opamp to the higher unity gain frequency and increase the bandwidth of the circuit. Now, what is the condition? For this to happen, we need to introduce  $\omega_0$  with the capacity  $C_f$  and adjust the value of  $C_f$ . According to the expression, we just derived which is  $C_f R \omega_u^2$  should be  $2 \text{CDR} \omega_u - 1$  divided by  $\omega_u^2$ . So, to increase the bandwidth of this particular circuit, what you need to do is choose a high bandwidth of opamp and choose the appropriate value of the capacitor, ok.

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So, this circuit is commonly used with photo diode trans impedance amplifiers. Photo diode invertible comes with some CD, where you can have some R and this CR. Now, quickly if you look at the loop gain here, what happens is that the feedback network from here to there as the loop gain that look likes starts with unity and low frequency, and as a pole at  $1 / (C_f + \text{CD} \times R)$  and also as a zero at  $R / (C_f \times R)$ . So, the gain become flat again at high frequencies and the phase shift also goes away. So, now if you

multiply with this, let say an opamp which has the unity gain frequency somewhere over here. Now, over all loop gain will look something like that. It will have a minus 20 degree per DB by slope here and minus 40; it comes back to minus 20.

So, the slope changes twice and when it reaches unity loop gain frequency, this slope is minus 20 degree per DB. So, this is possible to have the circuit to be stable even in the very high value of  $\omega$ . This is an interesting value where by using a 0, we introduce a space lead or a reduce delay and make the circuits stable and also the bonus that we get for the trans impedance amplifier is that when you give diode capacitor CD, and resistance R. You can still increase the bandwidth by using a faster opamp by using appropriate C f. Remember this was not possible with either of two circuits by connecting a resistor directly across the photo diodes by using a trans impedance amplifier.

Now, just have a last quick recap. This is technically also useful when we have amplifier. Now, there will always be some parasitic capacitance here if you reduce this combination to not an equivalent. You will have a current source at the input and that looks exactly the same as that one expect for a shunt resistor R1. So, in this case also, by putting the capacitor across R2, we can do something if you ran into stability problems because of this capacitance difference, I will call it CD. It is not due to any diode. It is just a parasitic capacity of the input. You can introduce it as 0 and fixes the situation. Now, when you have a resistor, you should have a freedom to simply use the resistor values, so that the parasitic to pole, the most of high frequencies. Now, that is for some reason is not permitted that you can introduce as 0.

Thank you. I will see you in the next class.