Analog Integrated Circuit Design
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Lecture No - 13
Two Stage Miller Compensated Opamp

Hello and welcome back, this is lecture 13 of Analog Integrated Circuit Design. In the previous class, we look at simple realization of an opamp at the level of the controls sources, we realize that using voltage control current source loaded by capacitor. We also saw that, the output resistance of the voltage control current source limits the dc gain and this consequently results in steady state error.

So, even after a long time, the output does not reach exactly the desired value, but will be a little bit away depending on the amount of dc loop gain. so, the dc loop gain has to be higher than a certain value and this requires us to have different opamp topologies, which can possibly realize higher and higher dc gains. In this lecture, we will look at one such opamp, which will perform better than one that we saw in the previous class.

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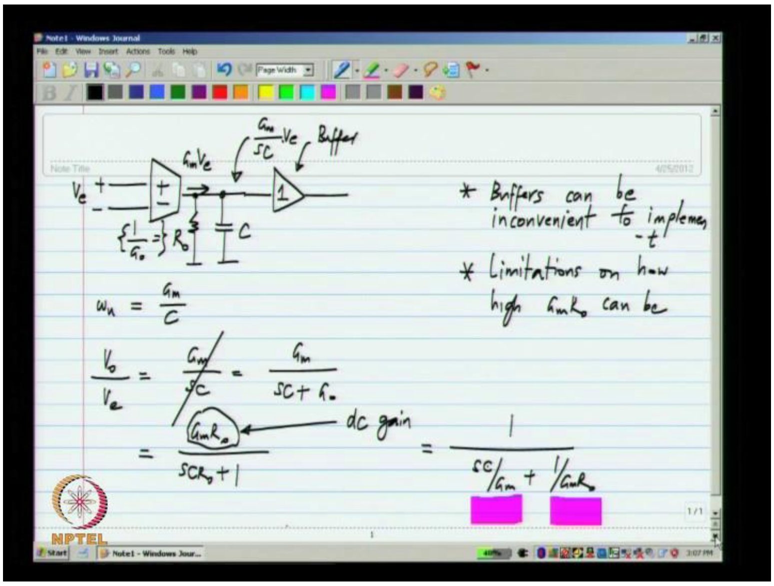

The opamp that we had was the voltage control current source or a trans conductor loaded by a capacitor and to isolate the external load, we can use a buffer, but it turns out that, buffers are not very easy to implement in CMOS technology. So, they can be implemented, but they bring with them their own limitations, we would like to avoid them. So, most of the time, opamps are used without explicit buffers in CMOS processes, this is the input voltage and a current G m V e is pushed out of it.

So, the voltage here will be G m by SC times V e, where this is the capacitor C, so the unity gain frequency of this opamp omega u equals G m by C. Now, even if the buffer isolates the external load, the trans conductor has an output resistance R naught or an output conductance G naught, which are reciprocals of each other. This is an inherent property of a voltage control current source, just like a current source as the output resistance which is not infinite, a voltage control current source also has an output resistance which is not infinite.

So, because of this, the transfer function that we will get, will be the output by input of the opamp, should have been G m by SC, but this is not what we will get, we will get G m by SC plus the output conductance. This can be written in various forms, we can write it as G m R naught divided by SC R naught plus 1, which makes the dc gain explicit. At dc or low frequencies, we have again equal to G m R naught and the pole which was at the origin in this transfer function has moved to a frequency minus 1 over C R naught.

It can also be written in a alternative form as 1 by SC by G m plus 1 by G m R naught, let me a write it on this side. In this case, this is the ideal part that we would like to implement, additionally we have a small non ideal number here, 1 over G m R naught. So, the higher the value of R naught, the higher the dc gain G m R naught and the smaller will be this non ideality. But, in general, we will never be able to make this in finite, so we will have live with some finite value of G m R naught.

And in fact, depending on the topology that we choose, the value of G m R naught may be limited to some modest value like 50 or 100 or so. Whereas, sometimes we would like to have a opamps with the gains of 10000 or even a million, so there are limitations on, how high G m R naught can be. Now, this depends on the topology, now depending on the topology, we could have either 25 or 250, but there will always be some limitation.

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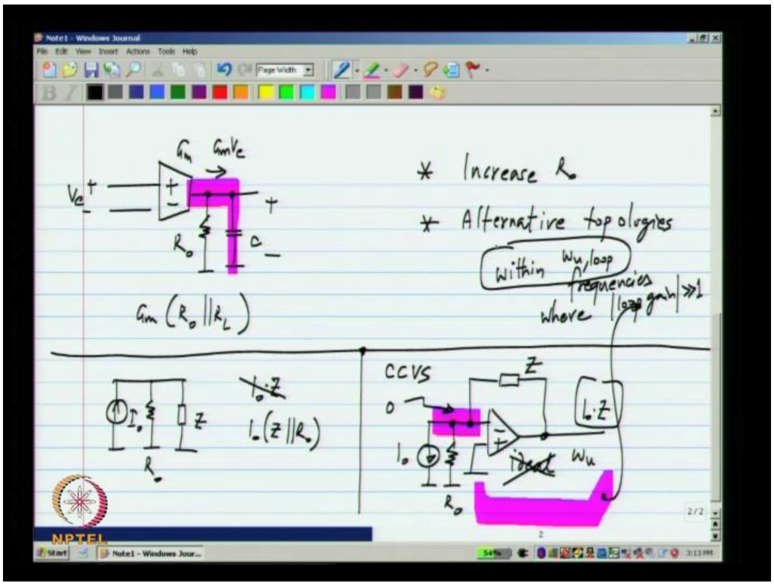

Additionally, what happens is that, sometimes that imagining a case, where the opamp is used without a buffer. Let us also assume that, there is a resistive load R L, now in this case, the dc gain of this opamp will be G m R naught parallel R L. And regardless of how high you make R naught, you will always have R L, so the dc gain will be limited to G m times R L. So, this is another problem that, sometimes when you have the resistive loads, the dc gain will be limited by that and we have to find ways of obtaining higher dc gains even with external loads.

So, basically there are two possibilities, there are different possibilities of trying to increase dc gain. First is increased R naught that is, we do something to the trans conductor so that, it is internal resistance is increased. Now, this clearly does not help when we have an external resistive load R L and we may have to use alternative topologies. So, during the initial analysis, we still assume that, there is no external load, we will derive the topology and then, show that, even with external resistive load, these things can work well.

Now to investigate alternative topologies, first we need to find out exactly, why this topology results in limited dc gain. It is quite simple, ideally we would have wanted all the current from this trans conductors G m to flow into the capacitor C. Now, what happens is, when you have R naught, a part of it flows into it, so that is why, we get a finite dc gain. As oppose to, when we had only a capacitor, we would have in finite dc gain and we had an ideal integrator.

Now, essentially what we are doing here is, converting the output current G m V e of the trans conductor into a voltage by passing it through a capacitor. Now, if we find the different way of converting this current to a voltage G m by SC without having some of the current going to some other component, we will make a better integrator, so that is the problem. Now, that is the well known problem, which also has a well known solution.

The problem basically is that, lesser you had a current I naught, going into an impedance z and the current source as some internal output resistance R naught, what happens is a part of this current is going to that. So, the output voltage, instead of being I naught times the impedance z, will be I naught times z parallel R naught. And the way to get around it, is not to simply try and pass the current through a load resistance by applying the load across the current source, but to make what is known as the current controlled voltage source.

And we have already seen the topology of a current control source using an opamp and for now, let us assume that, the opamp itself is ideal, I naught and I connect the same impedance z in feedback. And initially, let me assumed that, the opamp is ideal what does it mean, this voltage is 0 and the output voltage will be exactly equal to I naught time z, assuming that opamp is operating in negative feedback. Now, what happens if the current source is non ideal and we have resistance R naught.

Because this voltage is 0, no current flows through the resistance and all of the current I naught still flows through this impedance z. So, even in presence of R naught, the output voltage will be I naught times z, so that is why, when you want to convert a current to a voltage, it is better not to simply apply the impedance across the current source although that is possible, it is better to use a current control voltage source of the appropriate trans impedance value.

We can clearly see that, the problem at hand for us is also the same, we have a current G m V e, which has to pass through a capacitive impedance. Now, we were simply apply the capacitance to the output of the trans conductor, that is not what we should do. We should try to make a current control voltage source, whose input is the output current of the trans conductor and it is output voltage will be the output of the opamp, we will see how to do that.

Before we go there, one more thing to keep in mind is that, of course we will not have an ideal opamp, we will have a real opamp with some unity gain frequency omega u. Now, what is the range of frequencies over which, this behaves like a current control voltage source. Behaves like a current control voltage source over the range of frequencies, where the loop gain magnitude is much more than 1. This we have seen earlier, rather basically it is a range of frequencies within the unity loop gain frequency of this particular feedback loop.

This is very important point, this opamp will be a real opamp and it has to be such that, it is unity gain frequency is higher than the frequency of interest. Now, what is the frequency of interest for us, we would like our opamp to behave like an integrator over a certain range of frequencies and the unity gain frequency with the opamp, which is used to make the current control voltage source has to be much higher than the unity gain frequency with the opamp we are trying to realize that is, G m by C. We will see all of these things in more regress analysis later, but it is a good idea to get an intuitive feel for, how thing should be when we design that.

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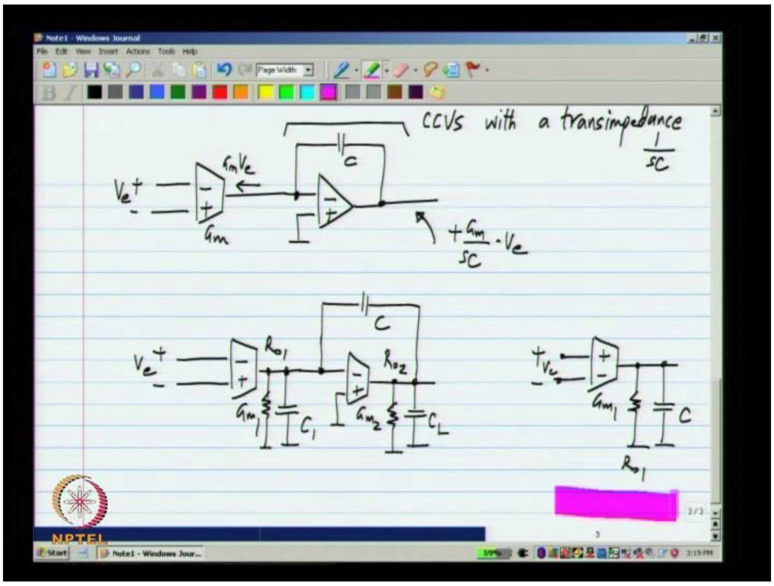

So, how do I make my opamp, instead of simply loading the trans conductor with the capacitor C, I will not do this, I will pass it through a current control voltage source, whose trans impedance is C with the trans impedance of 1 by SC. Now, with an ideal opamp, the voltage here will be 0 and the output voltage will be, I have a current G m V e here, so the output voltage here minus G m by SC times V e. The sign inversion comes, simply because the inversion through the second stage, so to get rid of that inversion, what I will do is, I will invert the signs of the trans conductor, I will have minus plus.

So that, I have G m V e flowing that way, it will flow in the same way through the capacitor and the output voltage will be plus G m by SC times V e. Obviously, this opamp will not be ideal, because if you had an ideal opamp, we would try to just use it and not make another opamp with it. So, this will be some real opamp and we have to make sure that, even with the real opamp, all our assumptions hold. Now, what is the opamp that we know, there is only one opamp that we know so far and that is, this particular opamp.

This is the opamp that we used in the previous lecture, discussed in the previous lecture and this is the only opamp that we know, so we will just use it in it is place. So, please understand, what is going on here, we want to make an opamp, so we will instead of loading the trans conductor with the capacitor, we will follow the trans conductor using a current control voltage source. But, to make the current control voltage source, we need some opamp., so we will use the simplest opamp that we know, which is basically a trans conductor, which is loaded by a capacitor.

So, if we do that, what do we get, this is the opamp used to make the current control voltage source and I will call the C L, C L will be any capacitance that is loading the trans conductor plus any external load that maybe applied to the opamp. So, that will always be present, so all of that is clubbed into a single capacitance C L and I have V e here, I will call this G m 1 and I will call this G m 2, just to distinguish between them. And also, invariably between any node and ground, there will be parasitic capacitance, between the output of the first ground, there will be some capacitance, which I will call C1.

In analysis, we need to include the effect of all of these things and finally, figured out what exactly happens. So, the reasoning so far I said that, this opamp will be better than using just that one that is, G m 1 loaded by a capacitor C, so that the reasoning, by which we derived all this. And also let us put the limitations in place, each trans conductance will have some output resistance.

Like I said earlier, you cannot make a current source with an infinite output resistance, you can also not make a trans conductor or it voltage control current source within infinite output resistance. So, this is the topology that we have two analyze and see, if it really better than what we started off with, which is that one.

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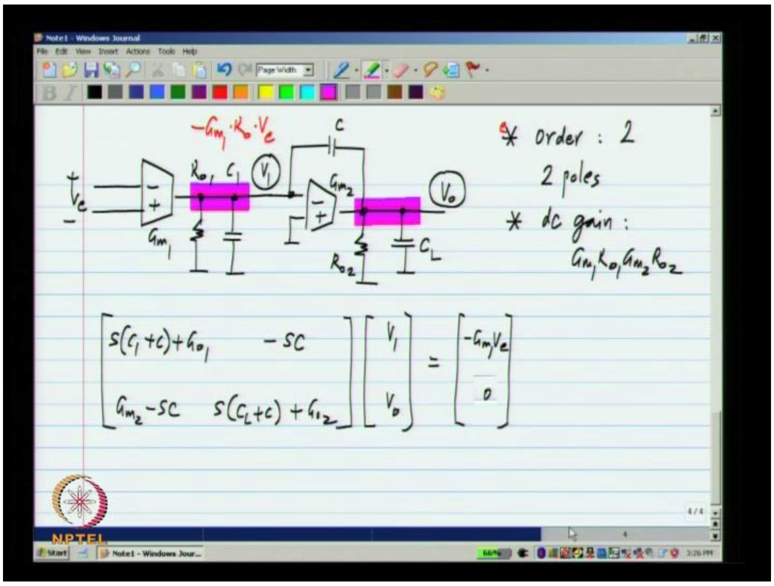

I will redraw my opamp here, we need to do analysis of this and find out the ratio of output voltage to the input to the opamp V e and that will be the transfer function of the opamp. Now, before we go and do the full fledged analysis, it is a good idea to look at this circuit and see, what the transformation might come out like, this will serve later as a sanity check for us. Now, what will be the order of this transfer function, how many poles will it have, the number of poles or the order is nothing but, the number of independent state variables in the circuit.

State variables are nothing but, capacitor voltages and inductor currents, here of course we do not have inductor, so it is only capacitor voltages and we have three capacitors connected like this. So, there can be utmost three state variables, but we also see that, the voltage on C1 plus the voltage on C equals the voltage on C L. The three capacitors are connected in a loop which means that, only two of them can be independently set. So, there are really only two independent variables so that means that, this will be a second order transfer function that means, there will be two poles.

Now, there can also be any number of zeros, that is a little harder to figure out, we will be later how to look at the circuit and try to figure out the frequency of the zeros. But, one guideline is that, whenever you have two parallel paths from, let us say the output of the first stage to the final output, there is a path through G m 2, there is also a path through the capacitor. In general, you can expect zeros in such a case and so, we will also expect that, there will be 0 here.

But, this is a little more vague and it is also possible that, you do not have a 0 or there may be 0 even without two parallel path and so on, but that is a some expectation that we have. And finally, what will be the dc gain of this, again when we find out transfer function and set the value of s to 0, we should get the dc gain. Now, we can also find the dc gain independently without doing the full blown analysis with Laplace transforms .If we do that then, after we do the analysis, we can compare it to this value and see, if it is satisfies sanity check or not.

That is again extremely simple, for dc all that happens is that, all this capacitors are open circuited C 1, C and C L. We have a current G m 1 times V e going into R o 1. So, the voltage here will be G m 1 R o 1 V e, negative of that and that is applied to the second trans conductor, which provides a current G m 2 times that voltage, which flows into the output resistance R o 2. So, the output voltage will be plus G m 1 R o 1 G m 2 R o 2 times V e.

It is basically a product of the dc gain of the first stage and dc gain of the second stage, this is something that you easily expect. When you have a cascade of stages, you will have the product of dc gains to be the total dc gain and that is, G m 1 R o 1 G m 2 R o 2 in this case. So now, let us do the analysis, this circuit has two nodes and by writing KCL equations of these two nodes, we can find out away all the voltages and currents in the circuit.

First of all KCL of this node, let me assign this node voltage to be some V 1 and this is of course, V naught. And I will write it in a matrix form, some admittance matrix times the vector of voltages V 1 V naught equal the vector of source currents at this current flowing into this nodes. First of all, the current flowing into this node containing v 1 is minus G m 1 times V e, this is provided by the first trans conductor and the matrix entries can be filled up.

This entry here is the total admittance, which is SC 1 plus C plus G o 1, G o 1 is the reciprocal of R o 1, is convenient to use the conductance directly, instead of writing it as 1 over R o 1 everywhere and this term is minus SC. Basically, the current through the capacitor is SC times V 1 minus V o, that is why we get plus SC here and minus SC here, that should be familiar to you from basics circuit analysis. Now, similarly the entry here is the total conductance at the node containing V o.

So, that is s C L plus C plus G o 2 and the entry here S, the current being drawn from this node due to the voltage on that node and that happens due to components, one is C and other one is G m 2. It turns out that, will get G m 2 minus SC, so this is the system with two nodes and there are two equations and by solving for this, we can find out the value of V naught in terms of V e and there are any number of ways to solve this, you can invert the matrix and so on. But, since we are only interested in the output variable, we will use Kramer's rule which says that, the output voltage V naught will be equal to the determinant of this matrix, when the second column is replaced by the source vector divided by the determinant of the admittance matrix, so let me copy this over.

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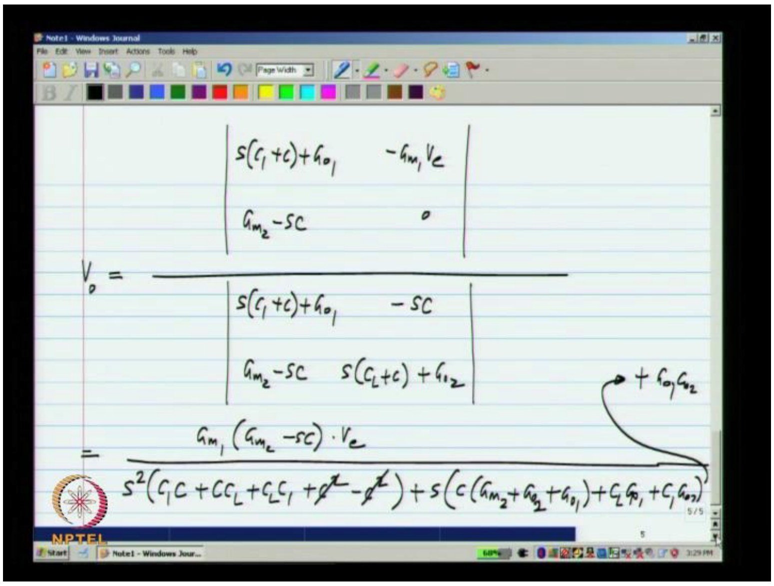

So, I will copy this over again, so as I said, V naught is nothing but, with determinant with the second column replace by the source current vector that is, the determinant of that divided by the determinant of the admittance matrix. And this gives you, it is the determinant of, what is on top, on the numerator and the determinant of the denominator is nothing but, will have number of terms containing s square due to product of this and that and also due to product of this and that.

This much is due to the product of this one and that one, minus I will have s square times C square and in fact, this cancel with that one and will have a number of terms containing as that is, due to product of this with ((Refer Time: 26:13)) this, product of this with that, the product of this with that one. And the terms turnout to be and in addition to this, there will be a constant and that only due to this one and that one there, so that will come out to be G o 1, G o 2.

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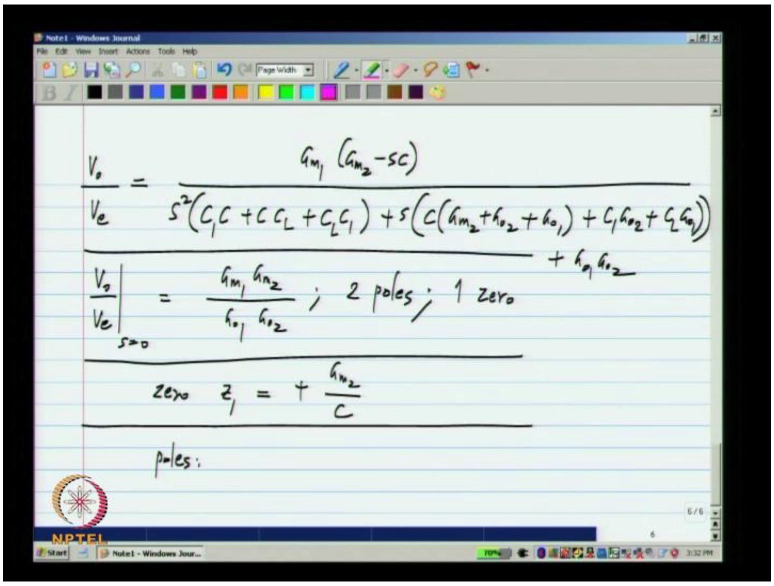

So, let me just rewrite it and also I will take V e to the left hand side so that, I get the transfer function V naught by V e. This will be equal to G m 1 G m 2 minus SC divided by the second order term first, C 1 C C C L plus C L C 1, first first order term which has first the constant term. So, this is the transfer function of the opamp, looks somewhat complicated, but it turns out that, we can make intuitive sense of this transfer function as well.

But, first of all the sanity checks, what did we say, the dc gain, what is the dc gain, here if I substitute s to be 0, this goes away and all these things go away. So, that is one thing and this is exactly the value that we got, we said the dc gain was G o 1 R o 1 times G o 2

R o 2 and that is exactly what we have here, except that it is written in terms of conductance that is all. And also we said that, it is a second order transfer function, because there are only two independent state variables and that is the case also, there are 2 poles.

And there is 1 0, we can see that in the numerator there is 1 0 and this also we guessed, because there were two different paths to the output, from the output of the first stage to the output of the second stage, there were two parts, one through the trans conductor G m 2, one through the capacitor C. And generally, even you have two parallel paths to the output with different phase shifts, different frequency dependences, you will end up getting a zero.

So, the next thing is to figure out, where the poles and zeros are and then, tried to make sense out of them. So, first of all the zero frequency is very easy, whereas the zero here, it is when the value of S, for which this term become 0 and zero, I will denoted by z 1 equals plus G m 2 by C, I explicitly write the plus, because zeros can be in the right half or left half plane and this happens to be in the right half s plane.

Now, the poles of course, can be obtain by solving this quadratic equation, but the conventional solution to the quadratic equation, the familiar one minus b by 2 plus minus square root of b square minus 4 a c by 2 a. That simply will not be able to do here, because each of the coefficient a, b and C are quite complicated and if I even managed to write down that expression, will not be able to make any meaning out of that. So, what will do is, we will find some approximate ways of solving the quadratic equation, it turns out that there is an easy approximation, which also in this particular case yields intuitive results.

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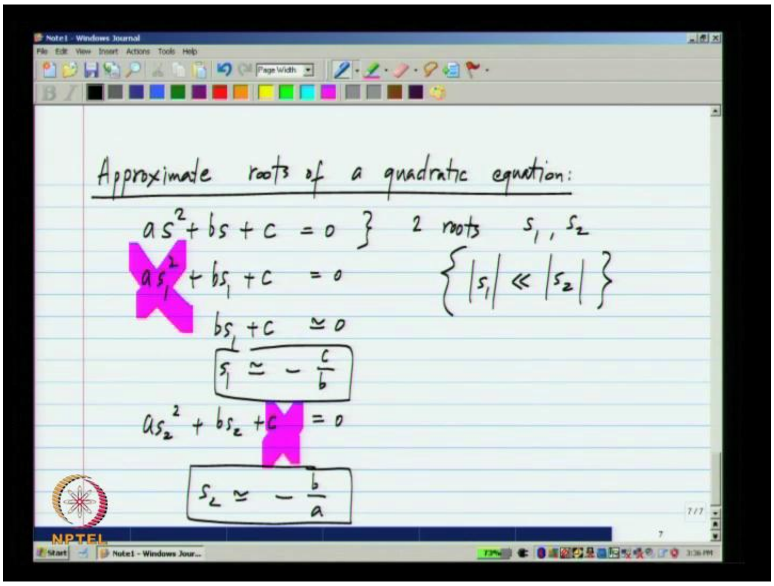

So, it turns out that, the quadratic equation of course has two roots, but when the two roots are very far from each other. That is, when the magnitude of one of the roots is very small compared to the magnitude of the other root, the following approximation can be used, that is why we have a s square plus b s plus c equals 0 and there are two roots s 1 and s 2. So, this clearly means, a s 1 square plus b s 1 plus c is 0 and also a s 2 square plus b s 2 plus c equals 0.

And let us assume that, magnitude of s 1 is much smaller than magnitude of s 2, we can verify this for yourself, you can write down the expression for the solution of the quadratic equation and see that, this is indeed the case. And in this case, it turns out that, first of all for s 1, this term will be negligible compared to be s 1 and c. So, this is approximately equal to 0 and we can easily determine s 1 to be approximately minus c by b.

What we have done is, to reduce the quadratic equation to a first order or a linear equation. Similarly for s 2, it turns that, this is much smaller than other two and s 2 can be approximated by minus b by a. Again we have to solve only a linear equation, now we have to keep in mind of course that, this is true only when one of the magnitudes is much smaller than the other. Now, in fact you can try solving every quadratic equation that you see approximately like this and see, if it is indeed true that, one of the roots has a much smaller magnitude than the other, if it is then, it is consistent, otherwise it is not.

Once you follow this procedure and find the roots, s 1 must come out to be much smaller of magnitude than s 2. Now, clearly this will not hold when the quadratic equation has complex conjugate roots, because when you have two roots which are complex conjugate is of each other, the magnitudes of the two roots is exactly the same. So, this will hold only for real roots, which are very far from each other. So, this at least looks manageable, given the complexity of the coefficient a, b and c.

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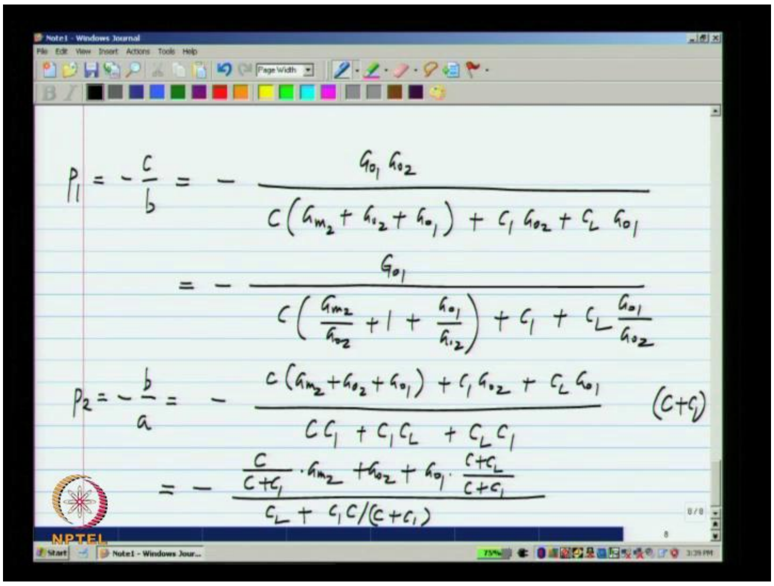

So now, let us find out the values of these, I will call this pole P 1 to be the smaller one and that is nothing but, minus c by b. And this if I substitute the coefficient from my quadratic equation, what do I get, so this is the expression we have, this is still somewhat complicated, but as you will soon see, you can make intuitive sense out of this one. I will just divide both numerator and denominator by G o 2 to put it in more meaningful form.

The reason I did this is to get the answer in the form of some conductance divided by some capacitance. As you can see, the numerator has some conductance G o 1 and the denominator has term, which represent capacitances, will later make sense of what this capacitances are. Similarly, the higher of the two roots, the higher frequency root P 2 will be minus b by a, which is minus and here we will have and in this particular case, I will divide both numerator and denominator by C plus C 1.

Again you later see, why this makes sense, so first of all I will have C by C plus C 1 times G m 2 and the numerator you see that, G o 2 is multiplying both C and C 1. So, will have plus G o 2 and plus will also have G o 1 C plus C L by C plus C 1 and in the denominator, I will have C L plus C 1 C divided by C plus C 1. So, again I have made some manipulation of the expression so that, in the numerator i have a conductance and in the denominator, we have a capacitance. So, that makes that easier to make sense out of the poles, now before we tried to do that, let us first quickly review, how one might be able to tell the values of the poles in a circuit intuitively without doing circuit analysis.

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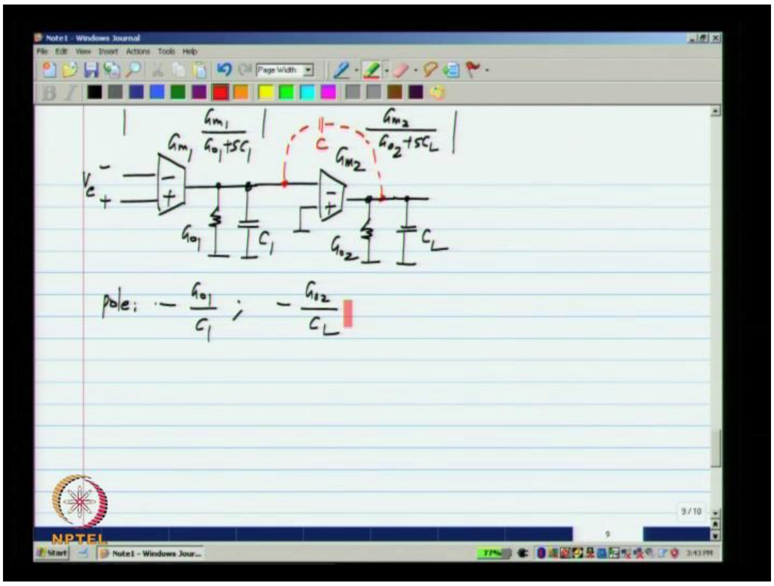

So, let us take the simple opamp we had earlier, let me just call the capacitance C 1 and conductance G o 1, that is simply the output resistance of this conductance G m 1. Now, you can work this out and see and you will find that, the pole is at minus G o 1 by C 1, I will not do the analysis here. And similarly, you can let add another stage here, let us say this is trans transfer G m 2 and this is the conductance G o 2 with a capacitance C l.

Now, you can work this out, it turns out that, we will have some transfer function from here to there and another one from here to there. And the two are independent that is, the first one as a transfer function G m 1 by G o 1 plus SC 1, second one has a transfer function G m 2 by G o 2 plus SC l. And the final transfer function is the product of the two and there will be 2 poles and the second pole is due to the second stage due to this combination, the first pole is due to that combination G o 1 by C 1 and there will be another pole at minus G o 2 by C L.

So, in circuits, here we have this R C parallel combination, which are isolated from each other, you can identify the poles to be simply minus the conductance divided by the capacitance across it. So, you identify capacitors, you find what conductance appears cross them, the ratio of conductance to capacitance gives you the poles. And you must done this basic circuit analysis also with simple R C circuit and exactly the same thing holds in this case.

Now, when you have capacitance and resistance connected in arbitrary fashion, this is not easy to do or maybe even impossible to do, but when you have isolated pieces of Rs and Cs, you can do this. Now, you also notice that, the example circuit I took is exactly the same as this opamp, except that I did not have this C. In my refine opamp, what I think is the refined opamp, I also have this capacitor C. Now, without that C, we can identify the pole very easily, now with C we have identified the poles to be this one and that one.

Now, will try to relate the case without C and with C, and see how it make sense, so also notice that, this pole has a conductance G o 1 and some capacitance and here it has a conductance G o 1 and a capacitance C 1, which is across it. Similarly, this has G o 2 plus some conductance divided by some capacitance, whereas here we have G o 2 divided by C L.

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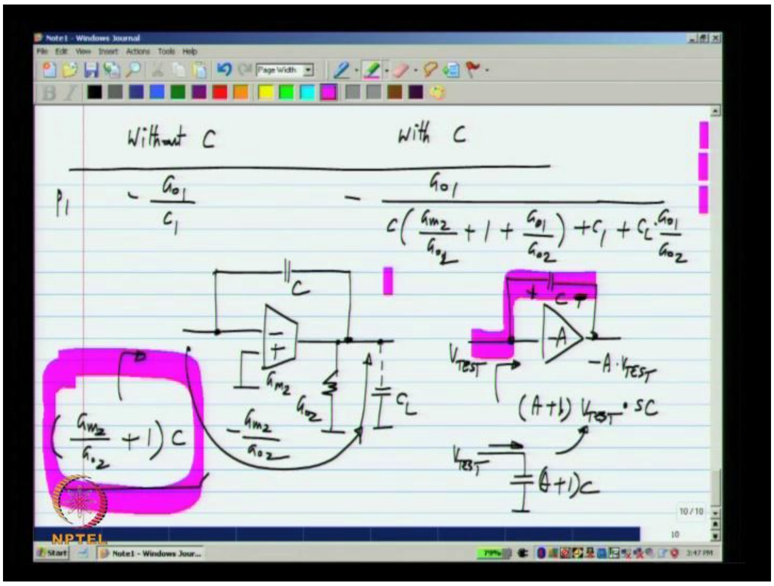

So, without C, I have minus G o 1 by C 1 and with C, I have minus G o 1 by C G m 2 by G o 2 plus 1 plus G o 1 by G o 2 plus C 1 plus C L times G o 1 by G o 2. Now, we can already see some relationship between the two, we have G o 1 by C 1 plus some capacitance and what is that capacitance that is, C times G m 2 by G o 2 plus 1. Why do we get a term like this, if you observe, the second stage look like this one, it also has this capacitor C L.

For the moment, let us ignore the capacitor, so basically from here to here, from here to the output, there is a gain of minus G m 2 by G o 2 and this capacitor C is connected across an amplifier, whose gain is minus G m 2 by G o 2. So, let us say, I have an amplifier of a negative gain minus A and I connect a capacitor across it, what happens and let us say apply some test voltage to the input. The output will be minus A times V test and the voltage across the capacitor in this polarity will be A plus 1 times V test.

So that means that, from this source V test, it will draw a current, which is equal to A plus 1 times V test times SC. So, simply looking into this block, it appears like I have a capacitance of A plus 1 times C, because if I apply V test to this, the current flowing here would be exactly same as that one, this phenomenon is known as the miller effect. If you connect the capacitor from the input to output of the negative gain amplifier, from the input it looks like a must larger capacitor.

And how much is it, it is equal to 1 plus gain times the capacitance value and this capacitor is also sometimes called the miller multiplied capacitor. Now, the second stage for a opamp as a negative dc gain of G m 2 by G o 2, we have a capacitor C connected across it, so looking in here, it approximately looks like a capacitance of G m 2 by G o 2 plus 1 times C. It is only approximately, so because the amplifier we have here is not a ideal, it is not an ideal voltage control voltage source of this gain, unlike this one. This is a ideal voltage control voltage source of gain minus A, whereas here, it is a trans conductor loaded by resistor, there is also a capacitor here as shown. So, only approximately it looks like a capacitor, so you do see that, in addition to C 1, which appeared across G o 1, you also have this particular term.

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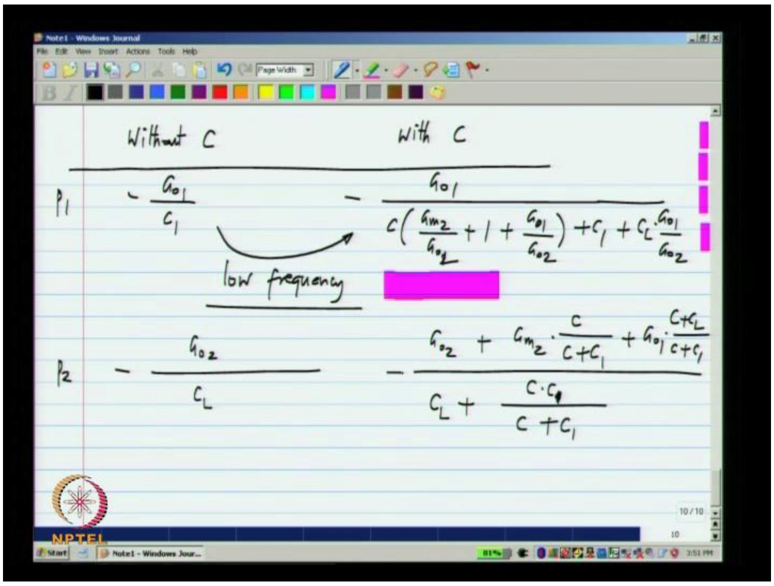

So, in other words, if I redraw the complete opamp, this is the first stage, C 1 appears directly across G o 1 and we have this capacitor across an amplifier, whose dc gain is G m 2 divided by G o 2 and there is also some capacitance C L. So, approximately looking into this miller multiplied capacitance, C times G m 2 by G o 2 plus 1. So, what do we have, we have a conductance G o 1, that is there, we have a capacitance C 1 that is there and we have a miller multiplied capacitance, which is there.

Now, there are also these other terms, this one and that one, they appear because, first of all this pole itself was obtained approximately and as an approximate root to the quadratic equation and secondly, this amplifier is not ideal, it has a finite output resistance I mean, non zero output resistance and so on. So, you also have this extra terms, but it turns out that, the significant terms are, what is highlighted here C 1 and the miller multiplied C.

So, although the expression was complicated, we were able to make intuitive sense out of it, which is good. So, what happens is that, we will have, across the output conductance of the first stage, we effectively have these two capacitors, capacitance C 1 and the miller multiplied capacitance C, because C is connected from the input output of the second stage. Now, it is also interest to see, what has happened to this pole frequency, as it increase or decrease, what do you think.

So, you can see that, first of all it is obviously reduced in frequency, because the numerator is the same, the denominator C 1 remains as it is and we also have this and if C is comparable to C 1, G m 2 by G o 2 is a number that is much more than 1, so the denominator has increased a lot. So, when you have no capacitor, so when you have a capacitor, it moves to low frequency.

Similarly, P 2 which was minus G o 2 by C L became minus G o 2 plus, there other terms like G m 2 C by C plus C 1 plus G o 1 C plus C L by C plus C 1 and divided by C L plus C times C 1 by C plus C 1. In the next lecture, we will go and see, interpret this and then, make sure that, it makes intuitive sense as well. In the next lecture, what we will do is, we will make sense out of this expression as well and see, how it makes intuitive sense.

Thank you and see you again.