Signal Processing Techniques and its Applications
Prof. Shyamal Kumar Das Mandal
Advanced Technology Development Centre
Indian Institute of Technology, Kharagpur

Lecture - 08
Discrete-time System

So, in the last lecture, we talked about Discrete-time Signals, and we will talk about Discrete-time Systems. Now, as you know, what is a system, whether it is a discrete system, analog system, or whatever. A system is nothing but a process or a device, a system where I can apply an input and I can get an output. So, that is a system. So, it may be a discrete system, it may be an education system, it may be a corporate office system.

So, a system is nothing but a procedure or an algorithm or set of rules or, I can say, a device where if I apply an input, I can get an output based on the algorithm, based on the procedure, based on the rule.

Now, what is a discrete-time system? Here, I can say discrete-time system. So, I said the signal is discrete-time, and now I will talk about the system is also discrete-time. So, what is a discrete-time system? So, a discrete-time system is a device or an algorithm that operates on a discrete-time signal.

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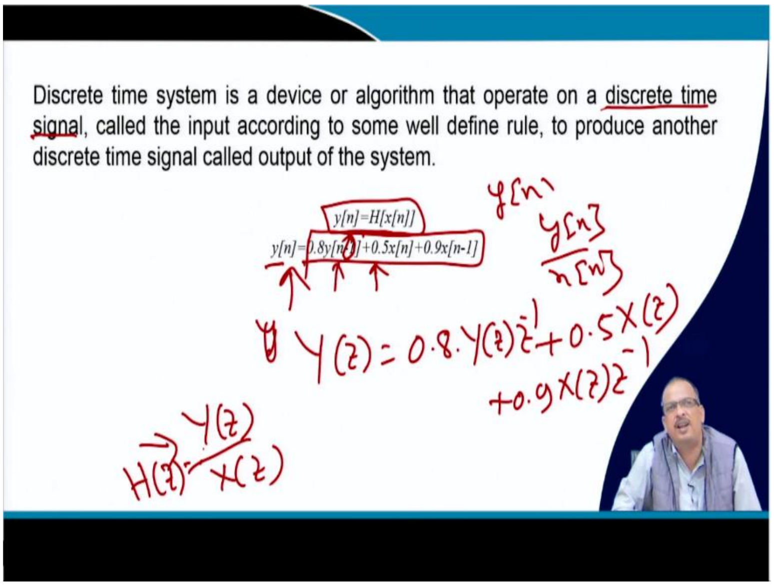

So, if the input signal is discrete-time the algorithm that will operate on that signal will be called discrete-time system. So, I can say that you know that y[n] is nothing but a H[x[n]], where H denotes the system, or I can say, let us say y[n] output is the combination of the input and previous output. So, that is also a system.

I can easily determine the transfer function of the system. What will be the H in this case? So, what is the H of the transfer function of a system? It is nothing but an output by input. Now, simply, if you know the Z transform, what is its meaning? That I can say if I convert it to the Z domain, I said Y(Z) is nothing but a 0.8 * Y(Z) * Z^{-1}. Z^{-1} denotes one sample delayed, n-1, one sample delayed + 0.5 X(Z) + 0.9 X(Z) * Z^{-1}.

I can easily calculate the value of Y(Z) by X(Z), which is nothing but a H(Z) transfer function of the system. So, discrete-time system input is the discrete-time signal. It may be a device algorithm, procedure whatever, and I will get an output that is also a discrete-time signal.

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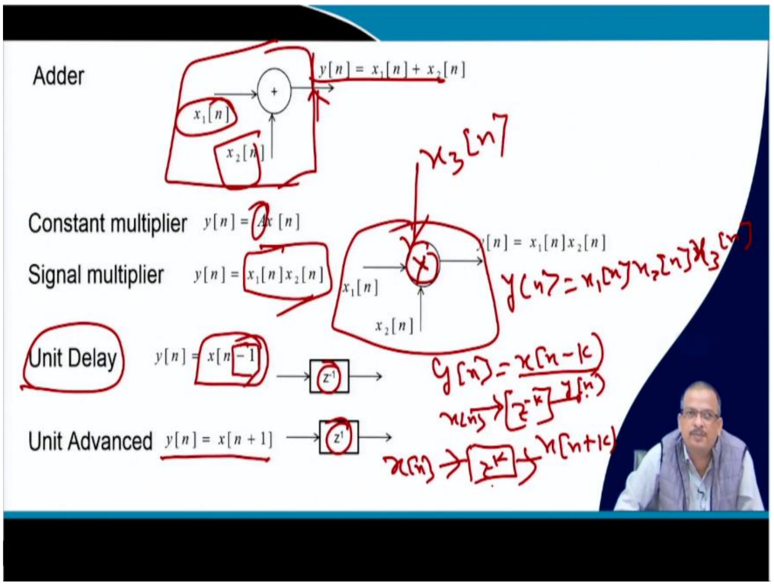

Now, suppose adder, the purpose in the system is that where the two signals will come together, they will add up, and that gives me the output. So, I can say it is represented by x_1[n], x_2[n] + them. So, this system is nothing called an adder. There can be three inputs or four inputs that do not matter, x_1[n] + x_2[n] + x_3[n] + x_4[n] + x_5[n] does not matter. A constant multiplier, or discrete amplifier, is a system that multiplies each input sample with a constant number A.

So, if I say this is a system, then I can say this system is called a constant multiplier. Signal multiplier: two signals will be multiplied together. So, if the block diagram looks like this, three signals can also multiply. So, I can put another x_3 here, and then I can say y[n] equals nothing, but an $x_1[n] * x_2[n] * x_3[n]$ does not matter. So, that is called a multiplier.

So, when I write the block diagram of a system, this circle and cross are used for the multiplier, circle and + are used for the adder, and circle and minus are for the subtractor; that means $x_1[n] - x_2[n]$. unit delay, as I said, the signal is delayed by one sample. So, here you can say it is delayed by one sample. So, if it is delayed, it is represented at a Z^{-1}; that means one sample is delayed.

Now, if I say y[n] is equal to n K number of delay x[n-k]. So, K sample delay. So, I can say it is nothing but a signal, and then I can say Z^{-K} and y[n]. So, this is the x[n], and this will be the y[n], and then I can say the output is x[n-k]. similarly, Unit advanced, I can say advanced by one sample.

So, when I say advance, it will be Z^{+1}. Similarly, if I say the K sample advanced, it will be x[n] applied on a system that is nothing but a Z^{K}. I get x[n-k]. Please put that arrow marking when you write the circuit block diagram or signal block diagram because the arrow is the flow of the direction of the signal, which is mandatory.

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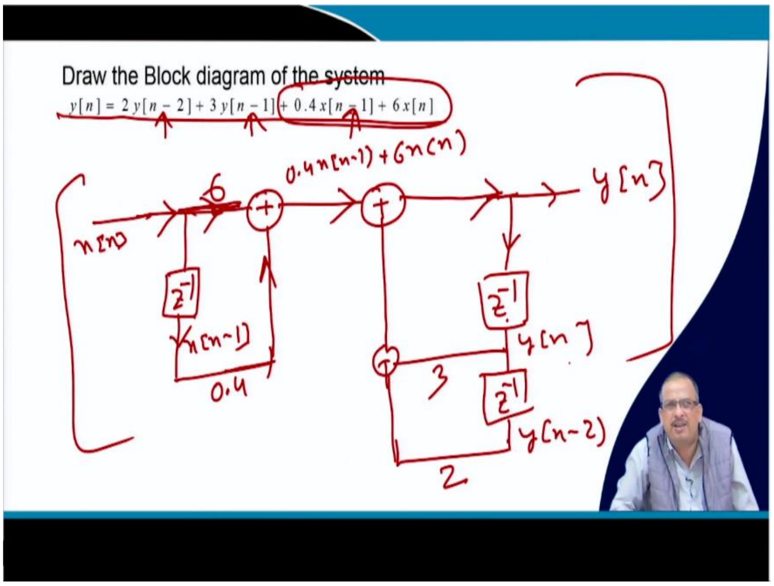

Then let us say I told you to draw the block diagram, so, elementary system block diagram, you know. Let us say I told you to draw the given system's block diagram. So, this is my system y[n] is equal to the combination of this one. So, if I say how many delay functions are there? One, two, three.

So, let us say suppose you do not know anything, do not remember when you write, and when you draw the block diagram, do not remember anything. See the first signal. Let us say this side is x[n] ok. So, the signal is flowing in this direction x[n]. So, I require x[n], and one part has to be delayed by one sample Z^{-1}. So, here I get x[n-1], then I know I must generate up to this.

So, there is an adder where x[n] will be multiplied by 6. Write the vector that will be added up with x[n-1] where the multiplying factor is 0.4. So, I get this one is nothing but a 0.4x[n-1] + 6x[n] this point I get. Now, what I want? is output y[n]. I want y[n] two sample delay signals. So, let us say this is y[n]; if I trap the signal here, then I get Z^{-1}.

So, I can say this is nothing but a y[n-2], one sample delayed, another one sample delayed. So, n is two samples delayed. So, y[n-2] and this has to be multiplied by 2, and I require y[n-1] also. This is y[n-1], and this has to be multiplied by 3, then both have to be added up here. So, that is the block diagram of the system.

So, any system people give you can write down the block diagram of the system. So, this is called a discrete system block diagram. Next, we come to how to minimize this delay in structure one implementation and structure two implementation, which I will discuss later on. So, any system given to you can draw the block diagram of the system without remembering anything. If you know the logic, you can do it in a minute, ok.

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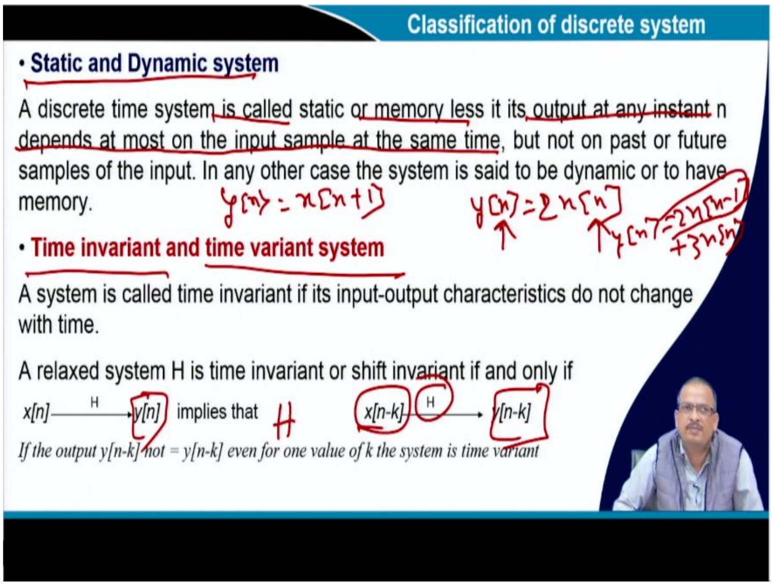

Next are properties of the system; similarly, signal properties – periodic, aperiodic, time-invariant, and time-variant. Similarly, the system also has a property. What is the property? Static and dynamic system. Sometimes, it is called memoryless, or with a memory system, a discrete-time system is called static or memoryless.

So, a static system means memoryless if the output at any instant n depends at most on the input sample at the same time; that means, if I require to remember, let us say y[n] is equal to 2x[n] I do not have to remember the previous sample, present output depends on the present input.

Now, suppose I have a y[n] equal to 2x[n-1] + 3x[n]. So, I know I have to remember the previous sample. So, when I say Z^{-1}, that means I have to store the previous sample value to get the current sample value. That is why it is called a memory system.

So, memoryless means I do not have to store the previous or future value with memory means when I have to be required to store the sample. So, suppose I said y[n] is equal to x[n+1], which also requires memory. So, a system will either have memory or without memory. If it is without memory or memoryless, then it is called a static system; if it is with memory, then it is called a dynamic system, clear?

The time-invariant and time-variant systems. Time invariant and time-variant signal, I have said. What did I say? If the properties of the signal do not change over the time range, if I say a system does not change its property over the time range; that means, in mathematics, I can say I have a signal x[n] if I apply and I get y[n] then if I apply x[n-k] I should get y[n-k] because properties of the system does not change.

If it is x is square n minus k, then it is not. If it is not y[n-k], then I cannot say it is a time-invariant system, like the speech signals the proper vocal cords. When I say something along the timeline, the shape of the vocal cord is changing; that means characteristics denoted by H are changing along the timeline. So, I cannot say this is a static system. This is a time-invariant system. This is a time-variant system. Along the timeline, property H is changing. That is why it is a time-variant system, understand?

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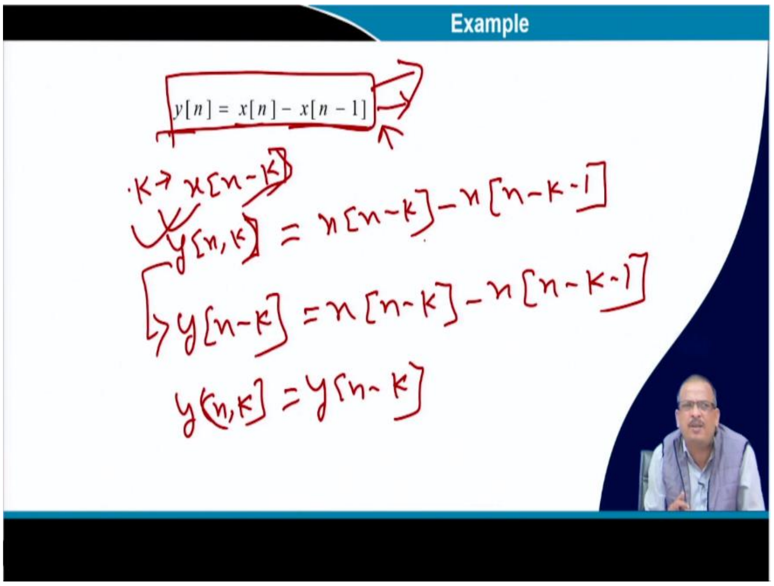

Let us take an example, let us say y[n] is equal to x[n] minus x[n-1]. Prove that the system is time-invariant or time-variant. Let us say I told that where the test whether the signal system denoted by this equation is a time-invariant or time variant. So, how do I do that? Very simple.

Let us say I apply a K sample delayed input. So, I apply x[n-k]; instead of x[n], I apply x[n-k]. So, I can say y at that point. So, y at delay when I apply the input K sample delayed input so, it will be x[n-k] K-sample delayed minus x[n-k-1].

Now, if I collected the K sample delayed output; that means, if I say y[n-k]. So, I collected the 10 K sample delayed output. So, that will be x[n-k] - x[n-k-1]. See that y[n. k]; that means the output due to the delay in the input signal and the output delayed output is the same.

So, I can say y[n,k] is equal to since y[n-k], then I can say the system is time-invariant because the properties of the system do not change with time, clear? So, I can give any system you can test whether it is a time-invariant or time variant.

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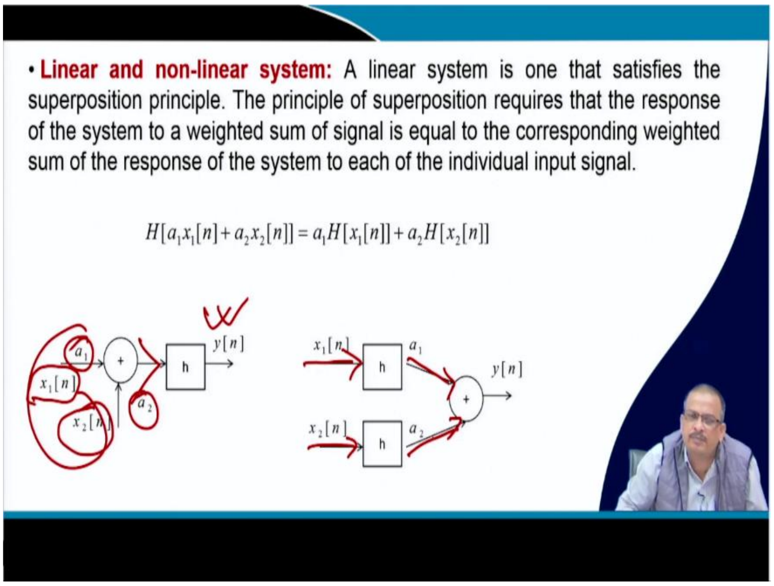

Similarly linear and non-linear systems, when I say a system is linear fine, this test gives very important properties of a system linear system and non-linear system. How do I test it? I am not reading the slides. So, what? think on your mind without seeing the slide; just think about it. Close your eyes and think. How do I say a system is linear? That means it should support the superposition principle.

What is the superposition principle? That means if I apply individual input and collected output and if I apply combined input and collected output, both will be the same. That means if I say I have a 2 signal $x_1[n]$ and $x_2[n]$, and their multiplying factor is a1 and a2.

So, if I add the input and apply it to the system, the output I get must be equal when I individually signal I apply and the output I add. Do you understand? So, I apply x_1 to take output to y_1, and I apply x_2 to take output to y_2. Now, I apply $x_1 + x_2$, I get an output of y_3. So, y_3 must be equal to $y_1 + y_2$. So, whether I individually apply the signal and combine the output or I combine input and collect the output, both will be the same if the system is linear.

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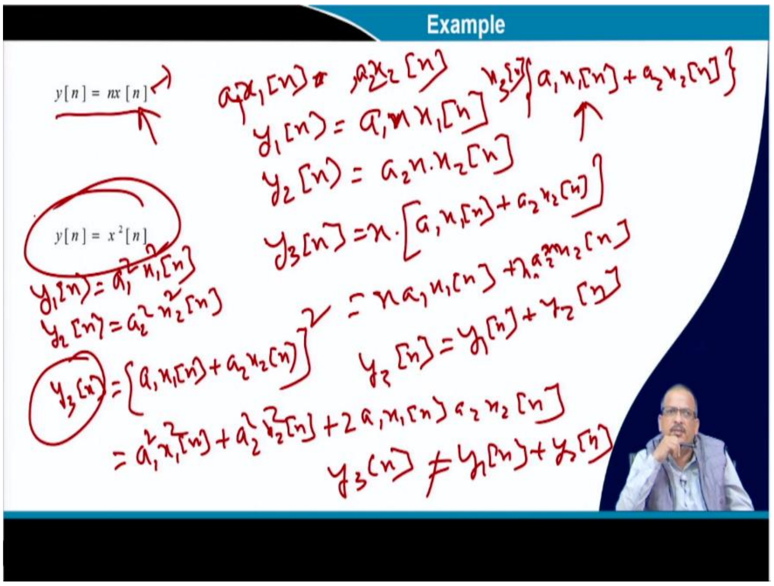

So, let us example, y[n] is equal to nx[n]; n is a number that varies from 0, 1, 2, 3, or minus 1, minus 2, let us say for all n. I have to test whether this is a linear system or a non-linear system. So, let us say I apply a signal which is $x_1[n]$ is equal to, let us say, apply a signal $x_1[n]$ and another signal is $x_2[n]$.

So, when I apply $x_1[n]$, let us say output is $y_1[n]$. So, $y_1[n]$ is equal to, let us say, I multiply x constant also $a_1x_1[n]$ and $a_2x_2[n]$. a_1 and a_2 are the multiplying factor. So, I can say $y_1[n]$ is equal to nothing but a1nx1[n]; $y_2[n]$ is nothing but a2nx2[n] because I applied a2x2n. So, I get n*x[n] as the output.

Now, if I apply a1nx1[n] + a2nx2[n] as a combined input, which is, let us say, $x_3[n]$, then what is the y_3? I said $y_3[n]$ is equal to I applied combine n. So, n into a1nx1[n] + a2nx2[n], which is nothing but an into a1x1[n] + a1x1[n] into. n will be multiplying both.

Now, I said, you can see that $y_3[n]$ is nothing but equal to $y_1[n]$ + $y_2[n]$. So, I can say the system supports the superposition principle. That is why I said this system is a linear system. Let us say take this system. The same input is applied. So, what is $y_1[n]$ here? in this case, it is nothing but $a1^2$ $x_1[n]$. What is $y_2[n]$is nothing but $a2^2$ x2[n].

Now, if I apply this combined input $x_3[n]$, then I get $y_3[n]$ is equal to nothing but a1x1[n] + a2x2[n] whole square. So, I get a 1 square $x_1[n]$ square + a2 square $x_2[n]$ square + 2 because a + beta whole square 2 a1x1n into a2x2n. So, I can definitely see that y_3 is not equal to $y_3[n]$ is not equal to $y_1[n]+ y_2[n]$. So, that is why I can say this system is not a linear system. So, what are the criteria for linearity? What about the system? It should support the superposition principle. That is the criteria, ok?

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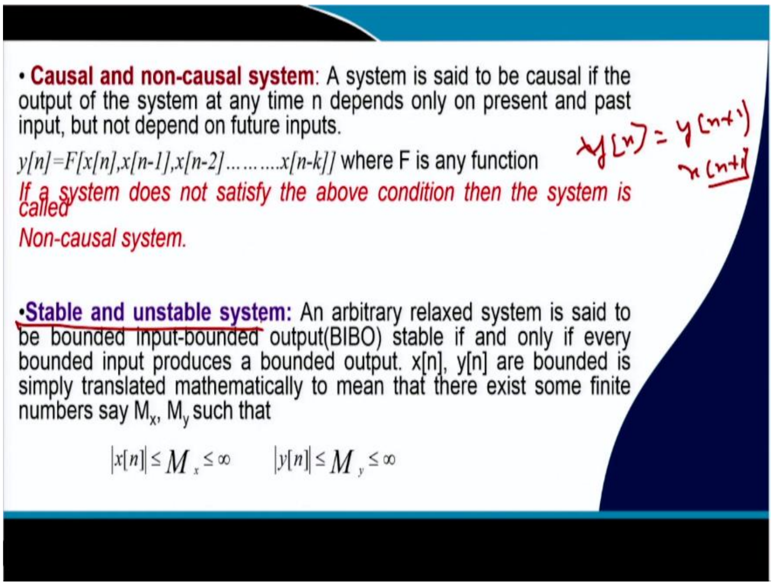

Next, causal and noncausal system: a system is said to be causal if the output of the system at any time n depends only on present and past input; that means if y[n] somehow depends on either y[n+1] or x[n+1], then I can say the system is not non-causal because futuristic input is also required. So, it is not causal, but if it only depends on present input and past input or past output, then I can say the system is causal rather than stable and unstable system.

So, you know that if I apply a bounded input, I should get a bounded output, then only I can say the system is stable; if I apply a bounded input and I get an infinite output, then I said the system is unstable. So, for a bounded input, it should give me the bounded output, then only I can say the system is stable, ok.

So, those are, you can say, the properties of the system. So, I can give you many equations of the system, and I can ask you to test whether it is causal, noncausal, whether it is linear, non-linear, whether it is a time-variant, or time-invariant system. So, you can do those things, okay?

Thank you.