Signal Processing Techniques and its Applications

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Lecture-07
Discrete-Time Signal

Ok. So, today, we will talk about discrete time signals. So, in the first week, we talked about digitization and then the concept of frequency in the digital signal, and we have what described that different kind of concept, and then we said that analog to digital conversion, digital to analog conversion, how this signal will change aliasing effect. So, all those things we cover in the first week. So, in this way, we talk about a discrete-time signal. So, what is a discrete time signal, and how is it written correctly?

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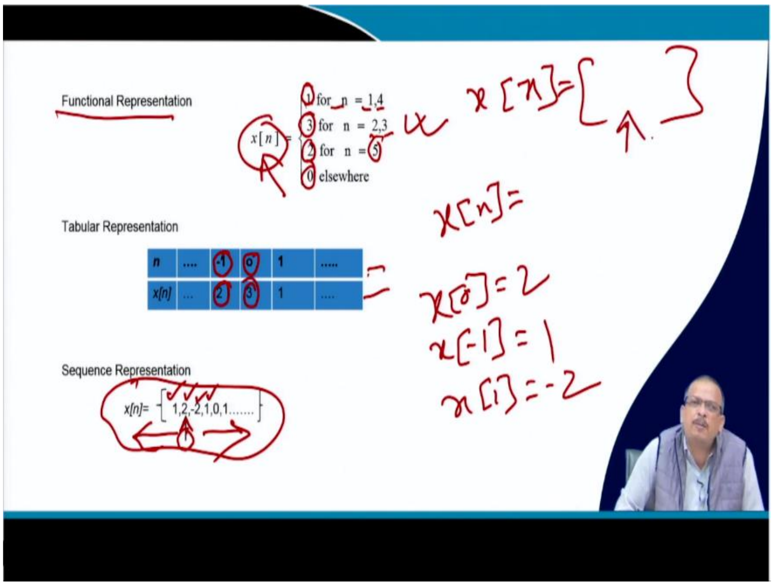

So, discrete-time signals can be represented in 3 ways. One is called functional representation, where I can represent that x[n] is like that 1 for n = 1 and 4; that means, if n = 1 and 4, the value of x[n] is 1. If n = 2 and 3, then the value of x[n] is 3. If n = 5, then the x[n] value is 2; elsewhere, it is 0. So, that is the representation of the functional representation of a digital signal.

As you know, the digital signal is nothing but a sequence of numbers because I have already discretized the time and quantized the value. That is why it is nothing but a sequence of numbers. So, I can represent that x[n] in terms of function or in terms of tabular form that n, x[n]. So, if the n value is-1, the value of x[n] is 2, and the n value is 0, then the x[n] value is 3. That way, you can also represent it, but the most popular representation is a sequence representation.

So, I said x[n] is nothing but a sequence of numbers. So, x[n] is a discrete-time signal. So, it is nothing but a sequence of numbers 1, 2,-2, 1, 0. So, all are nothing but the value of each individual sample. Then, if you see there is an arrow sign, this arrow indicates the 0th sample. So, I can say x[0] = 2 and x[-1] = 1. So, this side of n is negative, and this side of n is positive. So, I can say x[1] =-2.

So, those are the sample values, and the arrow denotes the 0th position, n equal to the 0 position. So, that sample value. That is why it is given in an arrow. So, when we give an x[n] in the form of a sequence of a number, we always indicate which is that your sequence 0 or n = 0 value by an array ok. So, this is a representation of a discrete-time signal.

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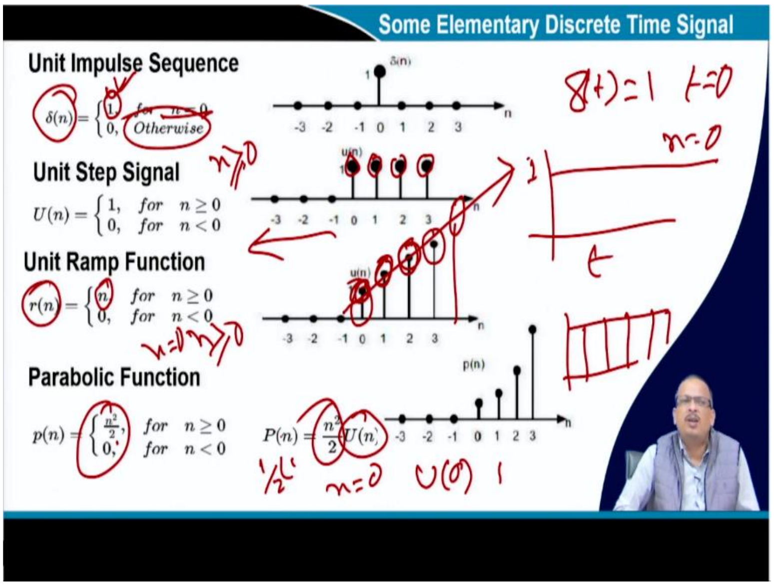

Now, some of the elementary discrete time signals are like that unit impulse. As you know, the signals and systems we have already studied. So, δ n. So, δ(t) = 1 and t equal to 0, which we have studied, but if we have a discrete-time system, t equal to 0 means n = 0. So, I said δ(n) = 1 when n = 0; otherwise, it will be 0, which is called the unit impulse sequence. The value of the impulse is a unit, and the sequence only exists when n = 0.

Then, unit step function: as you know, what is the unit step function in the time domain? Nothing but a straight line at value is one for all the time. Now, I have sampled it. So, that is why it is a sample value if you see it. So, for the positive, it is always 1. So, I can say the n is greater than 0; that means t positive will always be 1 or greater than equal to 0, some from 0th position to entire n it = 1 else it is 0. This side will be 0. So, negative n does not exist.

Unit ramp function: The ramp function means it is gradually improving. So, if I say that r(n) is a ramp function, r(n) = n. So, when n = 0, the value is 0. This will be not 1, if it is n equal to 0.

n equal to 1, 1; n equal to 2, 2 value; n equal to 3, 3 value; n equal to 4, 4 value. So, it is gradually increasing the value, which is why it is called the unit ramp function. Since it is a unit ramp function, I can say that if n = 0, it is 0. If n = 1, it is 1; if n = 2, it is 2. So, it will start from 0 to gradually unit ramp function. Then, the parabolic function.

So, many functions parabolic functions are defined as n^2 by 2. So, n equal to 0 means it is nothing but a U(0), and n equal to 1 means 1 by 2 into U(0). So, U(0) is nothing but the unit impulse. Now, it is multiplied by this factor.

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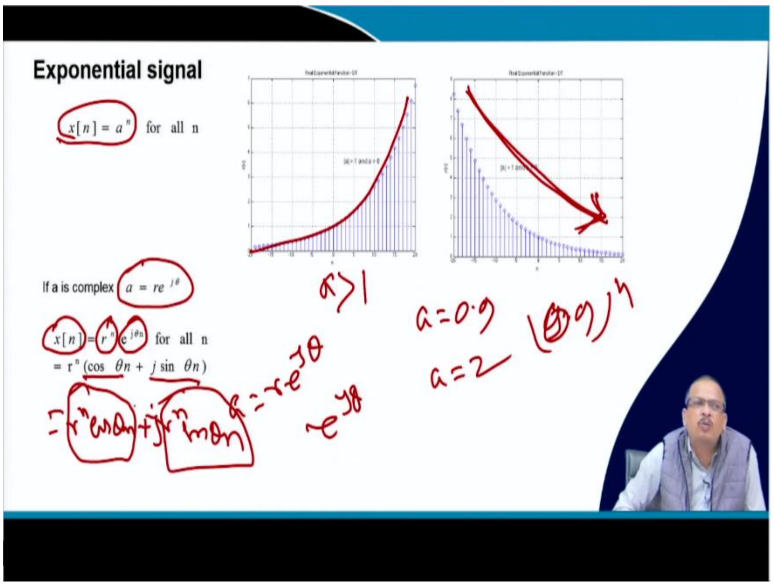

So, different kinds of function. Let us say the exponential function is a very important issue because we always look for the exponent's function in digital signal processing. So, in an exponential function, let us say $x[n] = a^n$ for all n; that means n = 0, 1, 2, 3, 4. Now, you know that if a is greater than 1 the exponential is increasing.

If a is less than 1 exponential, it will decay. So, using a x[n], you can take the value equal to 0.9, then 0.9^n generate the sample; n equal to 0, 1; n equal to 1, 0.9; n equal to 2, 0.9^2. So, like that way, I can generate the value; if I plot it, it will be decaying.

Similarly, if I take a equal to or greater than 1, let us say equal to 2, then I can say it will exponentially increase. Now, I come to that in signal processing, we said that yes, x[n] is an exponential function, and the value of $a = re^{j\theta}$.

So, it is nothing but a complex number, r is the magnitude and $e^{j\theta}$ is the direction. So, if I put $a = re^{j\theta}$, then x[n] will be $r^n.e^{j\theta n}$.

Now, I know that $e^{j\theta n}$ can be represented as a $\cos\theta n+j.\sin\theta n$. So, now, if I do that, I know $r^{n.} \cos\theta n+r^n.j.\sin \theta\ n$. Now, if I see this x[n], the exponential function is a two-part. One part is called the real part, and the next part is called the imaginary part. So, if I say that what is the plot of the real part, it is nothing but a cos function.

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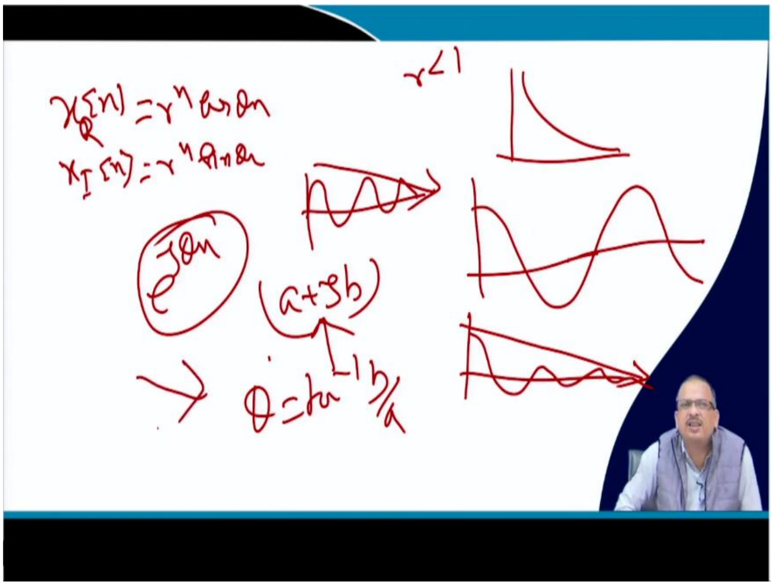

So, if I say the x[n] real part r part, it is nothing but a $r^n \cos\theta n$. Now, if the value of the r is less than 1, I know the rn will be exponentially decayed, amplitude will be exponential decade r^n and $\cos\theta n$, I know that $\cos\theta n$ looks like this.

So, a cos function, but exponentially decays the amplitude. So, I can say this will look like this. So, the amplitude of the cos function exponentially decays. Similarly, if I say xI[n] imaginary part, it is nothing but a r^n sinθn.

Similarly, it will be a sine function instead of a cos function. So, it will start from the sine function and exponentially decay. Now, if the value of r is above 1, I can say that it is gaining. So, those are the signal representation. So, $e^{j\theta n}$, I can say exponential signal n is the sample number.

So, n can be 0, n can be 1, n can be 2, and n can be 3. That way, I can vary n. So, if I told you suppose I have a complex signal and a complex digital signal a+jb, but it is a complex signal digital complex signal if I told you to draw the phase diagram.

So, a complex signal has a phase and an amplitude. So, if I want to let us say θ n, if I say that what is the value of θ is tan inverse b by a. So, if I turn the phase position, you know that it is nothing but a tan θ, but tan90 is infinite. So, it is a discontinue. So, I can plot this. So, when you go for the LTI system phase response LTI system magnitude response, we discuss those things, and that time is ok.

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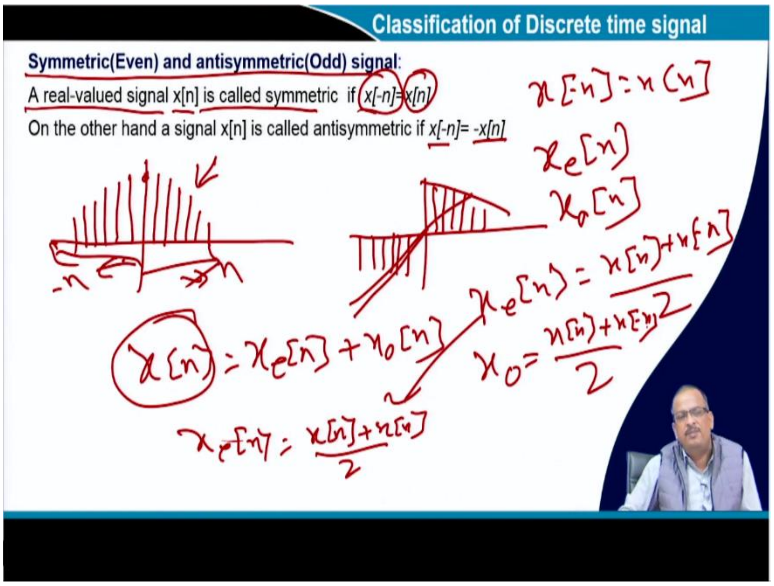

Now, I come to the properties of the signal. What is the classification of the discrete time signal or properties of discrete time signal? One is called symmetric and anti-symmetric, or you know that symmetric means even function, and anti-symmetric means odd function.

So, a signal a real-valued signal $x[n]$ is called symmetric if $x[-n] = x[n]$; that means, suppose I have a signal like this: this is the 0th amplitude, this is the 1, 2, 3, 4, 5, 6.

Similarly, this side also 1, 2, 3, 4, 5, 6. Now, if you see this is the negative n-n, this is positive n, and both side signals are symmetric. So, I can say $x[-n] = x[n]$. That is why I call it a symmetric signal. What is an anti-symmetric signal? If $x[n] =-x[n]$ like this, suppose I have a signal like this, and so, I can say this like this signal and this signal. So, this is an anti-symmetric signal.

So, it is nothing but a-$x[-n]$. So, one is called odd, and the other is called even. If it is symmetric, then $x_e[n]$ is called an even signal, and if it is anti-symmetric, then $x_o[n]$ is called an odd signal. So, you have heard about the even function and the odd function. You know that any real signal can be a composition of nothing but an odd signal and an even signal. You know that any real signal is nothing but an odd signal+an even signal.

So, I can say $x[n] = x_e[n]+x_o[n]$. You can prove that an odd part or even part is nothing but a combination of $x[n]$ divided by 2. So, I can say

$$x_e[n] = (x[n]+x[-n])/2.$$

&

$$x_o[n] = (x[n]+x[-n])/2.$$

Now, in this case, I can say it is $x_e[n]$ is nothing but a $x[n]+x[n]$ because-n which is the $x[n]$ and even signal, odd signal I can say minus. So, any signal is nothing but a composition of odd signals and even signals.

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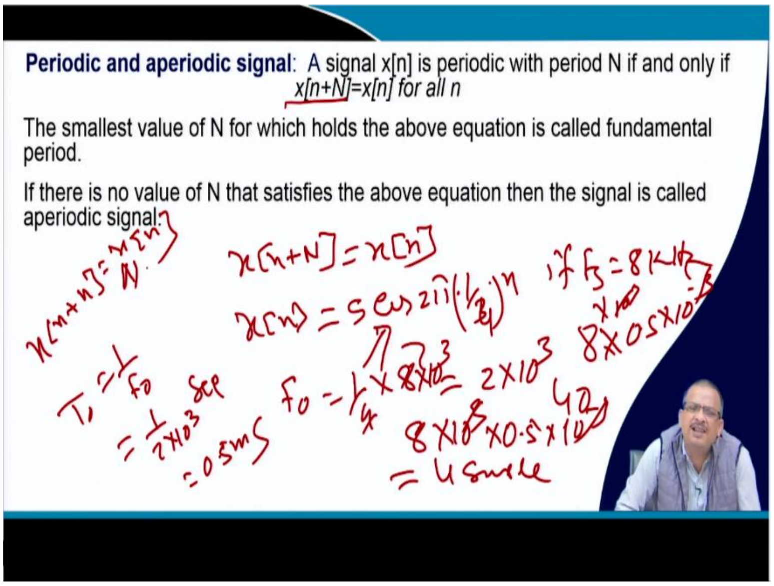

Now, I come to the periodic and aperiodic signals. A signal is said to be periodic when, say, in the time domain and also if the signal is repeated after a certain time. In the digital discrete time domain, if the signal is periodic, it should be repeated after a certain interval of n.

So, let us say I have a N in my period. So, after N, the signal will repeat itself. So, after N means x[n+N] should be equal to x[n], then only I can say that n is the period of the signal.

So, if I say the cos θ after 2π, the cos θ will be the same cos (2π+θ) = 2π is the period, as you know. Similarly, let us say I have a cos function within 2π. There are 10 samples. So, I can say within 10 samples, the signal will repeat itself. So, if I told you suppose I give you a signal, let us say I have a digital signal

$$x[n] = 5cos2\pi\left(\frac{1}{4}\right)n$$

How many samples will be there in 1 period of this signal? How many samples will be there within 1 period of this signal if the sampling frequency = 8 kHz? This 1 by 4 is the discrete frequency discrete frequency. So, I know the value of F_0; 1 by 4 into 8k. So, it is nothing but a 2 kHz.

Now, you know how many samples will be there within a period. What is the period? T_0 = $1/F_0$. So, it is nothing but a ½*10^3 second. So, I can say it is 0.5 milliseconds.

Now, you know how many samples are in 1 second; there will be an 8k sample. So, in 0.5 milliseconds, I can say $8*0.5*10^{-3}$ samples. So, $8*10^3$. So, this will be cancelled, 4 samples. So, 8 kHz. So, 8k samples in a second, so in 0.5 milliseconds? 10^{-3}.

So, I can say there will be 4 samples. So, I can say x[n]+4 will become x[n]. So, 4 is the period of the signal. If it is not satisfied, then I can see the signal is an aperiodic signal, not a periodic signal.

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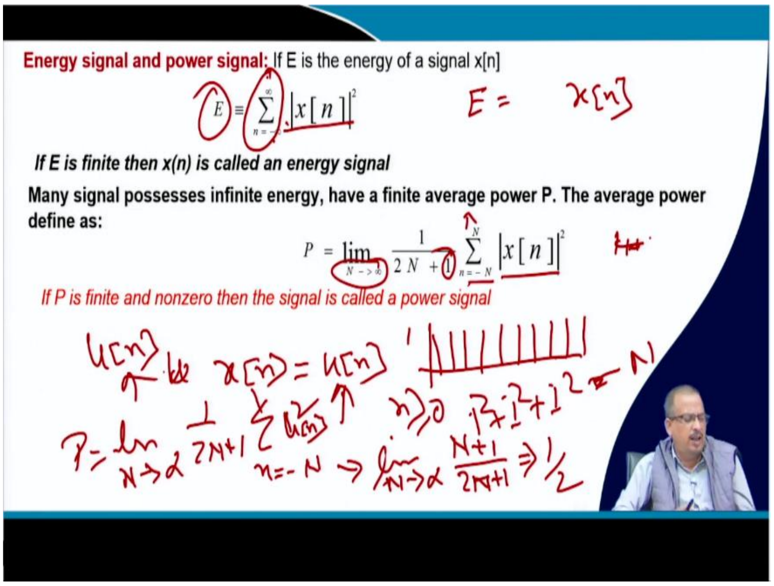

Now, I come to another type of signal. One is called an energy signal and a power signal. Sometimes, the energy of the signal is infinite. If the energy is infinite and the power is finite, then I call it a power signal. If the energy is finite and power is infinite, then we call it an energy signal.

So, what is the energy of a signal? If I have a signal of x[n], so, suppose I told you I have given a voltage V, what is the power? in the order of V^2. So, if I have a signal x[n] each sample square and sum over the entire signal is nothing but a power of the signal. So, it is the sum of the entire signal. So,-infinity to infinity, each sample square x[n] whole square mod. the whole square means that each sample would be square and sum, then I get the energy.

If the signal's energy is finite, then you call the signal an energy signal. Now, if the energy is infinite, but the power is finite, then I call it a power signal. So, what is the power?

Power is average. So, what is the average? So, it varies from-infinity to+infinity, and 0 will be in between.

So, if the n belongs to an infinite number, then I can say 2 N and 0 will be counted twice. So, I can; so, 0 will be counted 1. So, I put 1. So, 2 N+1; n equal to-N to+N x[n] whole square limit N tends to be infinite.

Now, if I told you to give an example of a power signal, let us say I have a signal called the unit step function. Let us say x[n] is nothing but a unit step function, which is nothing but a U[n]. That means the unit step function means at n greater than equal to 0, it will be 1 amplitude will be 1 n greater than equal to 0. Now, what is the energy? I can say energy is nothing but the square of the sample. So, it is nothing but a $1^2+1^2+1^2$ up to infinite, up to n varies from infinite.

So, I can say if I add up to infinite, so, it is infinite, sum will be infinite. In that case, I cannot say U[n] is an energy signal. Now, if I calculate the power of the signal, this is what will come? The sum is nothing but a 1+N. So, I can say the power will be limited, N tends to be infinite 1 by 2 N+1 and the summation of n =-N to N U[n] square; that means 1^2.

So, that is nothing but a limit that tends to be infinite. It will be N+1 divided by 2 N+1, which is nothing but a half. So, power is finite. So, in that case, I called U[n] a power signal, not an energy signal. So, if the energy is finite, then I call it the energy signal; if power is finite, I call it the power signal.

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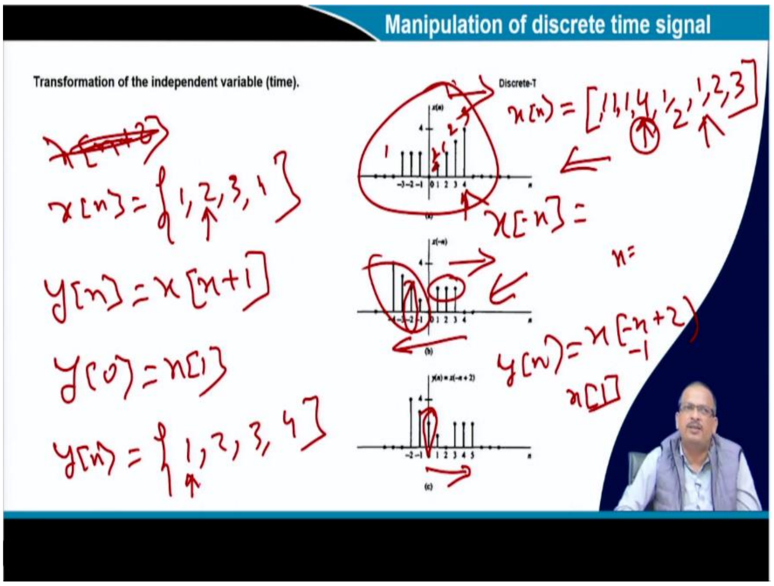

Now, manipulation of discrete time signal. So, a signal can be finite: energy signal, power signal, time. I can say periodic and aperiodic. I can say the signal is symmetric and antisymmetric, and I have already said that there is another type called time-invariant and time-variant. Signal: if the properties of the signal do not change with time, then we call the signal a time-invariant signal. If it is changed, then it is called a time variant signal.

Now, I have come to the manipulation of discrete-time signals and time scale manipulation. So, if you see this picture, if I told you what the sequence of x[n] should be, the x[n] has a signal value of 4 at 0 position and-. So, let us say this is a half, this is 1, this is 2, this is 3, let us say the value of the sample. So, I can say x[n] is nothing but a 1 1 1, and then there is a 4 at 0 position. I can write in that way sequence way.

So, the-1-2-3 value is 1 1 1. Elsewhere, it is 0. So, I am not writing that. Then I can say the next value is half. So, half, the next value is 1, the next value is 2, and the next value is 3. Now, I said, let us say I apply for a left or right shift. So, I can shift the signal this side, shift the signal this side, or change the x[-n]. So, it is nothing, but I am flipping the signal. So, the positive side becomes the negative side, and the negative side becomes the positive side.

Similarly, let us say I want to shift the signal of x[n] by 2 samples on this side, which is nothing but a x[n]+2. So, I said that x[n]+2 is my 0th sample. So, I just shifted this arrow to here, which is my starting shift. Then I can shift the signal to this side also. So, here I can say y[n] = x[-n]+2. So, this is the x[-n] signal, 2 index I make it positive.

So, 2 index positive means that I can say when n =-1, then this y[n] will be x[1]. When the index is-2, then it becomes x 0. So, I can say the-2 index comes to the 0th position, and the rest are shifted to this side.

So, this kind of x[n]+2 x[n]-2, so, suppose I have given a signal x[n] = 1, 2, 3, and 4, and I said this is my 0th sample, what will be the value of y[n] when y[n] = x[n]+1.

So, when n = 0, that means x[1] value. So, I can say y_0 is nothing but a x[1]. So, I can say the sequence of y[n] will look like this: 1, 2, 3, 4, and the arrow will be here. n equal to 0 means x[1]. So, the sequence will start from x[1]. So, I can shift the signal on the left side or right side depending on the requirement and represent it in this way.

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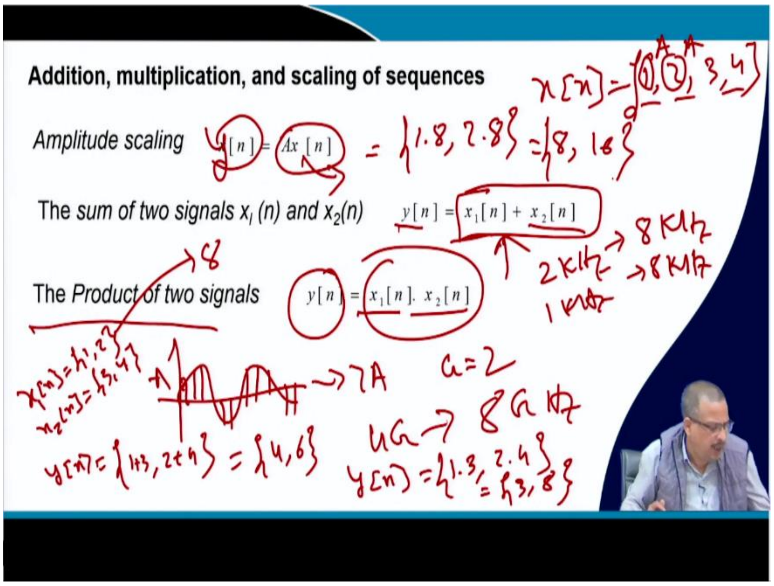

Similarly, I can add multiply or scaling of a sequence. So, x[n] is nothing but a sequence; x[n] is nothing but a number of sample numbers 1, 2, 3, 4. Those are the values of the sample, and those are the n-index. So, it is nothing but a sequence. So, if I want to amplify or use an amplitude scale, each sample will be multiplied by factor A. So, I can say x[n] or y[n] = A multiplied by x[n]. So, each sample will be multiplied by a constant factor A.

Similarly, if I say I want to sum up two signals, $x_1[n]$ is a digital signal discrete time signal $x_2[n]$ is also a discrete-time signal, I can say y[n] is nothing but a $x_1[n]+x_2[n]$. So, a superposition is nothing but an addition of the sample value. You can do it. Generate a 2 kHz sine wave with a sampling frequency of 8 kHz. Write down the statement, and you can do it in Excel or C programming; you can do it and see whether it is happening.

And 1 kHz sine wave with a sampling frequency of 8 kHz, then add them. Add them means sample level, add them, and again plot it. See how the plot is coming up. The product of two signals, I can say. y[n] is nothing but a product of two signals: signal multiplication.

If you have already studied electronics analog multipliers, how can we develop the analog multiplier using a circuit? You know that it will be very complex. But in the digital domain, a signal multiplier is nothing but a just multiplier of the two signals. Similarly, see that amplifier I want to amplify the signal. I have a signal whose amplitude is, let us say, A; I want to make it 2A.

So, I know in an analog way, I can do it. I can design an amplifier whose gain = 2, and then I can say yes, but that requires a circuit. Now, I want to make the amplitude 2 in a digital domain. So, I have a sample, and each sample will multiply by 2, nothing but an algorithm. Adder, I have to add two signals. It is very troublesome when I design in analog, but if it is digital, I can easily design it.

But again, I have a problem. The problem is that I required an analog-to-digital conversion because none of the signals in real life exists in digital; all are in the analog domain. So, the people who are working in a microwave, let us say I have a 4 GHz signal. You know that the sampling frequency required is 8 GHz minimum.

Now, in 8 GHz, you know what the processing time is for an analog to digital converter and what the sample difference is. 1 by 8 GHz is nothing but that much of a second and within that second, I have to process it. How do I do that? That is why the high frequency will not go for the digital domain, but the digital domain is very easy because I want to multiply ok.

I multiply by 2; I want to add two signals. I just add the signals, sample by sample. If I want a product, I product sample by sample, and I get a new sequence. So, suppose if I give you another example. Suppose I said x[n] = 1 and 2 and $x_1[n]$ and $x_2[n]$ =, let us say, 3 and 4. Then when I say $x_1[n]$ and $x_2[n]$, so y[n] is $x_1[n]+x_2[n]$, it is nothing but a 1+3, 2+4. So, it is nothing but a 4, 6.

Similarly, I said $y[n] = x_1[n]$ multiplied by $x_2[n]$. So, it is nothing but a product. I can say $y[n]$ = 1 into 3 and 2 into 4. So, it is nothing but a 3 and 8. If I say $x[1]$ is multiplied by 8, a constant factor of 8. So, every sample will be multiplied by 8, 1 dot 8 and 2 dot 8. So, it will be 8, 16. So that is why digital domain processing is very easy.

So, signal property, again, I summarize, is periodic, aperiodic, symmetric, anti-symmetric, time-invariant, and time-variant. Then, I can say energy signal or power signal. Then, I can say signal level manipulation. The time index can be changed anywhere, either on the right side or the left side. So, if there is a positive shift or a negative shift, then I manipulate the signal.

I can multiply each sample by a constant factor. I can add up two signals. I can take a product of two signals. So, these are the properties of discrete signals at first. Similarly, if I have a signal, which is a two-dimensional signal, let us say I have an image I have an image. So, an image is nothing but a two-dimensional signal. So, it is a two-dimensional signal. I have an x-axis, and I have a y-axis.

So, I have a sample of the x-axis and a sample of the y-axis. So, when you go to the market to buy a television, you have to say HD TV. So, for resolution, you have to say the x-axis how many pixels and the y-axis how many pixels. So, that is the resolution. So, that is nothing but a sampling frequency on the x-axis and a sampling frequency on the y-axis.

So, I know that each pixel is nothing but a number. Its friction is a sequence, and I can say the image is nothing but a two-dimensional array. So, I'll give you an example.

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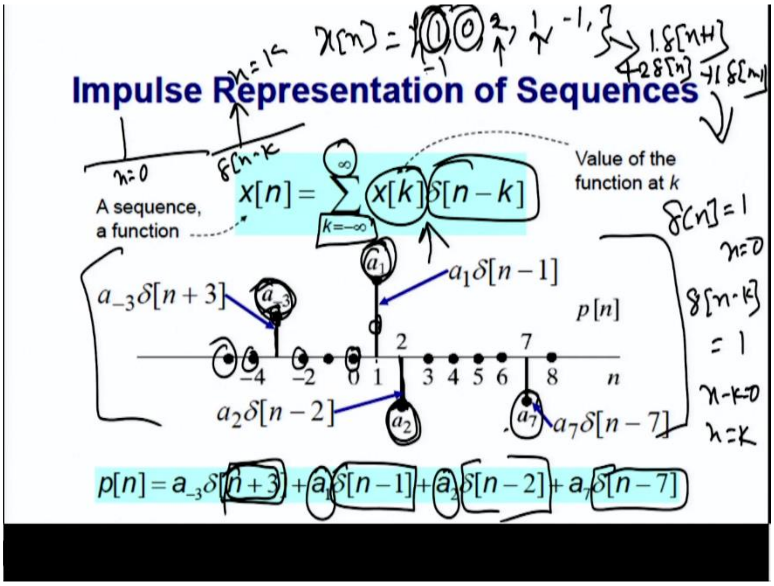

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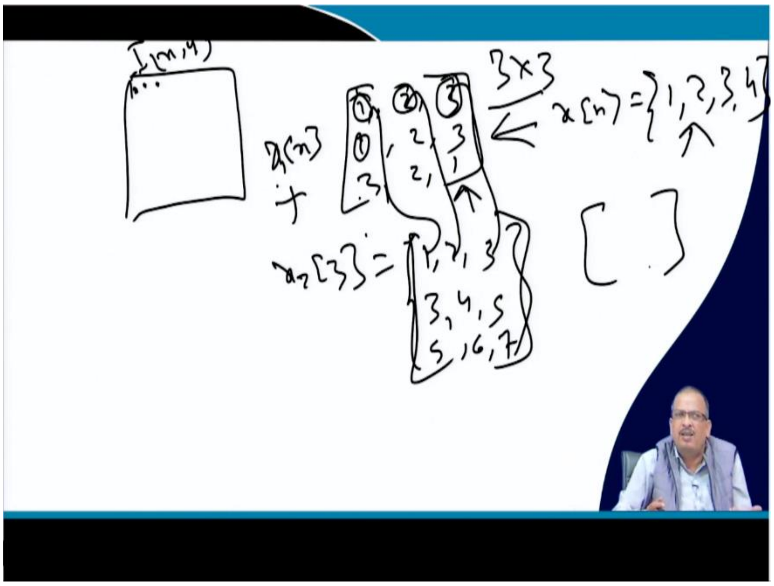

So, as an image, when I say any image, the image is nothing, but I have an image like this. So, every pixel has a coordinate x and y and has a value of I intensity. Let us say black and white image. If it is a color image, it is R, G, or B. If it is a black-and-white image, let us say 8-bit. So, I is quantized, I is nothing but a value. So, I can say an image is nothing but a matrix. Let us say 1, 2, 3, 1, 2, 3, 3, 2, 1. So, it is nothing but a 3 by 3pixel image.

The first-pixel value is 1; the first-row second pixel is 2, the third pixel is 3, and then the first-column second pixel is 1. I know the value. So, it is nothing but a matrix. When I say one-dimensional signal $x[n]$, it is nothing but a 1, 2, 3, 4 sequence of a number. Now, when I design a system when I design an algorithm, I have to operate either on a matrix if it is a two-dimensional signal.

If it is a signal dimensional signal, it will be in the signal dimensional array. I have to operate on this and write the algorithm. Add two images; suppose this is my $x_1[n]$. Let us say image 1 and $x_2[n]$ is another two-dimensional image, another two-dimensional matrix. Let us say 1, 2, 3, 3, 4, 5 and 5, 6, 7.

So, if I say $x_1[n]$ will be added up with $x_2[n]$, it's nothing but this pixel will be added with this pixel. This pixel will be added with this. This pixel, this pixel, I get a result in another two-dimensional matrix, multiply pixel by pixel multiply, scaling ok. I can multiply each pixel by a factor of A. So, the same thing can be done in a dimensional signal also. Now, how do I represent a sequence in summation form?

So, suppose this is my sequence. I have taken it from some book; this is my sequence. So, I know at n equal to 0, the value is 0; at n equal to 1, the value is a1; at n equal to 2, the value is-a2. a2 may be negative, a2 may be positive. Then a7, I have a value, n =-3, I have a value. So, at a equal to-2, 0 and a equal to-4, 0.

So, if this is my representation of the signal, I have a value only at 1, I have a value only at 2, only at 7 and only at 3; elsewhere, the value is 0. So, how do I represent it? So, I know $\delta[n] = 1$ when $n = 0$. Now, if I say $\delta[n – k] = 1$ when $n-k = 0$, that means $n = k$.

So, if I say this is my $\delta[n]$, let us say this is my $\delta[n]$ at n equal to 0 and dr is my $\delta[n – k]$, so, where $n = k$, this will be 1. So, now, I know I have to make this at one position I have to make 1 and multiply by a1. So, I can say δ a1 will be multiplied by the $\delta[n – 1]$. Then a2 I have to make n 2, a2 has to be multiplied by $\delta[n – 2]$, a7 $\delta[n – 7]$.

Now, I know $\delta[n – k] = 1$ when $n = k$. I have to make it at n equal to-3. So, k =-3. So, I can say δ is nothing but a n+3. So, the value of k is-3. So, n-k, n-(-k). So, it will become n+k. So, I can say n+3.

So, in a complex mathematical form, I can say k = -infinity to infinity $x[k]$ multiplied by the $\delta[n-k]$. So, any signal can be represented by a δ signal in this generalized formula.

So, if I give you a sequence signal sequence, let us say I give you a x[n] that = 1, 0, 2, 1, let us say-1 and let us say I given that arrow in here. Represent this sequence in a δ function. So, I know the value of x[n] exists at 1 at-1 sample. So, I can say the 1 into δ n--1. So, n+1 represents this sample; this sample is 0. So, 0 multiplied by anything becomes 0. I do not have to write.

Then this sample is 0 I know; 2 into δ n. This sample is-1. So, I can say 1 into δ n-1. So, that way, I can represent the signal using the δ function. Is it clear? So, we will do a tutorial; we will do some mathematics and some practice also so that you can be familiar with this, ok?

Thank you.