**Signal Processing Techniques and Its Applications**
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**Lecture - 07**
**Tutorial - 1**

So, let's take some Tutorials on what we have learned about sampling, the concept of frequency in sinusoidal, and then the multidimensional, multimodal signal, and multidimensional and multichannel signal. So, all those things will be contained in assignment number 1. So, today, I am taking a tutorial on those topics. So, what is the first problem?

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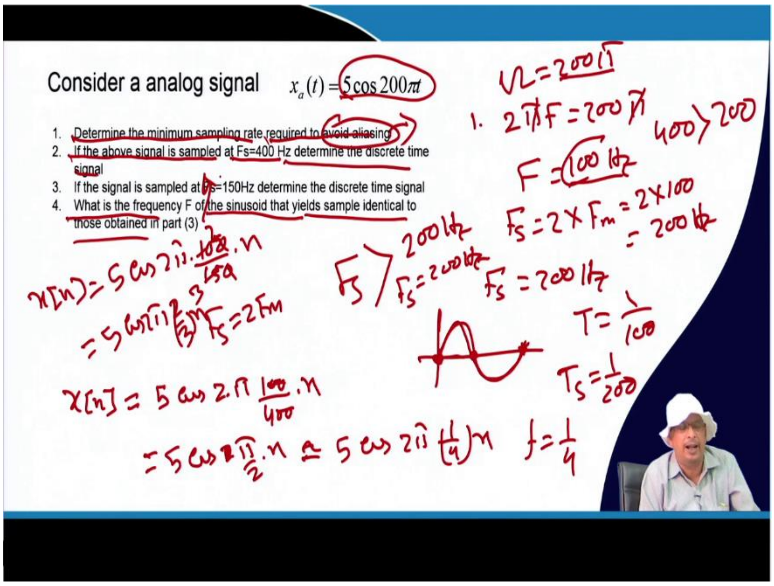


So, the first problem is to consider an analogue signal $x_a(t)$ =, let us say, 5cos200πt. So, the omega is = 200π. Determine the minimum sampling rate required to avoid aliasing. So, I have not said to determine the minimum sampling frequency required to sample so that it can preserve the Shannon sampling theorem, which means the signal will revert back ok. Let us first avoid aliasing. I will give you the answer.

So, avoid aliasing; what is the requirement? Fs must be = twice Fm. So, I will solve the number 1 problem. So, in this signal, it is nothing but an omega = 200π. So, I can say 2πF is = 200π. So, I can say F is = 100 Hz. So, my signal contains a frequency of 100 Hz. So, what should the sampling frequency be? Fs should be twice Fm, which is nothing but 100 Hz; 2*100 Hz =s 200 Hz, clear.

Now, instead of avoiding aliasing, what should be the minimum sampling frequency rate so that we can completely recover the signal from the digital signal? So, I have converted it to a digital signal and then again converted it to digital to an analogue signal. So, in that case, if Fs is = 200 Hz, since it is sinusoidal, I cannot recover the signal because, let us say, this is a 200 Hz signal.

So, Fs is = 200 Hz. This means that T is = 1 by 100, and Ts is = 1 by 200, so that means that if this is t, then 1/2 of T is my sampling time. So, this sample will be collected next T, which will be here. So, all sample values are 0. From the 0, I cannot get back the pure sinusoidal. So, if I said I have to get back that analogue signal again, I have to write Fs that are greater than 200 Hz; avoid aliasing, I can say Fs = 200 Hz clear.

The second problem, if the above signal is sampled at 400 Hz, determines the discrete time signal. So, I have given $x_a(t)$. I have to compute x[n]. So,

$$x[n] = 5\cos\left(2\pi\left(\frac{100}{400}\right)\right)n$$

So, this is =

$$x[n] = 5\cos\left(\frac{\pi}{2}\right)n$$

or I can write it down this way: 5 cos 2π * 1 by 4 in bracket n. So, small f is = 1 by 4 ok. If the signal is sampled, so, if it is 400 Hz since the frequency is 100 Hz, 400 Hz is greater than the requirement of 200 Hz. So, no aliasing is there.

Now, I said if the sampling frequency is 150 Hz, determine the discrete time signal. So, if it is 150 Hz then

$$x[n] = 5\cos\left(2\pi\left(\frac{100}{150}\right)\right)n$$

$$\text{Or } x[n] = 5\cos 2\pi\left(\frac{2}{3}\right)n$$

so, how can I do that?

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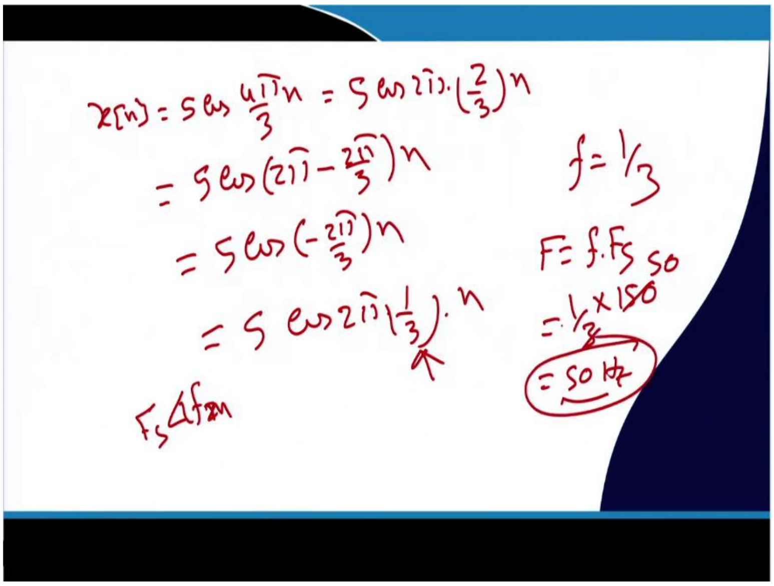


So, let us take another slide. So, what is coming? It is coming

$$x[n] = 5\cos\left(\frac{4\pi}{3}\right)n$$

So, if it is 4π by 3 n, then I can write down

$$x[n] = 5\cos(2\pi - (\frac{2\pi}{3})\,n$$

it is nothing but a

$$x[n] = 5\cos(-\frac{2\pi}{3})\,n$$

or

$$x[n] = 5\cos 2\pi(\frac{1}{3})\,n$$

So, F is = 1 by 3. Now, if this is my discrete signal, what should be the analogue frequency F; it is nothing but an F * Fs.

So, F is 1 by 3, and Fs is 150 Hz. So, I given a signal 100 Hz, and I get a 50 Hz signal. Understand or not? I give a signal 100 Hz I revert back the 50 Hz signal. So that means it is an alias with a signal of 0 to 4 100 Hz. So I cannot get back the signal. So, if my Fs is less than twice Fm, I will not get it after the digitization; the signal will be different, and I cannot return the original signal. So, that is the last question.

What is the frequency F of the sinusoidal that yields a sample identical to those obtained in part (3)? So, if I get the sinusoidal signal back again with a sampling frequency Fs = 150 Hz, then I get it 50 Hz sinusoidal instead oF100 Hz sinusoidal ok.

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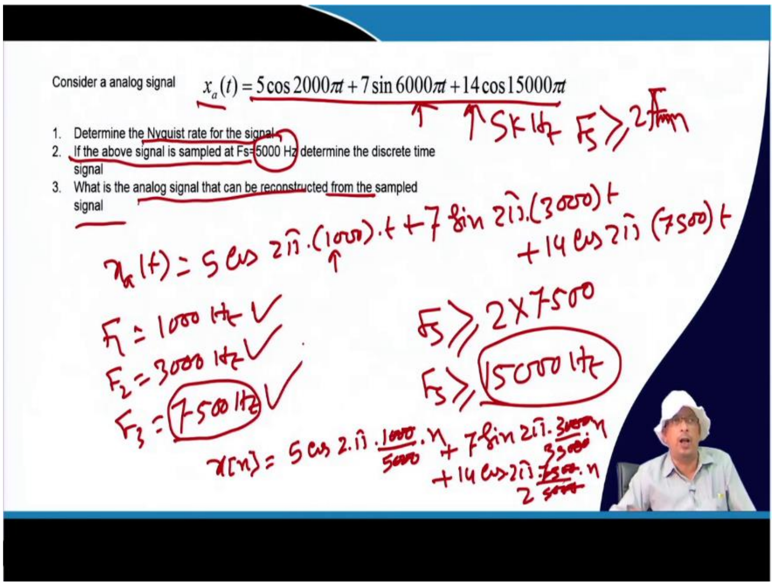


Next problem. Consider a signal $x_a(t)$; let us see this one huge long signal(refer to slide time 8:39). Determine the Nyquist rate for the signal. So, what is the Nyquist rate? Fs must be greater than = twice Fm capital Fm. Small f is discrete normalized discrete frequency.

So, now I have to find out the what is the maximum frequency content in the signal. So, if I say I can write down

$x_a(t) = 5\cos 2\pi(1000)t + 7\sin 2\pi(3000)t + 14\cos 2\pi(7500)t$

So, if this is F1 is = 1000 Hz, F2 is = 3000 Hz, and F3 is = 7500 Hz.

So, what are the highest frequency components in the signal? 7500 Hz. So, if I want to sample the Nyquist rate, Fs must be greater than = 2 * 7500. So, it is nothing but an Fs must be greater than = 15000 Hz or 15 kHz. So, the first part is entered. Clear? What is the second part? If the above signal is sampled instead of; this is my requirement sampling frequency requirement. If I sample this 5 kHz, I will determine the discrete time signal.

So, instead of 15 kHz, I sample the signal with 5 kHz; what should be my x[n]? So, I can say

$$x[n] = 5\cos\left(2\pi\left(\frac{1000}{5000}\right)\right)n + 7\,sin\left(2\pi\left(\frac{3000}{5000}\right)\right)n + 14\,cos\left(2\pi\left(\frac{7500}{5000}\right)\right)n$$

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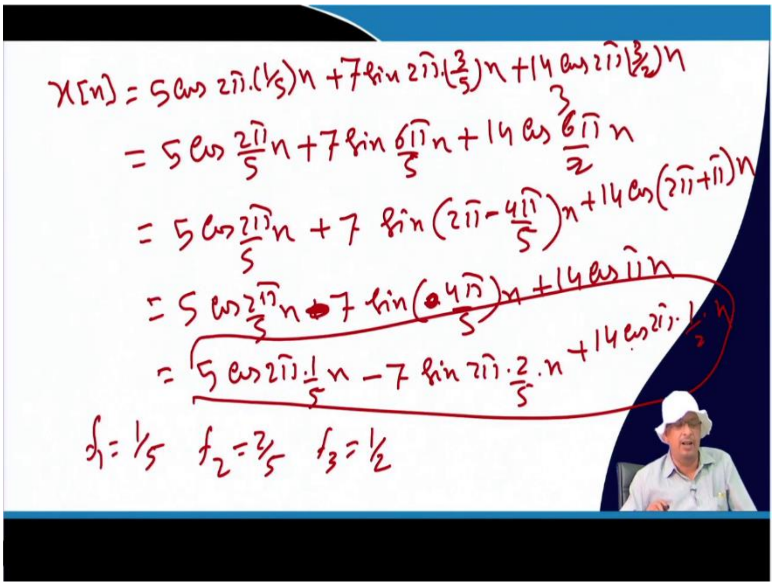


So, what should be the signal? I said x[n] = 5 cos2π 1000/5000. So, 1000 cancels with 1/5. So, 5cos 2π * 1/5 n + 7sin2π * how much is coming? 3000/5000 so 3/5 n +, how much it is coming? 14 cos 2π * 3/2 n. Now, I can convert ok. So, I can write down 5cos 2π/5 n + 7sin 6π/5 n + 14cos 6π/2 n. Ok or not? So, if it is that.

Then 5 cos 2π/5 n is ok, + 7 sin 2π - 4π by 5 * n + 14 cos 2π +π, how much? So, it says it is this will be 3π you know? So, it will be 3π. How much will it come? So, 2π + π * n. If it is 3π you can do it. It is 75 by 50; so, 3 2; 2 2 cancel, so 3π n. So, I can say 2π + 5 n.

So, then I can write down 5 cos 2π by 5 n + 7 sin - 4π by 5 n. The signal will repeat itself, that is this one, or I can write directly down -. This signal will be my sin - θ is = - sin θ + 14 cosπ n. So, I can write down 5 cos 2π * 1 by 5 * n - 7 sin 2π * 2 by 5 * n + 14 cos 2π * 1/2 * n.

So, F1 = 1 by 5, F2 is small f, this normalises discrete frequency 2 by 5, and F3 = 1/2, done. The next question is, what is the analog signal that can be reconstructed from the sample signal? So, what should be the analogue signal frequency if I reconstruct from this signal? I take x[n], and I convert digital to analogue. So, what I said? I take x[n] and convert digital to analog.

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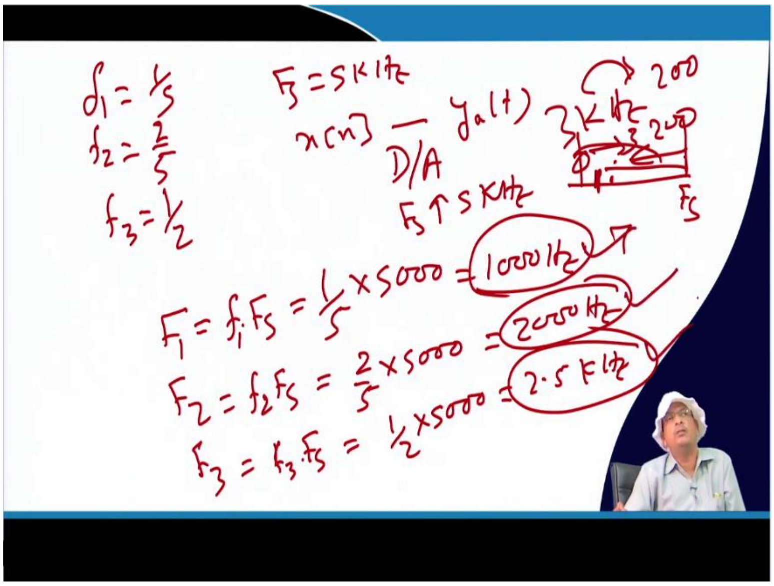


So, I get F1 = 1 by 5, F2 = 2 by 5, F3 = 1/2, and Fs is = 5 kHz. So, this x[n] is converted to again $y_a(t)$ using digital to analog converter whose sampling frequency is 5 kHz ok. So, what should be the F1 in the $y_a(t)$? F1 will be f * Fs. So, what is f? 1 by 5 * 5 kHz =s 1 kHz. What is F2 now? It will be F2 * Fs. So, F2 is 2 by 5 * 5000, 2 kHz. What is F3? F3 * Fs is = 1/2 *5000, 2.5 kHz.

So, I have applied a signal whose input F1 is = you said 1 kHz, F2 is = 3 kHz, and F3 is = 7.5 kHz, but if I sampled by 5 kHz and again digital to analog convert by 5 kHz then I get a sample with a signal which has a 1 kHz, 2 kHz and 2.5 kHz. So, 3 kHz is an alias, and it produced 2 kHz.

So, what is the; so I can say that 3 kHz component will be alias with a distance from Fs is 2 kHz and distance from 0 is 3 kHz. Similarly, the distance from Fs 7 is 2.5 kHz here and here it is 7.5 kHz. So, the 7.5 kHz alias with 2.5 kHz components, are the same. So, due to the aliasing, I will get a signal which contains 1 kHz, 2 kHz and 2.5 kHz.

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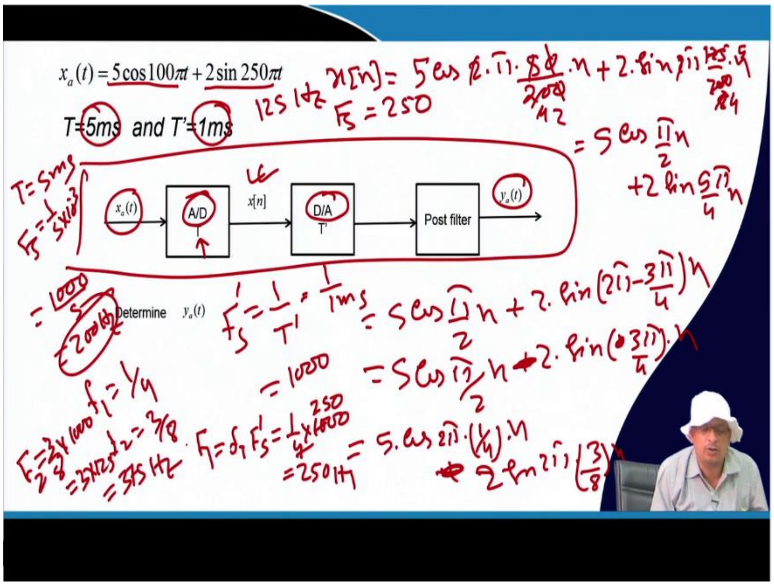


Another problem. Let us say problem number 3. $x_a(t)$ = 5cos100πt + 2sin250πt, and this is my signal processing circuit. So, $x_a(t)$ is applied to the analogue to digital converter with a sampling frequency t = our sampling time interval of 5ms, converted to x[n] and then again converted to digital to analogue converter with another sampling frequency or sampling time, which is 1ms. Then, pass through a filter, low pass filter, and I get $y_a(t)$ analogue signal. Determine $y_a(t)$.

So, if I say I want to find out x[n], what is Fs? If T is = 5ms, then what is Fs? Fs is = 1 by 5ms * $10^{-3}$ or I can say it is 1000 divided by 5. So, = 200 Hz. So, when I write x[n], it is nothing but a 5cos2π * 50 is the analogue frequency divided by Fs 200 * n + 2 * sin 2π. So, if it is 2π then 125. So, 125 divided by 200 * n.

So, if I simplify it, it is nothing but a 5 cosπ by 2 n + 2 sin. How much is it coming? So, it is nothing but a 5π by 4 n, ok. So, it is 5 cos π by 2 n and 2 sin 5π by 4 n. So, if I converted it, it will come to 5 cosπ by 2 n + 2 sin 2π; how much will come? 2π - 3π by 4 * n.

So, it can be 5 cosπ by 2 n 2 * sin - 3π by 4 * n, or I can say this will be negative, this will be 3π by 4 n. It will be aliasing because my highest frequency component is 125 Hz. So, I required a Fs, which is more than 2 * 120 Hz, but I sampled that 200 Hz. So, that will be aliasing. Now, what I said? So, I can write down this in the form of normal disk normalized discrete frequency 2π * 1 by 4 * n.

So,

$$F1 = \frac{1}{4} - 2sin2\pi\left(\frac{3}{8}\right)n$$

So, F2 = 3 by 8. So, what is the Fs now? What is F's? F's is 1 by T', which is = 1 by 1ms, which is nothing but 1 kHz. So, what is the sampling after digital to analogue conversion? What should be the analogue frequency F1? F1 * F's. So, that will be 1 by 4 * 1000.

So, F' is = 250 Hz one frequency F1. What is F2? F2 will be 3 by 8 * 1 kHz. I can calculate F2. So, then F2 will be how much? 3 * 125. So, it is nothing but a 375 Hz.

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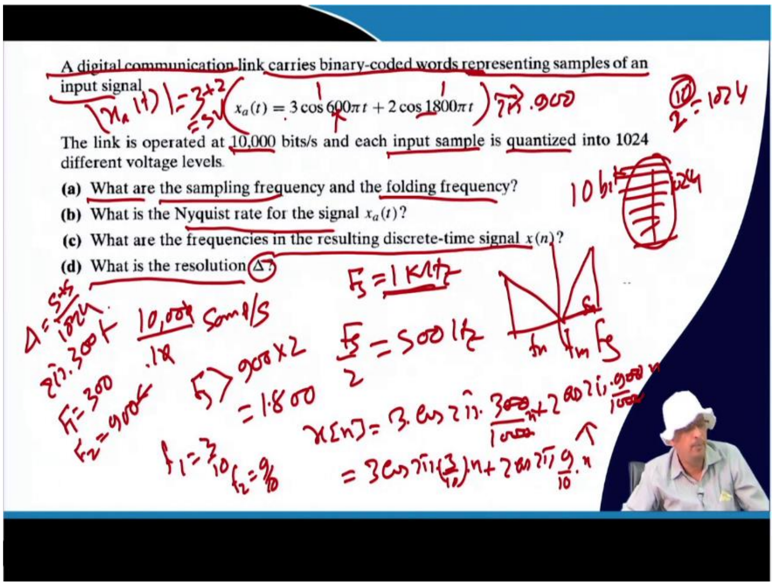


Then, if I say that, what should be the $y_a(t)$? So, what should be the $y_a(t)$?

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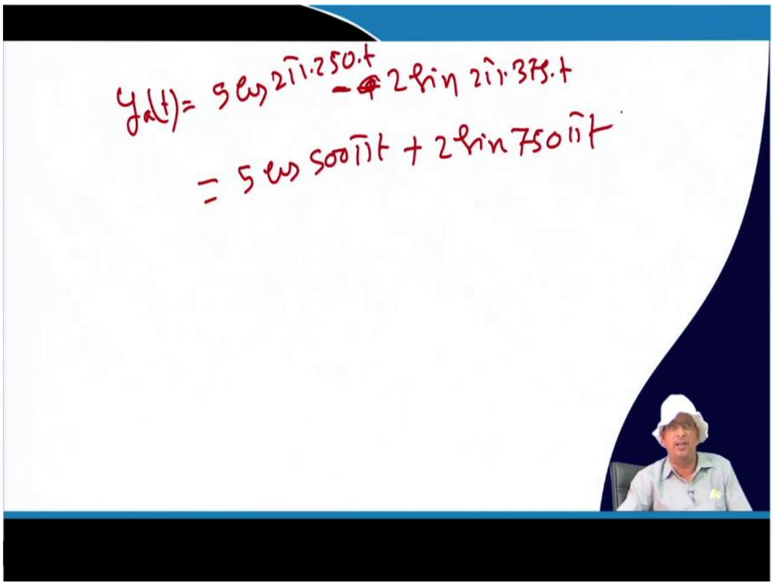


So, I can say $y_a(t)$ is = how much the amplitude? 5. 5 cos; what are the signals? 5 cos amplitude is cos F1 is 250 Hz. So, 250 Hz; 2π * 250 Hz * t + 2 sin, what is the F2; 375. So, it is nothing but a 2π 375 * t. So, there will be a - sign. So, it will be instead of + it will be -.

So, I can say the $y_a(t)$ = 5cos500πt + 2 sin750πt. So, I can calculate this $y_a(t)$ ok. So, these kinds of things you can do. Practice is very easy, you can do it, you can practice it. So, assignment 1 will be based on this kind of mathematics only. Another one, another type of example, this one. This one is related to, you can say, quantization.

I said a digital communication link carries binary coded words representing samples from an input signal like this (refer time 25:56). So, this is the input signal. This signal is digitized and transmitted over a digital channel. And what is the link? Link is operated at 10 kHz or 10 kilobits per second, and each of the input samples is quantized with 1024 different voltage levels.

So, the peak-to-peak voltage level is divided by the 1024 level. So, how much bit is required to represent all the levels? So, I said $2^{10}$ is = 1024. So, I can say that 10 bits are required to represent each sample. So, encoding is done by 10 bits. So, even if 1 second is 10 kilobit per second, then I can say how many samples each sample is 10 bit. So, I can say in 1 second, I have a 10000/10 sample per second.

So, in 1 second the channel transmitted 1 k sample. So, what is the sampling frequency? The sampling rate is 1 k. So, I can say Fs is = 1 kHz. So, what are the sampling frequency and folding frequency? What is the folding frequency? So, what I said? This is Fs, this is my Fm, this is my Fm. So, this is; so this side is Fm this side is Fm. So, as if it is folded here.

So, folding frequency is nothing but a Fs by 2 which is = 500 Hz. Understand? Next, what is the Nyquist rate of the signal? So, what is the highest frequency component in the signal? I have a 600π. So, 2π * 300 t. So, F1 is = 300. What is F2? 2π * 900. So, F2 is = 900. So, what is the maximum frequency content? 900. What should be the sampling frequency? Fs must be greater than 900 * 2.

So, I can say it will be 1.8 kHz ok. What is the frequency in the resulting discrete time signal x[n]? So, now, I require 1.8 kHz, but if I sample at 1 kHz, then what should be the x[n] first?

$$x[n] = 3\cos\left(2\pi\left(\frac{300}{1000}\right)\right)n + 2cos\left(2\pi\left(\frac{900}{1000}\right)\right)n$$

So, it is nothing but a

$$x[n] = 3\cos\left(2\pi\left(\frac{3}{10}\right)\right)n + 2cos\left(2\pi\left(\frac{9}{10}\right)\right)n$$

So, F1 3 by 10, F2 9 by 10. So, there will be aliasing in here. You can calculate that.

Then what is the resolution in Δ? So, what is Δ? What is peak-to-peak voltage? I can say the maximum value of xa(t) will be the max mod will be 3 only. So, when this one is = 1, this one is = 1? So, the 3 + 2, 5 volt ok. So, I can say Δ is nothing but a 5-volt peak-to-peak voltage + 5 - 5. So, volt v max is + 5 and v min will be - 5. So, I can say 5 + 5, which is a total swing of 10 volts divided by 1024. So, you can do it ok.

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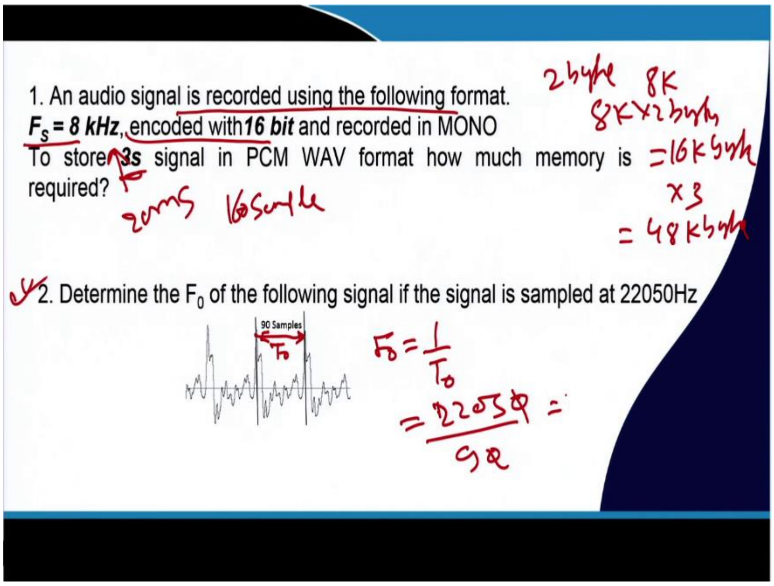


Let us say another this is the last two example. An audio signal is recorded using the following format. Fs is = 8 kHz encoded with 16-bit; how much memory is required? Let us say it is the uncompressed version is saved on a computer. There is no compression technique, only samples are stored. So, each of the samples is represented by 2 bytes. And how many samples are generated in 1 second? 8 kilo sample.

So, if I want to store 8k samples, I require 8k * 2 bytes. I can say I require 16 kilobytes. I have to store a 3-second signal, so that means 3, 48 kilobytes. Similarly I can say how many samples will be there in 20ms. You can calculate. 10ms in 1 second, 80k sample; 10ms 80 sample, so, 20ms; 160 sample.

Let us say the second question. So, this peak-to-peak distance is 90 samples, this peak-to-peak distance. So, what should be the $F_0$? So, this is nothing but a $T_0$. So, what is $F_0$? $F_0$ is = 1 by $T_0$. So, $T_0$ in time, then $F_0$ in Hz, and now $T_0$ is in the sample. So, I have to convert it to Hz. So, if it is nothing but a 22050 divided by 90, that much is the $F_0$.

So, you can do it. So, this kind of thing will be given in assignment number 1, and this you have to know because many times when you develop the application, this conversion is the maximum needed, and this conversion is required for that purpose.

Thank you.