Signal Processing Techniques and Its Applications
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Lecture - 12
Tutorial for Final Examination

So, I will use the two lectures I have given on spectral estimation, which will be the content of this week 12, week 12. Right now, I am discussing what kind of problem you will face in that exam and what kind of question patterns will be there. So, those things I am discussing right now are ok. So, as you expect, signal processing is a lot of mathematical things.

So, you can expect that all those questions will be related to numeric. So, there will be a 3 section, section a, section b and section c. So, section a contains 20 short questions, and 10 short questions carry 2 marks. So, 20 marks section b contains 10 short questions that carry 2 marks, that is 20 marks. So, 40 marks, and then section c contains 20 questions, and every question is 3 marks, which contains 60 marks; so, there is a total of 100 marks.

Section A has 20 marks, Section B has 20 marks, Section C has 60 marks, and Section 3 is divided into 20, so there will be 3 marks for each question and 20 questions. Section A and Section B require very low computation, maybe 1-line or 2-line computation, but Section C requires a computation. So, you may be required to compute, or you have to do that problem in pen and paper, and then you have to find out the answer, and then you have to give that correct answer.

Although it is a multiple-choice question, you must calculate those things; I can guarantee 90 per cent of cases. The question is numerical, and only a few questions are there where some basic importance theories are tested. I am also not asking for any formula for theory. So, the question does not say you derive this formula or what will be the formula for this no; many times, I have included the formula inside the question paper itself.

So, the formula is given in the question paper itself, but you have to feed the numerical things in that question in that formula, and then you have to solve it, and then you have to compute that thing.

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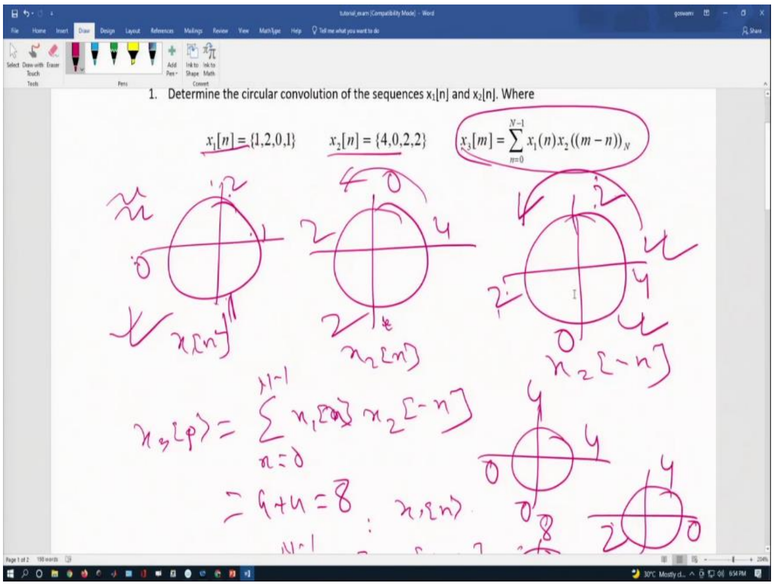

For example, let us say see that question number 1. So, determine the circular convolution of the sequence of x1[n] x2[n], where I can say x1[n] is given, x2[n] is given, and the formula is given. So, how do you compute? As you know, I can use circular things. So, you know the forward when I say x1[n]; this is 0. So, this is 1 2 0 1, and when I say x2[n], x2[n], this is nothing but a 4 0 2 2.

$$x_3[p] = \sum_{n=0}^{N-1} x_1[n] \cdot x_2[-n]$$

Now, when I say, let us say I want to draw the sample value for x2[-n], so it is reversed. So, I can say if it is anticlockwise, then it will be clockwise. So, if it is clockwise, then there will be 4 0 2 2, then what do you know that if I say what is x 3? x3[m] so, x3[0] is nothing but a n equal to 0 to N minus 1 x1[0] multiply by x 2 x1[n] multiply by x2[-n], n equal to 0. So, if you see that, what is x1[n]? This one. What is x2[-n]? I have to take the product and add it to this one.

So, I can say 4 multiplied by 1. So, the product is 4, 2 multiplied by the 2 4, 0 multiplied by the 2 0, then it is 1 2 0, there is 1, here is 1. So, 1 multiplied by 0 0, then what is the sum? It is nothing, but a 4 plus 4 is equal to 8; now, when I say x 3, x3[1].

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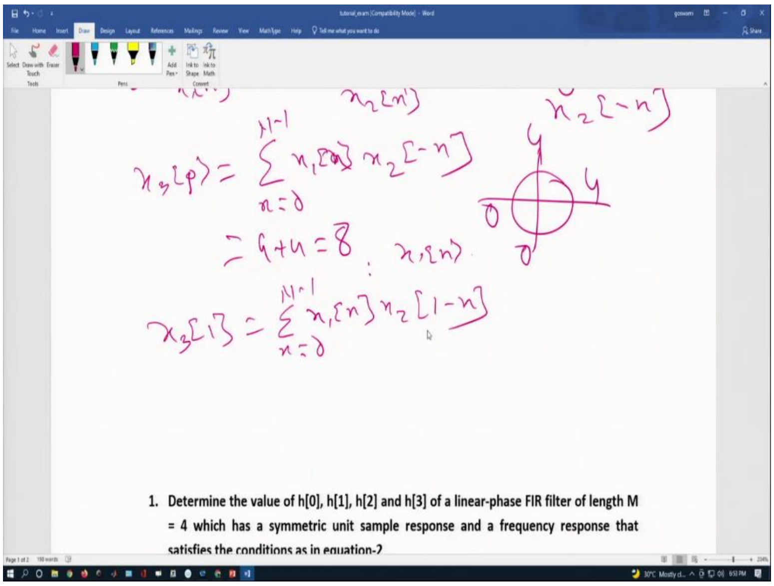

Then x3[1] is nothing but a x1[n] plus multiplied by so, x3[n] when I say x3[1],

$$x_3[1] = \sum_{n=0}^{N-1} x_1[n] \cdot x_2[1-n]$$

So, 1 minus means shifting by one sample, which you are shifting, which will be rotated by one sample. This x[-n] rotated anti-clockwise by one sample. So, I can say if it is rotated, then that 4 will come here, these 2 will come here, this 2 will come here, and this 0 will come here.

Then, it is the product, so 0 is multiplied by 1. So, what is the product sequence? 0 multiply by the1[0], 4 multiply by the 2 8, 2 multiply by the0[0], 2 multiply by the 1 2. So, it is nothing but an 8 plus 2 10. So, you have to compute it; then, you have to choose the correct option. So, in the question paper, you have to compute it and then choose the correct option.

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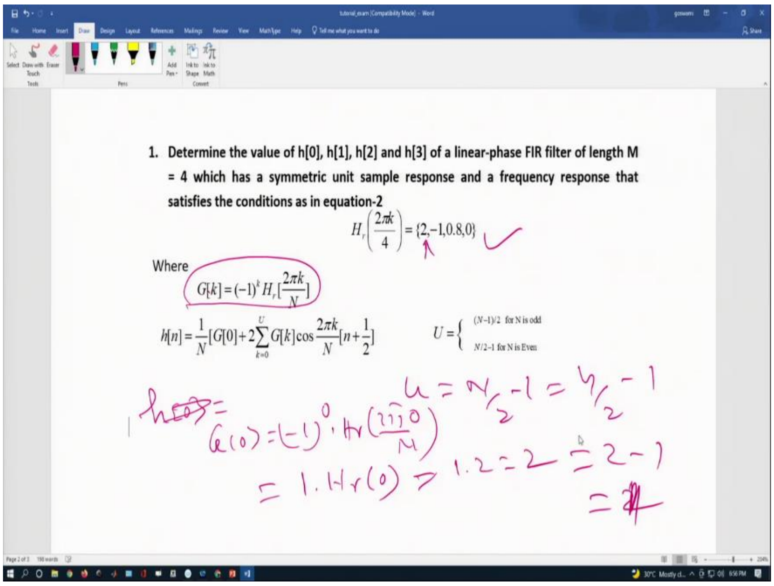

Similarly, now this question, let us say this question. I will come to the next page because that question and answer can be seen in the whole question, and the whole answer can be seen. So, let us say I said to determine the value of h 0, h 1, h 2, h 3 for a linear phase FIR filter M equal to 4, which has a symmetric unit impulse response frequency response is given; the formula is given.

So, this formula is given, the formula is given, and all formula is given. So, M is equal to 4; that means M is even. So, u is equal to N by 2 minus 1. So, 4 by 2 minus 1 is equal to 2. Sorry, 2 minus 1. Now, how do I calculate h 0? H 0 is nothing but a; so, first, you say what G 0 is? Let us say that

$$G(0) = (-1)^0 \cdot H_r\left(\frac{2\pi \cdot 0}{N}\right)$$

So, it is nothing but a 1 into Hr(0). So, what is the value of Hr(0)? It is nothing but a 2, so 1 into 2 is equal to 2.

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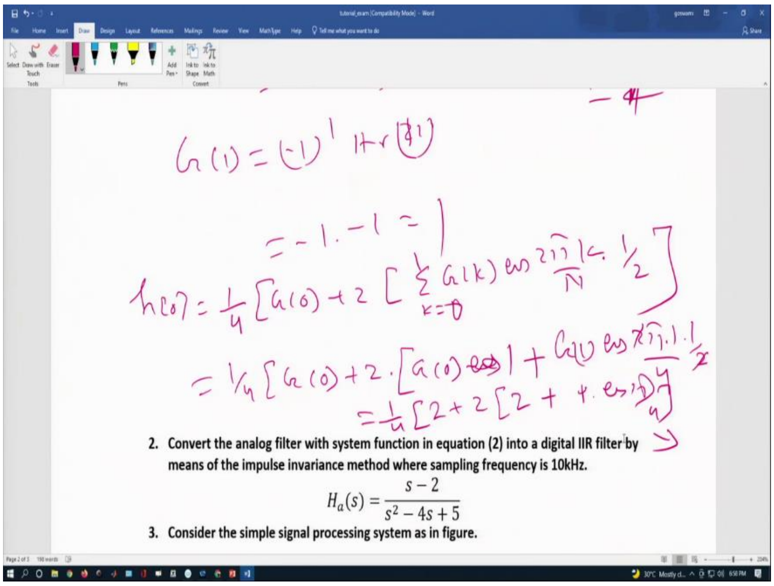

Then what is G 1? If it is nothing but a minus 1 to the power 1, then H r 2π; so, k equals 2 k equal to 1, so Hr(1). So, Hr(1) is nothing but a minus 1. So, I can say it is nothing but a minus 1 into minus 1 1. Now, if you see if I want to calculate h 0, what is the value of h 0? So,

$$h[0] = \frac{1}{N}\left[G(0) + 2\sum_{k=0}^{1} G(k)\cos\left(\frac{2\pi k}{N}\right)\cdot\frac{1}{2}\right]$$

so, I can say into half. Now, if you see k equal to 0 to 1, k equal to 0 means two parts will be there.

So, one part will be 1 by 4 G 0 into plus 2 into k equal to 0 G 0 plus G 0 cos k equal to 0, which means cos 0 1 1 plus k equal to 1. So, G 1 into cos 2π into 1 into half divided by 4. So, it is π by 4. So, I can say it is nothing but a 1 by 4; what is the value of G 0? Is G 0 value? What is the G 0 value? 0 value is 2. So, I can say 2 plus 2 into 2 plus G 1 is 1, 1 into cos π by 4. So I can calculate the value.

So, you have to compute the value and then give that answer. So, that is very much required; you have to compute the value, and then you have to give the answer.

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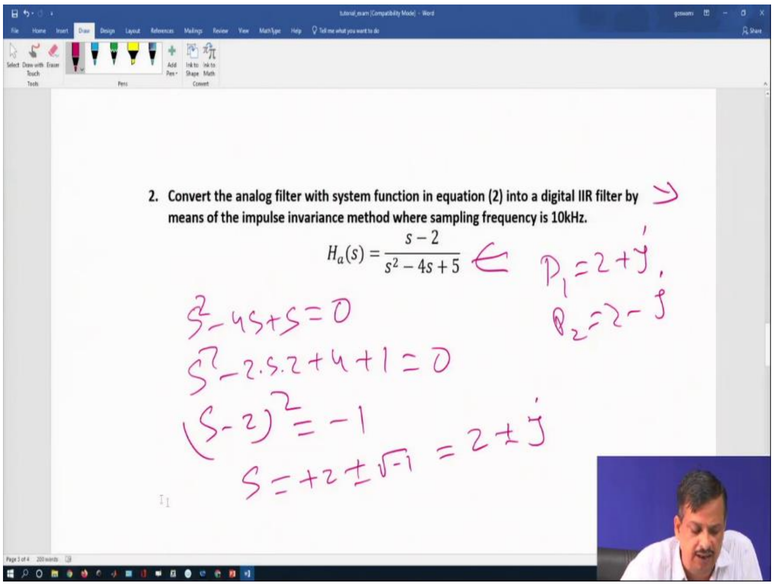

So, all the questions in section c are numerical, which is related to the value. Let us say the next question; the next question is to determine how to convert the filter system function equation 2 using a digital IIR filter by means of the impulse invariance method where the sampling frequency is 10 kilohertz. So, what do I have to do? First, I have to calculate the pole of the system. What is the pole value? s square minus 4 s plus 5 equals 0, s square minus 2 into s into 2 plus 4 plus 1 equals 0, ok or not.

So, s minus 2 whole square is equal to minus 1. So, I can say s is equal to plus 2 plus minus root over of minus 1. So, it is nothing but a 2 plus minus j. So, I can say this system has a 2 pole: p1 is equal to 2 plus j, and p2 is equal to 2 minus j. Now, what is required? I have to represent the transfer function in a fractional sum.

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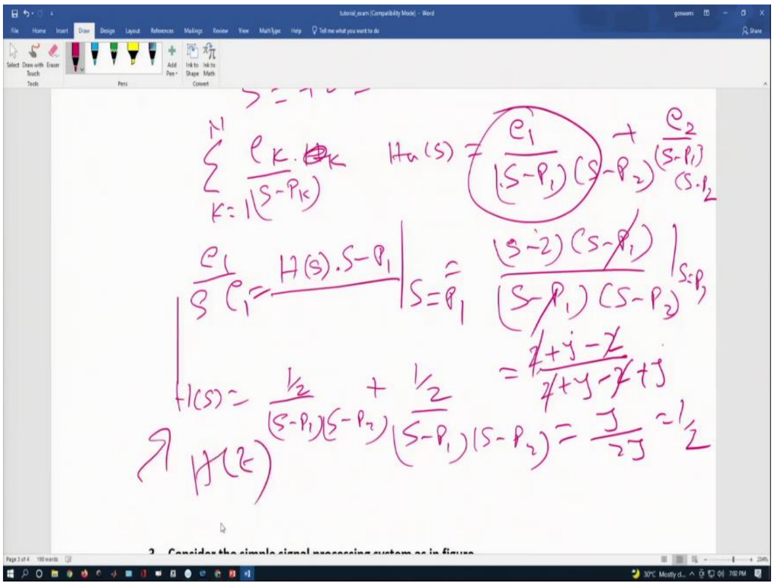

So, I can say the it is nothing but a k equal to 0 or k equal to 1 to N minus or N c k divided by c k c k into H k I can say that. So, or I can say this H s is nothing but a H of a s is equal to c1 by c1 into I can say that c1 by s minus p1 into s minus p2 plus c2 by s minus p1 by s. So, it is nothing but a c k divided by s minus p k ok. So, c1 c c1 p1 divided by s minus p2. Now, how do I get the value of c 1? I know that c1 by s 1.

So, I can say that H of s into s minus p1 at s equal to p1 gives me the c1 value. So, this is nothing but the H of s. So, it is nothing but a c1 equal to H of s is nothing but the s plus s minus 2, s minus 2 into s minus p1 divided by s minus p1 into s minus p2 at s is equal to p1. So, I can say this cancel. So, it is nothing but a s is s minus 2 s is p1.

So, it is nothing but a 2 plus j minus 2 divided by s minus 2, s is 2 plus j, and minus 2 is there, s minus p2. So, s is nothing but a 2 plus j. What is p 2? p2 is nothing but a minus 2 minus j minus 2 plus sorry plus j. So, I can say that these 2, 2 cancel 2, 2 cancels. So, it is nothing but a j by 2 j and nothing but a half. So, I can say c1 equals half, and c2 is half.

So, I can say H of s is nothing but a half of H of s is nothing but a half of H of s. It is nothing but a half of s minus p1 into s minus p2 plus half of s minus p1 into s minus p2. Now, I know what the value of s is. What is the value of z? So, I put that s and z I get that value. So, that way, you have to determine that H(z) because I have to find out if the H(z) is ok.

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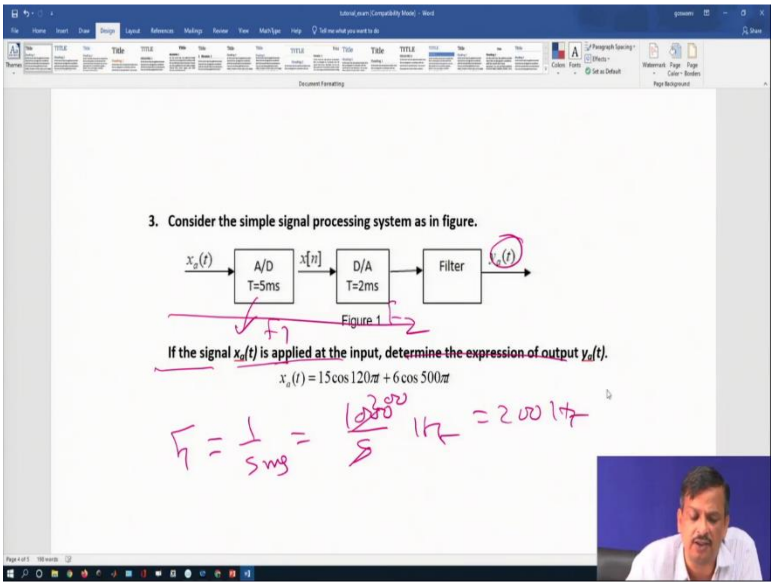

The next problem is, let us say this problem, this problem is related to the chapter number. I think 2 or 1, 1, not 2, or I can say the lecture number week number 1. So, this will be here, week number 1. So, what I said? I said that some simple signal processing systems are in the figure when considering a signal processing process. This one is the single processing system; if the signal xa(t) is applied at the input, it determines the expression of the output y a t.

So, there is an analogue-to-digital converter and a digital-to-analogue converter. So, what is that Fs in? Let us say this is F1 and this is F2. So, what is the value of F1 here, 1 minus 5 milliseconds, which is nothing but a tell me 1000 divided by 5 hertz, so 200 hertz? So, the sampling frequency of the signal is 200 hertz. So, what is the signal? This one is my signal.

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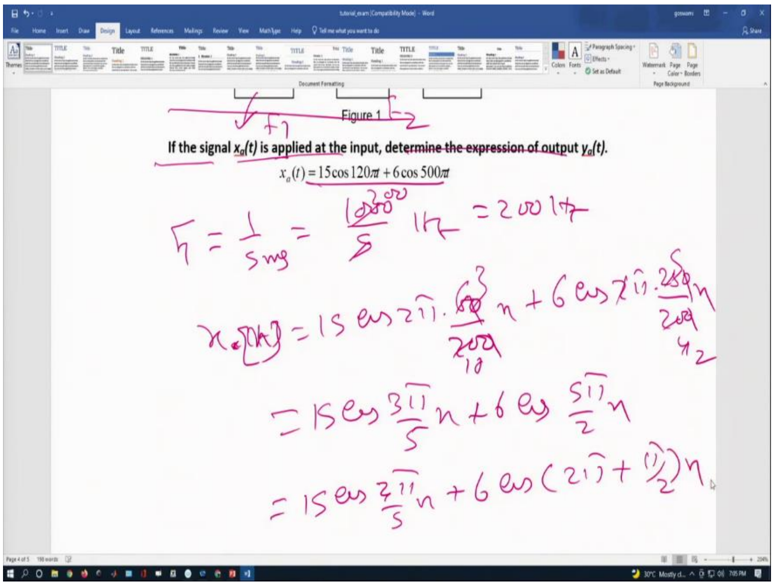

So, I can say xa(t) is nothing but a 15 cos 2π. What is the analogue signal? Analogue frequency is 60 hertz divided by 200 n plus 6 cos 2π; what is 500 mean? 250 divided by 200 n. So, that is nothing but an x[n]. So, I get the expression of x[n] 15 cos. So, I can say 3π by 5, ok or not 3π by 5 n plus 6 cos; so, 5π/2 n. Now, I know if the signal is separated by 2π, the signals are identical. So, I can say it is nothing but a 15 cos 3π by 5 n plus 6 cos 2π plus tell me; π/2 n 2π plus 5 by 2 n. So, I can say the signal is separated by 2π are identical.

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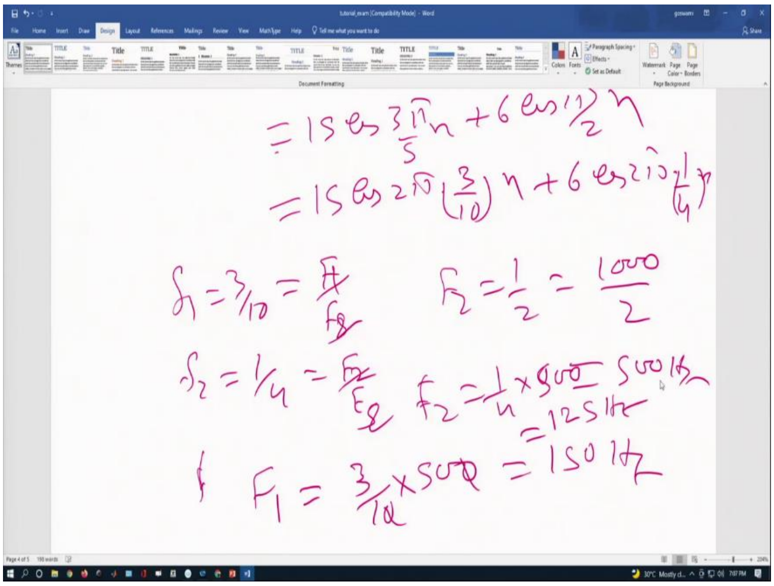

So, it is nothing but a 15 cos 3π by 5 n plus 6 cos π/2 n. Now, what is the discrete frequency F1? Discrete frequency is nothing but a cos 2π into 3 by 10 into n plus 6 cos 2π into 1 by 4 n. So, I can say that discrete frequency F1 is equal to 3 by 10; F1 is nothing but an F by F s.

F is the analogue frequency, and F2 is equal to 1 by 4, which is nothing but an F by Fs; Fs is the sampling frequency, and F is the analogue frequency. So, what is the sampling frequency in analogue to digital to analogue conversion? It is 2 milliseconds. So, I can say if let us this is Fs, this is Fs or F2, I can say F2 is equal to 1 by 2 milliseconds. So, a 1000 divided by 2 is nothing but 500 hertz.

So, I can say F is equal to so, this is F1. Let us say this is F2; F1 is equal to 3 by 10 into Fs 150 hertz, and F2 will be 1 by 4 into F s. So, I know it is nothing but a 125 hertz. So, if you see that the input analogue frequency is my 60 hertz and 250 hertz, here I get 150 hertz and 125-hertz analogue signal. So, I can get that ok. So, what is the expression? I can write down the expression easily.

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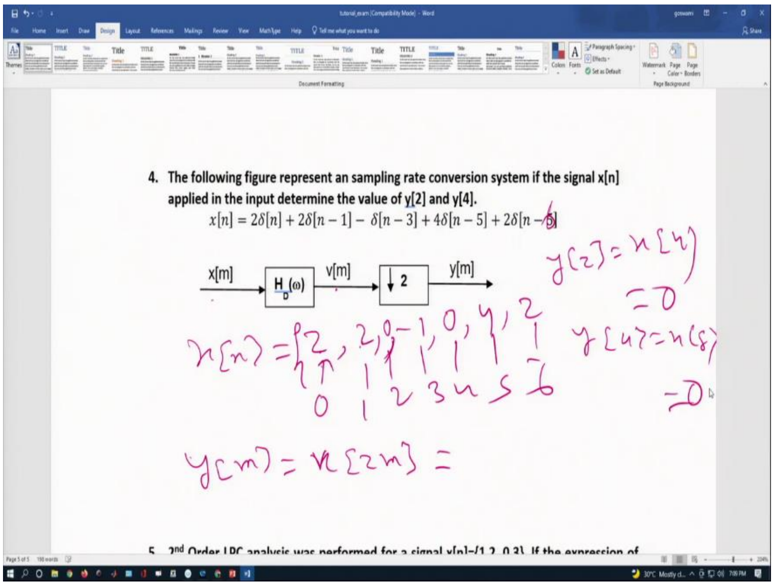

Now, let us look at question number 2 4. What did I say? The following figure represents the sampling rate conversion system. If the signal x n is applied in the input, determine the value of y[2] and y[4]. So, what is x[n]? x[n] What is the sampling repetition? 2 is the 0th sample, so I know 2 is the 1 sample, and minus 1 is the third sample, so that means 0 1. So, it is minus 3; that means 0 1, then 1 0 will be there, and then minus 1 is okay.

And, then minus 5, then 4 and 5; this is 0, this is 1, this is 2, this is 3, this is 4, this is the 5th sample. Then so, there will be another 5th. Sorry, this will be 6; let 6, then 2. That is the 6th sample. Now, what I said is dissemination by 2, so every alternative signal will be said. So, I can say y[m] is nothing but a, so x[m] is equal to v[m]. So, y[m] is nothing but a x[2m].

So, I can say that. So, y of 0. So, what I said is that y[2] is nothing but an equal to x[4]. So, what is x[4]? x[4] is equal to 0, y[4] is equal to x[8] which is equal to again 0. So, that way, I can do it ok.

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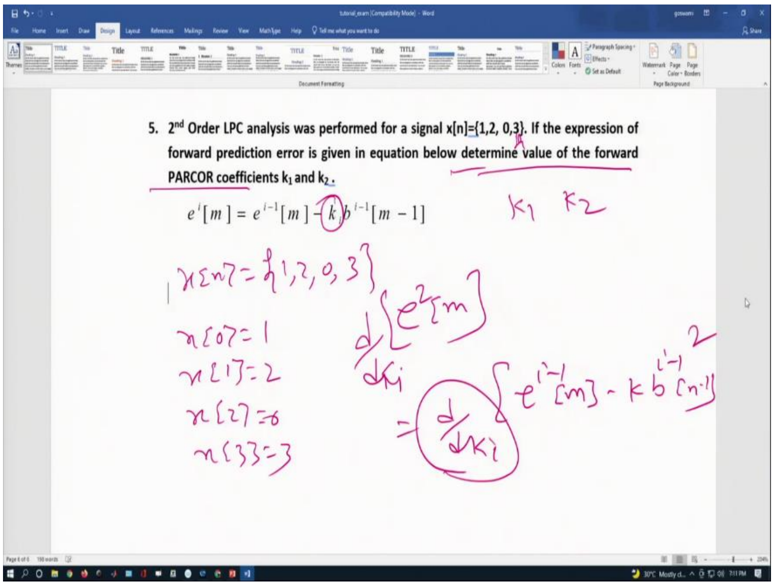

Similarly, question number 5. This is a good question; you can see that this required a lot of calculation. That is why I went to that page first and then did it.

So, 2nd order LPC analysis was performed using a signal x[n] is given 1, 2, 0, 3; that means x[0] is equal to 1 x[1] is equal to 2 x[2] is equal to 0 and x[3] is equal to 3 and partial reflection coefficient is k1 and k2 because the order of the analysis is 2. So, what do I have to find out? Determine the value of the forward prediction coefficients k1 and k2.

So, how do I get that? So, as you know, how do I get the minimum value of k1 mean square error? So, I can say

$$\frac{d\, e^2}{dki}[m]$$

So, I know that this is nothing but a

$$\frac{d\,[e^{i-1}[m] - kb^{i-1}[n-1]}{dki}$$

So, this is why I have to derivate it square.

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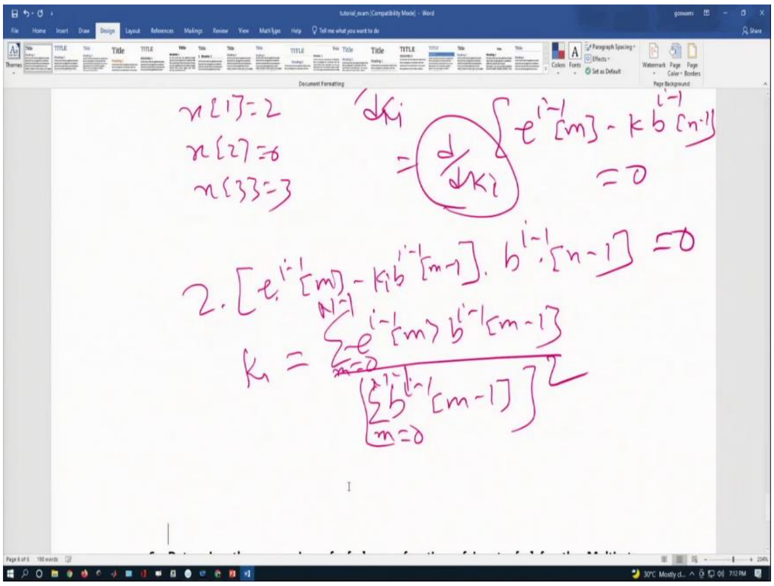

So, the square means it is nothing but a 2 into e^{i-1} m minus k b^{i-1} m minus 1 multiplied by b^{i-1} n minus 1. So, I have to find out that the mean square error is equal to 0. So, this is equal to 0. So, this is equal to 0. So, I can say that k i that k i this multiply by this and this multiply by this.

So, I can say.

$$k_i = \frac{\sum_{m=0}^{N-1} e^{i-1}[m] \cdot b^{i-1}[m-1]}{\sum_{m=0}^{N-1} (b^{i-1}[m-1])^2}$$

So, now, m varies, so there will be a sum m range from 0 to N minus 1. So, here also, m varies from 0 to N minus 1. So, what is N? N is the length of the signal. So, what is the length of the signal?

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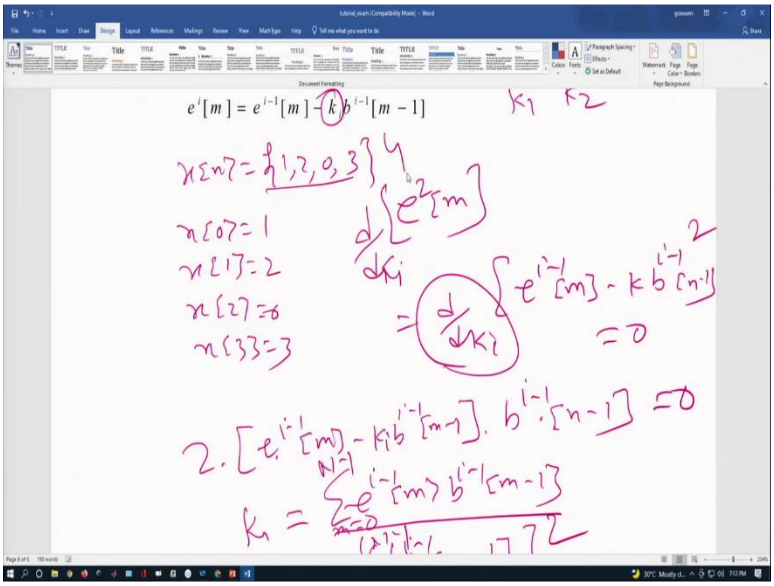

The signal length is 4. So, I can say that this is k was k, and this is m n equal to 4; 4 minus 1 means 3.

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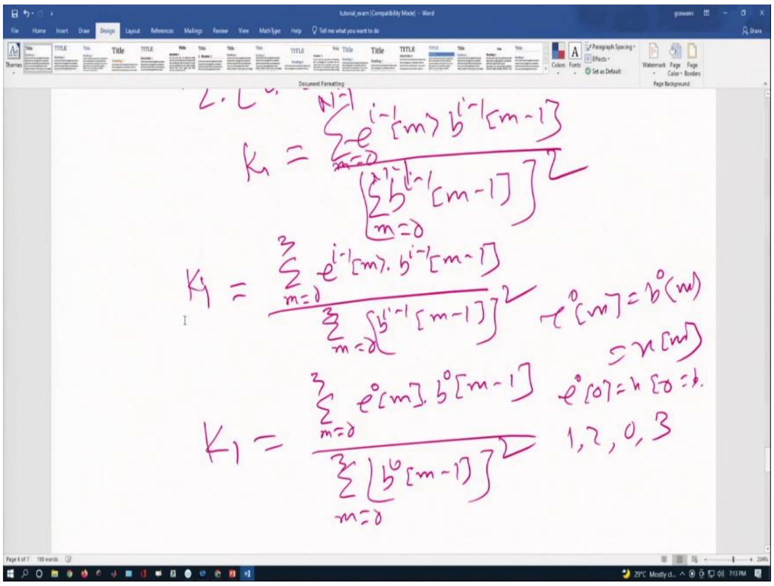

So, I can say

$$k_1 = \frac{\sum_{m=0}^{3} e^0[m] \cdot b^0[m-1]}{\sum_{m=0}^{3} (b^0[m-1])^2}$$

So, Now, as you know $e^0[m]$ is equal to b 0 m is equal to x m. So, I can say that e0[0] is nothing but a x 0 or equal to b0[0]. So, if my x x is 1 2 0 3; so, x is 1 2 0 3. So, I can say e0[0] is nothing but a 1.

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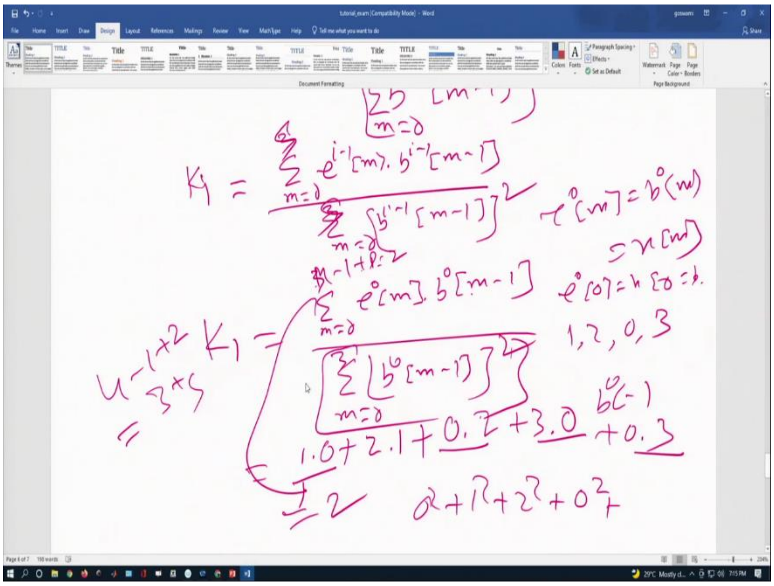

Multiply by e 0 b 0 minus 1, minus 1 means 0 plus u 1 2 multiply by the m equal to 1. So, b0[0] which is nothing but a 1 plus; so, 1 after 1. So, I can say 0 multiply by 2 plus 3 multiply by 0 plus 0 multiply by 3. So, I can say that it is nothing but this term 0, this term 0, this term 0, this term 0, it is nothing but a 2 2.

So, this term is 2. Now, what is this term? This term is nothing but a square. So, whether 0 will be not there, so, 0 square plus 1 square plus 2 square plus 0 square plus. So, sorry, this will be m minus 1 plus p, so p is equal to 2. So, that is why m minus 1 plus p.

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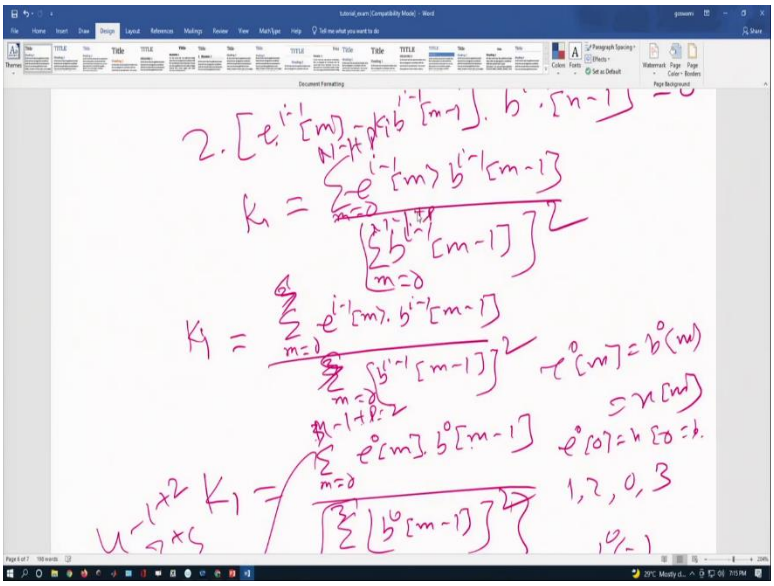

So, 4 minus 1 plus 2; so, this will be 3 plus 2 5. So, it is m varies from 0 to 5 because, 2nd order analysis is 0 to 5 N minus 1 plus p ok. So, now, I get this one will be plus 3 square.

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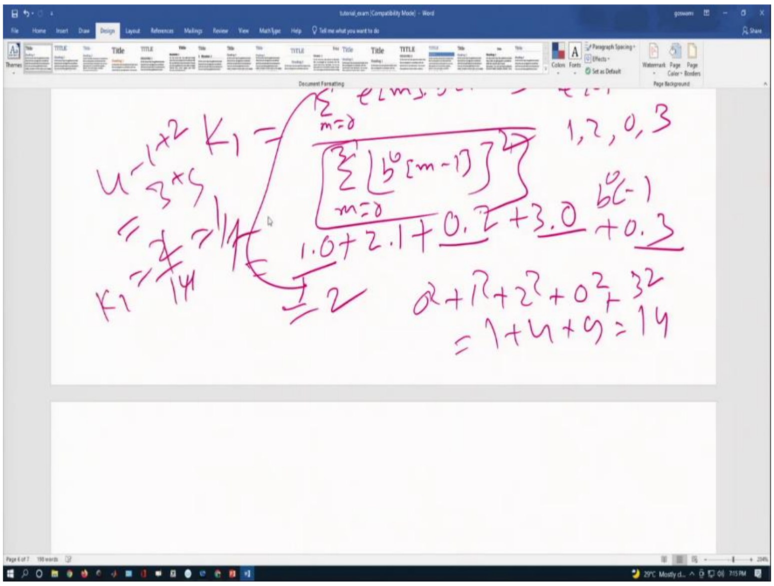

So, I will get how much? 1 plus 4 plus 9. So, it is nothing but a 14. So, I can say the k1 is nothing but a 2 by 14. So, it is nothing but a 1 by 7.

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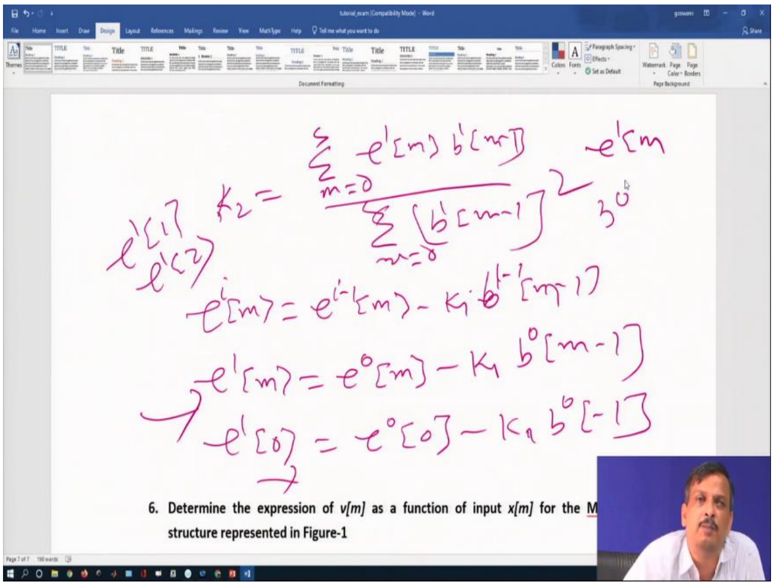

Then, I have to calculate k2. So, when I say calculate k2, k2 is nothing but a I have to set m equal to 0 to 5 e1[m] b 1 m divided by 0 to 5 b 1 m m minus 1 whole square. So, I have to determine e1[m]. So, what is e1[m]? So, how do I get the e1[m]? I know e i m e i m is equal to e^{i-1} m minus k i into e i b^{i-1} m minus 1.

So, I can say that e1[m] is equal to e0[m] minus k1 into b0[m-1]. From there, I can calculate that e1[0] is equal to e0[0] minus k1 and b 0 minus 1. So, from there, I can calculate e1[0], e1[1], and e1[2]. Similarly, I can calculate b0[0], b1[1], b1[2]; then, once I get that b 1 n e 1, I can calculate the value of k2. So, that required a calculation.

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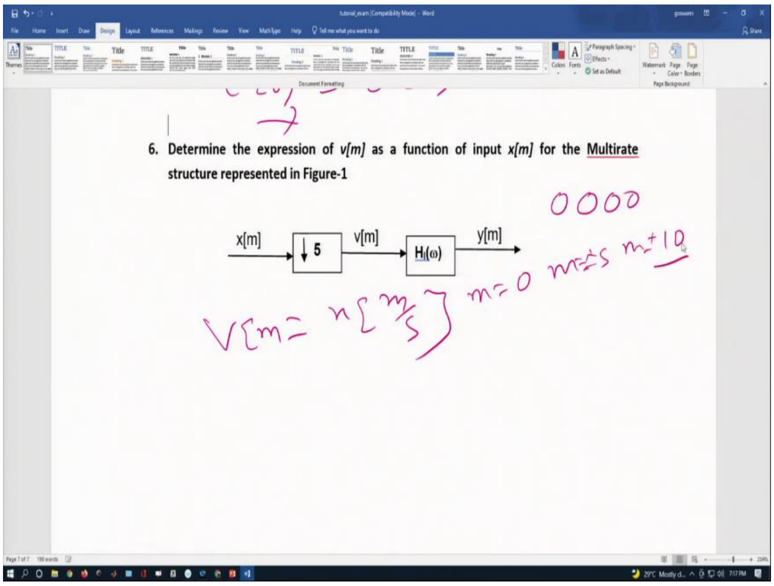

Then, I will determine the expression of a v[m] as a function of input x[m] in multi-rate signal processing. So, what is v[m]? v[m] is nothing but a x[m] m by 5. So, what is required? So, I have to 4 0 is inserted between 1 to 5th simple 1 1 to 2nd sample, where m is equal to 0, m is equal to plus minus 5, m is equal to plus minus 10 like that ok.

Then the so, this is the signal, this kind of mathematics you can expect from that question paper. So, I will say, please carefully look into the assignment and try to practice all the mathematics I have given in the assignment; that is very important. If you practice all the mathematics, I am sure you will be able to solve all the questions in the question paper.

So, thank you very much for attending the entire course. So, there is a review, and the feedback that the question is whether the course is too much mathematics; yes, signal processing is mathematics. But, I try to explain it as is your practical sense and give you some assignments that can use your brain to see what is happening practically because signal processing is the application of, or you can say, the physical phenomena expressed in a mathematical expression.

So, that is why this is too much mathematics, ok? So, thank you for your patience in attending the entire course. I hope that all of you will perform better in the exam. also if you have any questions or any queries regarding any point, my email id is available; you can send me an email, or you can drop a message in your NPTEL forum; so, that I will able to answer you. So, thank you very much for your kind patience.

Thank you.