Signal Processing Techniques and Its Applications

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Lecture - 55
Power Spectrum Estimation

Ok. So, this week is the final week of your course. So, I will not include other topics, new topics, or topic details this week, but I have to cover one topic. Because of that, if you say the signal processing course, this topic is very important. And so, I will take this one topic, not details, maybe some as in this course, which may be the introduction of these topics I will give you. Later on, you can study this topic in detail.

This is another topic that is very important for your signal processing, so that is what I will cover this week. Then I will talk about some problems like that question paper, what kind of question paper will come in the final exam, and some problems I will solve in this class. So, this week is basically a sub-topic, you can say, and which will cover mostly 1 on 1 or 2 lectures.

Then, in the 3rd lecture, there may be a tutorial around that exam, what kind of questions will come, what kind of mathematical problem you have to solve, and some problems I will present to you, and I will solve them for you that that whatever that solution will be there. So, you can get some essence of that, but mainly a final exam will contain that numerical problem; there is no theoretical part there, ok?

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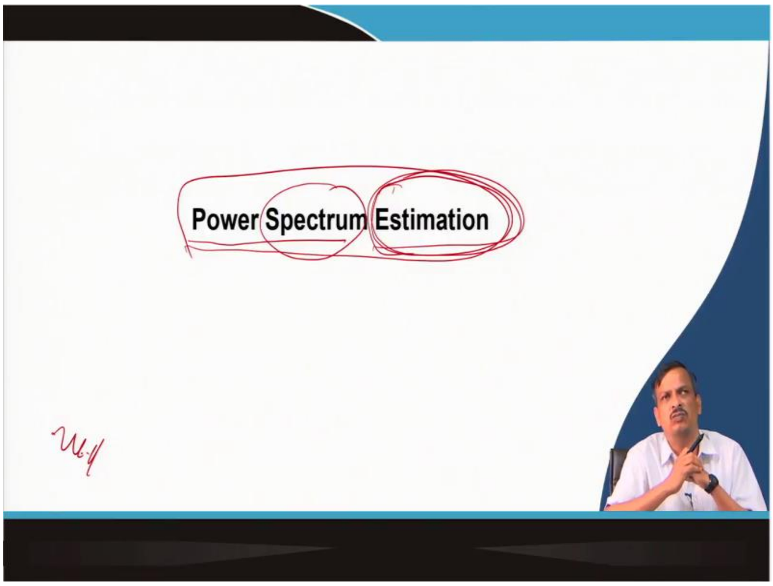

So, let us start with that topic, which I said is called power spectrum estimation. It is an important topic for signal processing, and there are many things you have to learn. So, in power spectrum estimation, there are two things: one is the power spectrum, and the other one is an estimation. So, the problem is two-fold: one is what the power spectrum is, what the energy spectrum is, and what the spectrum of the signal is, as I discussed during my class on the signal of the discrete-fourier transform.

That is how to calculate the spectrum, and then we would add another word called estimation. So, if I am able to calculate the spectrum, then why is this estimation involved, but commonly, the subject that topic is not at power spectrum estimation? So, where is it used? So, from any signal like that, say that you are developing a sensor, the sensor produces an electrical signal after conditioning the signal, and that signal is digitized.

Now, once you digitize that signal, then what you want to detect is something you have to measure or something you want to classify from the sensor signal. So, one of the parameters we said, the spectrum, yes, spectrum characterizes the systems, yes, no problem, but I have to estimate the spectrum, Power Spectral Density PSD or estimate the spectrum. So, what do you mean by estimation of the power system? Where is the error?

So, when is it estimated? The estimation comes into the picture when there is no deterministic. Something cannot be directly measured so that I can estimate those things. So, why does this estimation come? So, in the whole lecture, we will discuss what power spectrum is and what power spectrum estimation is, why the word estimation is there and if I want to estimate, then what I should do.

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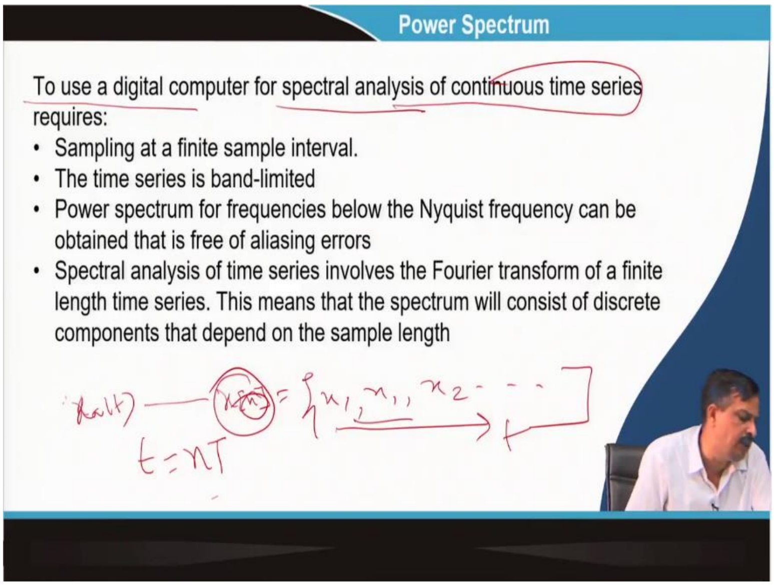

So, let us talk about the first power spectrum; what is the power spectrum? So, when I say the digital computer, I use a digital computer for spectral analysis of the signal of a continuous time series. I said continuous time series because if I say xa(t) is my analogue signal, then once I digitise it, I get that x[n]. So, x[n] is nothing but the series that contains the samples and the value of the sample. So, it is nothing but the time series along the timeline. So, n is the time index.

So, I can say continuous t is sampled using n into T. So, n is the index sample number 0, sample number 1, and sample number 2, so I can say this is time series. So, now, what do I want? I want that spectrum of this time series; if I have a time domain signal, suppose I have a time domain signal xa(t). So, xa(t) has a time domain signal.

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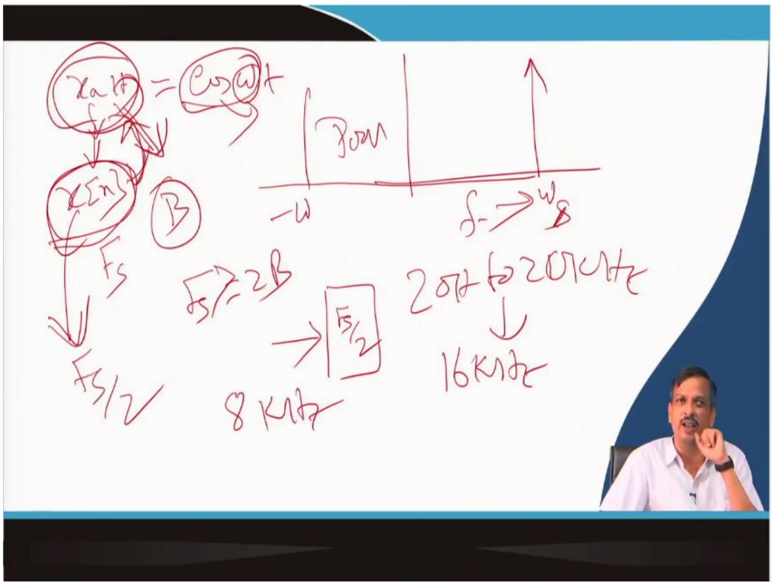

Let us say xa(t) is equal to cos (ωt). You know, it has any spectrum. Spectrum means the power across the frequency. So, this axis is power; this axis is frequency s. So, when I plot the power for different frequencies if I say cos (ωt) so that there is a distinct frequency in ω, then I know the power spectrum will be two distinct frequencies. I get which one is ω, and another is minus ω. So, I will get, that is, the representation of the spectral domain of the cos (ωt).

So, what I said is that I have power, so instead of an analogue signal, I have a time series x[n]. Now, I have to compute the spectrum of x[n]. There are some limitations when we digitise an xa(t) analogue signal to a digital signal. What is the first limitation? The first limitation is the Fs sampling frequency. So, I know x[n] that xa(t) or analogue signal must be band-limited.

I cannot convert to a digital signal if it is not band-limited. So, band-limited means it must have a frequency band. So, let us say the bandwidth is 0, and the bandwidth is B. So if I know from the Nyquist rate that Fs is equal to twice B, at least my Fs must be greater than or equal to twice B or on the other hand, I can say the x[n], the digital signal is a band limited signal I do not know whether that xa(t) contains much frequency.

But once I digitized it, I forcefully band limited it by Fs by 2, so that is called an anti-aliasing filter. So, when I designed that analogue digital converter, I had an anti-aliasing filter in the input of the analogue to digital converter, which band limited my input signal by Fs by 2. So, if the xa(t) contained a frequency that is above Fs by 2, due to the anti-aliasing filter, I cannot get the spectrum of that signal.

For example, we know we have a sound. Let us say the audio signal we know it is 20 hertz to 20 kilohertz, but let us say an audio signal is sampled at 16 kilohertz; that means I am band limited the audio signal to 8 kilohertz, although it contained after 8-kilohertz component, I cannot analyse it is in the digital version of the audio signal, which is represented by x[n] and the sampling frequency is 16 kilohertz.

So, when I say input signal, when I say spectral analysis, I only can analyse the spectrum of the signal, which is below Fs by 2. If the signal has any component that is greater than Fs by 2, I cannot analyze that. So, that is the one limitation.

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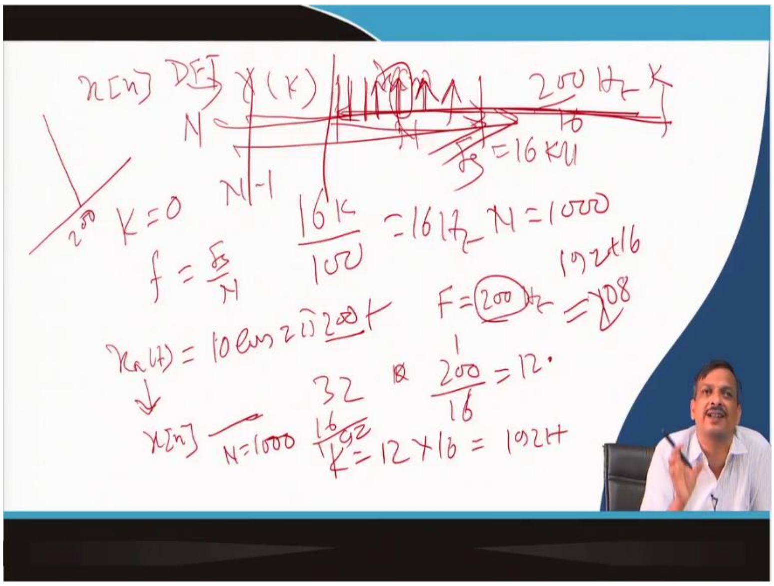

The second limitation is when I do the Fourier transform, let us say I have an x[n], how do I convert to the frequency domain?

When you compute X(k) using the Fourier transform, or I can say the Discrete-Fourier transform, I use it to convert x[n] to X(k) in the frequency domain. So, if I know the length of the DFT is N, then I am considering the signal x[n] is periodic with N. So, x[n] may be infinite length. Still, I have to take the finite length of the signal, which N. So represents, once I take the finite length of the signal. I know that frequency component, let us say it is sampled by, let us say, F s, then I know the resolution of this spectrum analysis, the frequency component I can get from k equal to 0 to N minus 1 with a frequency equal to Fs by N. That means, suppose I have a signal and sampling frequency of 16 kilohertz and N is equal to 1000s, then I know that the frequency component, which I will get, is that the spectral resolution is nothing but 16k divided by 1k. So, it is nothing but the 16 hertz.

So, what will happen? So, suppose I have a 200-hertz component in the signal I know, but I cannot get exactly a 200-hertz component in the sampled version due to the limitation of finite length N. So, what do I get? I get every 16 hertz. So, I have not received it since 2000, which is not divisible by 16. So, I cannot get a k value for which basically it is in 2000 hertz. So, suppose I have a signal xa(t) that is equal to, let us say, 10 cos 2π 2000 t.

That means analogue frequency capital F is equal to 2 200, 200 t, 200 hertz. Now, if I sample this signal by 16-kilohertz x[n], I will get and take the N point DFT n, which is equal to 1000. Then I cannot get exactly 6 200 hertz components because I will get k. What is the k value? 200 divided by 16. So, if it is 16, then 1, 4, 40, so, 12 points or something.

So, k equal to 12 is; that means 12 into 16. How much will come? 32 and 1 6, so 29; so 192 hertz. So, I will get a component at 192 hertz and a component which is 192 plus 16. So, this is equal to around, I can say, 8 0 208 hertz. So, ideally, if I am able to analyse the spectrum, I should get a component at 200 hertz only.

However, due to the band limitation or the finite length of the signal, I cannot get exactly 200 hertz. So, the power in 200 hertz in the signal power will be distributed among the nearby components, which is called DFT leakage. Once I say the finite length, there is another problem. So, how do I get the finite length of the signal? That means I have an infinite-length signal; I take a chunk of the signal, which is nothing but a windowing.

So, I know the signal's frequency response is nothing but a convolution of the window frequency response and the signal frequency response. So, exact signal frequency response I cannot get once I analyse the finite length of the signal. So, when I say a x[n] is x[n] equal to a cos (ωt), it is nothing but a deterministic signal.

Let us say non-deterministic, signal random process signal, so in that case, I do not have that x ω t, the non-deterministic point. So, in that case, I have to estimate the non-deterministic signal spectra. Now, you may say, sir, if I increase the N. So, that may come to the resolution is equal to 1 hertz. So, once I increase the length N, what will we be basically doing? If the N is very large, the frequency resolution is no doubt increased. I get every component; let us say I required a 1 hertz.

So, N is equal to 16k, in this case, 16000. So, if the signal is in a non-stationary signal, the signal in stationary signal I can take the large signal and if I take the significant signal, computational complexity will be increased. Let us say the computational complexity is there. I take a large signal but think about a non-stationary signal.

If my signal is non-stationary, that means, over the timeline, the signal property is changed; then, if I take a large window, basically, I lose the time resolution. So, I increased the frequency resolution, but I lost the time resolution. So, I cannot take a large number of samples and an ample number of sample evidence. So, I have to find out the spectrum of xa(t) or x[n] using the small number of evidence or samples. So, that introduces an error. So, this is the error in the case of a deterministic signal.

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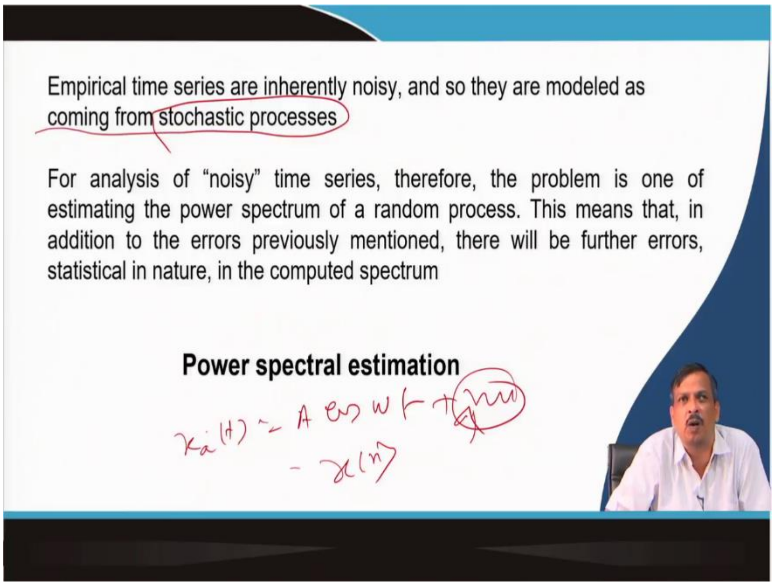

Now, say the empirical time series are inherently noisy and modelled as coming from a stochastic process. So, a deterministic signal, which means xa(t) is equal to A cos (ωt), is a deterministic signal, but generally practical, all signals are non-deterministic because there will be a signal along with some noise.

Now, if I want to find out the true spectra of the signal, then due to the noise, I have to estimate the power spectrum. So, the error due to the analysis is already there, the error due to the windowing, and the error due to the finite length already there, in addition to the noise, is a problem. So, I have to estimate the cos. So, I cannot say to determine the power spectrum of a random signal; I cannot do that; I have to estimate the power spectrum.

I can say that this may be the best estimation for the power spectra of the signal. So, that is why it is called power spectral estimation, which is ok.

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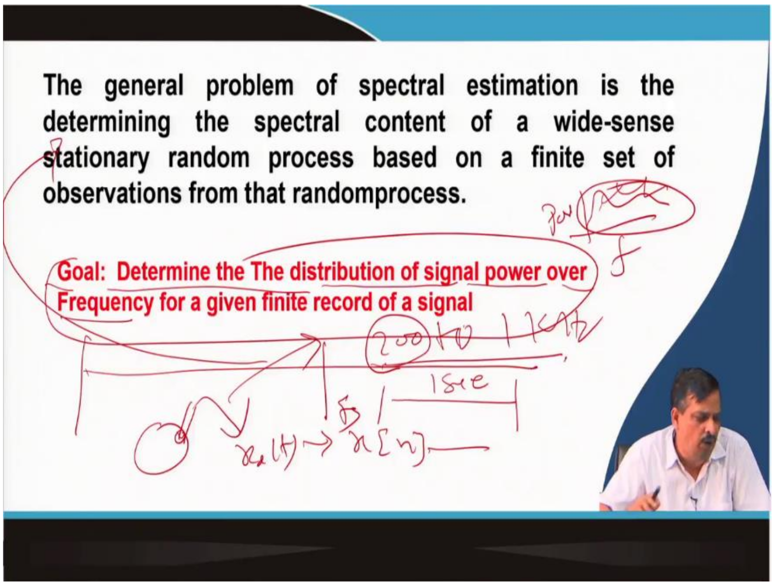

So, what is the problem? The problem general problem of spectral estimation is to determine the spectral content of a wide-sense stationary random process based on a finite set of observations. What did I say?

I have a signal in infinite length, but I cannot do that infinite length spectrum analysis because when I say the Discrete-Fourier transform, the signal must be deterministic, the finite length of the signal N. So, I have to take a finite length of the signal and estimate the spectrum on the random process signal. So, the goal of the problem is to determine the distribution of the signal power over the frequency for a given finite record of the signal.

So, what is power spectral estimation? Find out. So, suppose I give you a signal. I asked what components exist in the frequency component of the signal and their power value. So, suppose I said vibration analysis, I am doing an accelerator on a machine and finding out the signal. Now, what I am looking for is a particular frequency component. If there is something wrong, then some particular frequency component power or range of frequency component power will be increased.

So, what am I looking for? What is my problem? I have a machine. I put an accelerometer there. So, I am collecting a signal, which is xa(t) time signal, then I digitise the signal x[n] using F s; that means I am band limited the signal, done. Then I want to find out, let us say, what the frequency component power of 2000 hertz is to, let us say, 1 kilohertz.

So, what is the possible power of that frequency component, and what is its spectrum? How do I measure? Because I put the accelerometer there, along with the timeline, the signal is coming, coming, coming. So, how much signal will I take when I want to estimate that frequency component of 2000 to 1 kilohertz? Time is infinite. So, what I will say is I will take a deterministic approach, and I will take a finite length.

So, let us say I have a given finite length; let us say I told the 1-second signal I gave you. From the 1-second signal, you have to estimate the power of the frequency component from 2000 hertz to 1 kilohertz. So, determine the distribution of the signal power over the frequency for a given finite record of the signal. I cannot get an infinite record of signal, so that is my goal.

So, the axis will be the frequency, and the axis will be the power; I have to draw that spectrum. But I know that when I say the finite, it introduces an error. When I say the analogue to digital conversion, it introduces an error, and the signal will not only be there but also a noise. So, it is not a purely deterministic signal; there will be some signal. So, the signal will also be corrupted by noise.

So, then, from the noisy signal, how do I estimate that spectrum that is called power spectral estimation? Is it ok? So, I have defined what power spectral estimation is.

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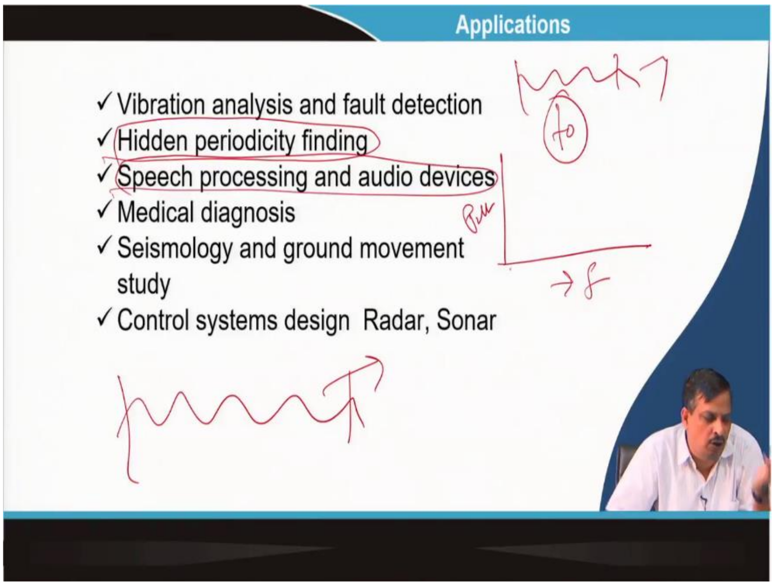

So now, when I say deterministic signal, there is some application; I have already explained one application, let us say accelerator. Then, the other application, hidden property periodicity, finds out the hidden periodicity of the signal.

We want to determine the frequency component for speech processing and audio devices. Let us say I want to find out whether the signal has a male or female signal. So, I find out the component's fundamental frequency and the signal's fundamental frequency. So, the fundamental frequency of power will be very high. So, I discovered that it is a fundamental frequency and calculated the main signal.

All spectral entropy can be a 1 parameter. Suppose I have a signal; this is a speech signal; I want to find out whether it is a curve or whether it is a saw. So, spectral features can be a parameter. So, I can say, find out the spectral spectrum of the signal. From the spectrum, I can easily say that whether it is a voice signal or a noise signal, noisy or the sibilant signal or fricative signal, that kind of thing can be done. Medical diagnostics is a very important power spectrum.

So, seismology, ground movement study, and spectrogram are all always nothing but a power spectrum. Have you seen that spectrogram like an x-ray plate, which I showed you during that frequency analysis of the signal, radar signal, sonar signal, and control system everywhere? What is the Bode plot? The Bode plot, When you make a control system bode plot x-axis is the frequency, the y-axis is the power, so nothing but a spectrum of the signal.

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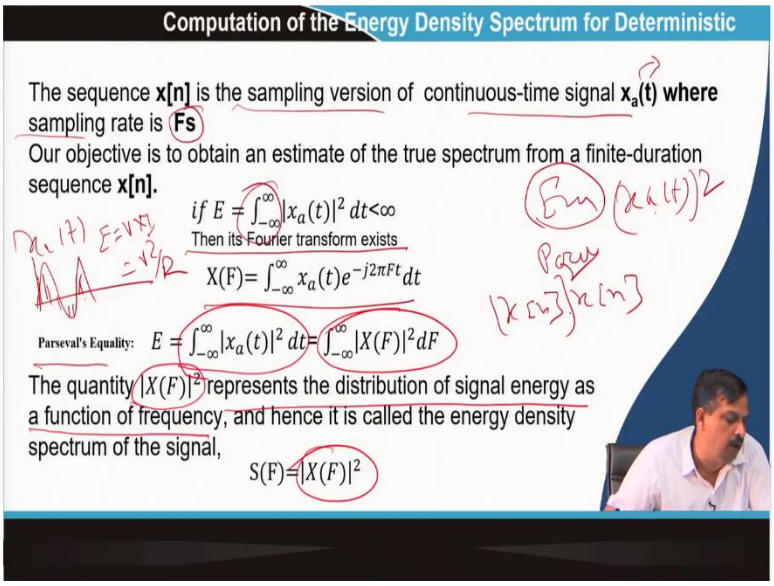

So, if the signal is deterministic, then we say energy density spectrum. When I say the signal, let us say the sequence of x[n] is a sampling version of the continuous-time signal where the sampling rate is Fs and xa(t) is a deterministic signal; it is not a random signal but a deterministic signal. So, you know a signal has two types of signal; one is called an energy signal, and the other is called a power signal.

When the energy so if the signal energy is finite, then I call it an energy signal. Signal energy may be infinite, but power may be finite, so we call it a power signal. So, how do I calculate the signal energy when the energy is finite? Let us say xa(t), t is infinite. So, I can say xa(t), the energy is nothing but a squaring of the signal. Now, if I say the sampled version of the xa(t) is x[n]. So, it is nothing but x[n] multiplied by x[n].

So, x[n] square is the energy because xa(t) is the voltage. What is that when I say xa(t)? Along the time axis, there is a voltage. So, you know the energy is equal to V into I. So, it is nothing but a V square by R, and R is constant, so V square means energy. So, how does the energy represent? The how much the signal contains area. So, if it is, a digital signal is nothing but the square of the sample value.

If it is the analogue signal, it is nothing but a square integrated over the minus infinity to plus infinity, which is the total energy. So, if the energy is finite, then the Fourier transform exists, and this is the Fourier transform for a continuous-time signal. So, what is the Parseval theorem? You said the energy can be computed in both domains; it can be computed from the time domain, and it can be computed from the frequency domain. So, what is the Parseval theorem? Basically, what it is said, it is said that I will take a slide here.

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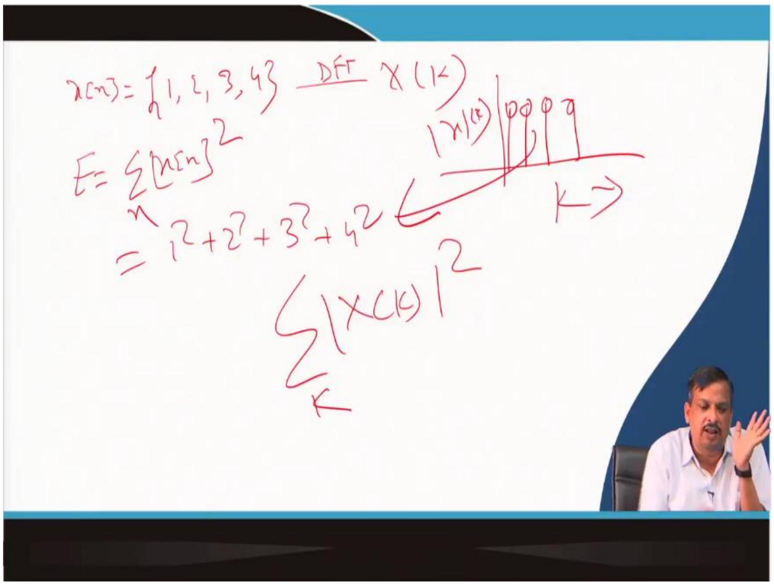

Let us say I show you from a digital signal; the slide is in the analogue signal. Suppose I have a signal x[n], x[n] is equal to 1, 2, 3, 4. Now, I take the Discrete-Fourier transform, and I get X(k). What the Parseval theorem says is that if I compute energy in the time domain, it is nothing but a summation of the square of the sample.

So, it is nothing but a 1 square plus 2 square plus 3 square plus 4 square, the energy which is equal. So, the same energy is also contained in the spectral version of the signal. So, what is the spectral version? I know it is nothing, but a k varies from this side and this side k and this side is x of the mod of X(k). So, if all frequency components and amplitude square up, then I also get the same energy.

So, I can say I can say the X(k), whole square summation over the k and here summation over the n. Energy cannot be destroyed, so this energy will be equal; that is the Parseval theorem. So, in the analogue domain, I can say this is the energy of the analogue signal; this is the energy calculated from the frequency domain representation of the signal. It is nothing but an integration; the analogue domain is nothing but an integration.

So, this X(F) whole square represents the signal energy distribution as a function of frequency; then I can say this X of the mod of X(F) whole square is called the energy density spectrum of the signal.

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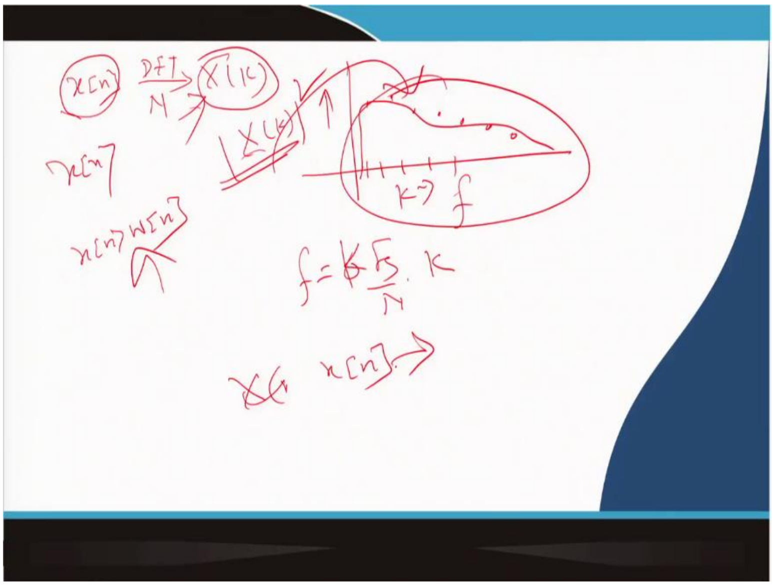

When I told you, suppose I told you to draw a spectrum. So, I have a signal x[n], and then I take the Discrete-Fourier transform, and I get X(k) of length N.

Forget about what the error will be introduced by n, how do I get that X(k)? For all those things, forget about that first; initially, forget about that. So, I know x[n] is a voltage signal, and X(k) is a voltage spectrum. So, I know one is going to the power or energy. I have to square it up. So, what is spectrum? Spectrum is nothing but an axis of frequency; this axis is the power. So, the power means X(k) mod whole square that is nothing but a power.

So, X(k) is a complex signal a+jb. So, power is nothing but a square plus b square root over will be off because of the square for a different frequency. Now, since it is the discrete domain. So, the frequency is discrete. This is because the k frequency varies from k. What is the relationship between the k and analogue frequency f? As far as I know, f is equal to Fs by N into k.

So, I calculate the f corresponding k and calculate the spectrum magnitude value to get a point. So, once for every k, I will get the point; I draw that I get the spectrum representation of x[n], that is the spectrum ok, so that is the spectrum. So, that spectrum I can calculated from the signal using Discrete-Fourier transform. So, what is the internal problem, the internal problem is that x[n] is an infinite-length signal.

So, I have to make the finite duration. So, x[n] is multiplied by a W n window function. So, the effect of that window will come into the spectrum, so that introduces an error. So, since there is an error due to the window. So, I have to estimate the true spectrum of x[n], which is why I call power spectral estimation okay. So, in the next class, I will talk about how this autocorrelation is also involved during the power spectral estimation.

Thank you.