**Signal Processing Techniques and its Applications**
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**Lecture - 54**
**Sample Rate Conversion by Stages**

So, as I am discussing the multi-stage, what are the advantages? So, let us say this is the problem.

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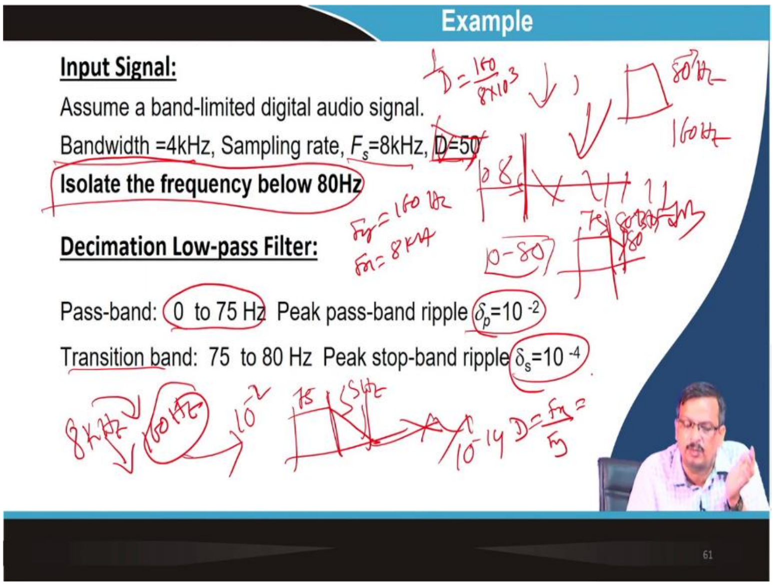


I have an input signal, a band-limited digital audio signal, bandwidth is 4 kilohertz, the sampling rate is 8 kilohertz, and I want to implement a down sampler D. The down sampler or, let us say, the problem D is not given. Let us say I isolate the frequency below 80 hertz. So, I have an audio signal. I want to isolate the signal, which is 0 to 80 hertz, which will exist, and after that, no signal will exist. That problem I want to implement. Can I implement using multi-stage sampling rate conversion?

So, if I say my bandwidth is 0 to 80 hertz, let us say I said 75. There is a 75 hertz. This is flat, and then up to 80 hertz, I allow a 3 dB response. So, up to 75, the response is flat, and after 75 to 80, it is 3 dB down ok, and the pass band ripple is $10^{-2}$ and the stop band transition band is, so then the transition bandwidth is 3 dB. So, I can say the transition band is 75 to 80 hertz.

And stopband ripple is $10^{-4}$. So, I am saying that I want a low pass filter whose transition bandwidth is 5 hertz and the passband is 75 hertz. So, up to 80 hertz, some signal will exist. After that, no signal will exist. The passband ripple is 10, with power minus 2, and the stop band ripple is 10, with power minus 4. And the transition bandwidth is 5 hertz. So, how can I implement it using a multi-stage decomposition or multi-stage downsampling?

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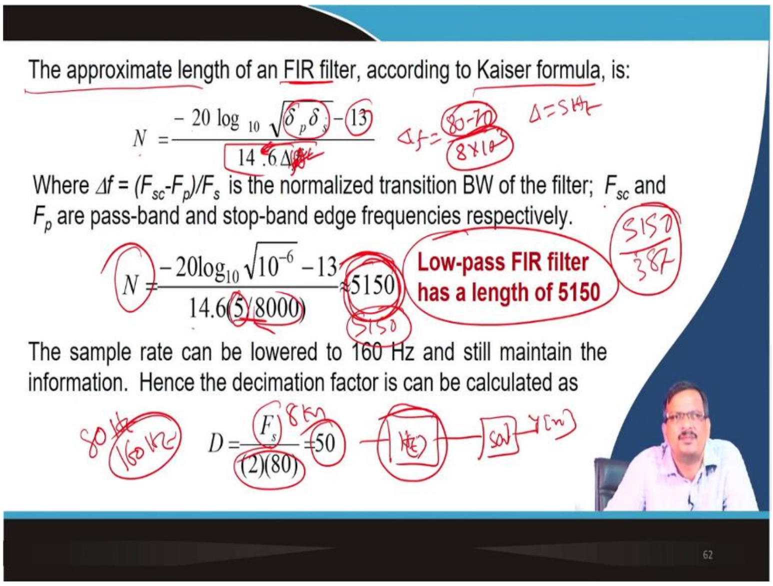


Let us say the approximate length. So, suppose if I want that, I wish to down-sample. So, what should be the D factor D if I want a signal to contain only 80 hertz? Does that mean how much sampling frequency I require 160 hertz? So, I have an 8-kilohertz sampling frequency signal. If I down it to a 160-hertz sampling frequency, then my purpose will not be hampered; the output signal will contain up to 80 hertz only. So, what is the D factor D. So, F y is equal to 160 hertz, and F x is equal to 8 kilohertz.

Then, what is the D? So, D is equal to F y divided by F x 8 into $10^3$, or I can say that it is 1 by D. So, D is equal to F x is the given and F y I required. So, it is nothing but an 8 kilohertz. So, 8 kilohertz divided by 2 into 80 or 160. So, this is equal to 50. So, I want to implement a down sampler that will pass through a filter, downsample by 50, and then get y n.

But what should be the length of this filter? The length of this filter is because the transition bandwidth is equal to 5 hertz only, and a ripple is given. So, if I use the Kaiser formula for

FIR filter length N is equal to minus 20log10 Δp into Δs minus 13 divided by 14.6 into Δf, this Δf is in radian. So, this Δf is equal to equal to transition bandwidth in hertz; that means 80 minus 70 divided by sampling frequency 8 into $10^3$.

So, 8 minus 70 is 5 hertz, and the sampling frequency is 8 kilohertz. So, that is my Δf, or I can say Δ ω instead of Δf you write Δ ω, Δf because f I am writing because π I have multiplied in here 2π I have multiplied in here. That is why I said Δ f. So, it is nothing but a transition bandwidth in hertz divided by the sampling frequency. If I put the values Δp Δf and Δf, then I get the order of the filter is 5150, which is the length of the low pass filter; see the complexity.

So, if I want to implement let us the simple problem given: Given that I have a signal which is sampled at 8 kilohertz, I want to design a low pass filter whose cutoff frequency flat cutoff frequency is 75 hertz and whose passband edge frequency is 80 hertz. So, the transition bandwidth is 5 hertz. So, I required a very sharp transition bandwidth. In that case, you know if the reduction of the transition bandwidth increases the filter's order. So, the order of the filter is very high because of the precision low pass filter I have to implement.

Now, I want to use the multi-rate signal processing to reduce the order of the filter; how can I do that? So, I know I required 80 hertz, which is my ultimate output signal. So, I know that if the output signal is sampled at 160 hertz, that is sufficient for my getting the signal up to 8 80 hertz. So, I calculate D, D equal to 50.

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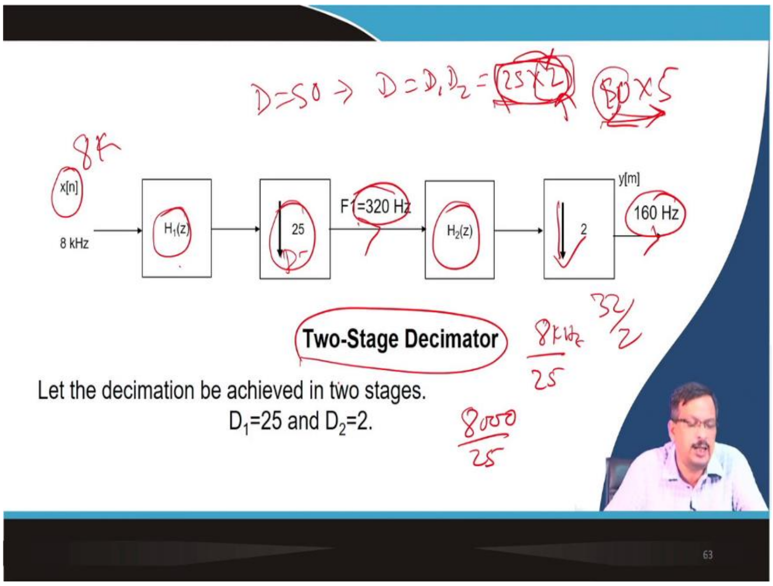


Now, once I get the D, it is equal to 50. I said this can be implemented so that D equals D 1 into D 2, which equals 25 into 2. So, I put 25 here because the next filter is the possible decomposition. I can do 10 into 5. I can say that 5 into, or I can say 10 into 5 also. If I do 10 into 5, the filter's order is not that much of a deduce because, in the second stage, I want the order to be very low, but here, the order will be distributed in the 2 filters.

So, let us say I want to implement 25 into 2. So, I first have an x[m], which is 8 kilohertz. Let us say H1(z) I want to implement, and then I pass through decomposition D is equal to 25. So, downsampling by factor 25. So, what is the sampling frequency here? So, I have an 8 kilohertz; 8 kilohertz is down sample by 25. So, 8000 divided by 25. So, I get a 320-hertz sampling frequency here. So, here, the sampling frequency is 320 hertz, and then that signal goes to another filter again, down the sample by 2. So, I know 320 divided by 2 is 160 hertz is my ultimate sampling frequency of the output.

So, I use two stage decimator or two stage down sampler to implement that filter. So, how can I implement it, let us say filter specification. So, I summarize the filter.

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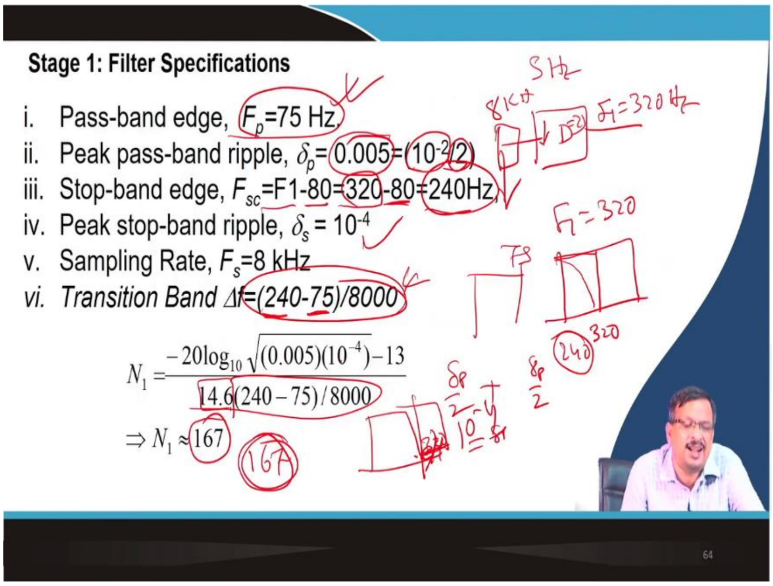


So, so I required a flat response up to 75 hertz. I require a transition bandwidth of 5 hertz. So, stage 1: I want to implement a filter that is equal to 320 hertz, an input of 8 kilohertz, and downsampling by D equal to 25. So, the filter specification of this filter specification pass band is simple 75.

Now, passband ripple since I am using two-stage filtering. So, I can say that in the first stage, the ripple will be Δp by 2, and in the second stage, it will be Δp by 2. So, the total is Δp. So, my Δp is $10^{-2}$. So, $10^{-2}$ by 2 is nothing but 0.005. Now, what is stop band stopband edge frequency? So, what is F1 is 320 hertz. So, I have a signal. first, there is a 4 kilo up to 4 kilohertz.

Now, I have a signal up to 320 hertz. Is that ok? So, I have a stop band, which is 240 hertz, understand or not. So, up to 240 hertz is a stop band, and I am sufficient. So, I can say F 1 minus 80. So, 380 hertz is my stopband edge frequency for the original filter, but I have 320 hertz. Up to 320 hertz, I have a signal. So, I can say ok will be before 80 hertz if we stop; I do not have any problem. So, I can say 320 minus 80 240 hertz and the stop band ripple is $10^{-4}$.

So, this portion ripple is $10^{-4}$. So, what is the transition bandwidth? Passband, stop band minus pass band divided by 8 k. So, that is my transition bandwidth. So, transition. So, I can put again the formula Δp Δs divided by the transition bandwidth in 14.6. I get the order is 167; if you calculate, it will come to 167.

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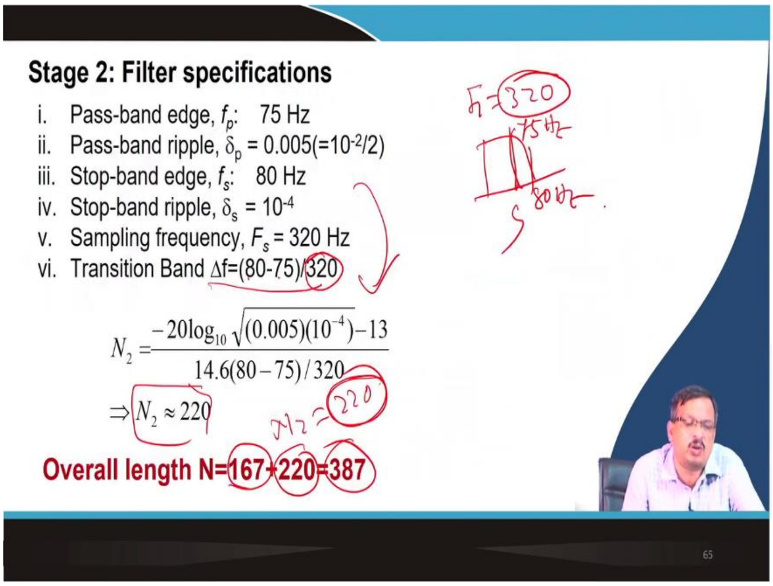


Now, in stage 2, in stage 2, F 1 is equal to 320, and I want a stop band that is 80 hertz, and I want a passband that is 75 hertz, and the sampling frequency is 320 hertz. So, what is the transition bandwidth of 5 hertz? So, what is the transition bandwidth when I divide by sampling frequency? It is nothing but 80 minus 75 divided by 320. So, again, I put all the values to calculate N2. So, N2 is equal to 220 in the order of the filter. So, the first filter order is 167, and the second filter order is 220.

So, the overall filter length, if I say, is nothing but 167 + 220 = 387, but if I simply implement the FIR filter without using multistage or multi-rate signal processing, the required order is 5150.

So, I can say 5150 divided by this one is that much I am reducing 387; I think 387 is the compression of the order. So, the reduction of the complexity almost 13 it was 13 times, reducing the length of the filter. So, when I implement a very narrow band filter, if I want to implement it directly, the filter is order becomes very complex.

However, I can use multi-rate signal processing using multiple stages of decomposition. I can easily implement the same filter with a smaller order of the filter. So, this can be an easy application for signal processing. Suppose I have to implement a very narrow bands comb filter or a narrow band filter; if I implement it using normal methods, then the order of the filter is too high.

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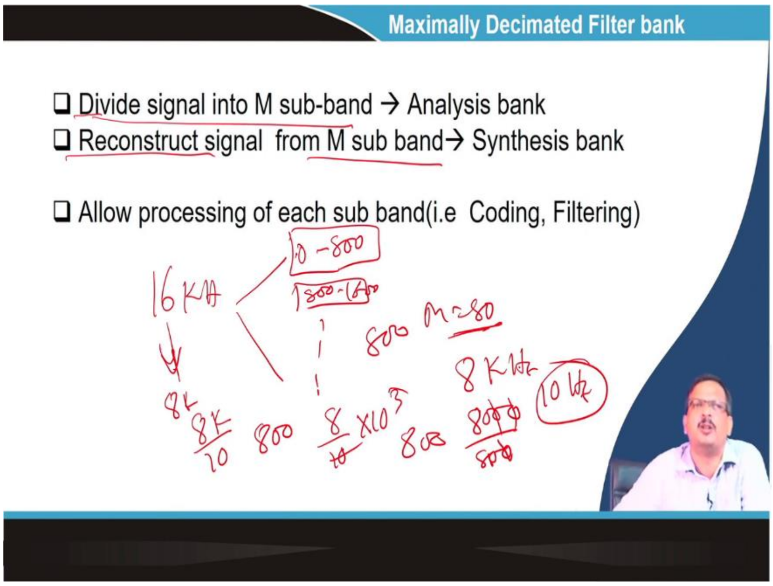


Then, there is another application which is called sub-band encoding or maximally disseminating the filter band, which is used in sub-band and coding kinds of things. So, what I do is divide the signal into the M subband, which is called the analysis bank. A reconstruct the signal from the M subband. So, I have a signal at 16 kilohertz; let us say I divided the sample signal into different, let us say, the 4 bands. So, 0 to 16 kilohertz.

So, what is the maximum signal content? This signal, signal is 8 kilohertz. So, I divided that 8 kilohertz frequency into 10 subbands. So, I can say what each sub-band is and what the range of each sub-band is. So, 10 8 k divided by 10, so 800 hertz. So, I can say 10. So, 0 to 800 hertz, then 10 8 divided by 10 into $10^3$.

So, it will be 800 such filters as sub-band 800 sub-band, 0 to 800, then 800 to 1600. So, there will be 800 such kinds of sub-bands. So, M is equal to 800. Sorry, M. If the M is equal to 800, then is 8 kilohertz your maximum frequency content? So, M is equal to 800, which means 8 kilohertz divided by 800 filters. So, each filter has a 10 hertz frequency. So, I can divide the signal into different subbands, and then once I divide it, I reduce the complexity of that subband by lowering the sampling frequency.

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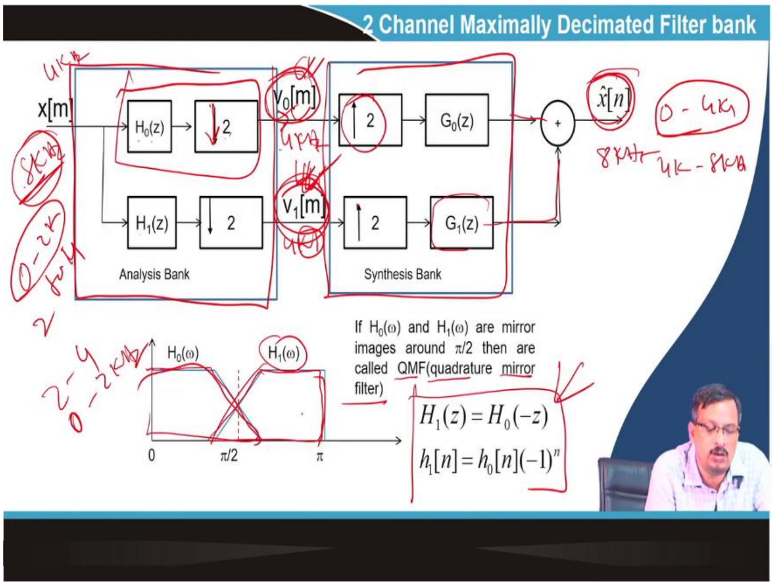


So, we will give an example of 2, 2 channels. So, 2 channel means. So, let us say I have a signal with 8 kilohertz. So, I divided it into 2 channels. So, 0 to 4 kilohertz and 4 kilohertz to 8 kilohertz. So, I have a signal with 8 8-kilohertz sampling frequency. I divided the signal into 2 channels: 0 to 4 kilohertz and 4 to 8 kilohertz. So, the maximum baseband signal here is 4 kilohertz. So basically, I want to divide the signal into 0 to 2 kilohertz and 2 to 4 kilohertz.

But when I say 8 kilohertz 0 to 4 kilohertz is there, that means I can reduce the sampling frequency by factor do. So, that sampling frequency here I require a sampling frequency instead of 8 kilohertz; I require 4 kilohertz because my baseband signal is 0 to 2 kilohertz. Similarly, here, I also require 4 kilohertz because my baseband signal is 2 to 4 kilohertz. Basically, if I consider it a 0 0 to 2 kilohertz,

So, V0 and Vm is reduced by the sampling rate. So, the complexity of the V0 and V m is reduced. So, all the processing I can do on V0 and V1, which is less complex.

So, like coding and filtering, all can be done here, but what is required is I have to get back the signal again, whatever the processing I will do on V0 and V1. Again, I have to combine V0 and V1, and I have to get the x[n] back, so that means, again, I have to upsample that V0 by 2 times and upsample the V1 by 2 times, and then I use a filter and combine them to get that synthesized signal.

So, this block is called synthesis; this block is called analysis. So, it is maximally disseminated filter bandwidth, now if it is 2 channels. If I say my H0 looks like this and H1 looks like this, this is like a mirror image of H0 is H1. This is called QMF quadrature mirror filter; QMF filtering means H0 is the mirror image, or H1 is nothing but the mirror image of H0, which is called quadrature mirror filtering. So, the requirement is this one. I will derive why this is the requirement.

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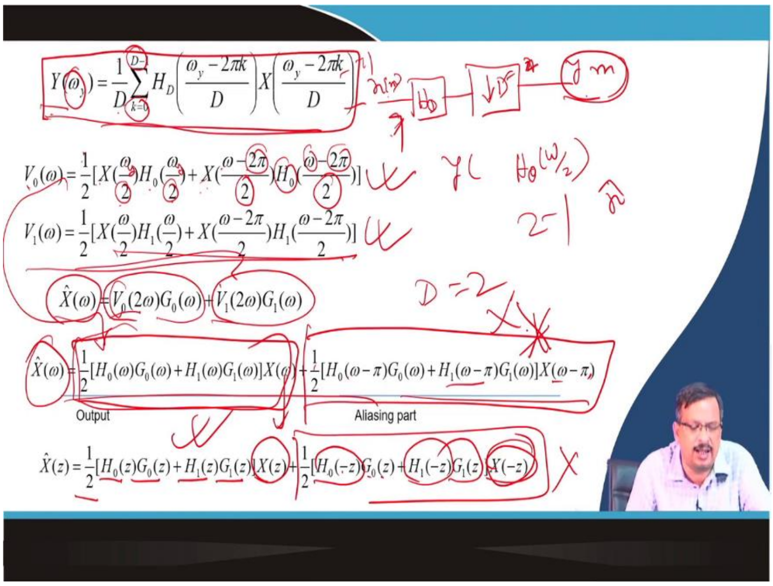


$$Y(\omega) = \frac{1}{D} \sum_{k=0}^{D-1} H_D\left(\frac{\omega-2\pi k}{D}\right) X\left(\frac{\omega-2\pi k}{D}\right)$$

So, let us say this is my requirement. So, this is my block diagram of the system, 2 channel maximally disseminated filter bank system; let us say then if I say what Y w y is so, that you know that. So, what do you know if I say that? Is that okay? So, here I am saying, suppose I have a filter or I have a disseminator D, and again there is a filter H D, and then this is y m, and this is x[m], and D is equal to 2. Let us say D. So, what is the expression of y m y? So, I can say the frequency domain representation of the y m is y ω y is nothing, but this one we already derived for this dissimulation factor D.

Now, if I say this is one of the disseminator factor blocks. So, H0 and dissemination by 2. So, I can say the V0. So, this is nothing but a y. So, V0 is nothing but a 1 by D. So, this D is equal to 2. So, 1 by 2 X(ω) by 2 H0(ω) by 2. I said it because H0 is the purpose of the H0(ω). So, the D is equal to 2. So, I can say that D equal to 2 means k is equal to 0 to D minus 1, and 2 minus 1 is equal to 1. So, I can say k is equal to 0. So, X of k is equal to put the k equal to 0 here.

So, I can say X(ω) y by D into H0(ω) y by D 2 D is equal to 2 into plus k equal to 1. So, it is X(ω) minus 2π divided by 2 into ω H0 into ω minus 2π divided by 2 k equal to 1 ok. Similarly, V1(ω) is the same, only the filter is H1(z). So, the filter is H1(z). So, this is V0 ω, this is V1(ω). Then, what is X(ω), what is synthesis ω, and what does synthesis mean for the sample filter? So, I can say V0 into G0, V1 into G1.

$$V_0(\omega) = \frac{1}{2}\left[X\left(\frac{\omega}{2}\right)G_0(\omega) + X\left(\frac{\omega-2\pi}{2}\right)G_1(\omega)\right]$$
$$V_1(\omega) = \frac{1}{2}\left[X\left(\frac{\omega}{2}\right)G_0(\omega) + X\left(\frac{\omega-2\pi}{2}\right)G_1(\omega)\right]$$

So, if I put the value of V0 here and the value of V1 here, I get this one. So, my synthesis output has 2 parts, one part, and another part for D is equal to 2; if D equals 3, I get 3 parts. So, this part is the mirror image or aliasing part. So, I only want this part to be removed. So, my synthesis signal will be the same as my input signal. So, instead of ω, if I take the z domain. So, it is nothing but half of H0(z), G0(z), H1(z), G1(z) into X(z) and half of H0(ω-π).

So, it is nothing but a ω 0 minus z, G0(z) ω minus π H1(-z) G1(z) ω minus π X(-z). Now, I do not want this part.

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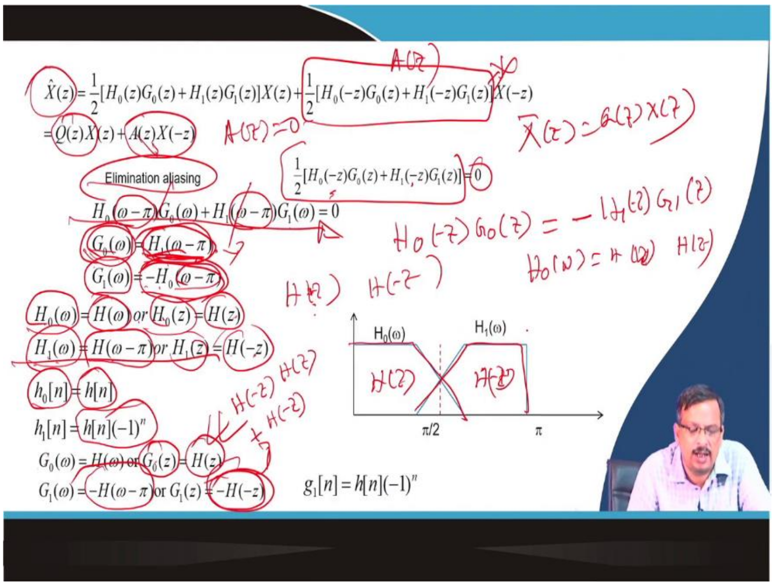

So, to avoid an antialiasing effect. So, I can say this is nothing but a Q(z) into A(z). So, to avoid eliminating aliasing, A(z) should be 0. So, A(z) is nothing, but this 1 is A(z). So, A(z) is equal to 0. Now, if I put A(z) is equal to 0, then what will happen that H0? I can say H0(-z) into G0(z) is equal to minus of H1(-z) into G1(z) or if I say ω in term of ω if I write.

So, minus z means ω minus π, and minus z means ω minus π is equal to 0. Now, when this will is 0, if my G0(ω) is equal to H1(ω-π) and G1(ω) is equal to minus H0(ω-π), then this 2 term will cancel each other and becomes 0. So, too. So, to avoid aliasing, the condition is G0(ω) should be H1(ω-π) and G1(ω) should be H minus H0(ω-π).

So, I know the frequency response of G0 and G1 from H0 and H1. So, if I design it that way, then the aliasing will be discarded, and I get the signal synthesis signal, which is nothing but a Q(z) into X(z). So, let us say H0(ω) is equal to H(ω). So, I told H0(ω) is equal to H(ω), or I can say H(z) H(z). So, if H0(ω) is equal to H(ω) or H(z) or H0(z) is equal to H(z), then what is H1(ω)? So, H1(ω) is equal to ω minus π. So, ω minus. So, H this is H0(ω), and this is H1(ω). So, this is a mirror image.

So, I can say H1(ω-π) or H1(z) is equal to H(-z). So, if this is H(z), this is H(-z) because this is starting from the π side, ok? So, if I do that. So, what is the meaning? So, H0(z) is equal to H(z); that means H0 n is equal to H n. When it is minus z, then it will be H n into $10^{-1}$ minus 1 to the power 1 n ok. Now, what is G0(z)? Now, I require G0(ω) to be H1(ω) minus π.

So, if G0(z) is equal to H(z) D, then I can say G1(z) is equal to minus H of ω minus π. So, minus H(z). So, if I put it H0(ω-π) H0(ω) minus π, that means H(-z) into G0(ω) H(z) plus H1(ω-π) H1(ω-π) minus; that means I can say it is nothing but a H(-z) into G1(ω) minus G1 H1(ω-π), no the there is a wrong.

So, H G0(z) is equal to H1(ω-π). So, what is H1, H1 is H(-z). So, H(-z) H1(ω) is H(-z). So, H of ω minus π is nothing but H(z). So, that is why G0(z) is equal to H(z) and G1(z) is equal to minus H0(ω-π). So, H0(-z) minus ok. So, if that is the condition for avoiding aliasing, then I can say X(z) is equal to Q(z) into X(z), which is my synthesis.

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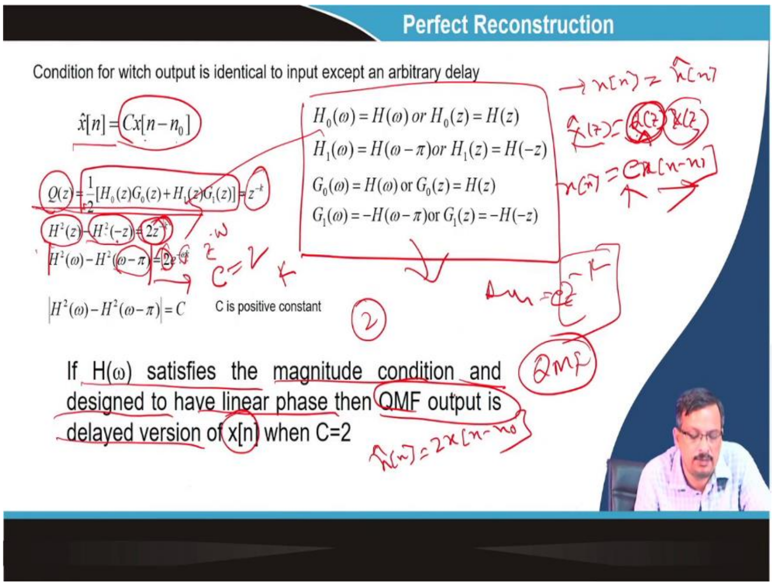


Now, when this X cap z and X(z) are the same, So, when I get the perfect reconstruction, what is the meaning? This means that I applied x[n], and I should get x[n]. Now I said x cap n I am getting. So, if it is exactly the perfect response, this will be equal. So, when will this be equal? So, I know X cap z is equal to Q(z) in X(z), and we have already done Q(z) into X(z).

So, if I say that Q introduces a constant C and delays the signal, so, if I say X[n] is equal to C into X[n-n0], then also say that ok, I am perfect, the required structure is possible, and only a signal delay will be there. So, I require a Q(z), which can only be a constant C, which may be multiplied. Some gain may be accumulated, and some delay may be allowed.

So, I said I do not want Q(z) to be equal to 1, which will be identical and very tough. So, Q(z) can introduce some constant gain and delay the signal, and then I can say I am perfectly reconstructed.

Let us say x[n] is equal to C into x[n] minus 0. So, now, you know, we have already derived this part for anti-enlarging. So, now, put this one into the equation of X(z) equal to Q(z) into X(z). So, this is the equation. So, this must so, if this Q(z) is a filter that introduced a delay. So, a delay is synthesised by $z^{-k}$. So, the z transform of the Q(z) should be in the form of $z^{-k}$, or I can say some constant C in the form of $z^{-k}$.

C into $z^{-k}$ minus k is nothing but a delay. Now, if I put the condition in here, we will get H(z) H2 z minus H2 minus z equals 2 into $z^{-k}$ because 2 will be coming here 2 will be the factor. So, which is nothing, but I can say if it is in ω form. So, ω minus π. So, $e^{-\omega kz}$ is equal to, let us say, ω $e^{\omega}$. So, $e^{\omega k}$. So, I can say the spectrum amplitude is nothing but a constant C. In this case, ideal case C is equal to 2.

If all aliasing is eliminated perfectly, then I know C is equal to 2, and the delay element is k, and 2 is positive. So, if H(ω) satisfy the magnitude condition and is designed to have a linear phase QMF output. So, QMF, the output is a delayed version of x n. So, I can say x cap n is nothing but a 2, C is equal to 2 2 into x[n-n0] is the delay. So, that is called QMF; how do we implement a quadrature mirror filter and how does it work for sub-band encoding? So, multi-rate signal processing is used much time used for sub-banding and sub-band encoding.

So, that can be implemented even in speech coding. We use multi-rate signal processing. So, instead of let us say I have a 16-kilohertz speech signal, the 16-kilohertz sampling frequency is 16 kilohertz, I can divide into the different frequency bands, but once I divide the different frequency bands, I may not require that much sampling frequency. So I can reduce the sampling frequency. So, the number of samples within a 20 milli second will be reduced, and my coding can be done.

So, sub-band encoding of the speech signal maxi we use in multi-rate signal processing. So, the design and filter I can complexity of the filter FIR filter can be reduced by multi-rate signal processing. Sub-band encoding can be used can be done using multi-rate signal processing. So, those are the applications of multi-rate signal processing.

So, in summary, I can say multi-rate signal processing, either up sampling or down sampling and aliasing or mirror image, and it can be, so filtering is the must. We cannot do any up-sampling or down-sampling arbitrarily. So, that is, I have to restrict the signal. Let us say I have a signal I can show you.

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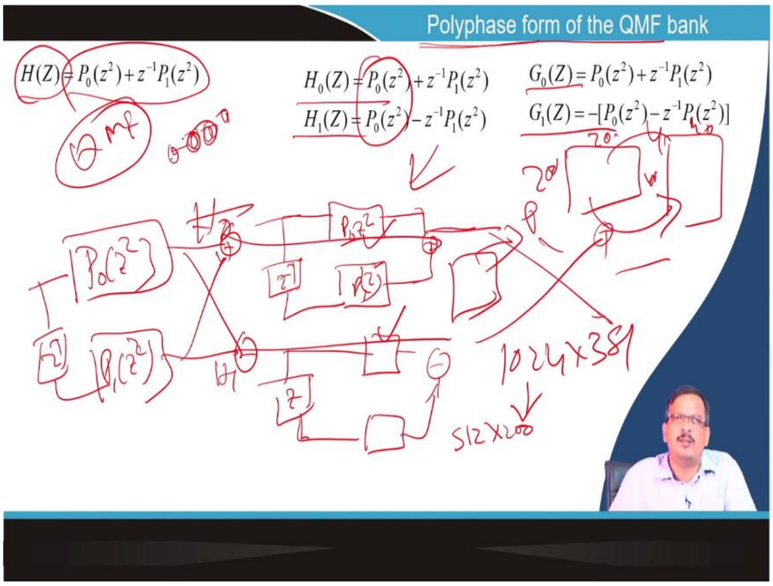


Let us say I required an ok, the diagram. I am not drawing the diagram. You can draw the diagram. So, the polyphase forms. So, QMF is implemented using poly-phase form. So, H(z) is equal to that form. So, now I know H0 H1 G0 G1 I know. Can we draw the signal flow diagram? You can do that. Only the signal flow diagram of this poly phase QMF filter signal flow diagram is available. So, there is a H0. So, first implement H0 p0($z^2$) p1($z^2$) ok p1($z^2$).

So, this is delayed by $z^{-1}$, which will be added together. I get H0 H0, then what is H1 again $z^{-1}$ instead of addition? I have a subtraction ok. So, H1 H(z) implemented, then use the 0 G0 G1 implemented, then you can do it, you can do the signal flow diagram, or I can say I implement. So, p0 this p0 is the same. So, why should I use 2 p0? I can replace it with a single p0. So, p0($z^2$) is used, and this is p1($z^2$).

So, the signal is fed to here, the delayed signal is fed to here, then I know H0 is nothing but an addition of these two and H1 is nothing but a subtraction of these 2, then I go to this side again this side also one is addition, and one is subtraction, I can get the output. So, that is multi-rate signal processing, as I am discussing in the summary. So, what is the use of multi-rate signal processing? Why do I do that many times? I may be required to sample the signal. Suppose I have taken a picture which is, let us say, 1024 divided by 381.

I want to reduce it, let's say, 512, into something else that, let's say, 200 pixels. So, I cannot cut those pictures or select some portion of the picture. I required the entire picture, but I reduced the size. So, it is nothing but a downsampling. I have a picture. Let's say 200 crosses 200; I want to make it 400 crosses 400. So, it is nothing but an interpolation. That is why you say a 2-megapixel camera and a 12-megapixel camera. So, when I make the picture more significant, it is called digital zoom because if the resolution is so high, I can interpolate.

So, instead of 2 times, I can do 4 times what interpolation means, I am puttinG0 in between 2 pixels. So, if I say that I put instead of 0, I interpolate that nearby 2-pixel value, an average value will be put there, and then what will happen is the picture will be a blur. So, if the resolution is very high, then the human eye cannot detect that blur. So I can digitally zoom the picture without hampering its quality. So, that is one of the uses of up-sampling.

Up sampling, down sampling. So, C D was recorded with an audio signal of 16 kilohertz. Now I have to make it 8 kilohertz; I can down-sample it using a filter because a filter is necessary unless what will happen 16-kilowatt signal is band-limited to 8 kilohertz. The sampling rate is 8 kilohertz; it should be band-limited to 4 kilohertz.

So, I have to design a filter whose bandwidth is 4 kilohertz. Then, I can reduce the sampling frequency, and the signal quality will remain the same. So, it has multi-rate signal processing. I will say that I have used many examples for calculation many times. Can you please do those calculations on your own? You take arbitrary dissemination factors, do the sampling rate conversion, and calculate the different kinds of up-sampling, down-sampling, factor frequency, and all those things.

So that when you do the assignment, that can help you to solve that assignment ok.

Thank you very much.