**Signal Processing Techniques and Its Applications**
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**Lecture - 49**
**Lattice Formulations of Linear Prediction**

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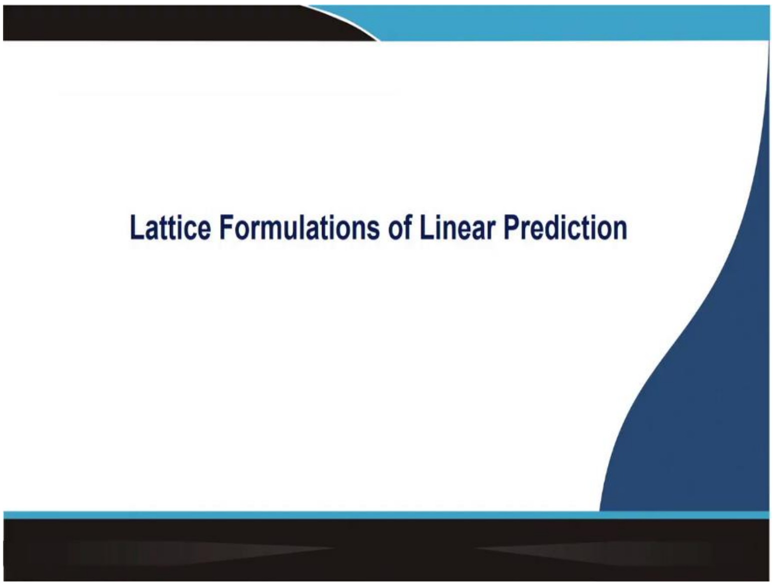


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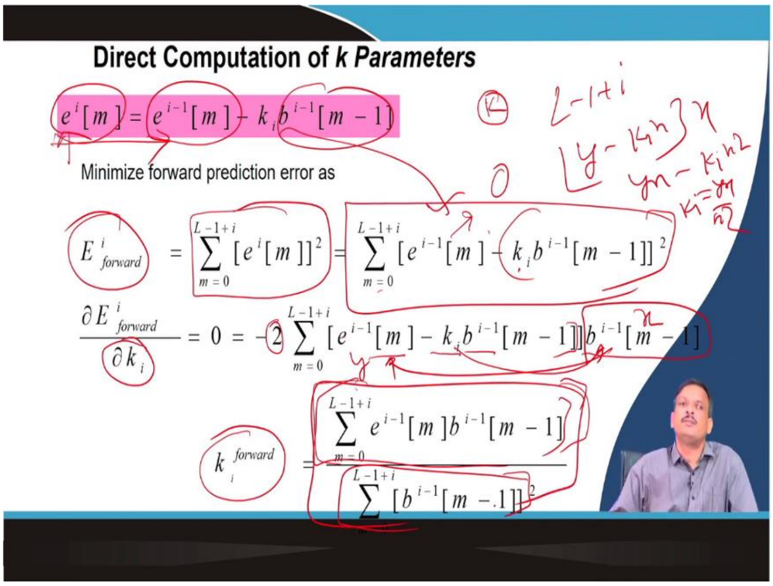

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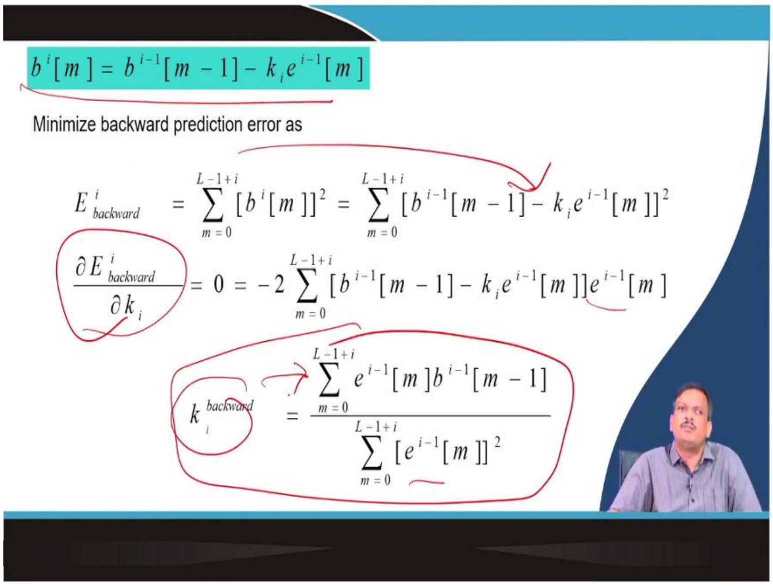


Ok. So, now, I go for the direct computation of the k parameter. So, what do we know? We know e1[m]. So, this is my forward prediction error. I write down the equation forward prediction error ei[m] is equal to $e^{i-1}$ $b^{i-1}$ m minus 1 ok. So, that is the forward prediction error. So, I am predicting forwardly. So, from the i-1 iteration, I am calculating the ith iteration ok. Now, I have to minimize the error.

What does minimize mean? With respect to whom I should minimize it with respect to k, only k is the gamma coefficient. So, I have to find out the value of k for which this ei[m] is the minimum. So, if I say the mean square error. So, I calculate the mean square error ith order prediction. So, what is the length L minus 1 plus i, ok? So, this is nothing but this equation: a whole square. Now, I take the derivative of this forward prediction error with respect to k i. So, the derivative with respect to ki x square means 2 into d x.

So, 2 into d/dx means that x. So, $e^{i-1}$ m minus ki $b^{i-1}$ m minus 1 multiply by. So, this part is 0, and this part will be k, I will be 0 1, and this part will be this one only, ok? So, b i minus 1. So, now, I am saying this is a ki forward, which is nothing but this one. Because this multiply with this will remain in upside and this multiply with this will be square which is coming downside. So, let us know if this one is x and this one is y. So, y minus ki x into x. So, it is nothing but a y x minus k ki x square.

So, I can say ki is equal to y x divided by x square, which is given ok. Similarly, I can calculate backward prediction errors. This equation minimizes the mean square error, and the derivative will come instead of k i; it will be e i 1 e i minus 1. So, this one is forward prediction ki minimizing the forward error, and this one is the ki value minimizing the backward error, ok or not.

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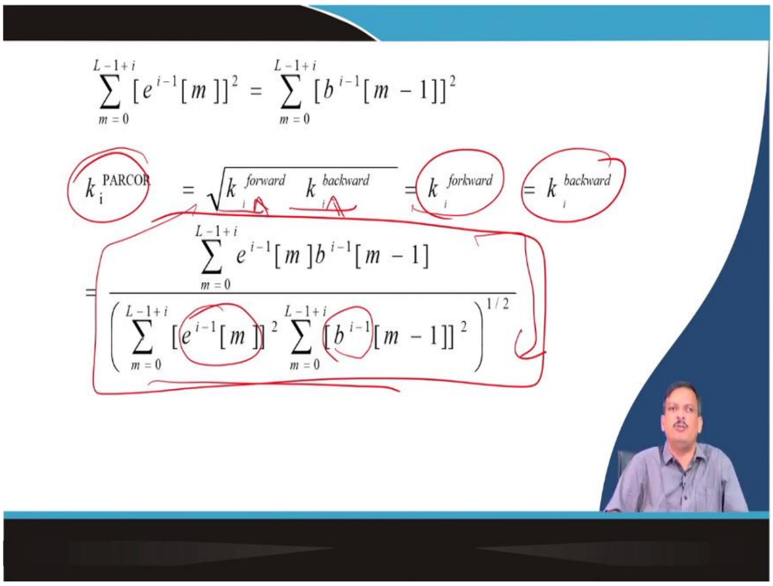


So, one is a forward error and a backward error. Now, ideally, both will be the same. So, I can predict that the Parker coefficient is nothing but a geometric mean of forward and backwards errors. In ideal cases, the forward and backwards are the same. So, it is nothing but either ki forward or ki backwards.

So, any one of the equations I can use, or I can use the geometric mean of ki and ki forward and ki backwards using this equation. I put the value of ki forward and ki backwards value. I will get this one because if you see in ki forward, I get $b^{i-1}$ in downstairs. In ki backwards, I get $e^{i-1}$ in down ok. So, this is one of the methods to calculate the ki value.

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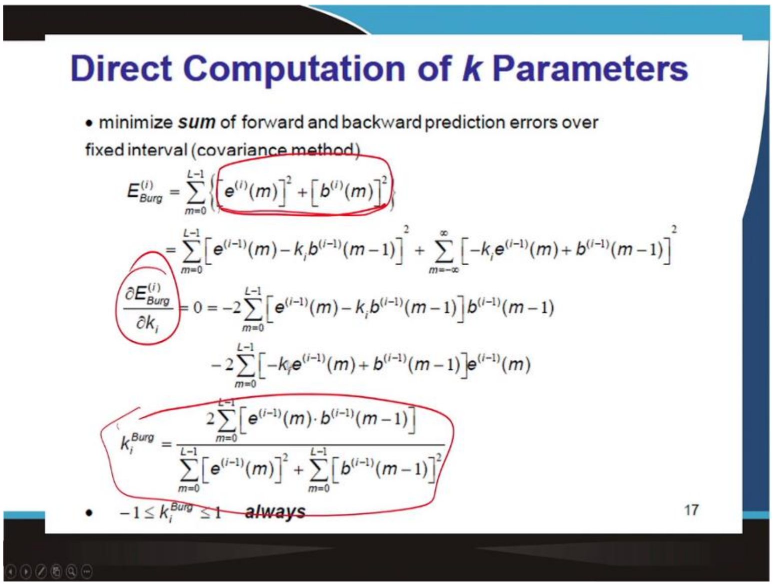


The next one is that I can use another direct k parameter calculation called the burg method. Instead of taking the geometric mean of the forward and backward error, they said that two errors, two mean square errors, are added up and then take the derivative. Two mean square errors are added up, and then take the derivative and then ki is equal to this one.

So, I can use either method to calculate the ki value directly. Let us say I can give you an example that will be clear.

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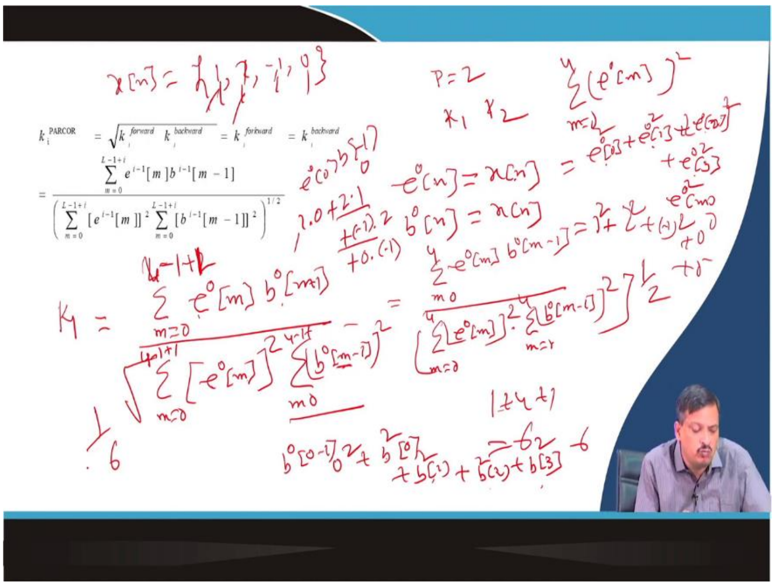

Let us say I took two signals. Let us say I have given you the signal x[n] is equal to, let us say, 1 2. Let us say minus 1 0. Let us say this is my signal. I told you to calculate the ki value partial reflection coefficient directly for order p is equal to 2. So, I have to calculate k 1 and k 2 i equal to 2 prediction error equals 2. So, what do I know? I only know x[n]. So, x[n] is the signal.

So, I know what e0[n] is and what b0[n] is, and n is the length of the signal. So, n depends on the length of the signal. So, e0[n] is equal to nothing but a x[n] and b0[n] is nothing but a x[n]. Now, if I say the value of k 1, k 1 is equal to m stands. So, m less is equal to 0 to L minus 1. So, what is the length of the signal 1, 2, 3, 4. So, 4 minus 1, what is the prediction length? 2.

So, 4 minus 1 plus 2 e. So, k 1 i is equal to 1. So, I equal to 1 means this will be plus 1, ok. So, i equals one; that means e i. So, e0[m] into b0[m] minus 1 divided by m equal to 0 to L is 4 minus 1 plus 1 e0[m] whole square again m equal to 0 to 4 minus 1 plus 1 b0[m] minus 1 whole square root of this one. Now, what are the values?

So, m equals 0 to 4 minus 1 plus 1 cancels e0[m] multiplied by b0[m] minus 1 divided by e0[m] whole square. So, again, m is equal to 0 to 4 e0[m] whole square into m equal to 0 to 4 b0[m] minus 1 whole square, square root. So, how do I get that e0[m]? So, what is the value of summation of m equal to 0 to 4? e0[m] whole square is nothing but a m equal to 0.

So, e 0 plus e 0. So, square sum or sum square then sum. So, I can say the square plus e 0 1 square plus e 0 2 square plus e 0 3. So, 0 1 2 3 and e 0 4 all are square. So, what is e 0? So, e0[n] is equal to 0 x equal to 0. So, x equal to 0 means the first sample. So, 1 square plus x equal to 2 means x equal to 2 means n equal to 2 means.

So, n is equal to 0 1 2 square plus minus 1 whole square plus 0 whole square. Let us say outside that signal is 0 0. So, I have not required. So, I can say it is nothing more than 1 plus 4 plus 1, which is nothing more than a 6. Similarly, this also comes so, this is m minus 1. So, b 0 0 minus 1, which is nothing but a 0 square, ok. b plus b 0 1 minus 1 b 0 0 square plus b 1 square plus b 2 square plus b 3 square.

Because m equal to 4 will be there. So, 4 minus 1 3 square will be there. So, e g b 0 is nothing but a 1 b 1 is 2 b 2 is minus 1 b 3 is 0. So, again, it is become 6. So, the root over of 6 into 6 is equal to 6. So, the downside is 6, and the upside is e 0 into b 0. So, e 0 is e0[m] equal to 0. So, e 0 0 into b0 minus 1 b0 minus 1 is 0.

So, 1 into 0 plus 2 into 1 plus minus 1 into 2 plus you can say minus 1 into 0 into minus 1. So, that is not required. So, 1 2 minus 2 cancels. So, 1. So, 1 by 6 k 1 is equal to 1 by 6.

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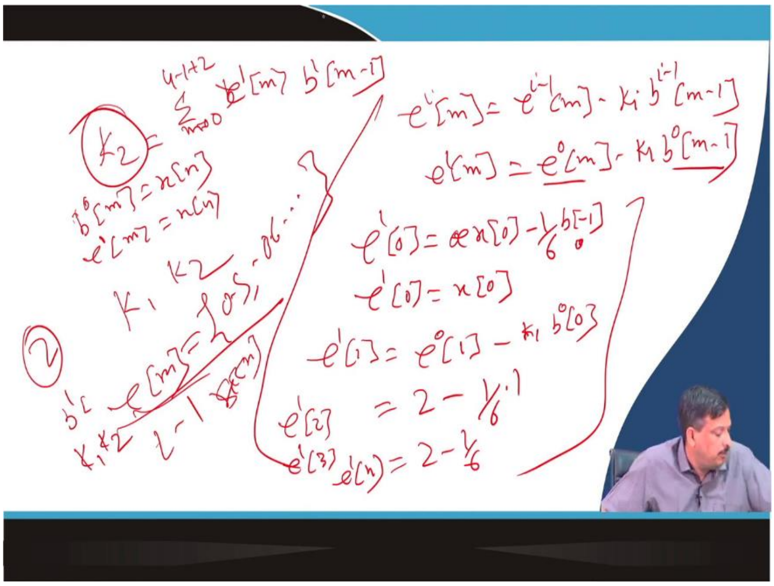


Now, what is k 2? Now, once I say k 2, then I say b[m] equal to 0 to m equal to 0 to 4 minus 1 plus 2 into b e i minus 1. So, 2 minus e1[m] into b 1 m minus 1. So, I have to calculate e 1 m, but I do not know e1[m]. So, how do I calculate e 1 m? So, I know that ei[m] equals $e^{i-1}$ m minus ki into $b^{i-1}$ m minus 1. So, I can say e1[m] is equal to e0[m] minus k 1 into b0[m] minus 1. So, what do I know? I know e0[m], which is equal to x[n]. I know that b0[m] and e0[m] x[n]. So, I can say e1[m] is e1[m] if this is known.

So, I can say that e 1 0 is equal to e 0; that means x 0 minus k 1 is 1 by 6 into I can say b 0 b[m] equal to 0 minus 1 b minus 1. So, I can say e 1 0 is equal to x 0 because this is 0 outside the less the signal is 0. Now, what is e 1 1 e 1 1 is this; that means e 0 1 minus k 1 into b 0 0.

So, what is e 0 1 e 0 one; that means x 0 1 x 0 1 is nothing but a 2. So, 2 minus 1 by 6 into b 0 0 is 1. So, it is nothing but a 2 minus 1 6. So, I can calculate e 1, I can calculate e 1 2, I can calculate e 1 3, I can calculate e 1 4, I can calculate that. Once I know e 1, then I can calculate b 1. Also, once I know e 1 b 1, then I can calculate k 2.

So, once I can directly calculate k 2, I will use lattice filtering methods only to calculate k 1 k 2. So, I only have to know the signal and nothing else. Once I know the signal and order of the prediction, I can easily compute the k 1 and k 2 values. Let us say k 1 and k 2 value is given, and then I told you I can implement 1 by A(z). So, here I am implementing A(z).

So, the I am signal is not given; let us say e is given, and e m is given. So, e m is given, let us say 0 0.5 minus 0.6. That way, e m is given, or length is L minus 1. I have to implement 1 by A(z) I have to implement 1 by A(z). So, in the same way, if I told you this is my error signal and the prediction is 2 k 1, and the k 2 value is given, I have given you the k 1 and k 2 values. Can you compute the s[m] or x[n] to do it? First, you do it for a small signal, then write down the program. first, you implement it for a small signal.

So, the problem is, suppose I give you the problem reverse problem instead of giving you the signal and telling you to compute that k 1 k 2 value.

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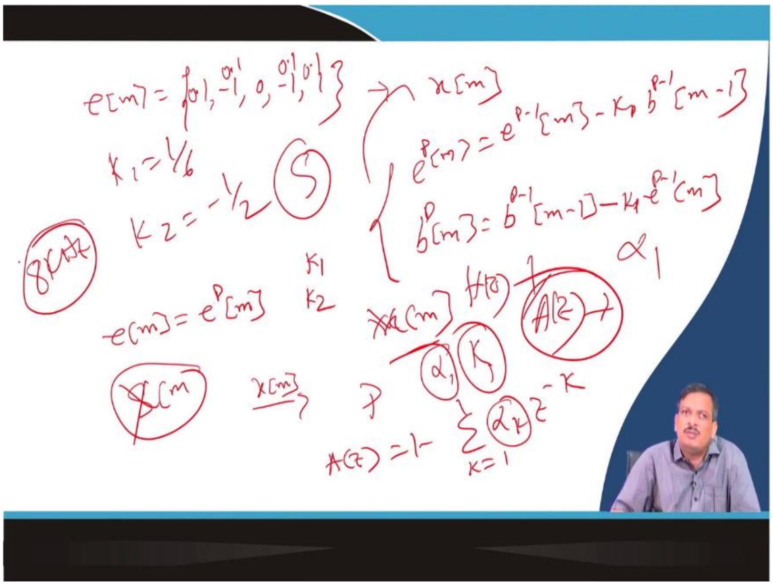

If I told you that suppose I given you that let us say I have an error signal e m is given e[m] is given e[m] is equal to let us say 1 minus 1 0 1 minus 1 1 let us say. Or let us say 0.1 0.1 0 minus 0.1 0 minus 0.1 minus 1 base j 0.1 like that. I give you I told you to calculate the signal x[m] if the k 1 is equal to, let us say, one-sixth and k 2 is equal to, let us say, minus a half. Calculate the value of calculate the signal x m. So, k 1 k 2 I have given I have given e m. So, e m is nothing but an ep[m].

So, you know ep[m], you know the value of k 1, you know the value of k 2. So, what is b p m? It is nothing but a b p minus 1 m minus 1 minus k p into ep minus 1 m. k is known, k p is known, and if I know ep[m] is equal to ep minus 1 m minus k p into b p minus 1 m minus 1. So, I am asking if I can calculate s[m] or x[m] as the input signal from these two equations.

So, how do you do that? There is an algorithm I have written. I think the algorithm is written here.

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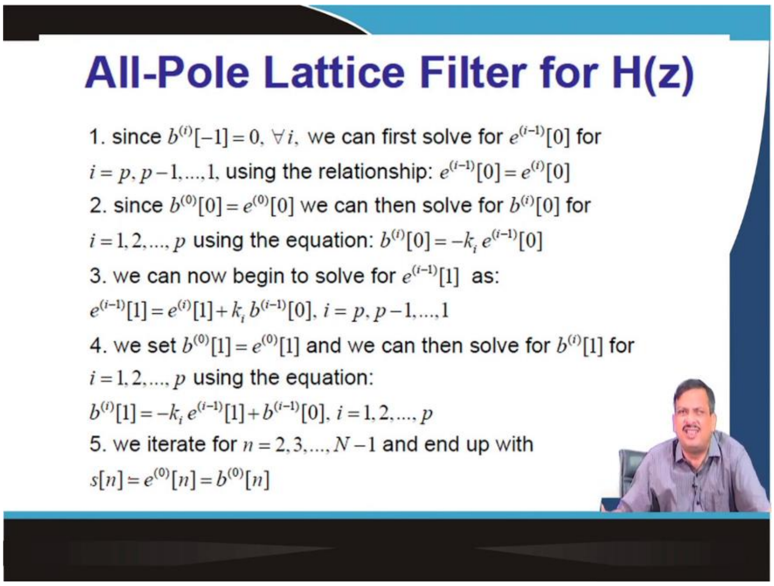


So, b minus 1 outside the let us say outside the signal is 0. So, minus 1 is 0 ok. So, now, you just think about it. I said I solved the first part, I solved the second part, you think about it, and I said you calculate x[m] calculate it, ok. So, this is a lattice formulation. Now the question is how do I decide the order I know x[m].

So, how do I know what the order of prediction should be? I have a given signal, and I tell what the order of prediction should be and what the relations between the prediction error and the order of the filter are. So, if you see that the general assumption is that if the order of the prediction increases, the error will be minimized. That is no problem. What is the physical representation of α 1 or k 1?

So, ki or α i ki is called partial Parker and k α i is the l p c coefficient, which basically the l p c coefficient represents, so, when I am implementing 1 by A(z), what am I basically doing? A(z), once I 1 by A(z), is nothing but an all-pole filter. So, what is A z? A(z) is nothing but a 1 minus k equal to 1 to p α k $z^{-k}$. So, α k is nothing but a pole position of the A(z) when I implement 1 by A(z).

So, α k basically represents the pole position of the signal. So, when I say anything, let us say linear prediction is a filter transfer function, and it is this: let us say h z, h z is a transfer function. If I know the pole position, what is the significance of the physical significance of the pole poles representing the resonance of the system?

So, suppose I have a signal that has a 5 resonance frequency and 5 natural resonance frequency; that means 5 formants since A(z) is a real pole. So, it is a complex conjugate pair. So, if there is a 5 pole, that means 5 complex conjugates, 5 formant or 5 resonance; that means 5 complex conjugate representations will be there; that means 10 complex poles will be there.

So, how much is required? Minimum, I require an order of 10; the minimum order is 10 for each pole. I require 2 p. So, 5 volts means 10 is my minimum order now in the case of the speech signal. When I say the speech signal, I am estimating the speech signal. So, if the speech signal sampling frequency is 8-kilo hertz, then what is the maximum frequency content in the signal 4-kilo hertz, which is f s by 2?

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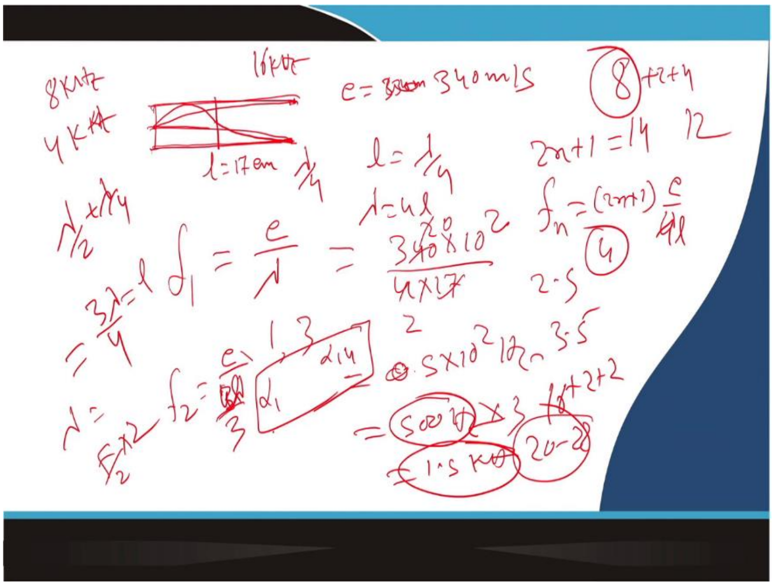

Now, as you know, the average length of the human vocal tract. So, let us say the human speech production system. So, this side is the glottis, and this side is the vocal tract. So, length l is equal to, let us say, 17 centimetres average length, and then the velocity of the sound in, let us say 3 334 meters per 340 meters per second, let us say 340 meters per second ok. Then how, what is the first formant first, then the first frequency component present that is nothing but a this will be minima. So, this will be λ by 4.

So, I can say λ by 4, l is equal to λ by 4, or λ is equal to 4 l. So, what is the f 1? The first formant f 1 is nothing but a c by λ, which is nothing but a 3 340. Sorry, how much is coming? C is equal. So, f 1 is equal to 340 meters. So, 340 into 10 to the power 2 divided by 4 into centimetres 4 into 17, 17 2 20.

So, I can say 0 5 into 10 to the power 2 hertz, which is equal to 500 hertz. What is the second? The second one is that there will be λ by 4, and here, there can be a stop. So, I can say this is nothing but this one and this one. So, I can say this is λ by 2 plus λ by 4. So, I can say it is nothing but a 3 λ by 4.

So, that will come around f the 2-second formant. So, it is nothing but a c by 3 λ, say 3 λ by 4 is equal to l. So, I λ is equal to 4 l by 3 4 l divided by 3. So, 3 will be multiplied here. So, it will be 1.5 kilohertz. What is that? 1 3 next one will be what? 5 because it is an.

So, it is nothing but a 2 n plus 1. So, the general formula is f of n is equal to 2 n plus 1 into c by λ c by 4 l c by λ is equal to 4 l. So, if I say that this is my formant frequency calculation. So, roughly, if I say the speech sample is sampling at 8 kilohertz, then I know the maximum frequency content is 4 kilohertz. So, if the fast resonance is over at 500 hertz, the second one is 1.5 kilohertz, and the third one will be 5th time.

So, 2.5 kilohertz, fourth one will be the then 7 time. So, 3.5 kilo hertz and the fifth one will not be there because the maximum of 4. So, I can say the 4 formant 4 resonance will be there. So, if there is a 4 resonance frequency, that means I require a 4 into 2 complex conjugate poles.

So, 8 is my minimum number for the LPC analysis, ok? Now, the glottis can also be modelled in the case of speeds seen. So, for the glottis, we take 2 and lip radiation is also in 20. So, in real cases, it is 8 plus 2 plus 4, or I can say 8 plus 2 plus 2. So, it is nothing but a 14 or 12-order lpc analysis for speech. So, if the sampling frequency is 16 kilo hertz, then I know this will become.

So, it is what? F s by 2 into 2, that much is required, or I can say it is f s. So, 16 kilos is 16 plus 2 plus 2 or 2 plus 4. So, either it is 20 or 22; that is the order. If I calculate the l effective, the l p c coefficient α 1 and α 14. If I take the frequency transform of this one, what will it represent? It only represents the envelope of the spectrum.

So, when you increase the order of the l p c analysis, the spectrum will be copied very minutely. So, minute variation will be added up. So, that will happen, ok. So, this is the l p c analysis I am not covering. If you are interested in more details, then you go for the speech processing course myself, which is called Introduction to Digital Speech Processing. There is an l p c analysis you can go through that YouTube video you can learn it from there also.

Thank you.