Signal Processing Techniques and Its Applications

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Lecture - 05
Digitization of signal (Contd.)

So, in the last class, I was explaining about the sampling. Now, I want to explain about how it is quantized.

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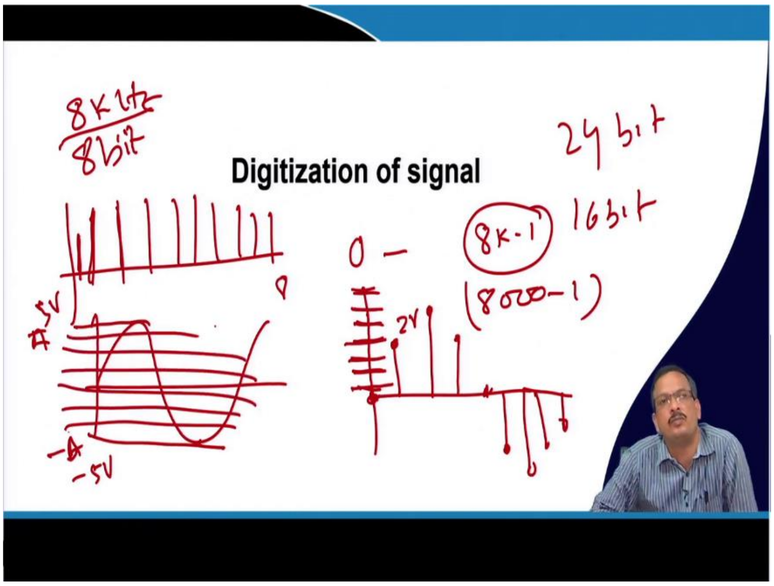

So, as I explained in the last class, you have heard about the 24-bit ADC and 16-bit ADC; what is that? 32-bit ADC, 8-bit ADC? Talking about the telephone channel signal, what is the sampling frequency for the telephone channel signal? 8kHz. What is the encoding bit size? 8-bit.

So, what is encoding bit size? So, 8kHz means in 1 second, I get 8k sample; 8kHz means in 1 second, I get 8k sample. So, if I recorded an analog signal for 1 second, I would get a 0 to 8k-1 sample., so I can say 8000 - 1 sample. So, in 1 second, 8k sample.

Now, for each of the samples, is there a sample? Sample means evidence of the signal at that time. So, if I have a sine wave and if I take the evidence in 0^{th} time, the evidence is 0. Let's say after T time, and the evidence is here; after T-2 time, evidence is here; T-3 time, in here; T-4 time, it is again 0, then I take the evidence in here, here, here, here like that.

So, I have the evidence. So, I have an amplitude of the sine wave A to -A or, let's say, 5 volts to -5 volts. So, what is the peak-to-peak voltage? 10 volts. The negative peak to the positive peak is 10 volts. Now, the samples that are time evidence at a particular time will vary between 0 to +5 and 0 to -5.

So, now, I have to discretize this y-axis. If I say this is 2 volts, that is continuous. So, it is a continuous value but discrete in time. Now, I have to discretize this value. So, how do I discretize it? Nothing but a quantization. What is quantization? So, what I say is that the volts in this 10-volt space can be divided into certain steps.

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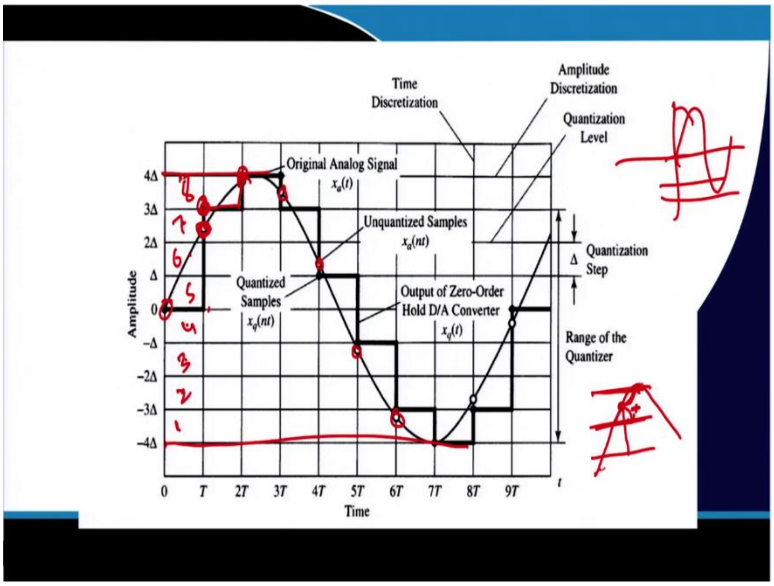

Let's say I have a sine wave; here is my sine wave. So, these circles are the time instant I have taken. So, I have a signal which varies from this level to this level. Let's say I divided the whole level into a number of 1, 2, 3, 4, 5, 6, 7, 8. So, I divided my peak-to-peak amplitude into 8 levels: 1, 2, 3, 4, 5, 6, 7, and 8 levels.

So, that means if my sample value is in here, I will assign the level of the sample value may be in here if it is above 50%. Suppose this is a step size, So this is the step and again, this is the 8th step. So, if the sample value is between two states, what is that mean? So, this is my step size, and I have a signal.

Now, suppose I have collected the sample here, so this sample either I can assign to this step size, or I can assign it to this step size. If I assign to this step size, I can say it is the below if the error is positive. If I assign it here, the error is positive, and if I assign it here, the error is negative. So, somehow, I get an error that is called quantization error.

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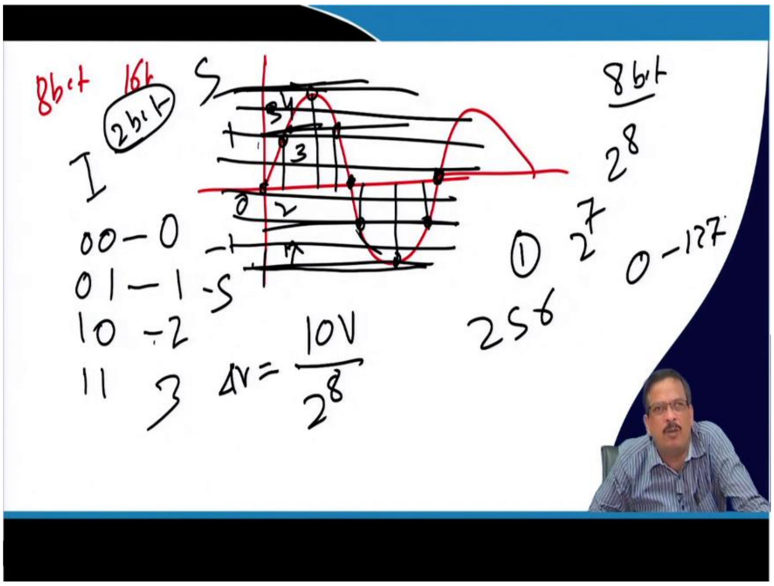

So what is this 8-bit, and what does 16-bit mean? So, let us say I have a signal, then I take the sample; evidence here, here, here, here, again here, here, here, here. So, that is the time sampling. Now I get a sample in here, here, here and here again, here, here, here and here.

Now, I say that I want to represent each sample in 2-bit instead of 8-bit. So, each sample has to be represented by 2-bit. So, how many possible numbers or decimal numbers are possible in a 2-bit; how many variations are there? A bit can be 00, 01, 10, 11. So, this is 0, this is 1, this is 2, this is 3.

So, I can say that the whole voltage, negative and positive, can be divided into four steps. So, I take four steps. So, this is the 1, 2, 3, 4. If my samples are within this step, I will assign this value. So, this is a positive one. If my sample is within this step, I assign this value. So, this is 0, -1 and +1.

Similarly, how many step sizes are possible when I say I have an 8-bit representation? 2^8. But 1-bit will be the sign bit. So, if I take the peak-to-peak value, then I know that the peak-to-peak value can be divided into 2^8 step sizes.

So, what should be the width of each step size? So, let us say it is a -5 volt and +5 volt. So, what is the total dynamic range? The total range is 10 volts. How many divisions are possible? I said 2^8 division. Each of the divisions corresponds to this Δ volt so;

$$\Delta \text{ volt} = \frac{10\ volt}{2^8}$$

Now, if I say that since the signal has a negative and positive, it has a negative and positive, then the sign is also important. So, 2^8 is 1 bit for sign, so 2^7 is the variation. So, I can get 256.

So, 2^8 is 256. So, I can say the value of the sample will be 0 to +127 and 0 to -127. Is it clear? So this is called quantization. So, I am dividing the whole sample space in a step. So, that is a step; each step depends on how many bits I have encoded in each of the samples.

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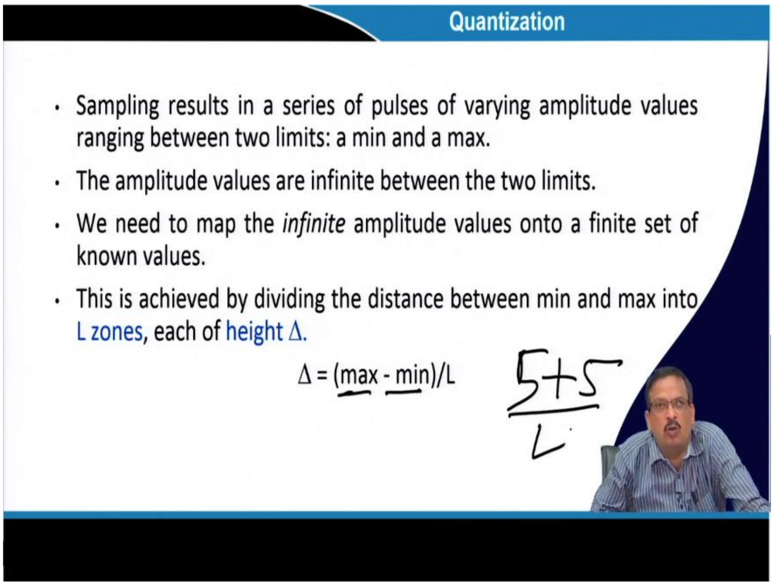

Let us say this one. So, max minus min. So, what is max? Max is +5 volts minus -5 volts divided by L; L is the zone.

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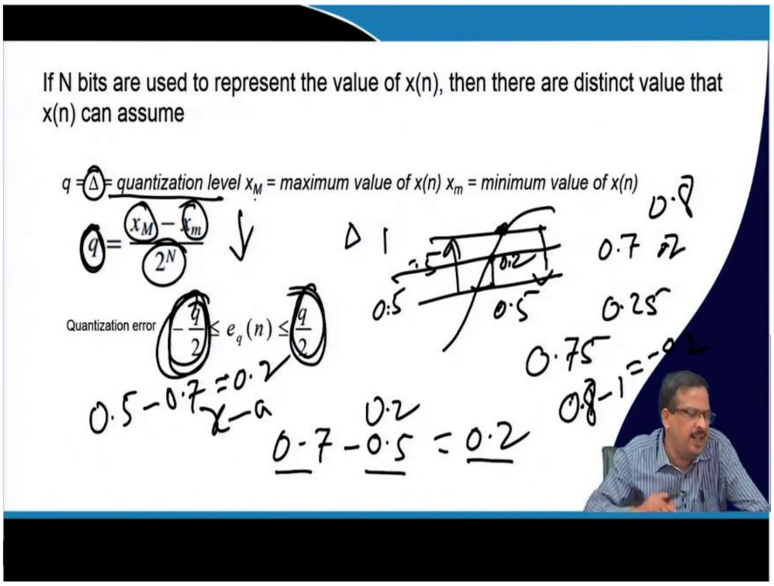

So, I can say because the quantization level is Δ , then I say the q quantization level is;

$$q = \frac{x_M - x_m}{2^N}$$

That is the Δ value. Ok or not? That is the Δ value and step size. Now, what is the quantization error? You see, I have a step size here and here, and now a signal is like this. So, I can say if the signal is above 50% of this level, suppose this level is 0.5, and this level is 1, and now I say my signal is 0.7. So, I want to find out the difference.

This level, so the signal is above 0.5, 0.7 in here; what is the difference? 0.2. So, I said, if it is 50%, what is the step size? It is 0.5. What is the 50%? 0.25. So, if it is 0.75, I should assign 1. If it is below 0.75, I should assign 0.5. Then, what is the error? When I assign 5, my error is 0.2; when I assign 0.8, I assign here, and my error is 0.2.

So, what is the minimum error? What is the maximum error? Whenever I say this, it is the original signal. So, an error is original minus assigned. So, the original is 0.7, assigned is 0.5. So, an error is 0.2. So, whatever you can take. So, I can say that somehow it is nothing but a maximum error and positive and negative on both sides, either q by 2. So, it is -q/2 to +q/2.

If I say this one, I take the original minus assigned error. Similarly, the original is 0.7, and my assigned is 1; if it is, let us say, 0.8. So, 0.8 assigned is 1, which is equal to -0.2. So, somehow, it will be only half. That is why the quantization error is q by 2 to q by 2. What is q? q is nothing but a step size. What is the step size? It is the maximum swing of the voltage divided by a number of steps.

So, 8-bit quantization, 2^8; 16-bit quantization, 2^{16}. So, if I increase the bit, the q's value will decrease. If I increase the q, if I increase the n, 16-bit. So, more precisely, I can also say the step size will be very low. So, a quantization error that is half of the step size is also very low.

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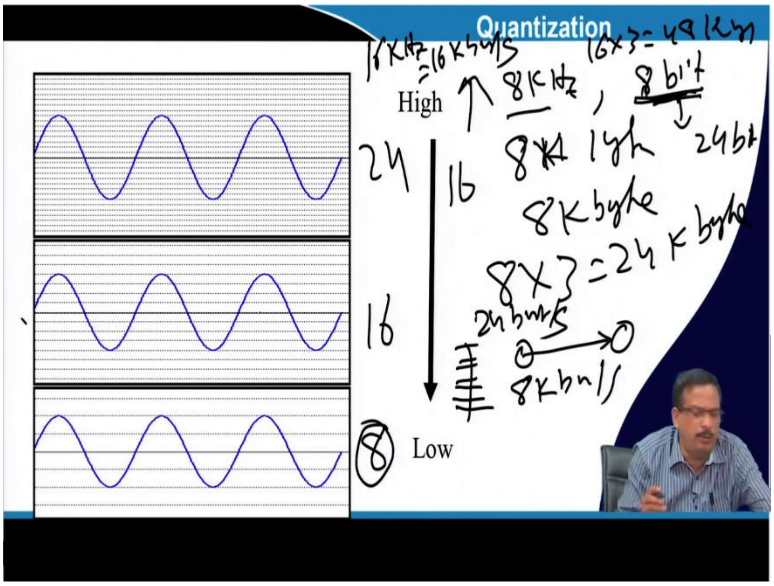

So, if I increase the bit size, the quantization error will be minimum; if I decrease the bit size, the quantization error will be maximum. So, let's say 8-bit, 16-bit, 24-bit. Which one is high error? 8-bit. But think about suppose I sample the signal at 8 kHz sampling frequency, and each sample is encoded with 8-bit, then how much memory is required to store that digital signal? 8 kHz multiplied by 8-bit.

So, I can say 8k * 1 byte. So, I can say 8 kilobytes. If I represent each sample by 24 bits, my memory will be 8 * 3, 24 kilobytes. So, when I increase the resolution in the vertical axis, I mean if I reduce the error in the quantization error, my memory requirement also increases.

So, when I say 24-bit, suppose I want to transfer a signal from this point to this point without any compression. So, if I want to transfer an 8-bit signal, this is 8k. So, a bit rate of 8 kilobytes per second is required; however, when I want to transfer 8k sampling frequency, but 24 bits is my encoding, then I require 24 kilobytes per second.

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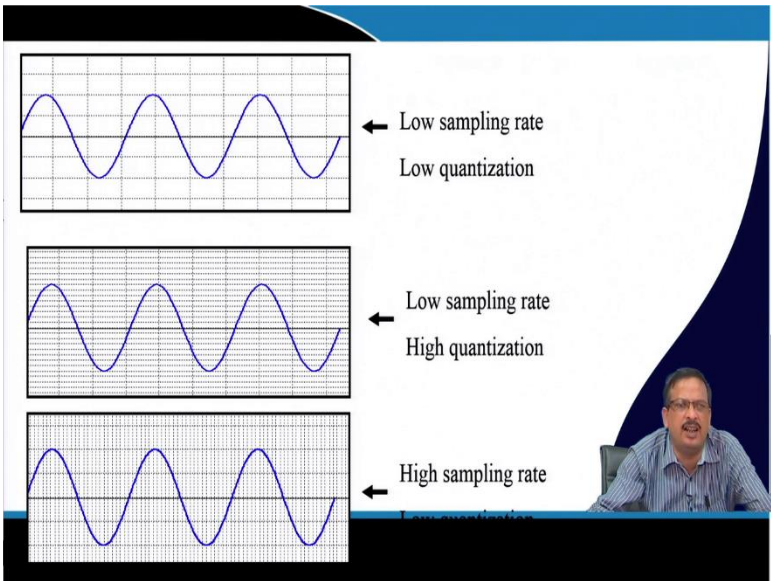

Similarly, when I talk about images, what is an image? First, I complete the sampling frequency; then, I come to the image. So, now, I said encoding bit. Now suppose I want to increase the sampling frequency; so, instead of 8 kHz, if I sampled at 16 kHz, then in 1 second, if it is an 8-bit encoding, I require 16 k kilobyte per second memory or transfer rate.

Now, I have to transfer if it is 24, then 16*3, 48 kilobytes per second. So, if I increase the sampling frequency, the bit rate required will also be high, or the memory requirement will be increased. If I increase the quantization, then also, in both cases, the error will be minimal because if I want to, what do you mean by increased sampling frequency? That means on the time axis, I get a higher resolution and closer to the analog signal.

When I say increasing encoding bit, that means the y-axis that the value axis, I have a higher resolution, so I get more accuracy. If you buy an audio cassette or audio CD, you will see that its sampling frequency is 44.1 kHz, and the bit rate, I think, is 24 bits per second per sample. The encoding bit size is 24 bits, which is why only 3 to 4 songs are there in 1 CD. Because it is an uncompressed version, an original audio CD. Now, when you go for the MP3, a complex version that separates different issues.

So, understand. So, now, I will just give you, first give you a one-dimensional signal example, and then I will come to the two-dimensional signal example.

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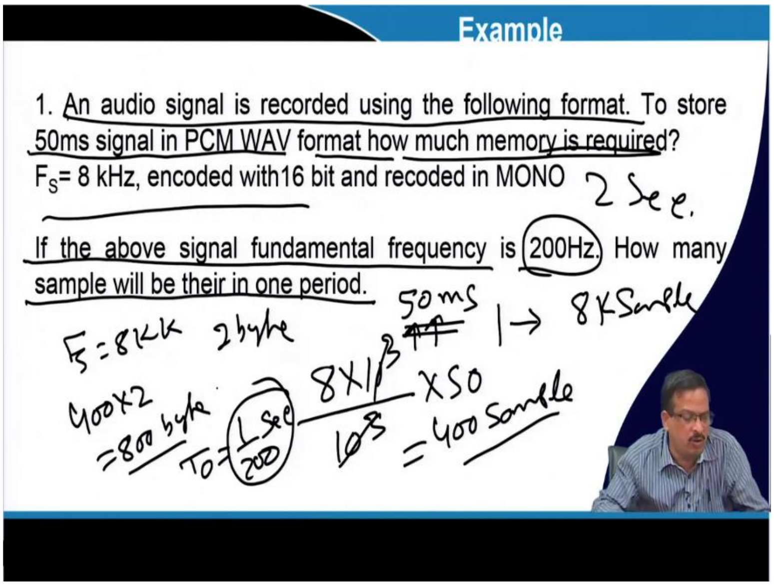

One example I have given. An audio signal is recorded using the following format. How much memory is required for a 50ms signal in PCM WAV format? PCM WAV format means it is stored sample by sample without any compression. So, what is the sampling frequency? 8 kHz. What is the encoding? Each sample is 2 bytes. What do I have to store? I have to store a 50ms signal.

How much memory is required? So, 50ms signal. How many samples will be there? In 1 second, I have 8K samples. So, in 50ms, how many samples will be there?

$$\frac{8 * 10^3}{10^3} * 50 = 400\ samples$$

I converted this second into a millisecond. So, both way you can do. So, I can say 400 samples.

So, I require I have to store 400 samples. So, each sample is 2 bytes. So, how many bytes is required? 400*2 = 800 bytes. Is it clear? If I say the same thing, I want to record 2-second signals. So, in two seconds, how many samples will be there? In 1 second, 8 k sample, in 2 seconds, 16 k sample, each sample is 2 bytes. So, 16 into 2, 32 kilobyte memory is required. Understand?

Now, another way. Let us say that the above signal's fundamental frequency is 200 Hz. How many samples will be there in one period? The fundamental frequency is 200 Hz. How many samples will be there in one period? So, what is the T_0? It is nothing but a one by 200 second. See, in 1 by 200 seconds, how many samples will be there? In 1 second, I have an 8k sample. So, in 1 by 200 seconds, how many samples will be there? Very simple.

So, those are the sampling byte, bit all conversion. You can become accustomed to those things. So, let us talk about that image.

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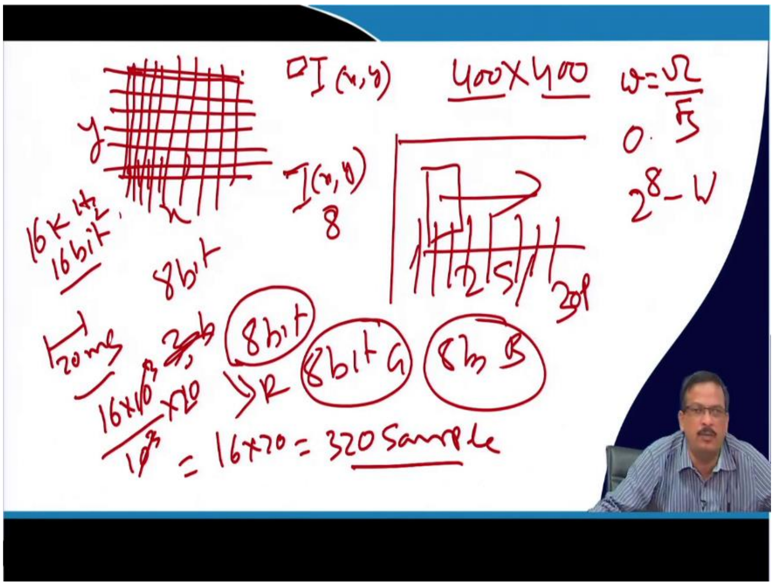

What is an image when you digitize it? The image is spatial. So, I have an x-axis and a y-axis, and let us have an image like this. So, what is digitization of an image? It is nothing but a pixel. So, I have divided the x-axis. So, I drew the x-axis and then divided the y-axis; I sampled both axes and then it created an x-y coordinate. The pixel shape is not rectangular; you do not know the pixel. Pixel shape is, let us say, a circle.

But there is a coordinate system of the pixel: x and y (x,y) and the pixel's intensity, so the I is a function of x and y ok. So, how many samples, I will say? The sampling frequency of the x-axis and sampling frequency of the y-axis is why you call 400 * 400-pixel image. So, 400 pixels by 400-pixel image. So, on the x-axis, there are 400 pixels, and on the y-axis, there are 400 pixels. So, that is an image. So, I is an x and y. Then, there is a quantization. What is quantization? I is quantized.

I can be represented by an 8-bit. It can be represented by a 3, 8-bits. If it is a colour image, you say the true colour is nothing but an 8-bit for R, 8-bit for G green, and 8-bit for B blue, understand? So, 8-bit; 1 byte required for R information, 1 byte required for G information, 1 byte required for blue information, 3 * 8, 24 bit. So, I is quantized using 24-bit.

Let us say if it is a black and white image, I is quantized into 8 bits. So, 0 means black and 2^8 means pure white. So, in between, there will be a grey. Understand? So, that way, when I say the video, is the video a continuous signal? When you look at it, it is a continuous appearance and a continuous signal, but if you see it exactly in the time axis, they are all steel images called frames. So, if I play 20 or more than 25 frames per second, then I say it is continuous due to the perception of the vision of the human being.

So, that is why if you watch it on television or when you see the video specifications, it is written in HD 60p, 30p or 30i. So, those are the specifications standard. Thirty frames, p means progressive; 30 progressive frames per second. So, that is another technology, progressive and inter alia. So, number of frames per second is the sampling of the time axis. Understand? So, that way, you can say that everywhere, it is digitization, and only when you talk about digitization, you have to think of what time instant I have taken and how I have divided the y-axis.

So, quantization bit and sampling things are important issues. Now, another problem: let us say I told you that I have recorded a speech signal with a 16 kHz sampling frequency, 16 kHz sampling frequency with a 16-bit encoding. I said I want to process only 20ms of the signal. So, how many samples will be there in 20ms? How many samples will be there in 20ms? So, in 1 second, I have a 16k sample So,

$$\frac{16 * 10^3}{10^3} * 20 = 320\ samples$$

So, sampling to memory, memory to sampling, sampling to time, time to sampling. So, you have to do those conversions and another issue I have said. I have said in analog, the frequency is called radian per second, but in digital, it is radian per sample. So, it is called normalized discrete frequency. So, what normalized discrete frequency omega is nothing but a radian per second divided by Fs radian per sample.

So, suppose I told you to write a c code or generate a 2 kHz sine wave with a sampling frequency of 4 kHz. Let us say I told you to write a program, C program, to generate a sine wave signal whose sampling frequency is 4 kHz and the frequency of the signal is 2 kHz.

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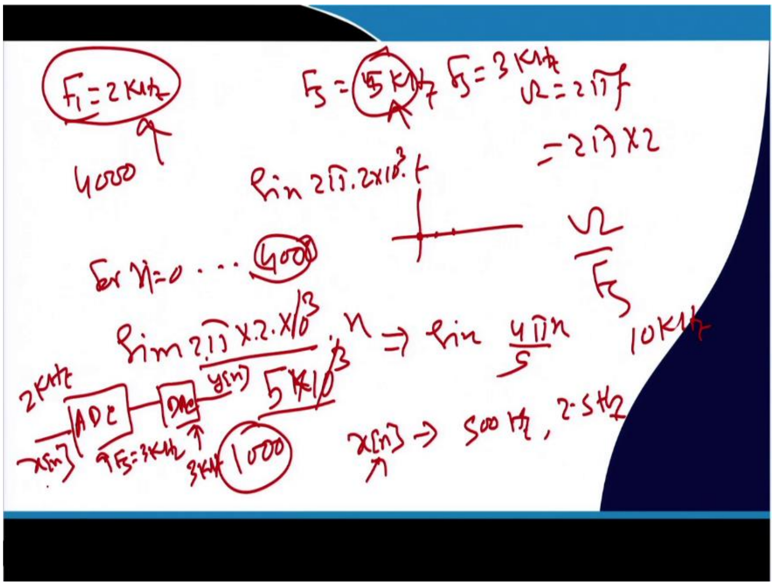

So, I said the sampling frequency of the signal F1 is 2 kHz, and Fs is equal to 4 kHz. Let us say or let us say 5 kHz. I told you to write a C function or write a program to generate a 4 k sample 400 sample of this sine wave using the sine function of using the sine function in a computer.

So, how do you do it? So, I know in sin θ, I know how to write the sin (Ωt). We know ;

$$\Omega = 2\pi f = 2\pi * 2\text{kHz}$$

then

$$\sin(\Omega t) = \sin(2\pi * 2 * 10^3 * t)$$

this is the time domain.

Now, I have to do it in the digital domain. So, I can write a for loop for I equal to 0 to 4000 or n equal to 0 to 4000. I will write

$$sin \frac{2\pi * 2 * 10^3}{5 * 10^3} * n$$

After simplification

$$sin\frac{4\pi n}{5}$$

and n varies from 0 to, let us say, 4000. So, I collected 4000 samples with a sampling frequency of 5 kHz and a 2 kHz signal.

If you plot in Excel, somewhere in MATLAB, you can see the sine wave. The sine wave in a computer is digital because it is not continuous. So, I have to define the sampling frequency. So, if I increase the sampling frequency. So, if I decrease the sampling frequency, t will increase. So, decreased sampling frequency means this t; when I increase, t will be less. So, as I increase it, the point will be more closer. So, the time resolution will be closer. Understand?

Now, if I told you to write a program to generate a composite x[n] with a two-frequency component or 2 monotones, one is 500 Hz, another is 2.5 kHz, and sampled with 10 kHz. Let us say you have to collect the first 1000 samples of x n. So, there are two frequency components: 500 Hz and 2.5 kHz. Now, I come to another problem. Suppose I said that instead of 5 kHz, the Fs is equal to, let us say, 3 kHz; then I told you what should be if I converted the digital signal to an analog signal; what should be the new signal?

I said I had an analog to digital conversion; I applied a sine wave whose frequency is 2 kHz. Then, my sampling frequency is Fs, equal to 3 kHz. So, which is less than twice b. So, less than twice, b is 4 kHz, and then I said I have applied a DAC; the sampling frequency is 3 kHz. What output signal should be, y[n], if I apply x[n] here? You think about it: what should the y[n] be? So, what new kind of signal will I get? What kHz sine wave will I get instead of a 2 kHz sine wave?

So, in the next class, we will discuss that, and I will pictorially show you how this aliasing affects here in the case of a sine, taking an example of a sine wave and a cosine wave.

Thank you.