Signal Processing Techniques and Its Applications
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Lecture - 46
Autocorrelation Method for Linear Prediction

So, what I said in our previous class is that I have to estimate α values.

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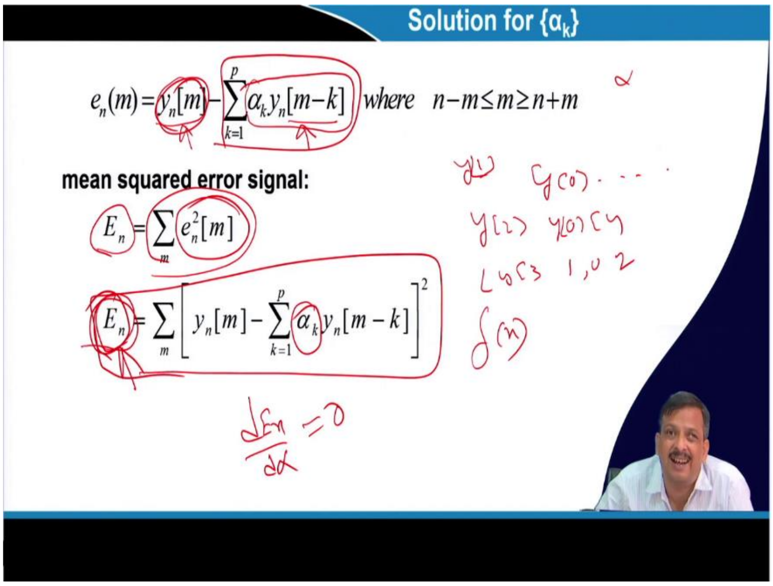

So, how can I estimate α values? So, what I said is that I have a present sample and a previous sample.

And I said the present sample can be estimated from the previous sample using a linear combination with α. A 1 into y m minus k ok. So, what is the error? Error is the current sample minus the estimated sample, so that is the error. So, what is the mean square error? The mean square is nothing but the En, and the mean square error is nothing but a square of the error for all over m.

So, I am, so I can estimate y[1] from y[0] and let other values, y[2] from y[0], y[1], y[3] for like, say, y[1], y[0], y[2] like that I can say p estimator p. So, I can say each time I calculate the error and take the average take the sum. So, I can get the mean square error. So, En square, so En square is nothing but a square of this one, ok?

So, now I want to minimize the mean square error. So, f x is a function. So, how do I minimize the function? With respect to what I want to minimize, with respect to I want those sets of α values where this En is minimum.

So, my objective is to find out those sets of α values for which my error is minimum. So, that means I can take the differentiation d En with respect to d α, and that is equal to 0, giving me the minimum error condition.

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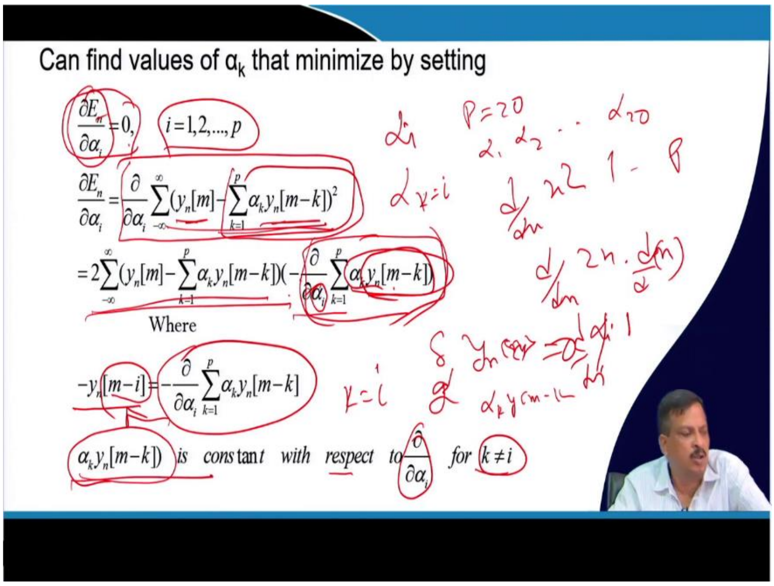

So, what will I do? I will take the d En divided by d αi is equal to 0 for i equal to 1 to p, p is the prediction number of prediction. So, let us say I want to design a predictor of p that is equal to 20; that means I can say that the current sample is a linear combination of the previous 20 samples.

So, there is a 20 α 1, α 2, dot dot α 20. So, the mean square error expression has to be taken, and the derivative has to be derivative should be 0 to minimize the error with respect to the α. So, dEn/dαi is equal to 0. So, I take the derivative of this E d E n; En is nothing but a y m minus the whole square ok. So, it is nothing but a take the derivative means d/dx of x square, it is nothing but a d/dx of 2 x into d so, that into d/dx of d/dx of x ok or not.

So, I can say 2 into this one into minus this one. Is it ok or not? Because yn m does not contain any α, so d/dα. So, d α of yn m is equal to 0, so only this part has an α that is why this part is there. Now, if you see what is d/dα of k equal to 1 to p α k yn m minus k. So, α k y m minus k is a constant with respect to d/dα except k equal to i.

I am taking the derivative with αi, so except this α k; k equal to I; that means that it is αi; otherwise, it is constant and not in his purview. So, I can say only α k y m k exists when k is equal to i. So, I can say this differentiation is nothing but a minus m minus i. Because of that time, the αi d/dαi by αi will be 1, and m minus i will exist. Is it clear?

I take the square and take the derivative; the derivative of the x square is nothing but a 2 into x into 2 into x into d/dx of x. So, now when I say the x is this one, it does not contain any α, so this will be 0, this will be minus this one, but α k varies from 1 to p. Now, except k equal to i, that others are constant with respect to αi ok.

So, that is 0 derivative of a constant is 0. So, I can say this is nothing but a minus m minus i.

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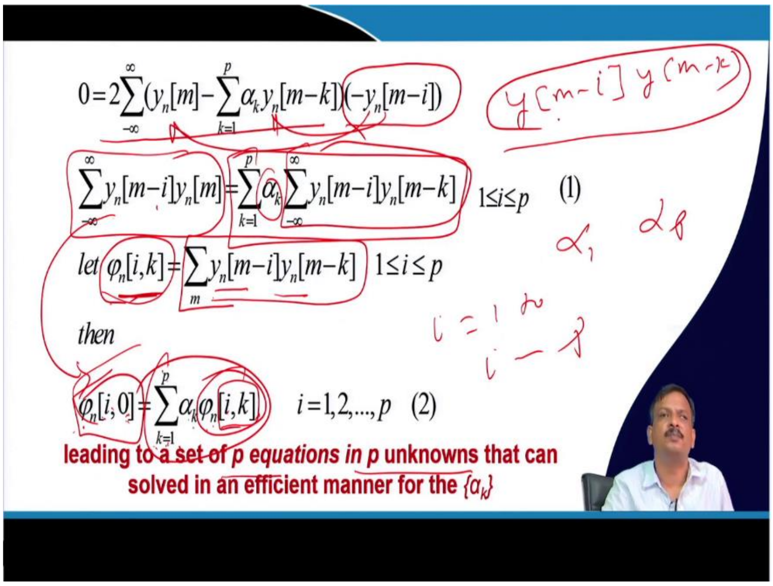

So, I can say the final derivative becomes 2 into this one into yn m minus i. Now, I can say this is y m into this one, which is equal to y m into this one. Let us say m is a signal m that varies from minus infinity to infinity, but k varies from 1 to p. Is it clear?

Now, I can say that let us φ I k is defined as this one; this is nothing but a φn I k. So, this is i is there k is 0, so I can say this site is nothing but a φ I 0 φn I 0 and this site is nothing but an α k will be there, and this site is nothing but a φ I k.

So, I can say φn i 0 is equal to k equal to 1 to p α k φn i k, so i also varies from 1 to p ok. Now, if you see, I have a p equation in p unknown, so I have an α 1 to α p unknown, and I have a p equation i 1 to I p.

So, I can say i φn 1 0 is equal to k equal to 1 to p α k φn 1 k. Now, I can solve it, so I have a p number of unknowns and a p number of equations. So, if I want to write in matrix form, how can I write in matrix form?

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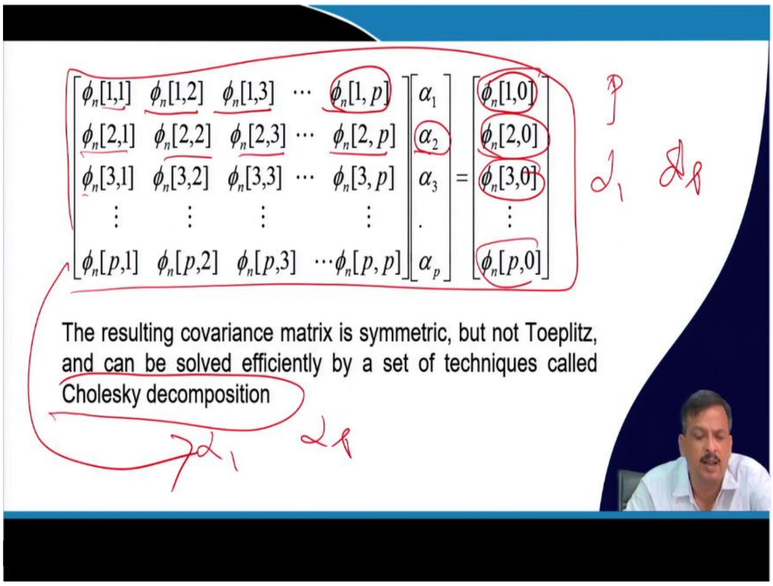

On this side, what is there α k, so k is equal to 1 to p. So, i is equal to 1. So, if I say φ 1 0, this is i equal to 1. So, I put i equal to 1, k equal to 1 to p. So, I can say i equal to 1 1, φn 1 2, 1 3, φn 1 p.

Because k varies from 1 to p, next i equals 2. So, second row 2 1, 2 2, 2 3, 2 p α 2 multiplied by α 2 φn 2 0 i equal to 2, i equal to 3, i equal to p. So, this is a matrix form of this equation. Now, I have to solve this matrix to find out α 1 to α p. Now, can I say this is a symmetric matrix?

So, what is φ 1 1? So, φ 1 1 is nothing but a what is φ I k is nothing but a signal y m minus I into signal y m minus k, which is nothing but a covariance of the signal. So that is why it is called a covariance matrix. So, this covariance matrix is symmetric but not trapezoidal; Toeplitz it is not that and can be solved using Cholesky decomposition. So, this is a matrix solution. How do I solve this matrix? Ok.

So, I get a p number of p set of the equations; I have to solve for α 1 to α p. So, there should be some method to solve it; let us discuss that method, ok?

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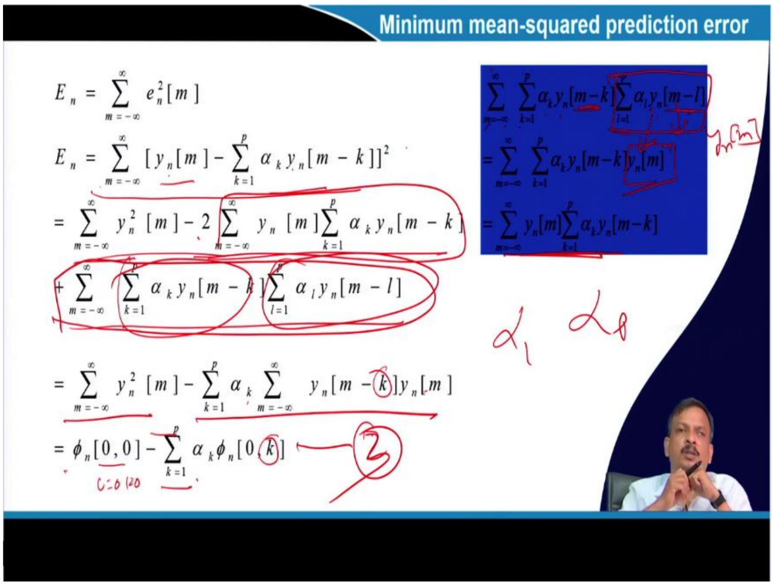

Now, before that, what is the minimum mean square prediction error? What is the minimum mean square prediction error? So, En is equal to this one. I calculate the minimum mean square prediction error. So, a minus b is the whole square, a square minus 2 a b plus b square, summation of m equal to minus infinity to infinity a minus b the whole square. So, let us say the signal is infinite in length.

So, a minus b square a square minus 2 a b into b square; b square means 2, this is one b and this is the second b ok. So, now it can be shown. So, now what is the how? What is this? So, if I say m equal to minus infinity to infinity k equal to 1 to p α k yn m minus k and i equal to 1 l equal to 1 to p α l yn m minus l. So, when I say that, then can I not say this one is nothing but an estimation of yn m.

So, instead of that, I can put yn m as the minimum mean square error. That means the error is 0, let us say, considering 0, so my estimation is correct. So, I can say this is nothing but a y m. So, I can say that this product is nothing but a product. So, I can say 2 minus one of these. So, that is nothing but a y square minus this one.

Now, if I want to express in terms of φ is nothing, but a φn 0 0, i equals 0, k equals 0 minus k equals 1 2 p α k. So, it is nothing but a 0 k, i equal to 0, but k is there. So, that is the minimum mean square prediction error.

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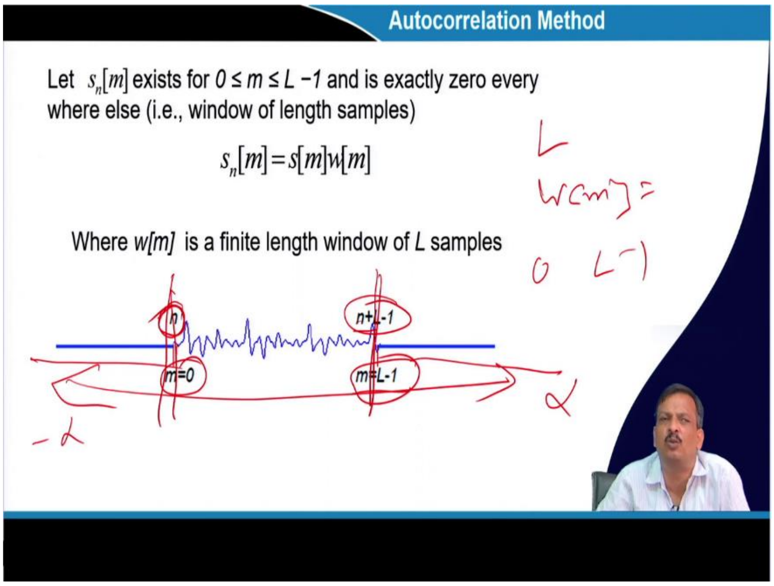

Now, if I say how do I, so minima. So, this is an equation number 2, and this is an equation number. Sorry, this is an equation number 2, and this is an equation number let 3.

So, from these two equations, how do I calculate the value of α 1 to α p? So, what method exists to solve for α 1 to α p? Let us describe two methods; the first method is called the autocorrelation method.

I will give the example of a speech signal. Let us say speech signal; let us say I have a signal. I have a signal. I cut a signal of a window length of L. So, n is minus infinity to plus infinity long signal I have n. So, I took the signal portion of a signal that is cut by a window length w m, which is 1 for 0 to L minus 1. Elsewhere, it is 0, so if I say the index of the signal m is equal to 0 to L minus 1.

If the static index is n, it is nothing but the n plus L. So, if the static index is n, it is nothing but a n plus L minus 1 in infinite sample space. If the static index is 0 so, it is m minus L minus 1. So, what I am saying is I have a long signal. I take a portion of the signal, which is length L. So, that means the index of the signal varies from 0 to L minus 1.

So, the finite length window of the L sample ok. So now, what should be the length of the prediction error?

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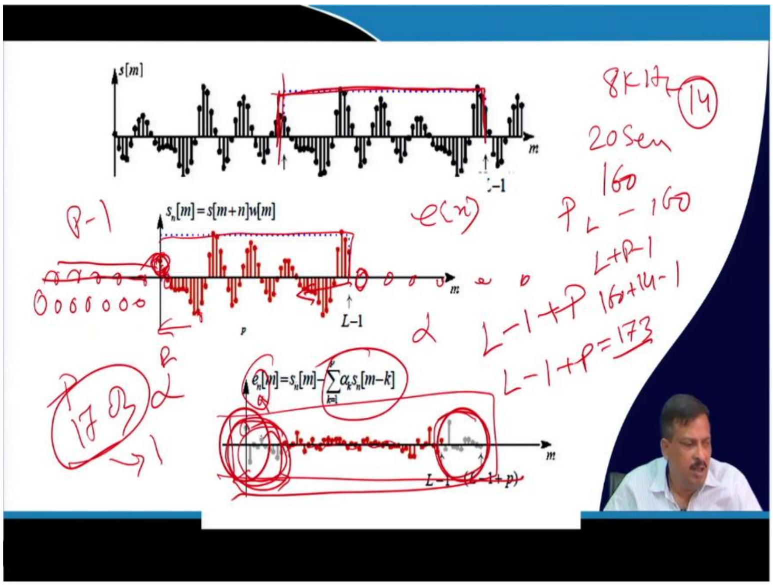

Here is an example. So, I have taken this portion of the signal let us say this portion of the signal I have taken. So, this is the portion of the signal, so outside that signal is 0. Now, as I said, I want to run the Pth order predictor. So, that means I said the current sample can be predicted from the P number of the previous sample.

So, all the previous samples are 0. So, I can say that the first sample can be predicted from the P number of samples whose values are 0. So, I want to predict the first sample from the previous P number of samples whose values are 0.

So, what is the prediction error? It is nothing but a sample size whole error. So, a linear combination of the 0 is nothing but a 0. So, the prediction error will be the maximum value of the sample value.

For the second sample, I predict from the first sample and the previous P minus 1 number of 0. So, an error will be maximum because all the coefficients are 0. Only one coefficient is contributing.

So, I can say up to P number of sample the error will be maximum. So, I can say the beginning of the window the error will be maximum after that I am predicting a sample where the P samples are there.

So, the error will be minimal. So, then I get the error. Again, when I come here, I predict 0 from the previous P number of the sample value. So, the prediction error will be again maximum. So, I can say the prediction's length is nothing but a convolution equation. So, the length of the prediction is nothing but an L minus 1 plus P.

So, En is nothing, but you can say this is a convolution kind of equation. So, I can say the length of En is nothing but an L minus 1 plus P. Let us say I told you a real-life example; suppose I have a speech signal with an 8-kilo hertz sampling rate. If I say my window length is 20 seconds, how many samples will there be? 160 samples will be there.

So, my L varies from 0 to 160, 160 minus 1, 159. Then, if I told you what it would be like if you ran a 14th-order prediction, then what should the length of the prediction error be? So, the length of the prediction error is nothing but an L plus P minus 1. So, 160 plus 14 minus 1 is nothing, but 173 ok.

So, if I cut the signal, the problem is the prediction error in the beginning, and the prediction error in the end is maximum. Now, when I say how many sets of αi will get if I run the 14th-order prediction.

So, for each sample, I get a set of α values. So, if there is a 160 sample, I get a 160 set of α value; out of 160 sets, I can say I have to my job is to find out the one set that will minimize my error.

So, I can say out of 173 error positions, all αs are there. So, 173 sets of αs are there α 1 to α 14 173 sets are there out of 173 sets, I have to find out 1 set for which my error is minimum. So, that is my job, ok.

So, at least I required the 14th equation, I require at least the 14th En I required, but I will get 173 of E n. My job is to find out what the minimum is and which is the set of α, and find out the optimal value of the set of α for which error is minimum ok.

Then I said, autocorrelation methods. So, why is this called the autocorrelation method? I am not yet discussing the auto-correlation method. I have only said I have a long signal; I have a cut portion of the signal and using that portion of the signal, I want to find out the value of α 1, α 2, α 3, which will represent that portion of the signal ok.

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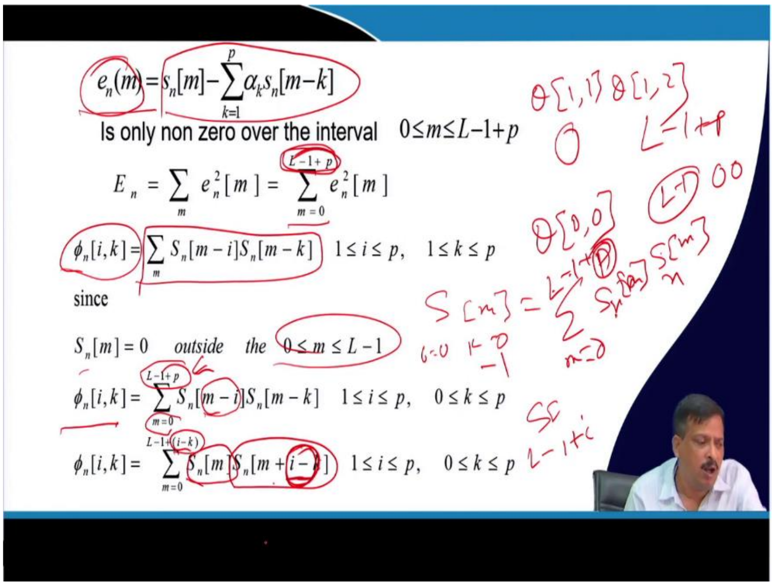

So, now you can say that En m is nothing but a signal minus an estimated signal. The signal is only non-zero, so I can say the error is only non-zero over the interval of 0 to L minus 1 plus p.

So, for the mean square error, I do not have to compute infinite samples; I only compute to m equal to 0 to L minus 1 plus p because elsewhere it is 0. So, it is not computing that contributes to the mean square error. So, the mean square error is contributing only L minus 1 plus p in errors. I understand. So, I know that φ i k that we have already derived φ i k is represented this way, where i equal to varies from 1 to p and k also varies from 1 to p.

And S n m outside that L minus 1 is 0, S m the signal is outside that L minus 1 is 0. So, I can say φ i k. So, this is the error contributing only to L minus 1 plus p. So, L minus 1 by plus p m equals 0 to this one. So, when I say φ 1 0 0. So, I equal to 0 k equal to 0; it is nothing but an m equal to 0 to L minus 1 plus p S n 0 multiplied by S n m multiplied by S n m.

So, which is nothing but each sample will be multiplied with each sample up to L minus 1 plus p number of sample and added together that is nothing, but a φ 0 0. So, φ 1 0, 1 1, φ 1 2 I can calculate easily.

That is nothing but a covariance of those window signals. Now, if you compute it, then when you say that outside the L minus 1, the signal is 0. So, if I say here S of n m, although it is L minus 1 plus p, but outside the window, the L minus 1 signal is 0. Let us say I consider m minus i as an m; m minus i as an m.

So, now it becomes L minus 1 plus i. So, I can say that L minus 1 minus 1 plus i minus k into S n m plus S n m plus I minus k; I just change the index. So, i minus k is an absolute value, so if i is equal to 0 and k is equal to 1, that value is minus 1. If I take that there is some value that will be there, let us say mod of I minus k.

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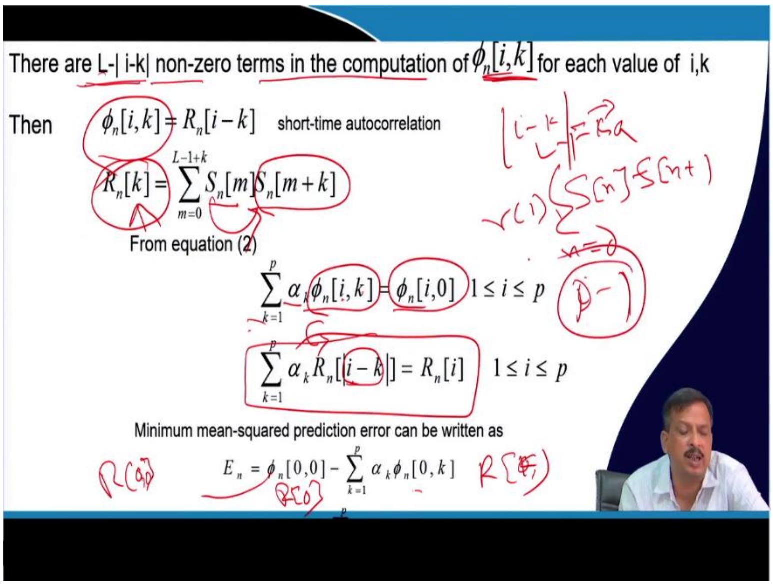

So, I can say the mod of L minus i minus k non-zero term in the computation of φ i k. So, if I say that minus k has a value, that means whether it may be negative, it may be positive, but it has a value.

So, if I take the mod of that part mod of part as a value, let us say the value is a. So, once I said the value is a or k whatever, I can say that this value is k, let us say. So, it is nothing but a signal S m plus k.

So, what is the correlation? Correlation is nothing but a signal r 1 is equal to S 1 into S n plus 1 mod n varies from 0 to L minus 1. So, I can say φ I k is nothing but a correlation autocorrelation. Why is it auto-correlation? Because the signal correlate with itself.

So, it is nothing but a φ I k that can be replaced by an autocorrelation value. So, the equation k equal to 1 to p α k φn i k φn i 0 can be replaced by i k can be replaced by R n i minus k.

So, it is nothing but the autocorrelation and minimum mean square error again φ can be replaced by an R 0; 0 0 and it is nothing but so, I this is nothing but an R 0 k, R k and this is nothing, but an R 0.

So, which is the energy of the signal, ok?

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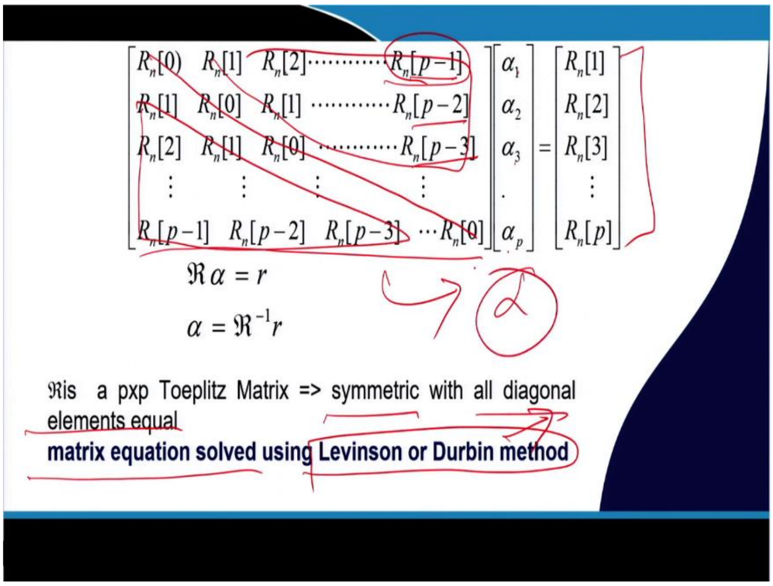

Now, I can say. So, I can say R 0 if I want to repeat this equation as a matrix form. So, I can say R 0, R 1, R 2, R n p minus 1 i minus k. So, if I say k equal to 1 to p., there will be an i minus 1 term will be there. So, if I when it is p; p minus 1, it will be there. So, I can say it is p minus 1, p minus 2, p minus 3 like that α 1, α 2, α 3, α p and this is R 1, R 2, R 3, R p.

Now, you can see all the diagonal elements are the same; the lower and upper triangular matrices are the same. Now, this is a Toeplitz matrix form, which is symmetric. All diagonal elements are equal. This matrix can be solved using Levinson and Durbin's recursive methods.

So, in the next class, I will talk about this recursive method and not go into detail about mathematics like mathematics. You can search in mathematics Levinson recursion, and you can get it, but I will show you how this matrix is solved for the computation of α value and how you can write a program to compute the α value if you know the signal S n ok.

Thank you.