Signal Processing Techniques and its Applications
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Lecture - 43
Chebyshev Filter Design Method

So, now I will talk about the Chebyshev filter Design. So, Butterworth filter design I have done. So, I have to now go for the Chebyshev filter design. How do I do that?

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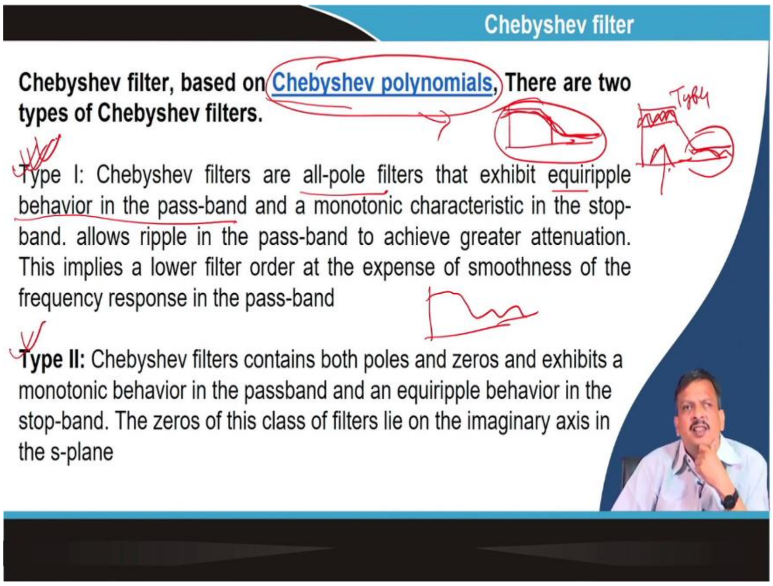

So, what are Chebyshev filter design methods? So, the Chebyshev filter is based on the Chebyshev polynomial. Actually, this algorithm is a polynomial used to curve approximation, approximation of a curve. So, now, if you see when somebody gives you the filter specification, let us know this is the 3 dB down Ωp, and then it is attenuated further, and this is the attenuation.

So, I just want to copy this graph. So, basically, the Chebyshev approximation is used to copy the desired frequency response of the filter. So, there are two types of design. The first is the Chebyshev filter, which is an all-pole filter that exhibits equiripple behaviour in the pass band.

So, if I allow ripple in the pass-band and the stop band is monotonic, that means no ripple, then I call type I. So, what are the properties of this type of filter? This is an all-pole filter. It is equiripple in pass-band and the stop band is monotonic.

So, since the stop band is monotonic, there is no ripple, so the attenuation in that stop band is very good. So, when I require a stop band, attenuation is very good. I want to design an application that requires a stop band of almost 0. Then, I used the type I Chebyshev approximation and type I filter.

And what is type II? It contains both the zero and pole, but it does not have a ripple in the pass-band, but it has a ripple in the stop band. So, the type II filter has a ripple in the stop band, but there is no ripple in the passband; it is a pole-zero filter. So, type II is a pole-zero filter, and type I is an all-pole filter. So, how do I design this type I and type II filter?

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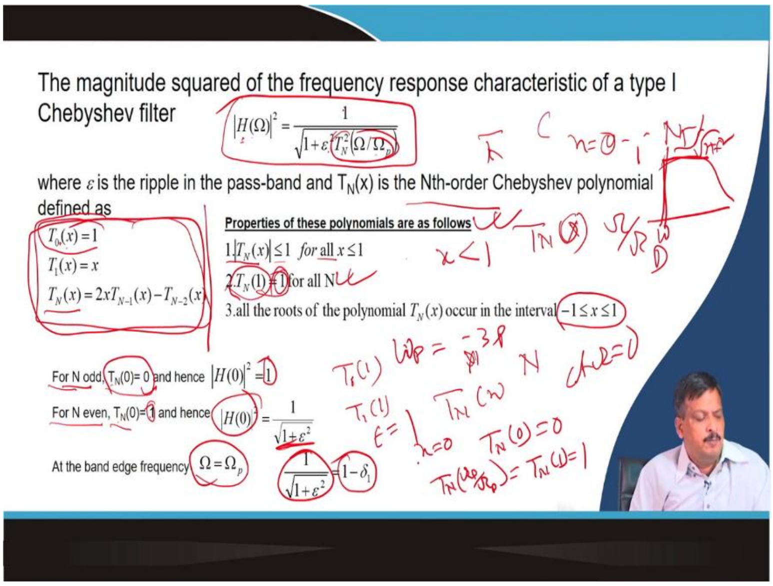

Let us say go for the type I first. So, let us say this is the square magnitude response of the filter. So, what if you see what extra there is in butter worth filtering here, that is, ε **Ω** by **Ω**p? Here, it is not; it is **Ω** by **Ω**p, which is a function of T_N square.

So, $T_N(x)$ is called Nth order Chebyshev polynomial, Nth order T_N Nth order Chebyshev polynomial is used here. So, what is the polynomial? So, it is nothing but a $T_0(x)$, and x is the inside variable. So, x is nothing but a **Ω** by **Ω**p. So, x is equal to 1 when N is equal to 0.

So, N is the Nth order, which means if I say small N, it varies from 0 to minus 0. So, a small N equal to 0 means T0, T1, Tn.

So, this is a recursive kind of polynomial that is used to approximate the desired frequency response of the filter ok. So, what are the properties of that polynomial? Number one $T_N(x)$ is less than one always for all x. If the x is less than 1, then $T_N(x)$ also will be less than 1, and the magnitude of $T_N(x)$ will also be less than 1.

T_N if x is equal to 1, then TN 1 is nothing, but a 1 maximum amplitude is 1 when x is equal to 1. So, if x is less than 1, the polynomial value is also less than 1; if x is equal to 1, the polynomial value is 1 for all N. Whether it is T 0 1 or whether it is T 1 does not matter. It is always 1.

So, all polynomial roots occur at intervals minus 1 to plus 1. So now, I come to the these are the properties, and this is my polynomial Chebyshev polynomial. So, I use these two properties to design that filter. How do I design it?

So, for N odd, TN 0 is equal to 0; that means, if x is equal to 0, then what is the value of TN 0? So, T_N 0 is equal to 0 if N is odd. So, if the order of the polynomial is odd, then T_N 0 is 0. If the order of the polynomial is even, then T_N 0 is 1. So, when T_N 0 is 0, that means the whole thing is 0. So, it is nothing but a 1 by 1, so it is nothing but a 1. Now, when TN 0 is equal to 1, that means, at Ω equal to 0, so at Ω equal to 0. So, this H 0 whole square is nothing but a 1 by ε square, ok?

So, at Ω when I say that when I approximate that filter. So, at Ω equal to 0, here is the maximum is 1 in case N is odd. In case N is even, this is nothing but a 1 by root over of 1 plus ε square. E is the pass-band ripple if N is even.

Now, at Ω equal to Ωp. So, if I say at Ω equal to T_N Ωp by Ωp, so, Ωp by Ωp is equal to T_N 1. So, T_N 1 is always 1. So, I can say this factor is 1, so it is nothing but a 1 by ε square root over of ε square, which is equal to your passband ripple.

Now again, you can prove that if the pass band and frequency are defined in 3 dB minus 3 db, then the passband ripple ε will equal 1 ok, same principle.

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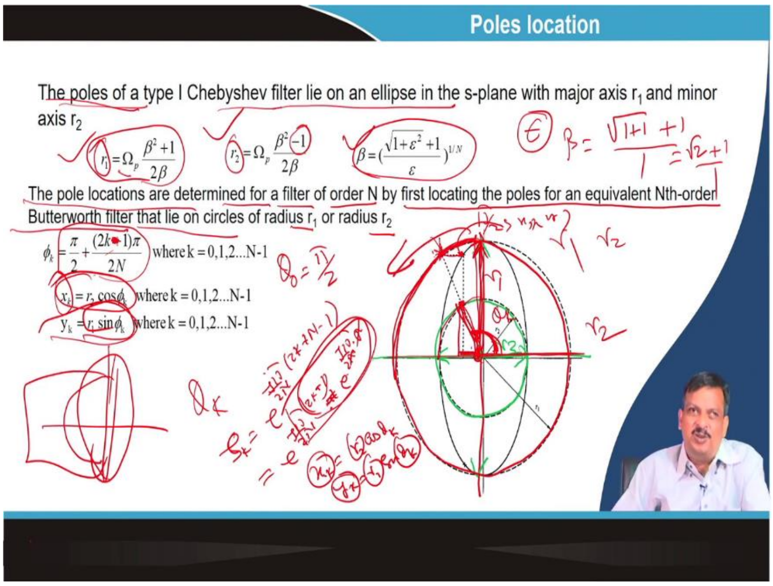

Now, I come to the pole location. So, in the case of the Butterworth filter, the pole location r s k is defined by N, and we have derived that equation. So, I also have to find out the location of the pole here.

Now, if you see the Chebyshev polynomial in type I, the Chebyshev filter that all poles lie on a ellipse in a s plane. So, it is not like the Butterworth filter. All poles lie on the left-hand side of the s plane here and also on the left-hand side, but it is not in the entire left-hand side plane. It will lie on an ellipse, ok?

So, if it is an ellipse, an ellipse has two axes: one is called the major axis, and one is called the minor axis. So, the major axis lets us know this one is the major axis, and this one is the minor axis. So, this is r2, these two, this is r2, and these two, this is r1.

So, now, if I say these two, this radius is r2 here centre point to here. So, if I take an r2 as a radius, I can draw a circle. Similarly, I can take an r1 as a radius, so here to here is in r1. So, r1 is a radius, and I can draw a circle.

Then what is the value of r2 and r1? So, r2 is equal to Ωp into β square plus 1 divided by 2 β where β depends on the ε s; ε is the pass band ripple ok.

If ‘is equal to 1, then β is equal to root over of 1 plus 1 plus 1 divided by 1. So, it is nothing but a root 2 plus 1 by 1. Once I know the β, I can calculate r1 and r2. also, I can calculate that this is minus 1. That is why r2 is less; r2 is always less than the minor axis, which is always less than the major axis.

So, if I know ε , then I calculate β, I can calculate r1, and I can calculate r2. Then, where is the pole? The pole location is determined for a filter of order by first locating the pole on an equivalent Nth another Butterworth filter that lies on the circle of radius r1 and r2.

So, first, I used the Butterworth process to find the location. So, the location will be on the radius of r1 and r2 circle of the radius of r1 and r2 ok. So, φ k, let us say, is equal to π by 2 plus 2 k plus 1 π divided by 2 N. So, φ k is the angle.

So, if you see the Butterworth filter $S_k = e^{\frac{j\pi}{2N}(2k+N-1)}$. What is the meaning of e $^{(j\pi/2N)*2k}$ Let us say 2 k minus 1 divided by into in plus e to the power or into e to the power j π by 2N into N, N cancels, so j π by 2.

So, I can say the pole position if I want to, so this upper side is the angle. So, pole positions are determined from that angle. So, φ so φ k. So, φ k equal to 0 means φ 0 is equal to π by 2, why it is π by 2, I offset is required. Because all poles are on the left side, this is π by 2.

So, π by 2 plus this side π by 2 plus 2 k plus 1 π by 2_N where k equal to 0 2. It may be I think it will be minus 1, but there is a mistake. It will be minus 1 because this is the Butterworth filter, things ok. So, once I know φ k, I am locating the pole on the radius in here, ok? So, if you look at this axis, it is my X-axis. So, X-axis is related to r2, and Y-axis is related to r1.

So, if I say the poles are located here, I can say that x value x value is x k. So, xk is equal to r2 cos phi. So, let us say this is φ k angle is φ k. So, φ k angle is this 1. So, I can say it is nothing but a r2. So, this portion is nothing but a r2 cos φ, and y 1 is nothing but a r.

So, if this is φ, then y can, in case of sin φ, is it ok? So, r2 cos φ k and r1. So, this is φ k. Think about π by 2 plus this 1. So, r this portion r2 up to this portion is nothing, but I can say it is nothing, but an x is nothing, but an r2 this is φ k. So, this is the radius r2 cos φ k is the x k.

Then what is yk in this axis? In an ellipse, this is an ellipse, and this point is in an ellipse x k, yk. So, this is nothing but a r1 sin φ k. So, I know that in the case of type I filter, a pole position is xk is equal to r2 cos φ k and yk is equal to r1 sin φ k. So, if I know φ k r2 and r1 I can find out the value of xk and yk ok.

So, this is the pole position, and I will show you one example. So that you understand, let us give an example right now and then go for that Chevy. Let us say cover the type II and then give you the example, ok? So, the pole position is xk yk. So, how it comes to the pole, I will show you, ok?

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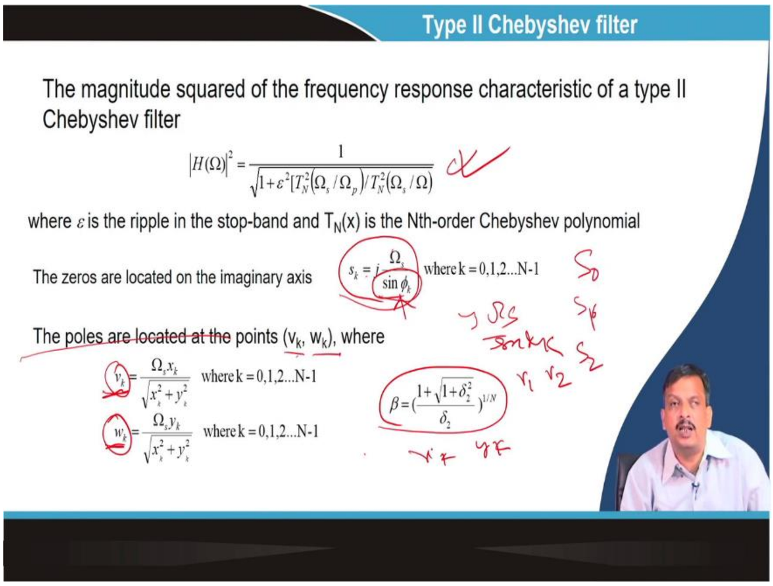

Then, use the Chebyshev filter type II, type II filter. Type II filter magnitude response is this one ok and again all are same as here xk is equal to j π k by j Ω s by 0 r located. So, it is a pole-zero filter 0 located at s k equal to j Ω s by sin φ k. And poles are located at the point of vk w k, where vk is this one and wk is this one Ω s k divided by root over of s k yk root over of xk yk square.

Again, you can say β is equal to this one. So, I can say that I know r1 and r2 from the β, and once I know the r1 and r2, I know xk yk. Once I know xk yk, I can say vk wk calculated ok.

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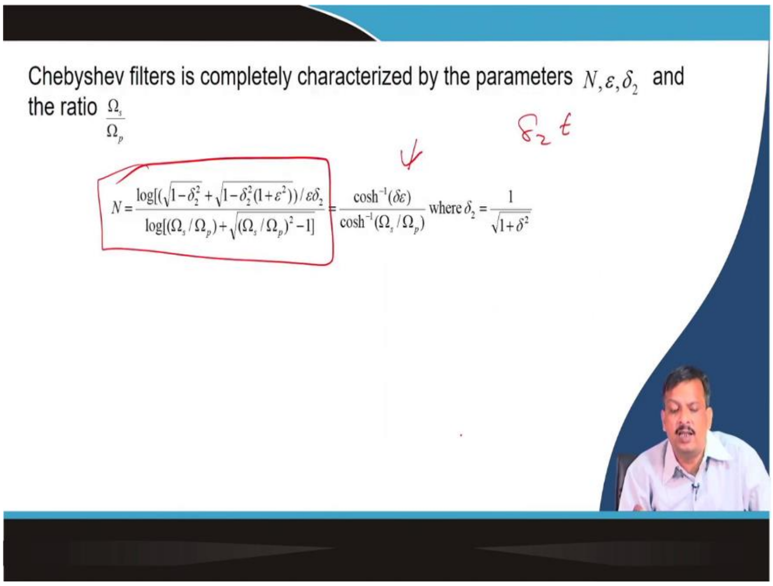

Then again, how do I determine the order? So, the order equation is this one. So, if I know delta 2 and ripple pass-band ripple or stopband ripple, I can design the; I can find out the order of the filter. This is the relationship. Let us give an example.

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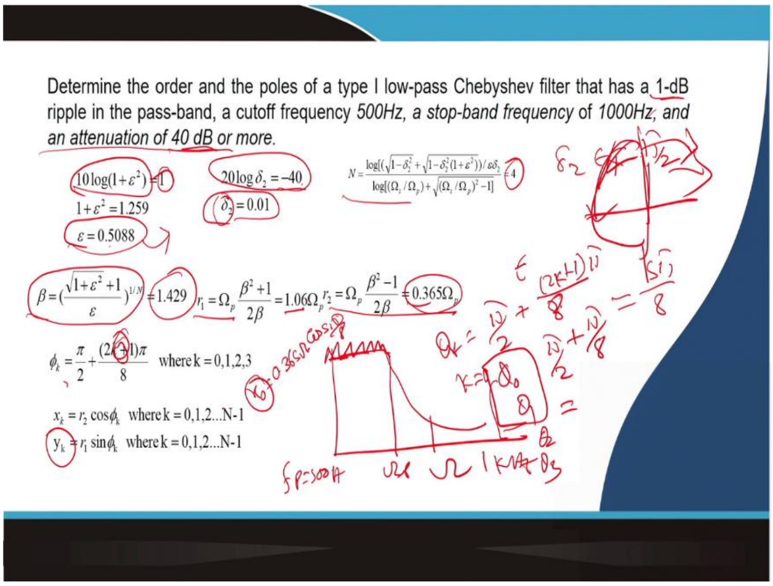

Determine the order and pole of the type I low pass Chebyshev filter, which has a 1 dB ripple in the pass band, a cut-off frequency of 500 hertz at a stop, a stop band frequency of 1-kilo hertz and an attenuation of 40 dB.

So, first, from those verbose things, first you draw the filter. So, this is my Ω axis, and this axis is my in dB let us in db, so let us say this is my 0 dB let us say. Then I say determine the time of low pass filter has a 1 dB ripple in the pass band. So, 1 dB ripple in the pass band means ε I have to calculate.

So, there is a ripple in the pass-band one dB ripple, and Ωp is 500 hertz. So, f p is given 500 hertz and stopband edge frequency is 1-kilo hertz; I have to determine the order and pole position of the filter ok. So, what is the order of the first find-out that ε?

So, ε is nothing but a 10 log 1 by ε square is equal to 1 db so that I can calculate ε. Once I calculate ε, I can calculate ε is known, then attenuation at 40 dB or more. So let us say 40 dB. So, if it is 40 dB, then delta 2 is equal to 0.1, which is okay. So, delta 2 is equal to 0.1 and order I put the delta 2 value and ε value, I calculate the order 4 ok.

Now, once I calculate the type I. So, β is equal to this equation. I put the value of ε 0.5008, and here, I find out the β value is 1.429. Then I can calculate r1, I can calculate r2, I can calculate φ k. So, what is φ k? Is φ k is equal to π by 2 plus 2 k minus 1 or plus 1 whatever. Let us say plus 1 divided by 2 into N equal to 4 means 8 then k equal to 0.

So, φ 0 I calculate φ 0, φ 1, φ 2, φ 3. If it is the real filter, I can say φ 0, φ there will be a complex conjugate. So, if I only compute two values, others will be complex conjugates, I can write directly and write down ok. So, φ 0, what is the value of φ 0 π by 2 plus π by 8? So, it is nothing but a; it should be plus, it will not be minus; if it is minus, then what will happen? The pole may come to the first coordinate because this is π by 2 in the s plane.

So, if the pole is all, the pole will be on the left-hand side, not the right-hand side. So, if it is minus, it may come in the right-hand side, which is why it will always be plus. So, this will always be a plus.

So, it is nothing but an, I can say, 5 π by 8. So, 5 π by 8. So, it is nothing but a xk x 0, which is equal to r2 0.365 π 365 π into cos 5 π by 8. So, you can calculate the value of xk, and you can calculate the value of yk. So, that is the pole position on the left-hand side of the s plane. So, I have an explanation. So, how do I determine the transfer function let us say I take a blank slide here. So, how do I determine the transfer function?

So, what I said is that all the poles are on my left-hand side. So, let us say this is my s plane, and this is my ellipse.

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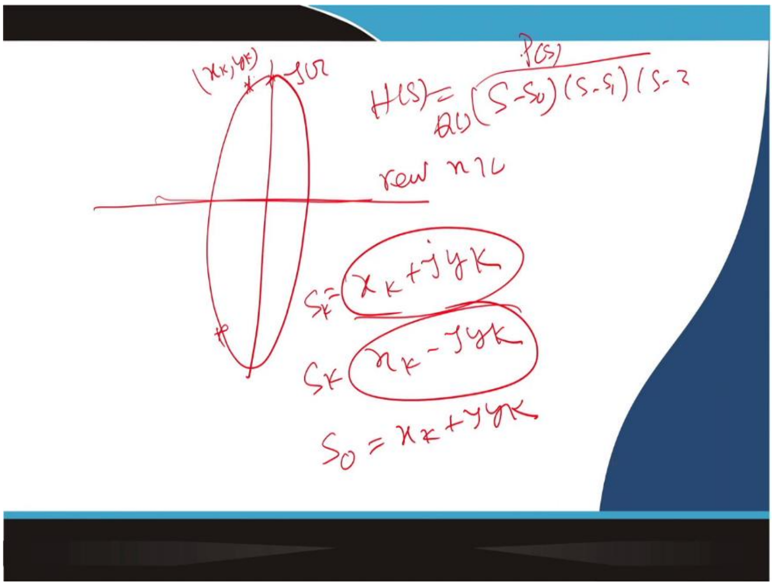

Let us say, and so all the poles let us a pole in here. This is nothing but a xk yk. So, this is the real axis. So, the real axis is related to xk j, and **Ω** axis is related to yk. So, I can see the pole is nothing but an xk plus j yk, and another one is a complex conjugate xk minus j yk. So, this is 1 pole this is another pole.

So, once I know the value of xk and j yk, I know S minus; let us say this is x S k, and S k means pole position. So, S 0 is xk plus j yk. Let us say so: I can say S minus S 0 into S minus S 1 into S minus S T. I can go H S, and what is the zero location? Zero location is always S k is equal to j by **Ω** s divided by φ k.

So, once I know φ k, I can zero location, I can find out S 0, S 1, S 2 I can find out and I can put it in here zero location in the transfer function. So, P S by Q So, that is transfer function is designed. That is called the Chebyshev filter design.

Is it clear? If you have any confusion, then you can comment on your forum so that I can clear it up.

Thank you. Thank you very much.