Signal Processing Techniques and its Applications

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Lecture - 42
Traditional Analog Filter Design

So, in the last class, we talked about pole-zero placement methods for designing that IIR filter, but we have seen there are a lot of limitations to using pole-zero placement methods. So, the best possible way to design an IIR filter is called the analogue filter design method.

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What are analog filter design methods?

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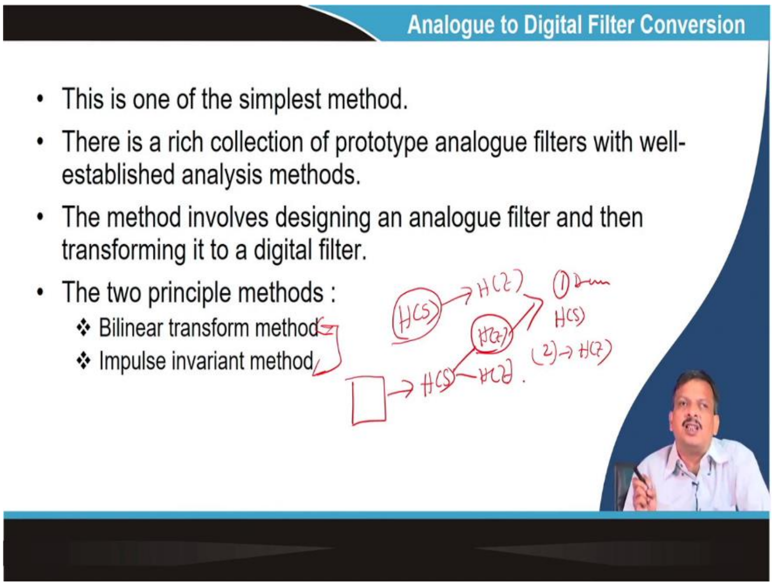

If you look at the analogue filter, this method is the simplest method because analogue filter design methods were available long before the digital filter came out. So, design methods for the analogue filters are always established. So, there are a lot of prototypes for designing analogue filters: Chebyshev, Butterworth, and elliptical. So, all kinds of prototype filter design methods are available and have already proven to be very good methods.

So, analogue filter design methods of IIR filter design, what do you do? First, we designed that analogue filter to meet the given specifications. So, whatever the filter specification is given, I design that analogue filter for that specification. What are the design methods? An analog filter means H(s); an H(s) means a Laplace representation of the transfer function of the analogue filter.

So, we design H(s) using analogue filter design methods, and then we will convert H(s) to H(z) using any one of these two methods, either bilinear transformation or impulse invariants methods.

So, the analogue filter design method can be summarized like that. First, you design the desired analogue filter transfer function. Or I can say from the given specification, derive the analogue filter transfer function H(s). Once I get the H(s), I can convert H(s) to H(z) using any one of the methods either bilinear transformation or impulse invariants methods.

Once we get that H z, I can implement it using discrete structure one and discrete structure two methods. I can use any one of those and implement them on the computer. So, analogue filter design methods of IIR filter design summarise basic 3 points; one is to design the desired analogue filter and derive the transfer function H(s). Once you get the steps to convert H(s) to H(z), step 3, once you get the H(z), implement H(z) using a computer using structure 1 or structure 2 methods.

So, let us first talk about how to design analogue filters. In this lecture, I will not cover details on that, but I will summarize what is there and how this can be used for design and given analogue filters for a given specification and an analogue filter.

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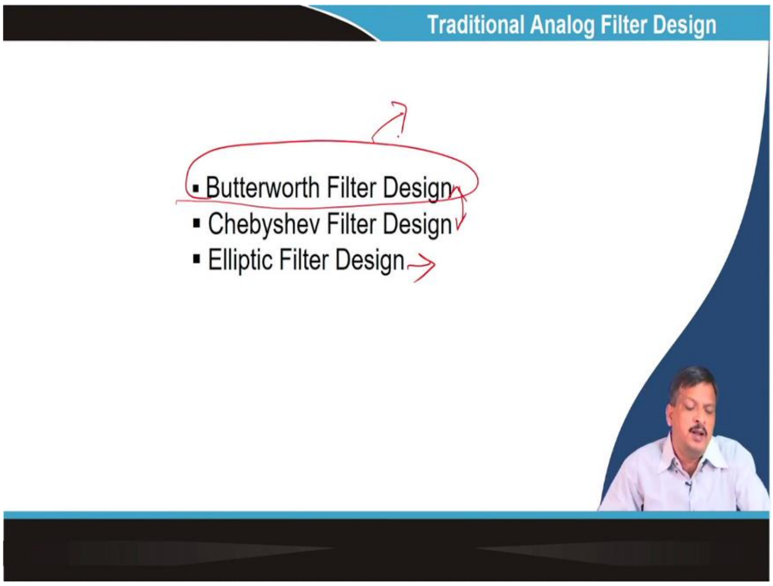

So, there are 3 methods are there Butterworth, Chebyshev, and elliptical filter design. I will cover these two methods. You can see this method in the book because analogue filter design methods are taught in many classes. I do not want to shorten it because unless this is done again, it will require another week of lectures. So, here also, I will not cover the details of the Butterworth filter design method; I will cover only the point that is required to derive the transfer function of the filter from a given specification.

Then, we tested how to design the IIR filter. So, let us talk about the Butterworth filter design method.

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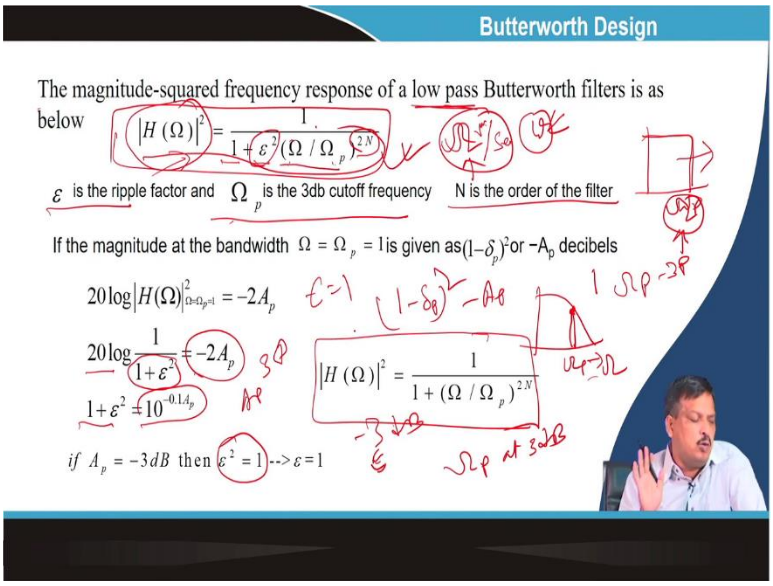

So, what is that now? Let us thus use the magnitude^2 frequency response for the Butterworth low pass filter. So, we design either the low pass filter and then shift it for the desired kind of high pass filter or band pass filter. So, let us design a low pass filter using Butterworth design methods, where Ωp is the pass band edge frequency, and Ωp is the pass band edge frequency. The magnitude response using the Butterworth design will be given like this: $H(\Omega)^2$; this is a magnitude response.

So, magnitude response means the frequency I have taken that inverse transform H N is converted to the frequency domain, and I have taken that magnitude of that frequency domain. So, there is a phase and magnitude. So, I plot the magnitude part. So, if I take the magnitude response, it will come to 1 by 1 plus ε^2 into Ω by Ωp; this Ω is the analogue frequency. So, this is radian per second, and this is not radian per sample; that is why I write this Ω, not write this Ω.

$$|H(\Omega)|^2 = \frac{1}{1+\epsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}}$$

This Ω is radian per sample. So, this is the difference. This is digital, and this is analogue. So, this is discrete frequency normalized discrete frequency, and this is analogue frequency. So, if the Ωp is my pass band age frequency in analogue pass band edge frequency, then that magnitude response is 1 by 1 plus ε^2 Ω by Ωp^{2N} ok. Where ε is the ripple factor, Ωp is the 3dB cut-off frequency or passband edge frequency, and N is the order of the filter.

So, if the order of the filter is N, the pass band ripple is ε, and Ωp is the pass band edge frequency, then the magnitude response^2 magnitude response is this one ok. Now, at Ω equal to Ωp. So, if you see the filter looks like this, let us say this is my 3 dB down, so this is my Ωp. So, at this axis, it is Ω. So, at Ω equal to Ωp then, I can say this 1 Ω by Ωp is equal to 1, and that is nothing but a 1 minus δp whole^2, or I can say minus Ap decibel ok; δp is the ripple in the pass band.

So, if I convert into the dB 20, log this one. So, Ω by Ωp becomes 1. So, this is nothing but a 1 by ε^2. $20 \log |H(\Omega)|^2_{\Omega=\Omega_p} = -2A_p$. Now, I know 1 by ε^2 is equal to 10 to the power 0.1 A p. Now, if the Ap is 3 dB down. So, Ap is 3 dB minus 3 dB. So, if Ap is 3 dB down, then ε^2 is equal to 1. So, ε is equal to 1.

So, the transfer function becomes this one. So, is Ωp is defined at 3 dB down. Now, let us say Ωp is not defined 3 dB down 4 dB down, then I can get an ε the passband ripple is it ok. So, I know that the Butterworth transfer function (Refer Time: 08:23) filter design is the magnitude response of the Butterworth filter.

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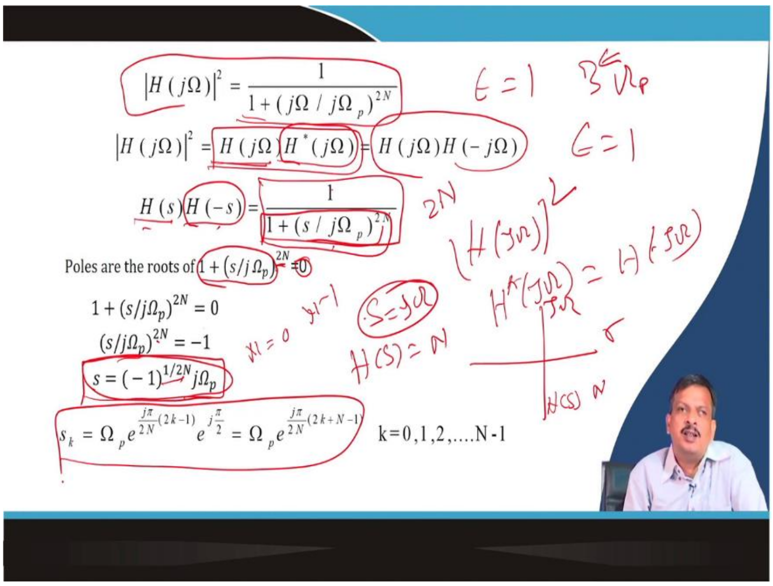

So, now, I know. Let us say that Ω is replaced by jΩ. So, it is 1 by 1 plus the ε is equal to 1; I said my filter specification is designed at 3 dB down Ωp is designed at 3 dB down. So, ε is equal to 1. So, I put that ε is equal to 1, so this is my Ω H omega^2. So, what is $H(j\Omega)^2$? It is nothing but a, so it is a complex function. So, the^2 is nothing but the product of complex conjugate.

A magnitude^2 is nothing but a product of complex conjugate. So, if I do the complex conjugate product, then I can write down H(jΩ) into H complex conjugate of jΩ. Now, you know the complex conjugate of the jΩ from the frequency transform property is nothing but an H(-jΩ). So, it is nothing but a -jΩ.

Now, what is Laplace domain? What is a Laplace transform? Laplace transform is nothing but a complex plane called a plane. So, s plane this is jΩ, and this is some real part, so s if I replace s by jΩ.

So, jΩ is replaced by s; that means the real part of the Laplace domain is 0. So, in that case, I can say it is nothing but a H s into H of minus s is equal to 1 by s by -jΩp^{2N}. Now if I see that this is my transfer function of 2 H s. So, the order of the is 2 N, then I know the order of H s is N, H s into H s is order of 2 N; that means order of the filter transfer function H s is nothing but a N ok.

Now, how do I get the pole of this H s? So, the pole is nothing but a solution to this equation. So, I have to solve this equation, so 1 plus s plus -jΩp^{2N} is equal to 0. So, if I want to solve it, jΩs by -jΩp^{2N-1}. So, s is equal to minus 1 to the power 1 by 2 N into -jΩp.

So, those are the pole position, the solution, and the pole position. Now, H(s) is an N number of the poles because the order of the filter is N. So, 1 minus 1 to the power 1 by 2 N this small N varies from 0 to N minus 1.

So, the order of the pole is N, ok. Now, if I want to write down the generalized form of this part, what is the generalized form of this part? I am not writing this one because this one I will take a slide here, and then I will show you how you can draw the generalized form.

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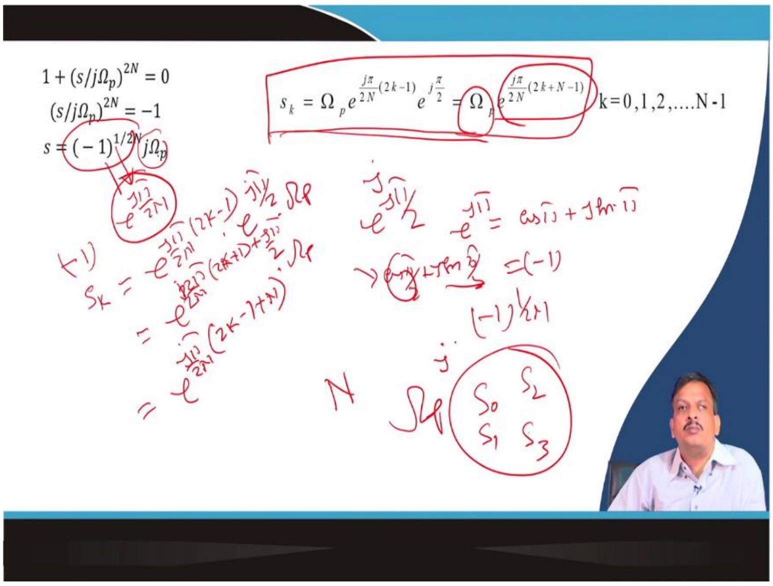

So, I said this is the generalized form. How do I derive it? Unless you know that you may not be, you may not understand how this kind of expression will come about. So, what I said minus 1, so minus 1, can I implement using e to power something? So, if I say $e^{j\pi}$. What is $e^{j\pi}$? $e^{j\pi}$ is nothing but a cos π + jsin π, so which is nothing but a minus 1.

So, minus 1, can we say $e^{j\pi}$? So, I can say $e^{j\pi/2N}$, that is this one; this portion can be synthesized like this one. Now, since I have to for N is varying, N is the order of the pole, so it is if the Nth pole is there. So, N varies, so where are my other poles? So, for that purpose, I said let us s k, k equal to 0 1 2. So, if I say that, then I can say it is nothing but a jπ by 2 N into 2 k minus 1.

If you see k equal to, see all the time it will be minus 1 to the power 1 by 2 N, k equal to 0, same thing k equal to 1, k equal to 2 ok. Now, what is -jΩp? So, Ωp is constant ok the cut-off frequency; what is j? How can I synthesize j? So, $e^{j\pi/2}$, what is the value? Cos π by 2 plus j sin π by 2. So, if I say this is 0, and this is 1, it is nothing but a j. So, I can say $e^{j\pi/2\Omega p}$ ok.

So, now, if I add this one, so $e^{(j2\pi/2N(2k+1))+j\pi/2\Omega p}$. Now, if I said 2 N would be there if I say that $e^{j\pi/2N}$, I have taken out. So, it is nothing but a 2k minus 1, and this plus is nothing but an N. So, it is nothing but an in the form of Ωp into $e^{j\pi/2N(2k+N-1)}$. So, those are the pole positions I can determine if I know N if I know N, the order of the filter I know known, and Ωp I know. Then I can calculate s 0 k equal to 0, s 1, s 2, s 3, and I can calculate all the poles, if any.

So, when I design the Butterworth filter from the given specification, if the Ωp is defined at minus 3 dB, then I know ε is equal to 1; that means the passband ripple is equal to 1, and once the passband ripple is equal to 1, I know how to define the pole position from this equation. This is the equation to define the pole position.

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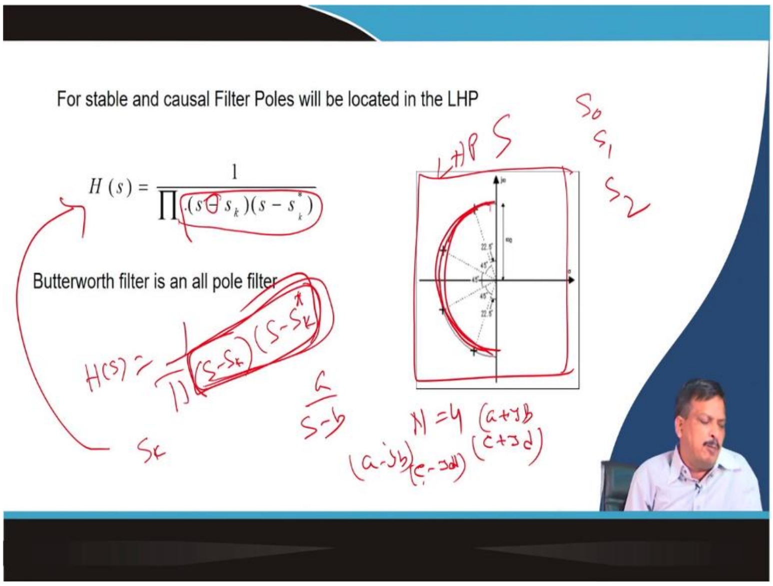

Then what is the relationship? How do I design the transfer functions? So, for a given order of the filter, I know the pole position. Now, pole position sk I know. So, what I know, I know s0, I know s1, I know s2. Now, if the filter wants to be a stable filter and causal filter, then all the poles must be located in the left-hand plane of the s plane, which is the s plane.

So, all the poles must be located on this side and be stable. So, I can say this side means I can say it is nothing but an H(s), which is nothing but a 1 by-product of s minus sk into s minus sk star.

Why did I give it a star? If the filter is real, in the poles, the filter transfer function is a real transfer function, real transfer functions, then this pole will occur in the complex conjugate. That means, let us say, a by s minus b. So, is it real, s? In that case, that pole will occur in complex conjugate form; that is why I said it is a product of a complex conjugate pole.

So, when I say a real filter with order N equal to 4, order equal to 4, that means there will be two complex conjugate pairs will be there. So, if the first one is a + jb and the second one is c plus j d, then the third one will be a minus j b, and the other one will be c minus j d. All will occur in a complex conjugate manner. So, a product of this will be the transfer function of H s. So, once I know sk, I can derive the transfer function H s, and it will always be minus because the poles are located on the left-hand side of the s plane.

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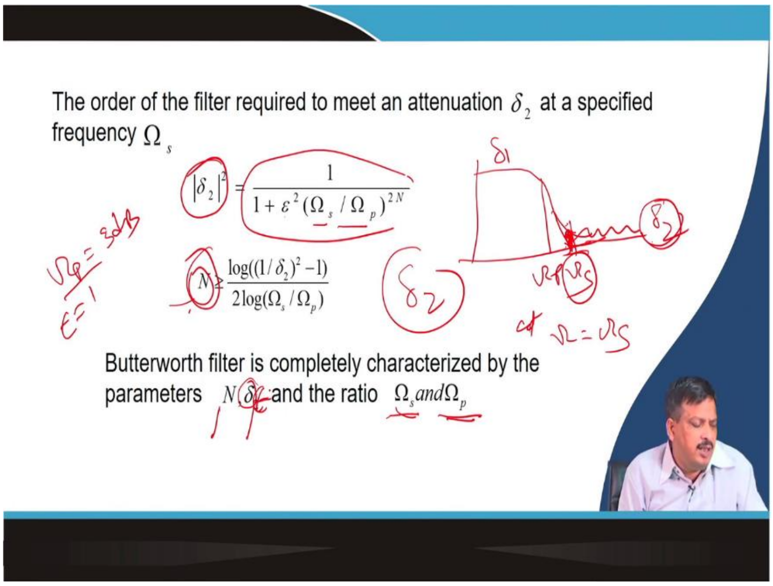

Now, I come. The order of the filter is required. So, how do I define the order of the filter? How do I define the order of the filter from the given specification? So, what is given in the specification? It gives passband edge frequency Ωp, given stopband edge frequency Ωs, given stop band ripple.

So, the stopband ripple is δ2, passband ripple is δ1. So, stopband ripple is given, and stopband edge frequency is given. So, what is the square magnitude at the stop band edge frequency? Because H(Ω) I know $H(\Omega)^2$, $H(\Omega)^2$ $H(\Omega)^2$ is nothing but a 1 by 1 plus ε Ω divided by Ωp 2^N.

So, at Ω equal to Ωs. So, at Ω is equal to Ωs; that means, at this point, stopband edge frequency. So, the amplitude is δ2. So, I can say

$$|\delta_2| = \frac{1}{1+\epsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}}$$

Now, if I say that Ωp is defined at 3 dB down, then I can say ε is equal to 1. So, I take ε to be equal to 1 unless I have to calculate the ε value.

From there, I can calculate the N. So, if I know δ2, I can calculate the N. So, I can summary I can say that N δ1 ε and qs and qp characterize the Butterworth filter design requirement. So, if I will be able to determine the Butterworth filter transfer function, if I know Ωs, if I know Ωp if I know δ and if I know ε. N I can derived, or if the N is given, then I can say what should be the passband ripple and vice versa.

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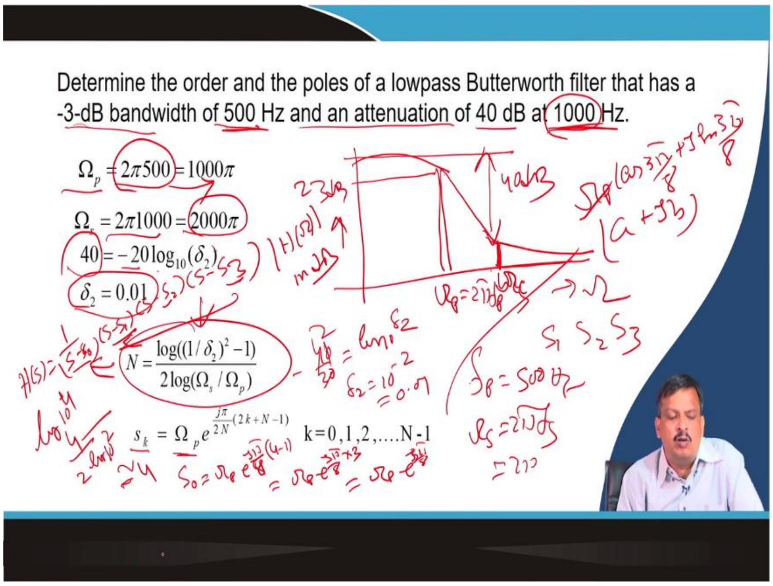

So, let us get it. I think there is an example. Let us take an example. I said to determine the order and pole of a low-pass Butterworth filter that has a minus 3 dB bandwidth at 500 hertz and an attenuation of 40 dB at 1 kilohertz. So, what is it? First, you draw the picture. So, I can say there is a pass band at 3 dB; Butterworth, let us say this kind of thing, at 3 dB down, let us say this is 3 dB down.

So, this is Ωp is equal to 2π into f p. So, f p is equal to 500 hertz. So, I can say Ωp is equal to 2π into f p is equal to 1000 π so, if this axis is Ω.

Now this is, let us say, stopband edge frequency, this attenuation so this is the attenuation that attenuation is 40 dB, this axis is mod of H(Ω) whole^2 in dB ok. So, this is 40 dB down; this is nothing but a Ωs. So, what is Ωs? Ωs again, it is nothing but a 1 π f s. What is f s? At 1 kilohertz, it is nothing but a 2π into 1 kilohertz, it is 2000 π. How much attenuation?

40 dB. So, what is δ2? So, 40 is equal to minus 20 log 2 δ2. So, I can say δ2 is equal to 0.01 because of this. So, I can say 40 is equal to 40 by 20 minus is equal to log 10 δ2.

So, I can say this is nothing but a 2. So, it is nothing but a δ2 equal to 10^{-2}, which is nothing but a 0.01. So, once I know the δ2, I can put δ2 in this equation. So, it is nothing but a log 1 by δ2 $whole^2$. So, 1 by δ2 means 10 to the power 2, and the $whole^2$ means 10^4. So, 10^4, minus 1, I can neglect. So, 10^4 means 4 divided by 2 into log Ωs 2000 divided by 1000 log 10 2, 2 into log 10 2. So, 4 by 2 into log 10, how much will come? You can determine and let us say this is equivalent to 4.

So, m is equal to 4. So, once I said 3, whatever it is coming, N is equal to 4. So, the order of the filter is 4. So, I know sk is equal to Ω. So, I can say s 0 is equal to Ωp $e^{j\pi/2}$, jπ by 4 4 into 2 means π by 8 into k equal to 0. So, it is nothing but a 4 minus 1. So, it is nothing but a Ωp $e^{j\pi/8*3}$. So, it is nothing but a Ωp $e^{j\pi/3\pi/8}$, which is nothing but a Ωp into cos 3 π by 8 plus j sin 3 π by 8.

So, Ωp, you know you can get that complex pole, let us say a + jb. So, I will get a + jb, then put k equal to 1, k equal to 2 and k equal to 3; you calculate s 0 s 1 s 2 s 3. Once you calculate it, what is the transfer function? What is the transfer function H s? H s is nothing but a; I can write down here that H s is nothing but a 1 by s minus s 0 into s minus s 1 into s minus s 2 into s minus s 3.

You can find out that s 0 s 1 you calculate, and again s 2 and s 3 are the complex conjugate of s 1 and s 0 you can find out also. So, I have designed ok. So, this is the Butterworth filter design. Let us take another example.

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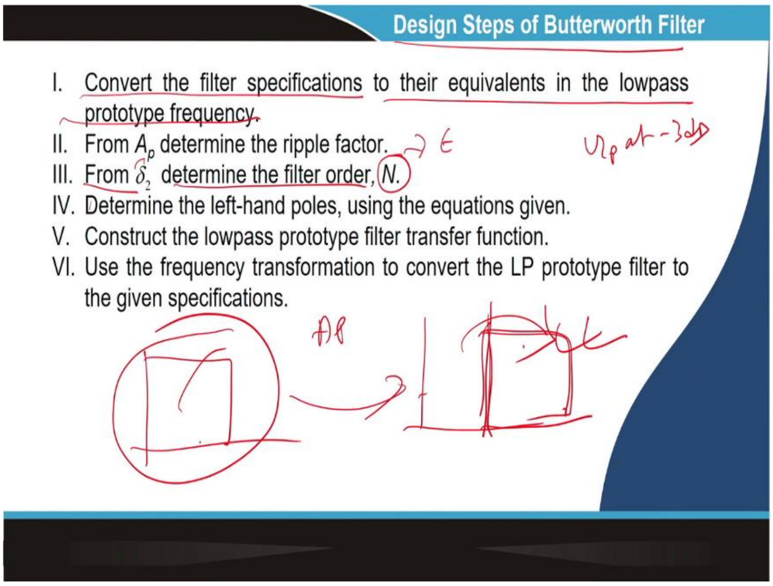

Example: first give, then I come to the step.

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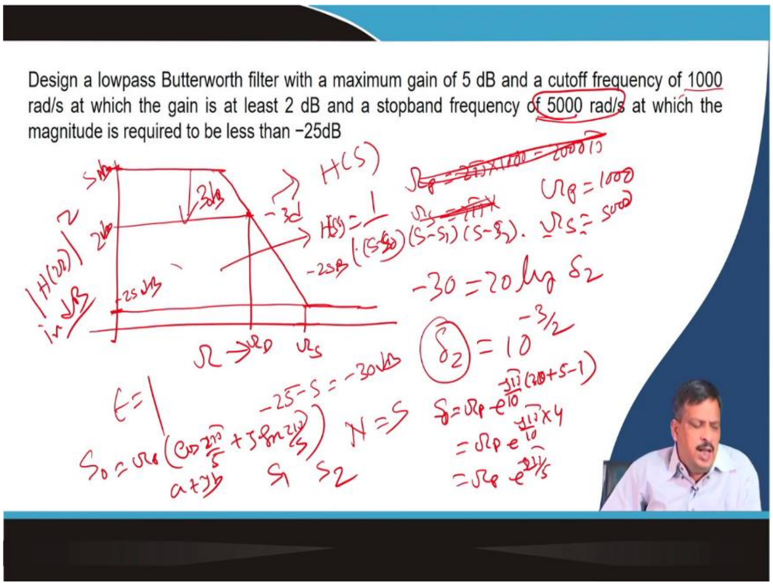

So, what is the example? Design a low pass Butterworth filter with a maximum gain of 5 dB and a cut-off frequency of 1000 radion at which the gain is 2 dB. So, let us first draw the frequency response of the filter. This axis is the **Ω**, and this axis is the mod of H(**Ω**) whole^2 in dB ok. So, I what I said? I said this is 5 dB, gain of 5 dB, then it is decreased to 2 dB. So, it is here it is 2 dB, so this is 5 dB. So, how much is decreased? This decreases by 3 dB.

So, here it is Ωp, and then again it is decreased to minus 25 dB, minus 25 dB. Here, it is minus this axis minus 25 dB, which is Ωs. I have to design this filter using Butterworth. So, what do I have to find out? I have to find out H(s). So, what is Ωp? Ωp is equal to 2π into fp. So, what is the fp? fp is 1 kilohertz. So, it is into 1 kilohertz. So, it is nothing but a 2000 π. What is Ωs? Ωs 2π into f s; f s is equal to 5 5000 radians, 5000 so it is given in radians per second. So I am not required to multiply.

So, it is already given Ωp is equal to 1000, and Ωs is equal to 5000; this is given in radians per second; if it is given in hertz, then I have to multiply 2π ok. So, Ωp is given, Ωs is given. Now, if you see, since it is minus 3 dB down, I can say ε is equal to 1, ok? Now, how much is the total dB down? Minus; so here it is, therein 5 dB, and it comes to minus 25 dB. So, total attenuation is minus 25, and this is also 5. So, minus 5 is equal to minus 30 dB in the total attenuation.

So, I can say minus 30 is equal to 20 log $\delta 2$. So, $\delta 2$ is equal to $10^{-3/2}$. So, you can calculate $\delta 2$. Once I know the $\delta 2$, I can calculate N; N is equal to, you know, the formula for calculating N;

$$N = \frac{\log\left(\frac{1}{\delta_2^2} - 1\right)}{2\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

So, I can calculate N; let us say N comes around 5, and then what is the transfer function? So, the transfer function is nothing but a s k; I have to find out s k.

So, s 0 is equal to Ωp into $e^{j\pi/2N}$. So, if it is N equal to 5, it is 10 into 2 k, 2 k equal to 0, 2 into 0 plus N into 5 minus 1. So, it is nothing but a Ωp $e^{j\pi/(10*4)}$. So, it is nothing but a Ωp e; 2π by 5. So, which is nothing but a s 0 is equal to Ωp into cos 2π by 5 plus j sin 2π by 5. So, I get a + jb format. Similarly, I will calculate s 1, I will calculate s 2, and then I said this H s; transfer function is H s is equal to 1 by s minus s 0 into s minus s 1 into s minus s 2 dot dot dot how many poles are there as per the order of the filter.

So, I get that H s ok. So, once I get H s using the Butterworth filter design, I can convert it to H z, and I can design the IIR filter ok. Now, the designed step for the Butterworth filter is summarized. So, convert the filter specification to their equivalent in the low pass prototype. So, what I said I have, suppose I have to design a band pass filter. Bandpass filters also can be designed using a low-pass filter.

So, this is the band pass filter; I can say this is a low pass filter here, so I see. So, if I design this one as a low pass filter and then shift it to a certain frequency, I get the bandpass filter, and I will detail it. I will discuss at the end of this week's lectures how this can be done,?

So, first, design the prototype and determine the equivalent prototype of the low pass filter. So now, from Ap attenuation in the pass band from there, if this is the find out ε most of the cases ε will be 1, if I if the Ωp is defined at minus 3 dB down, then you determine the δ from δ to determine the filter order. So, if the stop band attenuation is given, you calculate the filter order. So, determine the left-hand poles using the equation and construct the low pass filter prototype and transfer function ok.

So, the Laplace domain poles are always on the left-hand side, not the right-hand side. If it is right-hand side, filters become unstable. So, all the poles are positioned on the left-hand sides, which is why it is s minus s k, s s minus sk, and if it is the transfer function is real, I can say the poles will occur in a complex conjugate manner, ok? So, this is the summary of the design of Butterworth filters and analogue filters. Then, in the next class, I will talk about the Chebyshev filter design analogue filter design.

Thank you.