Signal Processing Techniques and Its Applications
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Lecture - 37
Linear symmetric and anti-symmetric filter

So, in the last class, we discussed the Linear phase condition for the FIR filter.

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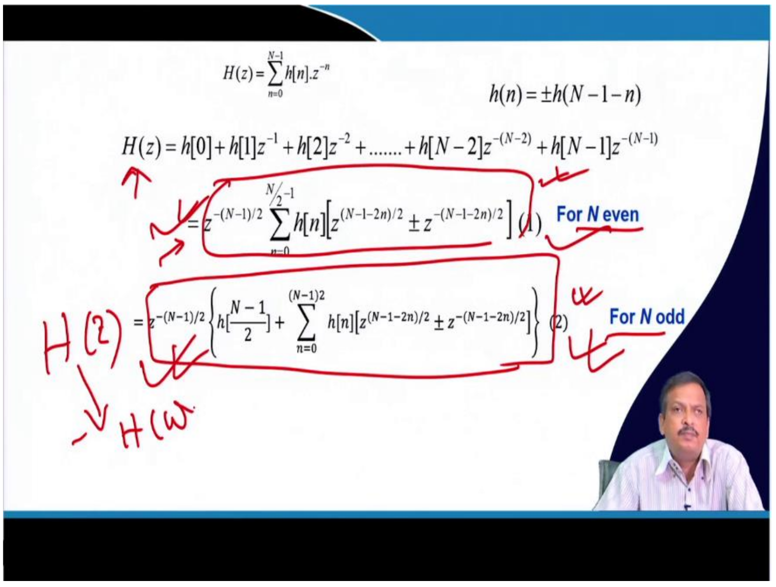

So, in this slide, we are showing what that FIR filter equation should be if the N is even and N is the order of the filter. So, the order of the filter is even, or the order of the filter is odd. So, in both cases, we have shown that H(z) can be written in this form if N is even and in this form if N is odd. You can verify this using, let us say, N equal to 5 in the case of odd or N equal to 6 in the case of even. You can see whether this satisfies your H(z) or not.

So, this is the form of N even, and N odd N is the order of the filter. So, a number of coefficients exist in the filter.

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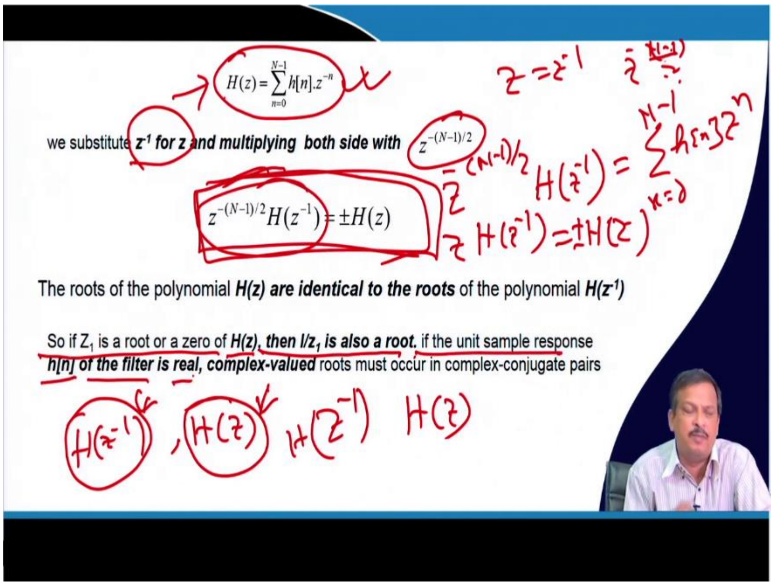

Now, if you see, let us suppose this is your H(z) and H(z) can be written as H(z) can be written as this one H(z) can be written sorry H(z) can be written as this one for N even and this one for N odd. Now let us say if I consider that I multiply this equation on both sides, replace z with z^{-1}, and multiply $z^{-(N-1)/2}$ on both sides.

So, what will happen? This side will be $z^{-(N-1)/2}$ H(z) minus. I replaced z by z minus, then

$$H(z^{-1}) = \sum_{n=0}^{N-1} h[n] z^{n}$$

Now, if I say that if z^{-N}, is this one? So, let us say minus N, let us say for N even.

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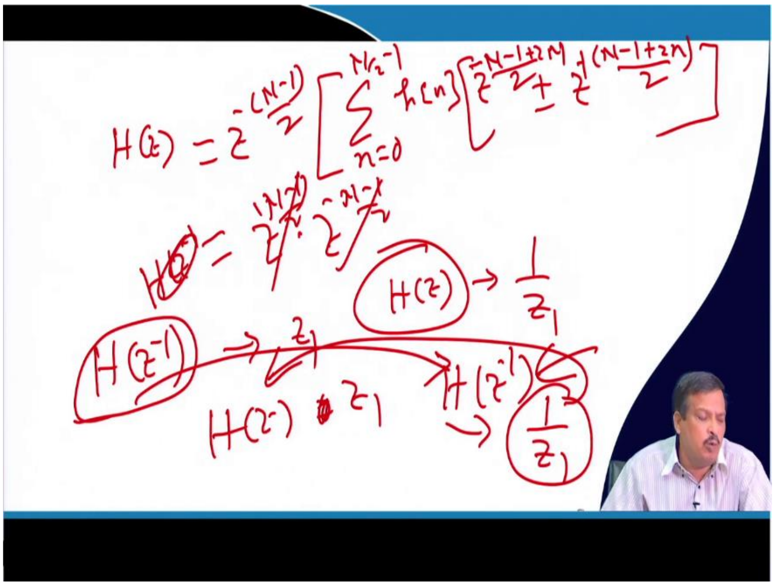

So, for N

$$H(z) = z^{-\frac{N-1}{2}} \left[\sum_{n=0}^{\frac{N}{2}-1} h[n] z^{\frac{(N-1)+2n}{2}} - z^{-\frac{(N-1)+2n}{2}}\right]$$

So, this is your, let us say, H(z). So, what I said, I replaced z by z^{-1} and then multiplied $z^{-(N-1)/2}$. So, I can say once I replace z by z^{-1}. So, what will happen? So, if I replace it, then it will be plus N minus 1 divided by 2 into z^{-N} minus 1 divided by 2. This thing will cancel each other, ok or not.

And once I replace it z^{-1}, this will be negative, and this becomes positive, ok or not? So, I can say if that is z^{-1}. So, I can say if I multiply both sides with $z^{(N-1)/2}$ and replace z with z^{-1}, I will get the equation. $z^{(N-1)/2}$ $H(z)^{-1}$ will be equal to plus minus H(z) ok.

So, if it is plus minus H(z), you see interestingly if you watch this equation. So, I can say the H root to the power minus z^{-1} is equal to plus minus H(z) with a multiply factor z n minus 1 minus 2. So, what can we observe? We can observe that the pole is a transfer function, and this is also a transfer function, and both the transfer functions are equal; that means, the root of this transfer function is also the root of this transfer function. 0 of this transfer function is also 0 of this transfer function.

So, I can say if z^{-1} transfer function and H(z) transform function, both roots are equal, and both 0 are equal; let us consider that I have an H(z), $H(z)^{-1}$, I have a root z1. Now, what will be the root for H(z)? In that case, H(z) will be 1 by z1, or if I say H(z) has a root z1, then I can say $H(z)^{-1}$ has a root at 1 by z1. Now, if I say 1 by z1 is the root of z^{-1}, then this is also the root of H(z) because I said the root of this one and the root of this one are the same.

So, if z1 is the root of H(z), then z1 is also the root of $H(z)^{-1}$. Similarly, if 1 by z1 is the root of z^{-1} $H(z)^{-1}$, then it is also the root of H(z). Is it clear? So, if I want to draw that, let us take a slide here.

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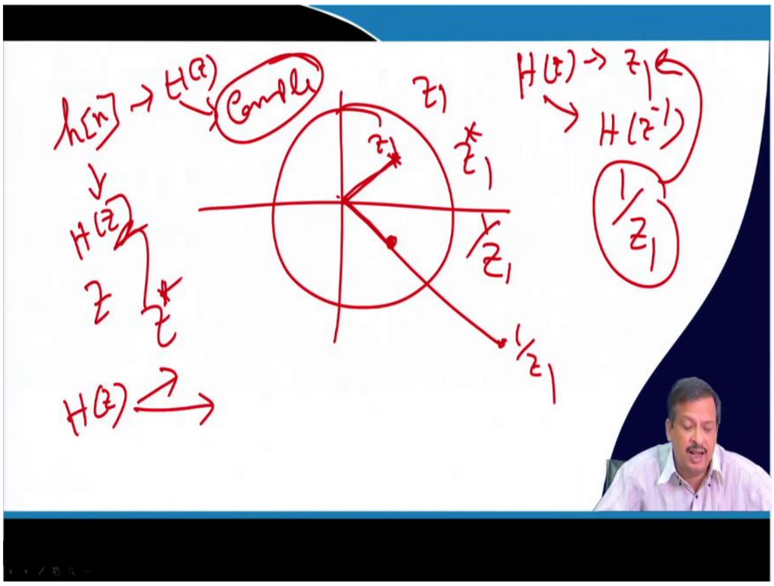

So, let us say this is my z plane, and this is the unit circle. So, let us say I have a root, which is z1 for H(z). H(z) has a root z1. See, say, if the H(z) is equal to $H(z)^{-1}$, then I can say 1 by z1 is also the root of H(z).

So, if z1 is a root, then I can say 1 by z1 is also a root or not. So, 1 by z1 is also a root 1 by z1 is outside the unit circle. So, what can I conclude? If z1 is a root of H(z), then 1 by z1 is also a root, and the unit impulse response of H n of the filter is real. Now, I consider if I have an h[n], which is a real filter; let us say I have an h[n], which is a real filter. All filters h[n] are real; h[n] is a real filter, so I can say h[n] exists.

So, I can say H(z) is a real filter. If the roots are complex, then it will exist in a complex conjugate manner because, let us say, z is a complex number. If z is a root of H(z), then I can say 1. If the complex conjugate of z is also a root of H(z), z is a complex number.

So, if z is a root, then complex conjugate is also the root of H(z). So, if my H(z) is real, then I can say it exists in the root, which is in complex conjugate nature. So, if I say I have a z1 as my root, there will be another z1, which is z1 by z1 is also a root. So, now, if the I have a complex root z1 and z1 is a complex number, then the complex conjugate is also a root. For example, let us say I have given a real example.

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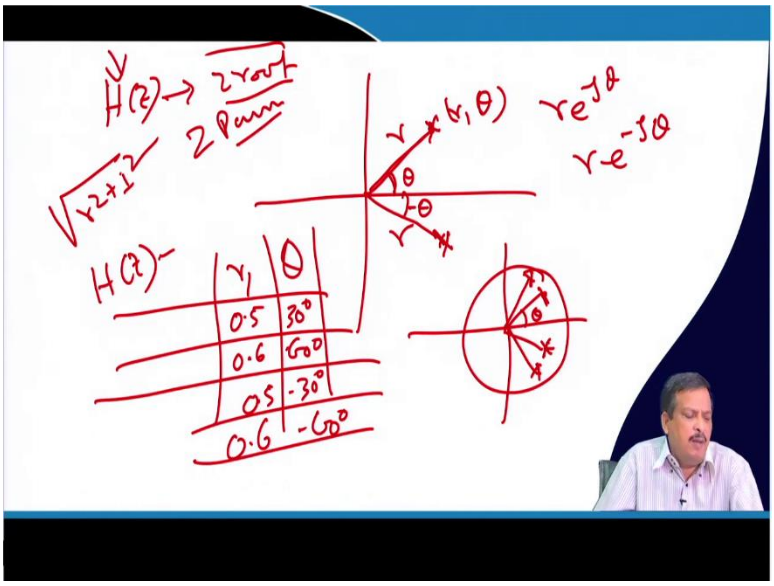

The human vocal tract has a transfer function H(z) ok. The human vocal tract has a transfer function, H(z). Now, the human vocal tract has a 2 roots. Let us say 2 roots are there. So, since the human vocal track is a real transfer function, then if I say 1 root. So, I can say that 1 root is, let us say, this one: the value is r1 and θ. So, you know that the r1 is the distance, and θ is the angle.

So, if 1 root is $re^{j\theta}$, then there will be a complex conjugate root that is what which is nothing but a $re^{-j\theta}$. So, there will be another root, the distance will be the same, and the amplitude will be the same, but it is the complex conjugate because amplitude is nothing but a real square plus an imaginary square. So, the amplitude will be the same, but there will be a complex conjugate.

So, if I say I have 2 pairs of complex conjugate roots if H(z) is real. So, if I suppose if I give you a problem, the problem is that I am a transfer, I have an H(z) is a real transfer function, and I said H(z) has a root in the following tables, where the table said that r1 and θ value. So, the r1 value is 0.5 θ is 30 degrees r2 value r1 value is 0.6 θ is, let us say, 60 degrees. So, if H(z) is real and if r θ is a complex root.

So, what are the other 2 roots of this transfer function? So, the other 2 roots are nothing but the same 0.5 minus 30 degrees and 0.6 minus 60 degrees. So, if I want to draw it, it will be very simple that this is positive r, and there will be a complex conjugate here. If there is another r, let us say 60 degrees here, then complex conjugate. Here, it is clear if it is a real pole, but if it is a real transfer function, if it is imaginary, then I know if z1 is a root, then 1 by z1 is also a root of the transfer function because I have proved it ok.

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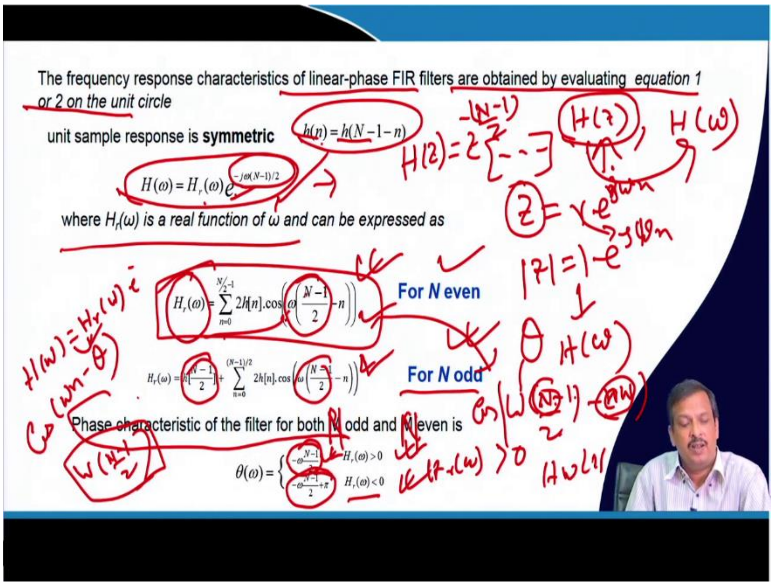

Now, I come to another condition that suppose the frequency response characteristics of the linear FIR filter are obtained by evaluating equations 1 and 2 in the unit circle. So, how do I get the frequency response? So, this is equation 1, and this is equation 2. So, H(z) is this one. So, I know H(z) I 1 H(ω). How do I get it?

So, H(z) I know from equations 1 and 2 that I know H(z). I have to know H(ω), which is the frequency characteristic of the FIR filter linear phase FIR filter because the transfer function H(z) support the linear phase condition, which is why we derived it.

Now, if H(z) is a linear phase FIR filter, then how do I get this H(ω)? Frequency characteristics of that FIR filter. Now, what is H(z)? What is the z domain? So, z can be written as $re^{j\omega}n$. So, z is a complex number; it has a value magnitude and it has a frequency ω. So, now, if I say the mod of z is equal to 1, that means a value of r is equal to 1.

So, magnitude is a unit, and I can say it is generated for every $j\omega n$. So, when I say that, then it becomes $H(\omega)$. So, z is replaced by a magnitude is 1, only ω will be there, $j\omega$ will be there. So, if I do that, then I can say $H(\omega)$ is nothing, but a $Hr(\omega)$ into $e^{-j\omega(N-1)/2}$ because there is another term if you see H(z) has a 2 term H(z) is a term $z^{-N-1/2}$ into some term is it clear or not? Let us say not clear.

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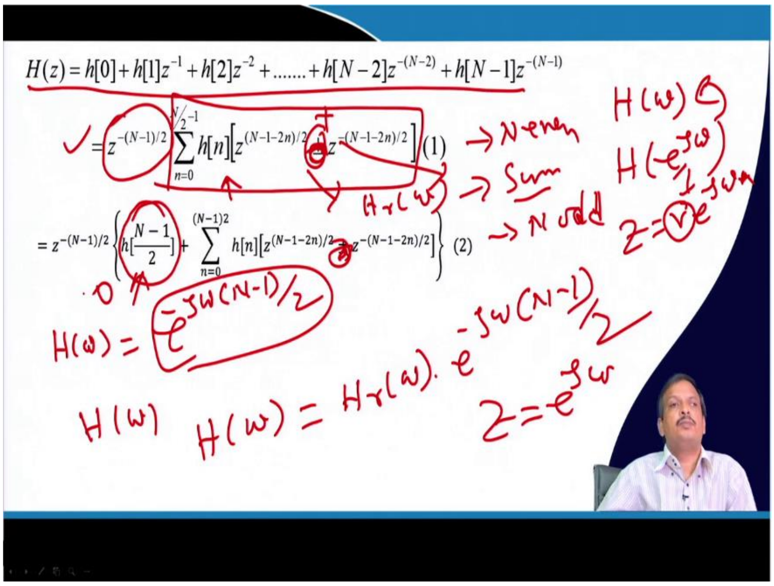

So, what am I saying? This is my H(z), and I have derived these two equations: this is for N even, and this is for N odd. Now, what did I say? I want to find out $H(\omega)$. So, once I know H(z), I know if the H(z) is replaced by H $e^{j\omega}$, this is nothing but $H(\omega)$. So, what is the meaning? This means that z is equal to $re^{j\omega}n$. So, in that case, I can say n is very different from ω. So, I can say if r is equal to 1, then it is nothing but a frequency response. This becomes the frequency response.

So, if r1, then I can say $H(\omega)$ is equal to for this term $r = e^{-j\omega\frac{(N-1)}{2}}$. Just replace by ω let us say instead of N ω N write $j\omega$ replace by ω. Then, this is one term, and the next term is ω of this function. I have to evaluate for e to the power, putting $e^{j\omega}$ in this function. So, I can say $H(\omega)$. Let us see this is called $Hr(\omega)$. The ω representation of this portion is $Hr(\omega)$.

So, I can say that H(ω) for N equal to even is nothing, but a Hr(ω) multiplied by $e^{-j\omega(N-1)/2}$, which is this one ok. Now, if I evaluate this one, z is equal to $e^{j\omega}$. If I evaluate this one, I will get this expression. So, expression of Hr(ω), I will get this one expression of H ω for N odd, I get this one.

So, what is my total H(ω)? My total H(ω) is nothing but a Hr(ω) multiplied by e to the power of this one. For N even and for N odd. Now, if you see both cases, the phase response of Hr(ω) is nothing but a ω into N minus 1 by 2. So, cos ω into N minus 1 by 2 minus n ω this is nothing but this one. If I come to here ω. So, N is a fixed number. This small n varies from 0 to n by 2 minus 1, but capital N is my fixed number.

So, I can say that this is nothing but a θ and this is my n ω. So, I can write down this one very easily cos of ω into n minus θ minus cos θ is equal to cos θ ok. So, this θ is my phase. So, what is the phase portion? It is nothing but a ω into N minus 1 divided by 2.

So, this is my θ ok where this is not M, and this is N; for both N is odd and both N is even. So, if the N is odd, the same term is N is even, and the same term is there. So, whether N is odd or N is even, you can say this is nothing, but a phase is nothing, but a ω N minus 1 by 2.

So, I can say it is a phase that is a linear equation. So, θ ω in both cases is the same, but θ ω is this one when Hr(ω) is greater than 0. Now, if the Hr(ω) is negative, then what is the meaning? This means that I have to add another π phase. Do you understand or not?

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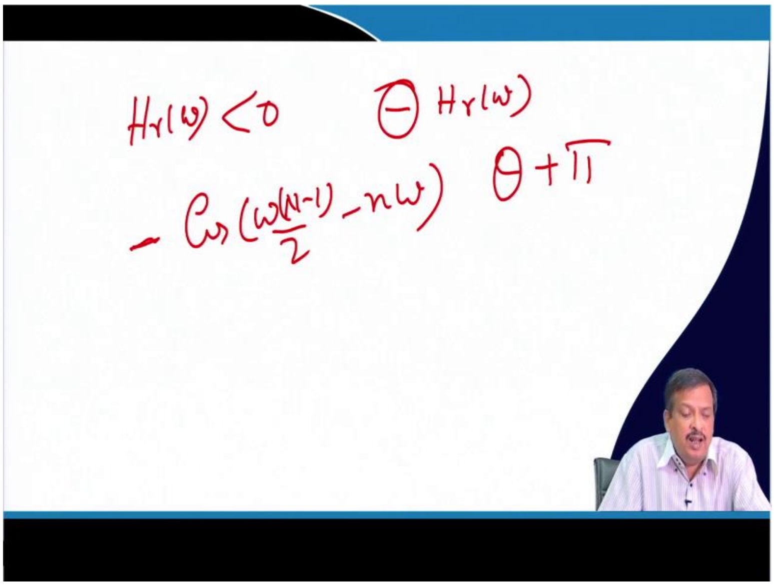

If you say Hr(ω), Hr(ω) is less than 0, that means I can say it is minus Hr(ω). What do you mean by minus? So, I have a $\cos\left(\omega\frac{N-1}{2} - n\omega\right)$ as my function. Now, what do I say minus cos? That means the θ is added by phase is added by π.

So, that is why if Hr(ω) is negative, this θ will be phase shift will be more π plus ω into N minus 1 by 2. If the Hr(ω) is positive, then I can say it is nothing but a ω N minus 1 by 2. So, in both cases, N even or N odd, both the case phase characteristics of that filter are like this, which is nothing but a linear phase filter. It is a multiplying θ multiplying by the constant. It is not square. There is nothing square term is there.

So, since I designed the FIR filter supporting the linear phase, I can prove that the linear phase is maintained here, and I can find out the phase shift of the filter. Is it clear? So, when we implement it, I will show you the physical implications of this phase shift. Why do I call this phase shift is there? You can see that the whole signal will be delayed by N minus 1 by 2 term whole signal. When you get the output first N minus 1 by 2 term, you have to leave if it is a symmetric filter or if it is, you can say that it is linear phase filter ok.

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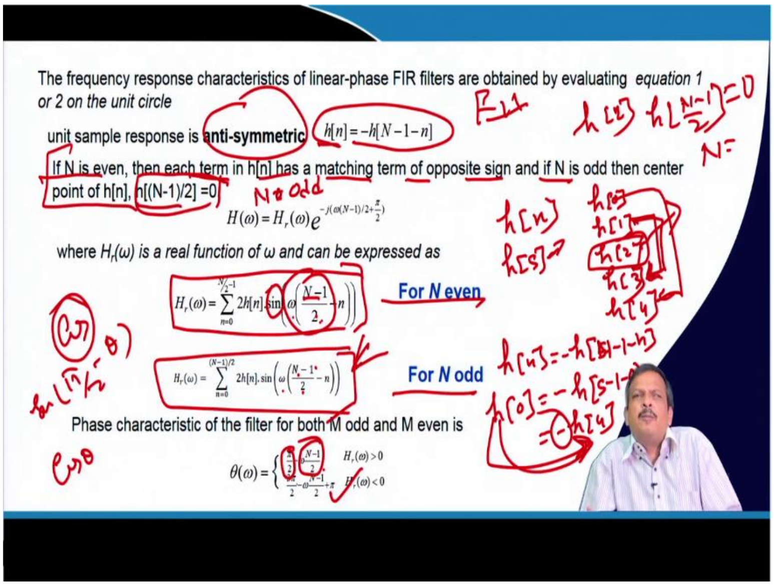

Now, I come to another topic, which is anti-symmetric. So, in the case of symmetric, I get this 1 because I consider h n equal to. So, I consider only the plus term here if you see there is a plus minus so when I consider only the plus term, that is nothing but a symmetric condition.

Now, when it is an anti-symmetric condition, that means, h[n] is equal to minus this one. So, if I consider the negative term only, only the negative term if I consider then what will happen? Now, in that case, if N is even, then each term in h n has a matching term of the opposite sign of N. Do you understand or not? Suppose I have an h n.

So, n is odd. Let us say I have an h 5. So, what is nothing? What is there? h 0, h 1, h 2, h 3, h 4. Now I said h[n] is equal to minus h[n] minus n minus 1 minus n. So, now, if I say h(0) small n equal to 0 minus h(5) minus 1 minus 0. So, I can say it is nothing but a minus of h 4. So, I can say h 0 matches h 4, and h 1 matches h 3, but what is the matching of h 2? N is odd.

So, h 2 does not have any matching. If it is an even filter, then I can say every pair has a matching and equal here. The h 0 difference between the h 0 and h 4 is only the negative sign h 0 is equal to h 4 minus h 4. So, if we see that h 0 has a pair with h 4 and h 1 has a pair with h 3, but h 2 does not have any pair. So, what do we mean by h 2? h 2 is nothing, but a h[n-1] divided by 2 5 minus 1 4 divided by 2 is 2 only. So, it is h[n-1] divided by 2.

So, if it does not have any pair, then do I want to design it symmetrically, or what is antisymmetric? Now, what is the logic required? It must be 0, then only this side 2 equal to the minus of this side. So, if this side is positive, this side is negative, but it is a symmetric function in which the negative is negative, but this side is equal only in negative. So, I can say this terminal term has to be 0.

So, in the case of odd or odd numbers and an anti-symmetric filter, this N minus 1 by 2 is equal to 0, but if it is even, if my N is even, let us say N is equal to again, I can say the take another slide here.

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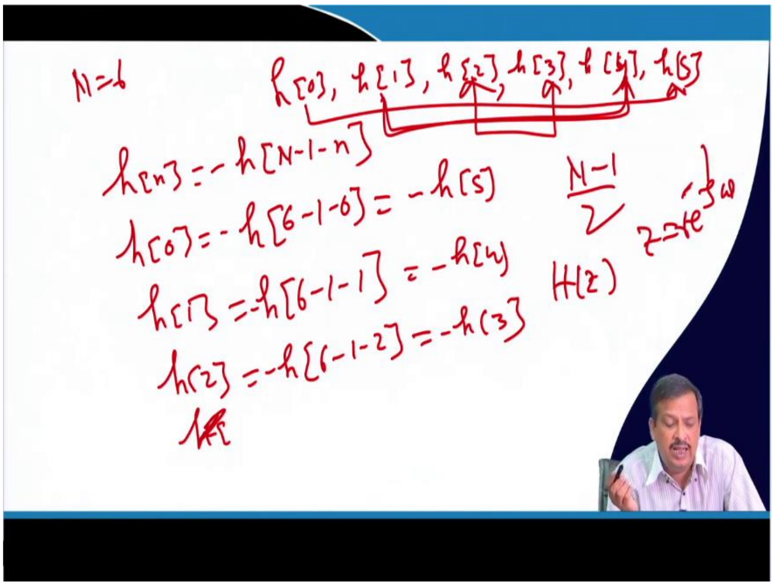

So, when my N is even, let us say I have taken a capital N, which is equal to 6. So, I have an h 0, h 1, h 2, h 3, h 4 and h 5. Now I know h[n] is equal to minus of h N minus 1 minus n. So, I can say h 0 is equal to minus h 6 minus 1 minus 0 is equal to minus h(5).

So, 0 matches with 5, now h of 1 is equal to h of 6 minus 1 minus 1 is equal to minus h of 4. So, 1 matches with 4, then h(2) is equal to minus h 6 minus 1 minus 2 is equal to nothing, but a minus h(3), then I can say h(3). So, h(3) is already there. So, I have covered that signal. So, 1 0 matches with 5 1 matches with 4 and 2 matches with 3. So, there is nothing is left.

But when it is in odd you can see the h 0 will be matches with that one, but there will be a one term which is nothing, but a n minus 1 by 2 term which will be no pair. So, in that case, I can say h[n-1] by 2 is equal to 0 when N is equal to odd, and N is odd when N is odd. Now again, I know what I know. I know H(z). So, I again put z is equal to $e^{j\omega}$ and r equal to 1, and on that same equation only I consider the negative sign.

So, here I only consider the negative sign only. If I only consider the negative sign and I put the z value, I get this form for N even and this form for N odd because if you see in case of odd, this term is 0. In the case of symmetry, this term exists, understand or not, but here this term can exist, but it is negative, which is why this term does not exist.

So, anti-symmetric filter, when I say H r FIR linear phase FIR anti-symmetric filter, then Hr(ω) is only this one, and this one is the N even, and this 1 is for N odd and again, if you observe the phase ω multiply by N minus 1 divided by 2 and ω multiply by N minus 1 divided by 2. Now, if I say that, what are the differences between the cos? This is a sine function; instead of the cos function, it becomes a sine function.

So, if I say that in the case of a symmetric filter, the phase is π by 2 minus this ω, I know the sin π by 2 minus θ is equal to cos θ. So, I can say π by 2 minus ω N minus 1 by 2, and again, if I say that it is less than 0, then this will be nothing, but this one ok. So, this is the θ. So, I have designed a symmetric and anti-symmetric linear FIR filter. Now, you may ask, sir, why is it required? Where should I apply it?

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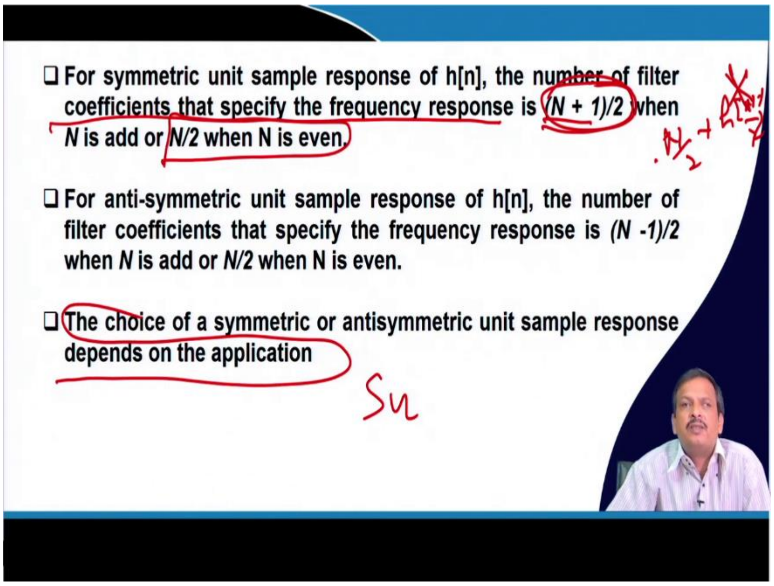

Suppose I give you a problem: what is the summary of this filter? So, when I use anti symmetric filter and when I use symmetric filter that depends on your application. So, the choice of symmetric and anti-symmetric unity impulse response will depend on what kind of application I want to bend for symmetric unit impulse response h n the number of filter coefficients that specify the frequency response is N plus 1 by 2 because N by 2 plus h[n-1] divided by 2.

So, N plus 1 by 2 if N is odd and N by 2 for n is even for an anti-symmetric unit response, this is reduced by 1. So, it is N minus 1 by 2, N is odd and N by 2 when N is even. Now, when I use symmetric or anti-symmetric unit impulse response, it depends on the application.

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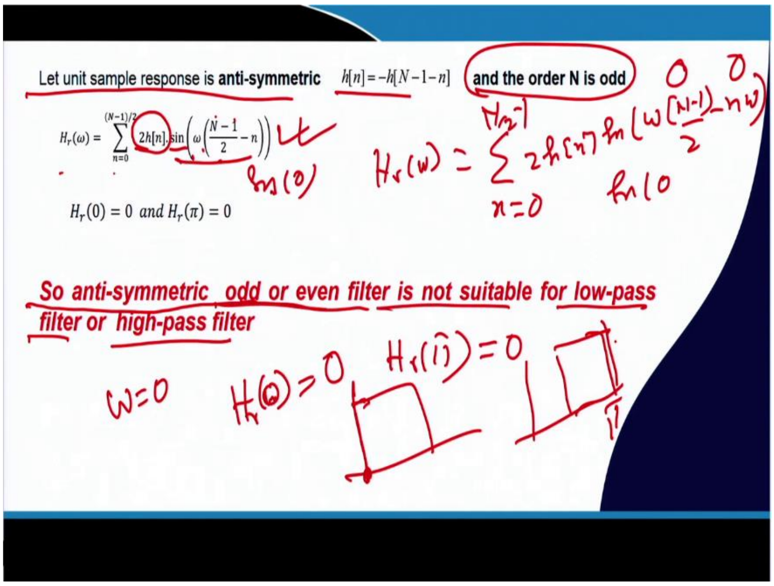

For example, suppose I have a unit impulse response that is anti-symmetric and N is odd, then

$$H_r(\omega) = \sum_{n=0}^{(N-1)/2} 2h[n] \sin\left(\omega\left(\frac{N-1}{2} - n\right)\right)$$

So, this is, and if it is, N is even, and this is the same. I think so; this is the same, only the summation sign will be different. So, in case this is nothing,

$$H_r(\omega) = \sum_{n=0}^{(N-1)/2} 2h[n] \sin\left(\omega\left(\frac{N-1}{2} - n\right)\right)$$

Now, if I say ω equal to 0. So Hr(ω) 0 means ω is equal to 0. So, ω 0 means this is 0 this is 0. So, sin 0 here is also multiplied by ω sin 0. So, sin 0 0. So, Hr(ω) is 0.

Then, Hr(π), if I put the value of π again, it is 0. If the whole value is π, and the ω value is π, then again, it is 0 because n minus 1 divided by 2 minus n is nothing but the integer multiple of π. So, sin π is 0. So, I can say Hr(ω) π equal to 0. So, anti-symmetric odd and even filters are not suitable for low-pass filters or high-pass filters. Low pass filter: I want a low portion to be passed, but what will happen it will make Hr(0) is equal to 0.

So, at this point, it becomes 0, and in the case of a high pass filter, what is required? It should exist up to π, but in the case of π, it becomes 0. So, I cannot design high pass and low pass filters using anti-symmetric odd or even. Filter response impulse response, which is designed as anti-symmetric odd or even both in both cases, is not possible.

So, whether you use symmetric or anti-symmetric depends on your application. For example, can this anti-symmetric case not be used clearly for a low-pass or high-pass filter design? So, in the next class, I will talk about how to design that FIR filter or what the common methodology used for designing FIR filter is.

Thank you.