Signal Processing Techniques and Its Applications
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Lecture - 04
Digitization of signal

Ok. So, in this lecture, we'll discuss analog to digital conversion. So, I will not talk about the circuit for analog to digital conversion; I will talk about the information that is required for digital signal processing purposes, such as analog to digital conversion.

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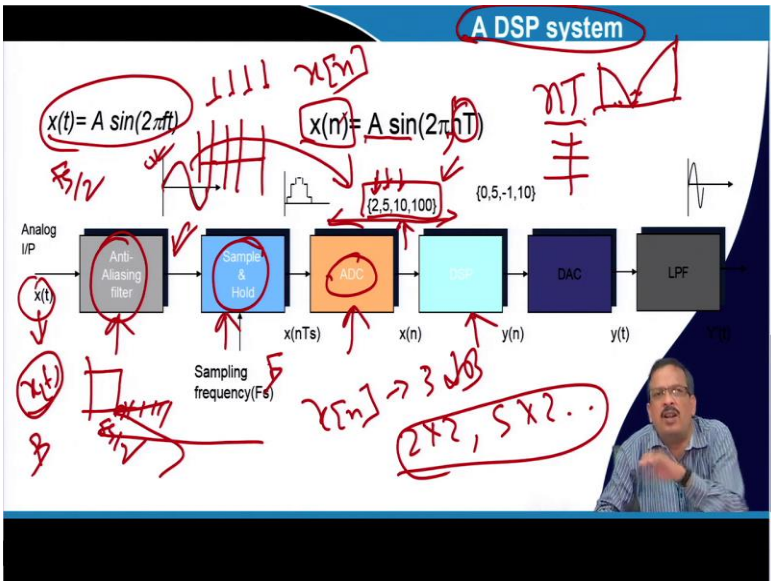

So, let us start. If I see a Digital Signal Processing (DSP) system, people may ask what the DSP system is. So, if you see an analog signal;

$$x(t) = A \sin(2\pi ft)$$

let's say f is the frequency, and t is the time. So, this is an analog signal.

Now, you apply the analog signal to an analog-to-digital converter, so ADC analog to a digital converter. So, to do that analog to digital conversion, there is an anti-aliasing filter, sampling and hold; I will come later. So, my purpose is that I have an analog signal that varies in analog, and I have to convert it to a digital signal; what is the digital signal? The digital signal is a sequence of numbers 2, 5, etc., called samples.

So, which is nothing but the x(n), sometimes it is written x[n] is written this way. So, instead of the first bracket, you can use the third bracket to represent the digital signal x(n).

$$x(n) = A \sin(2\pi nT)$$

So, if you see continuous T this. So, 2πf and f will be there nT continuous T is divided by nT. T is the fixed distance we'll come. So, a digital signal is nothing but a series. So, how do I convert the analog signal to a digital signal?

And once I get the digital signal, suppose I told you to multiply x amplify x(n) 3 dB, 3 dB means double. So, I can double that signal sample value. So, 2*2, 5*2 like that way I can do. So, it is nothing but an algorithm. So, it is nothing but digital signal processing. so any DSP system input is an analog signal that has to be converted to a digital signal.

So, how do I convert an analog signal to a digital signal, and what are the limitations? That is the discussion for analog to digital conversion. So, how do I convert? Let us say you do not know anything about analog-to-digital conversion.

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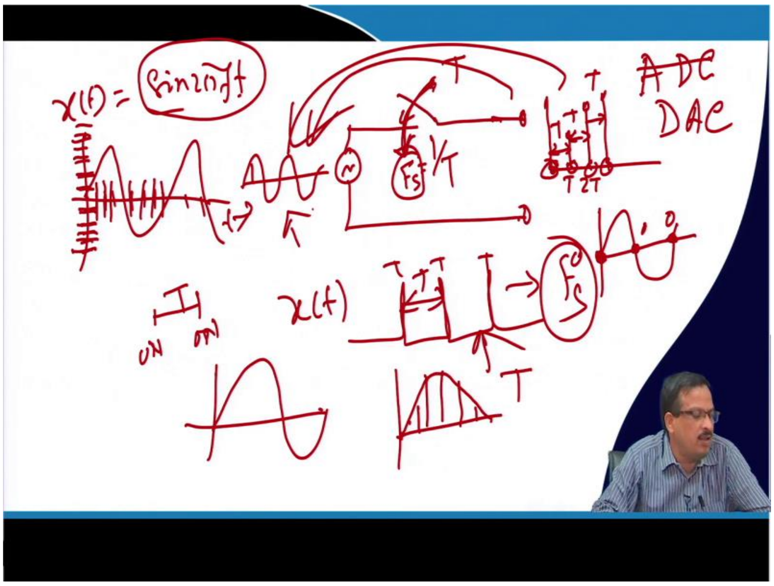

So, what do we have? We have x(t), which is nothing but a, let's say sin(2πft) continuous t. So, I have a signal like this. Now, what do I want? I want a sequence of samples. So, there are two things I have to do; one is that you know that discrete time signal, I have to discretize the time axis first. So, this digital signal is a discrete-time discrete value. So, I have first to digitize this time axis and then I have to discretize this amplitude axis.

So, when I discretize the time axis, that is called sampling. Now, when I discretize this amplitude axis, this is called quantization; you know that a discrete-valued signal is nothing but a quantized signal, and a discrete-time signal is nothing but a time axis, is discretized. So, how do I discretize the time axis? Let's say I have a circuit like this, and here I am applying the signal; I am collecting the signal on two points. So, if I operate this switch on and off. So, when the switch is on, I get the signal when the switch is off, I should not get the signal.

So, if I say I apply a sine wave in the input, I will get like this in the output. Let us say initially that the switch is on and then off. So, at 0 value, I get a value; after T time, the switch is on again. Then I get a value, and after another T time switch is on again, I get another value. So, it is T, it is 2T like that way I can get.

So, I am collecting the evidence of the signal at 0, T, 2T, 3T, so the distance between two samples is T. So, what am I doing? I am taking a sample of the signal. So, if you see in a mandi when a farmer comes with a lot of rice, trucks of rice. So, it is impossible to check all the rice packets. So, what do you do? We take some samples.

So, a sample means on a continuous space, I am selecting a certain time, so that is sampling. So, how do I do a sampling? So, along the time axis, I take as the evidence of the signal at a certain interval. So, that interval is nothing, but that interval actually operates the switch. So, if I say I am operating the switch with the time interval T, then what is the frequency of operation of the switch? It is nothing but a 1 by T if the time interval is T, so it is on and off.

So, initially, it is on, then off, then again on after T time. So, on-to-on is nothing but a period. So, that on-to-on is T time. So, I know the frequency of this operation of the switch is 1 by T, which is nothing but a, let's say, Fs, which is called sampling frequency. So, if I want to sample an analog signal that is continuous in Time, I have to discretize the time axis I have to multiply, or I have to do some operation by which I can take the evidence of the signal after a certain time interval.

So, if I ask you if I want to do it mathematically, what must I do? So, I have to operate this switch. What do you mean by operating that switch; that means I am multiplying x(t) with the impulse whose time interval is T. When the impulse comes, the switch is on, and when the impulse goes, the switch is off. The distance between the two impulses is T. So, that is called sampling frequency or sampling signal and the frequency of this impulse is called sampling frequency. Ok, I know that.

So, now suppose I have a signal, let us say sine wave signal; now, if I take the evidence of the signal, let us say here, here and here, can I get back the signal again? Because, after the digitization, after the processing of the signal, I have to convert it to the analog signal again. So, the main purpose of the signalling is that I have to reconstruct that analog signal again, which is called digital to analog conversion DAC. So, I have to revert back the signal again.

So, let us operate the switch in a sine wave. I take the evidence here, here, and here; all are 0. Can I get back the signal? No. So, there is a problem with how frequently I should operate the switch such that I can revert back to the input signal at the output from the output signal, you know. Instead of taking this entire signal, if I take, let us say this point, this point, this point, this point, this point, this point, I can make an interpolation, and I can get back the continuous signal.

So, the limitation is at after what interval I should switch on or switch off, or I can say what should be the sampling frequency such that at the output from the discrete signal, if I apply digital to analog conversion, I will revert back to the analog signal, that is called sampling theorem, so, Shannon sampling theorem.

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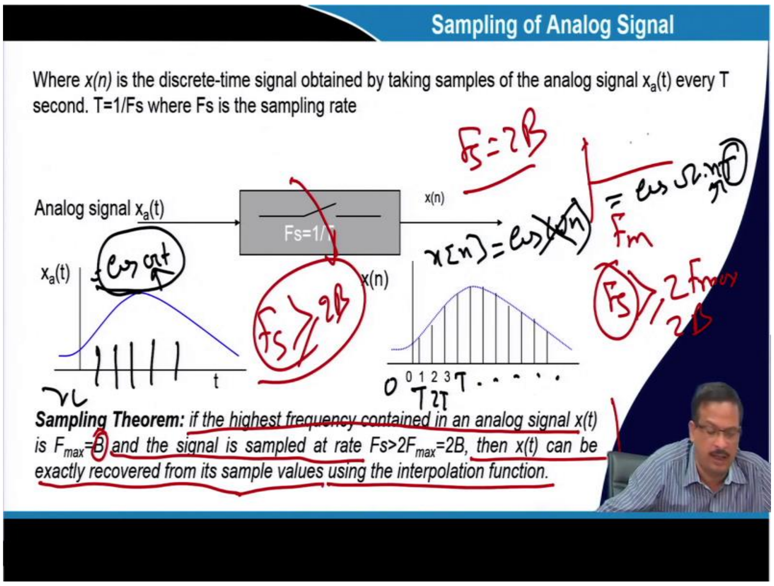

Sampling theorem so, if the highest frequency content in the analog signal, an input signal is oscillating, let us say Fm highest frequency component ok, of F_{max}, is equal to B. B is the bandwidth of the input signal and the signal is sampled at the rate of $Fs \geq 2F_{max}$ or $Fs \geq 2B$

then x(t) can be exactly recovered of its sample value using the interpolation function.

So, what sampling theory is limited to that? What should the frequency of the switch be? At what rate should I operate the switch so that from the discrete-time instant, I can get the continuous instant again back? So, that is the limitation of selecting my Fs. So, what will happen if Fs is not greater than or equal to twice B? What will happen if the Fs is equal to twice B and the signal is pure tone?

So, what will happen, and how can I explain those things?

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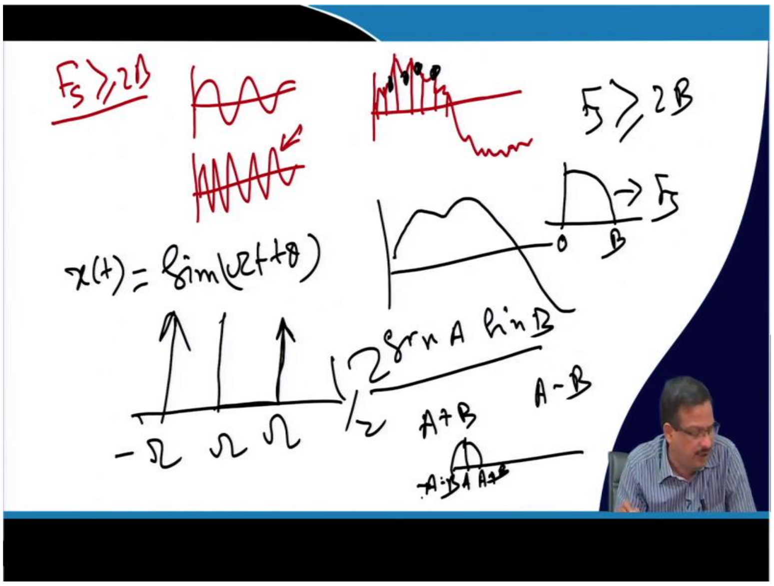

So, let's say I have taken slides here. So, I said that if my Fs is not greater than or equal to twice B, the B is the input signal's bandwidth; what will happen? Forget about the frequency domain representation. That part I’ll come later. First, you think about the frequency of a signal. So, suppose I draw a sine wave like this, and I draw a sine wave like this which one has a high frequency, this one. So, frequency means more oscillation.

So, if I have a signal that looks like this, it means this signal contains more frequency, and frequency components are high because there are a lot of small oscillations. Now, say this is my signal and my completely oscillated signal. I want to sample that signal this way: What do I miss? I missed the point; which point did I miss? I miss the point of this one, this one, this variation, this variation. So, if my Fs is not greater than 2B, I miss that small signal variation. So, when I reconstructed the signal, the signal would not be that variation signal and would be smooth out.

So, that means I am losing information, but what is the requirement of the sampling theory I want to get back the original signal exactly? So, that is the time domain representation; now, I come to the frequency domain. So, what is the frequency domain? If I say if I have a sin $x(t) = \sin(\Omega t+\theta)$ is my signal, then what is the frequency domain representation?

So, if this axis is Ω, I know it has a peak in here, and it has a peak in here, which is positive Ω, this is called minus Ω, this is 0. So, that is the frequency domain representation of a sine wave. Suppose I have a baseband signal whose bandwidth is B. So, this is my baseband signal up to B bandwidth. So, the input signal as a frequency component from 0 to B Hz, which is my bandwidth.

What should I get if I multiply that signal with an impulse whose frequency response is Fs, whose frequency is Fs? You know, suppose I told you I have a sin A. What should I get if I multiply a sin B? 2 into half, 2 into sin B and A into B? So, I can convert it to the cos A plus B minus cos A minus B that way. So, I get two components: one component is A plus B, and another component will be A minus B. So, if the A is high, I can say under the A, there will be a B, A plus B and A minus B. So, this is A, then it is A plus B, and this will be A minus B.

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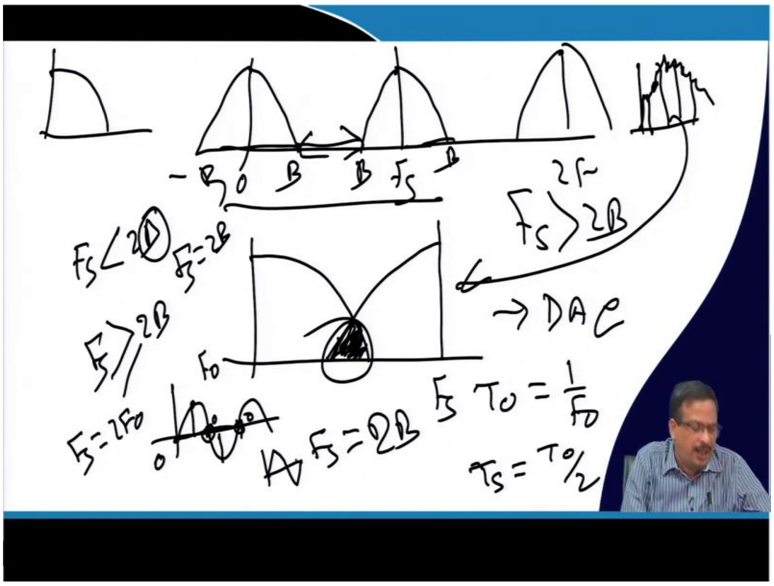

So, I have a baseband signal like this when I multiply with Fs. So, this is my Fs. So, I will get this one, and I will get this one, this is (Fs + B), this is (Fs – B), at the beginning or at the 0 also I get, this is 0, B, minus B then I get twice Fs. Now, if Fs is less than twice B; that means I am just drawing the positive side; this side, there is a B. Let us say this kind of thing I will get. So, here, Fs is less than twice B, my baseband signal bandwidth is B, and Fs is less than twice B.

So, what will happen? There will be some overlap in this case. If Fs is greater than twice B, then I can separate it, ok? So, if there is an overlap, then when I process it, I convert it from digital to analog conversion. So, that information will be missing overlap, which is called the aliasing effect, and in time domain representation, what I said that I have a signal that has a small variation if I sampled it in which it is not greater than B twice B, then I will miss this small variation then also this is also called aliasing.

And if I explain it in the frequency domain, this is exactly this. So, what is the requirement? Do I have an analog signal? I have to guarantee that Fs is not less than Twice B. It should actually be greater than Twice B, or at least it should be equal. Even if it is a periodic signal, a monotonous signal like the sine wave, even if Fs is equal to twice B, then also I cannot extract the signal.

So, if I have a sine wave signal, let us say the frequency is F_0, then what is T_0? Is 1 by F_0? Let us have a sampling frequency of Fs; Fs should equal twice F_0. So, I can say Ts equals T_0 by 2 because you know T_0 is 1 by F_0. So, Ts is 1 by Fs. So, if I convert, I can get Ts equal to T_0 by 2. So, the difference of the sample is the 0th position. I take T_0 by 2 here, and the next one will be here. So, this is 0, this is 0, this is 0, I cannot get back the sine wave again. So, if it is a pure tone, I cannot say the Fs will be twice B; Fs must be greater than twice B.

So, in that case, I required one sample here, one sample here, and one sample here, and then I could also get back the signal. So, that is the limitation of sampling. So, the sampling frequency is what sampling is; sampling is nothing but a discretised signal time axis. So, if I go for the mathematical representation. So, I have a sampling means x[n], x[n] is equal to actually if it is let us say $x(t) = \cos(\Omega t)$ this is continuous frequency, x[n] will be $\cos(\Omega n)$ or I can say it is nothing but a $\cos(\Omega nT)$.

Forget about this part; this part I will come later on. So, it is nothing, but I am discretising the time axis. So, n is the index. So, it is 0 index, it is T, it is 2T, it is 3T like that. So, n is the index, and T is the interval between the samples. So, in mathematics, if I have an analog signal, which is $\cos(\Omega t)$ in the digital domain, it will be $\cos(\Omega nT)$ ok or not. So, now, take another slide.

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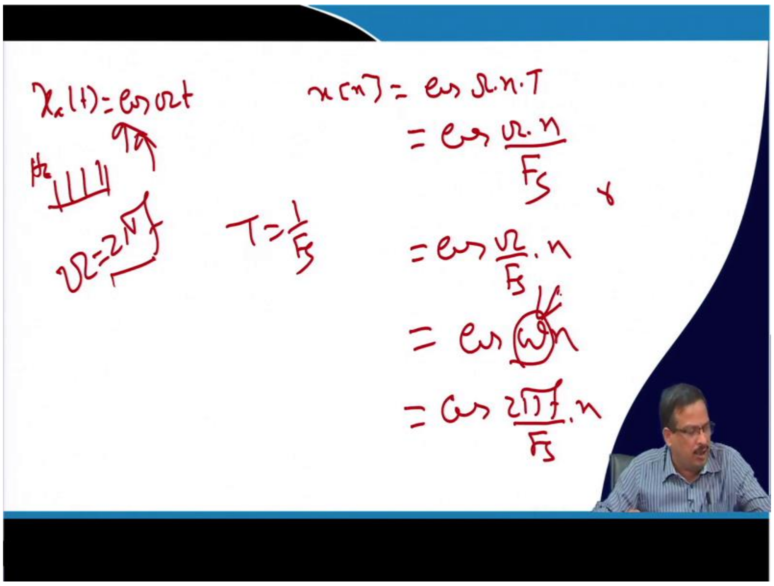

So, I say $x_a(t)$ = cos (Ωt); now I say x(n) = cos (ΩnT). So, let us say this Ω is called analog frequency; what do you mean by analog frequency? It is only a radian per second or hertz, and hertz is radian per second. Now, the second is not there because I have digitized the time axis. So, this Ω, what is this Ω? It is nothing but a cos (Ωn) divided by Fs, because T is only a 1 by Fs.

So, now it comes cos Ω by Fs into n, it is written as cos Ω n, this Ω is called digital frequency, this is radian per sample, and this is radian per second. So, once I digitize the signal, the sampling frequency is divided; it is radians per sample. So, this Ω is nothing but a radian per sample. If I say this $\Omega = 2\pi f$, f is the frequency in hertz. So, if it is like this, then I can say this will be cos ((2πf)/Fs) n. So, there is an f by Fs term, will that be ok?

So, radian per sample, radian per second, is sometimes called discrete frequency. So, I will come to what the highest rate of oscillation will be. That part I will come later on. So, now, as per this analog to digital conversion from sampling theory, what limitation does it put? It puts the limitation on the Fs being greater than equal to twice B.

Unless I cannot correctly reconstruct the input signal analog signal again, because if you see all real-life signals are analog signals, suppose I said I have a DSP processor or I want to store my voice digitally, what is there in your internet channel or when you downloaded this MPEG 4 my digital audio is there. So, how do I get the digital audio?

So, my voice is recorded by a microphone. The microphone produces an electrical signal, which is analog and continuous in time, and then it is converted to a digital signal using an ADC analog for digital conversion.

So, when it starts converting from analog to digital, it is multiplied by an Fs sampling. So, the sampling theorem has to be held there, and if you see, once I recorded it, then once I want to play it again, what do I require? I must convert the digital signal to the analog signal again and apply it to the loudspeaker. Then only you can listen to it. So, if Fs does not satisfy those criteria, then the digital signal cannot be converted to the analog signal again. So, one my voice you cannot get from the loudspeaker. So, that is the sampling theory.

So, when I want to design an ADC, I have to restrict the bandwidth of the signal if it is B, or I can say if the sampling frequency of the ADC is Fs, then the bandwidth of x(t)must be restricted to Fs by 2. So, the bandwidth of the ADC is Fs. If the sampling frequency is 4 kilohertz, then I have to restrict the input signal bandwidth to 2 kilohertz. So, how do I restrict it? It is called a filter; which filter it is? It is nothing but a low pass filter that would cut off the frequency of Fs by 2.

So, I have to restrict the input signal to Fs by 2 by a filter. What kind of filter is this? It is a low-pass filter, and high frequency should not be there. So, that filter is called an anti-aliasing filter; what it is restricted that Fs must be greater than or equal to 2 B. So, that aliasing part in the frequency domain, whatever when I explain that this part should not be there, maybe no aliasing no overlapping section in here, is it clear. That is why this anti-aliasing filter exists.

Then, there is a sample and hold. So, after this anti-aliasing filter, what do I get if I multiply with a sampling frequency sampling frequency impulse? I get a discrete time instant. So, each is called sampled; each is called sample, and each sample is then required to be quantized. So, how do I quantize the sample that requires? At some time, I have to quantize that sample; the value of the sample must be converted to a binary number because computers only understand binary.

So, how do I convert it to a binary number? There are a lot of processes to convert into binary numbers that require time. So, for that time, I have to hold the signal. So, I required a sampling and hold for that purpose; then I quantized the sample and got a binary value representation of the sample. So, in a digital signal, every sample is nothing but a number, such as the index number of sample 1, sample 2, sample 3, sample 4, and sample 5.

So, I have explained how I convert and discretize the time axis, In the next lecture, I will explain how I discretize the amplitude axis, which is called quantization. So, how do I quantize a signal? So, that is called quantisation. So, you can say you have heard about 8-bit ADC, 16-bit ADC, 24-bit ADC, and 32-bit ADC; what is that 8-bit ADC, and what is 24-bit ADC that is linked with the quantization ok?

Thank you.