Signal Processing Techniques and Its Applications
Dr. Shyamal Kumar Das Mandal
Advanced Technology Development Centre
Indian Institute of Technology, Kharagpur

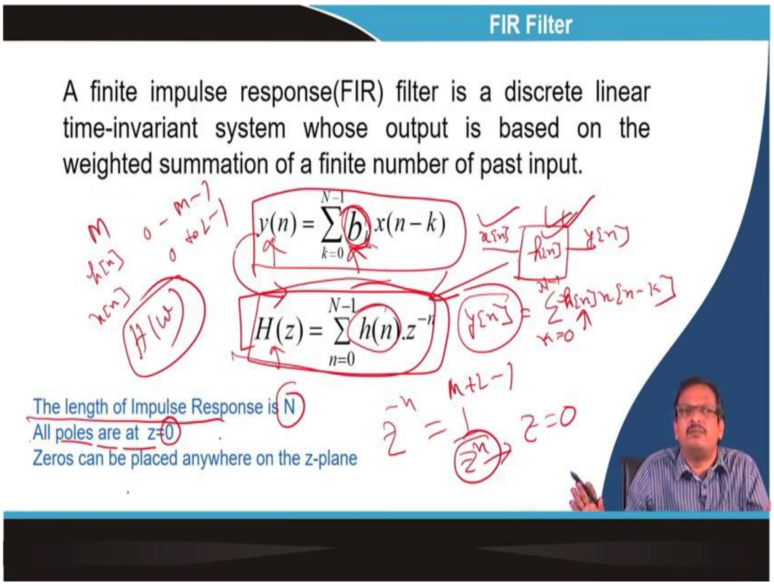
Lecture - 36
FIR Filter

Ok. So, we are discussing about the linear phase. We will discuss how this linear phase will be implemented in FIR Filter. First, let us start with that FIR Filter, and then we will go for that. What do you understand about linear phases in a system? What is the linear phase? Why is it required, and what is the condition for a linear phase in the FIR filter? How is it possible to implement a linear phase in an FIR filter? Those are things we will discuss.

(Refer Slide Time: 00:49)

So, let us start with the FIR filter.

(Refer Slide Time: 00:51)



So, as I said, the FIR filter is nothing but a finite impulse response filter. So, this b k, I can say, is nothing but a filter coefficient that is represented by h[n] understand or not. So, this is the H(z). So, what is the output? Output is the filter coefficient. So, if I say a filter is a system, this is h[n], let us say this is x[n]. So, the output is y[n]. So, when I say what is y[n]? It is nothing but a convolution between x[n] and h[n] convolution between x[n] and h[n].

So, it is nothing but the convolution k equal to 0 to N-1 x of the h[n] x[n-k]. So, h[n] is nothing but the filter coefficient. That is why I can write a value filter coefficient using the b k filter coefficient. So, if I know the filter coefficient from the given specification of the filter, then I can implement the filter, which is nothing but a convolution of the signal with the filter coefficient.

Let us say I have I want to implement a filter where the length of the filter is, let us say, M, M order of the filter is M. So, in that case, I have an h[n], n varies from 0 to M minus 1. Let us I have a signal x[n] whose length is, let us say, L varies from 0 to L minus 1.

Then what is the length of y[n] or the length of the convolution? I repeatedly said that the length of the convolution is nothing but an M plus L minus 1. During convolution and also during frequency response of the DFT, I have discussed this one. Now, if I take the z transform of that z transform of the filter coefficient filter h[n], which is nothing but this one.

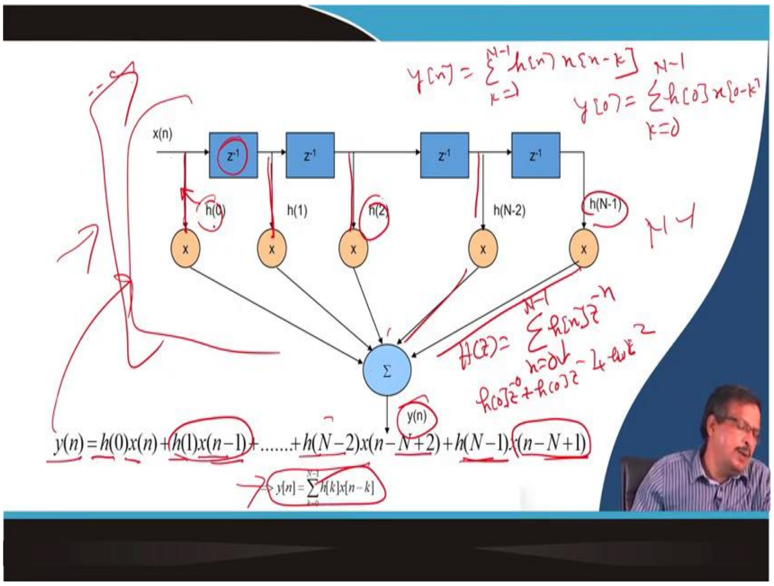
This is the z transform of h[n], where n is the order of the filter. So, the length of the impulse response is N, all poles are at 0, all poles are at 0, z^{-n}. So, it is 1 by z^n. It is nothing, but all poles are at 0, and 0 can be placed anywhere in the z plane.

So, this is the property of this z transform z^{-n} 1 by z n. So, I can say that all poles are located at 0 at z equal to 0 because the solution of z^n is nothing but the z equal to 0; there may be an n number of poles, but all are in origin, and 0 can be placed anywhere because 0 is not there 1 by z^n is only there.

So, 0 can be placed anywhere. This is the z-domain response of the impulse response of the FIR filter. So, ultimately, our aim is to find out b k from the filter coefficient and compute convolution between b k and x[n-k] or find out h[n] from the given filter specification, and I have to compute convolution.

What filter specification is given to me in the frequency domain? H[ω] that is given to me this is not given to me h[n]. So, I have to convert H[ω] to h[n] based on the specification. Once I get h[n], my filter is nothing but a convolution between the input signal and h[n] ok.

(Refer Slide Time: 05:07)



Now, if you see, so, this is x[n]. So, this is a pictorial representation of this function that y[n] is equal to k equal to 0 to N-1 h[n] into x[n-k]. So, now, if you see when I say y[0], y[0] is nothing but a sum of k equal to 0 to N-1 h[0] into x[0-k] ok.

So, x[n] is delayed by 1 sample; so, I can say this since this if you this is the z domain, what is H(z)?

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

Now, if I say what is l equal to 0. So, if I expand this summation, it is nothing but a h[0] multiplied z^{-0} plus h[0] multiplied z^{-1} plus h[0] multiplied z^{-2}.

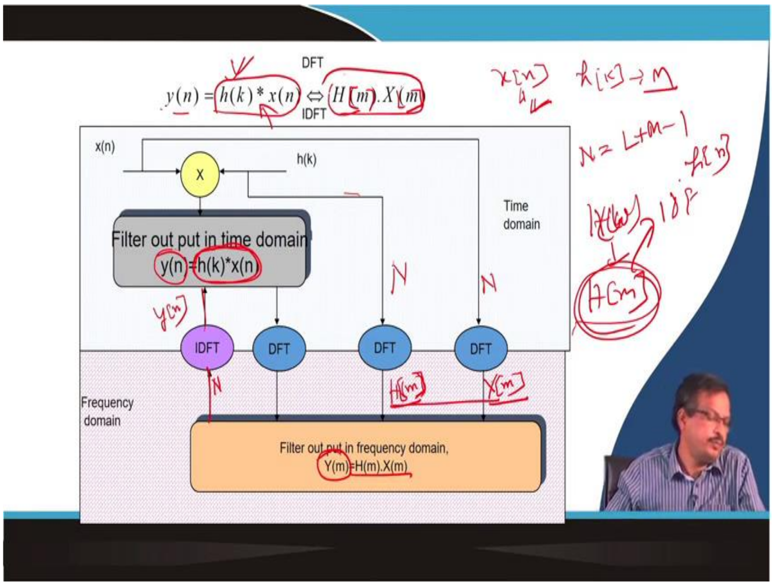
So, z^{-1} means one sample delay minus 2 means two sample delay minus n means n sample delay. So, it varies from N-1. So, it will be n minus N+1 and N-1 sample delay, ok? So, I get this 1 y[n] is equal to h[0] into x[n] h[1] into x[1] sample delay of the x then here h[n-2] N minus 2 sample delay of x[n] and h[n-1] N-1 sample delay of x n.

So, I get this one convolution function. So, if I want to represent in a figure, how do I do that? So, x[n] multiplied by h[0], h[0] multiplied with x[n] x of 0 and product and come to sum h[1] it will be multiplied after this delayed by one sample. So, h[n] without delay x[n] multiply with h[0], one sample delay multiplies with h[1], two sample delays multiply with h[2], three sample delays multiply with h[3] N-1 sample delay multiply with h[n-1] and all are summed together to give me my y[n].

So, I can say this is my structure: one implementation x[n] is applied delayed by one sample, one sample is multiplied with the filter coefficient, and by summarising them, I get the y[n]. So, this can be written in a program x[n] I if I x[n] is known, h[0] is known, h[1] is known, and h2[n] is known.

So, I can represent this one in nothing but an algorithm, which is this algorithm you represent this one computation of the convolution. So, now, my filter is an algorithm. Now, what is important? How do I get this h value? What should be the value of h 0? What should be the value of h[1]? What should be the value of h 2 from the filter specification, which is called FIR filter design?

(Refer Slide Time: 08:51)



Now, if you see another view, let us know if this can be implemented in the frequency domain. So, as I know, the frequency y[n] is a convolution in the time domain. So, in the frequency domain, it is nothing but the multiplication ok or not? In the time domain, it is convolution. So, the frequency domain is multiplication.

So, instead of saying I can compute this convolution, I can say I have an x[n], okay? So, this is h(k). Let us say h(k), h(k) is the impulse response of the filter, and I have an h(k) because the order may not be the same. This may be the L number order, which is L, or the order is M.

Then, if the order is L here and M here, then what should be the length of the DFT? L minimum length is L+M-1, which is equal to, let us say, N. So, I can say I can do the N point DFT of x n. So, I get what I will get? I will get capital X[m]. I can compute N point DFT of h(k). I get capital H m. The third bracket or first bracket is generally preferred because discrete signals are generally written in the third bracket.

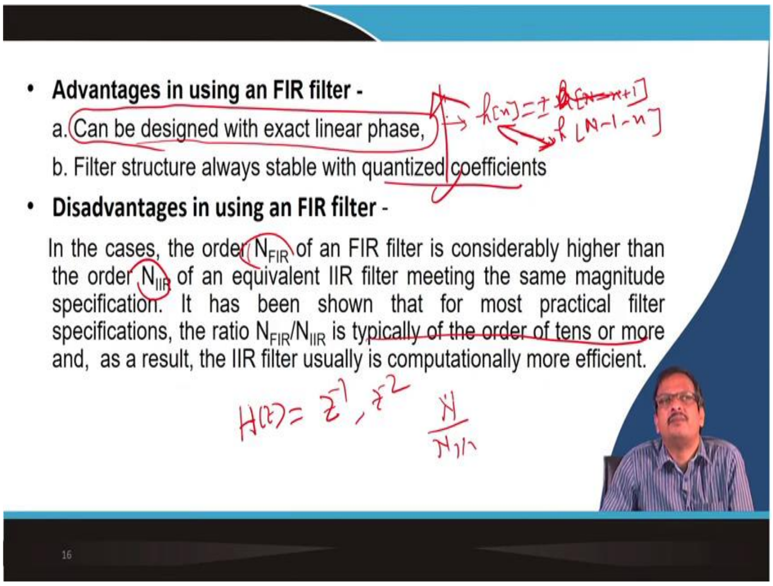
So, I can say that once I get H[m] and X[m], I can multiply them and get Y[m]. So, I multiply them in the frequency domain, then I can do inverse discrete Fourier transform N point inverse discrete Fourier transform to get y[n], or in the time domain, I can directly convolute the I directly I can compute the convolution I get y[n].

So, I can implement the filter in the frequency domain; let us say that H[ω] is given. So, from H[ω], I derive H[m] discrete frequency response, and then I compute the DFT of the input signal and multiply with H[m], then compute inverse Fourier transform I get y[n]. So, the complexity is that 2 times I have to compute DFT complexity first is this DFT, and the second one is inverse DFT because H[m] will be given in the frequency domain itself. So, I h(k). I do not require h(k), which is not given. I have given only H m.

So, once I am given in H[m], I can directly take that I DFT of x[n] and I can calculate X[m] and multiply them and take the inverse DFT I get y[n] because anyhow H[ω] is given I will compute H[m] and from there I will be compute I DFT and find out h[n] that is the procedure. So, anyhow 1 DFT I am doing for designing the filter inverse Fourier transform, or I can do the inverse Fourier transform offline and calculate the filter coefficient.

So, when I apply online if the filter is applied in real-time, it does not take much time or computation of the filter coefficient, but if you change the specification, then again, you have to compute the filter coefficient and store it. In MATLAB, you see that you have stored the filter coefficient. Then you take the X[m] and just take the convolution, and you get the output of the filter. So, this is the implementation strategy: whatever you want, you get to use it.

(Refer Slide Time: 13:07)



Now, I come to the advantage. Why are we talking about the FIR filter most of the time? Many times, we have not talked about the implementation of IIR filters in digital filters.

We are mostly talking about the FIR filter. Why? The advantage is that the design with an exactly linear phase can be designed with an exactly linear phase.

So, what is the condition for the linear phase? Linear phase is h[n] must be equal to plus minus h[n] minus N+1 or n sorry N minus h[n-1] minus n plus minus h[n-1] minus n symmetry or anti symmetry filter function. So, that is required exactly linear phase.

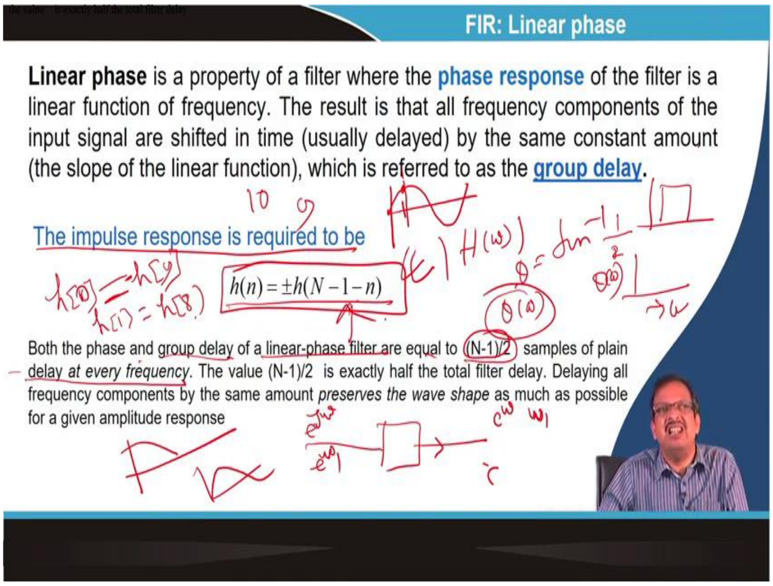
So, the linear phase is an important issue. We have to explain why this linear phase we are looking for. If it is not, what could be the reason? What could happen if it is not an exact linear phase? I have to understand that, and then I will only talk about the linear phase.

The filter structure is always stable with a quantized coefficient because here, the pole 0 concepts only all zeroes are all poles at the origin. So, within that unit, the circle is guaranteed. So, the filter structure is always stable. Now, what is the disadvantage? The complexity in the case of N, N is the FIR filter order FIR filter is considerably higher than the order of N IIR filter. So, the number of coefficients in the computational complexity in the case of an FIR filter is much more compared to an IIR filter.

You see, when I put an H(z) second-order polynomial, how much delay is required z^{-1} and z^{-2}, but think about 350 coefficient FIR filter. So, the order is 350. So, the order of FIR is divided by the order of IIR, which is typically an order of 10 or more.

So, 10 times is much more complex than the IIR filter, but the advantage is that it can be easily designed for linear phase, and is always stable. So, that is why we are looking for an FIR filter for the first two advantages of linear phase and stability, but yes, the FIR filter is much more computationally complex compared to the IIR filter.

(Refer Slide Time: 16:30)



Now, we come to the linear phase. What is a linear phase? I am not reading the slide. You can read it. So, linear phase is a property of a filter where the phase response of the filter is a linear function of frequency. So, you know the magnitude response H[ω] mod of H[ω] magnitude response and tan inverse, you know that tan inverse imaginary part divided by real part is the θ.

So, θ is also a function of ω. So, θ is called phase response. So, as I draw a magnitude response like this, similarly, I can draw the phase response also θ ω θ versus this axis is ω, and this axis is θ ω. So, this phase response must be a linear function of frequency. If it is not linear, what will happen?

So, the filter is nothing but a system if the system does not support the linear phase criteria. Then if you see if I apply a signal $e^{j\omega}$, ω is a particular frequency, and another $e^{j\omega 1}$, which is another frequency if the phase is different. So, at the output, if the phase is not predictable ω and ω 1, if the phase is an unpredictable phase, then I should not know what the output component may be completely out of phase or completely in phase I do not know.

So, that means the result is that the frequency component of the input signal is shifted in time by the same constant amount. So, I can say 1 kilo is the amount of sorry. What is phase? First of all, you have to understand what the phase is. When I say a sine curve, let us start from here, start from here. So, this starts from 0 if the sine curve starts from here.

So, it is pi by 2, which is the phase shift. So, that is the phase. So, if I say a signal consists of a sinusoidal component at every frequency.

So, if every frequency is shifted in an equal amount, then I know it is a linear phase. If it is not, then the phase is not linear, and the frequency response will be phase will be different for different frequencies. So, what is required for the group delay? The first order derivative is linear y equal to m x plus c where the m is the slope is the first order derivative of the phase response, which is nothing but a group delay.

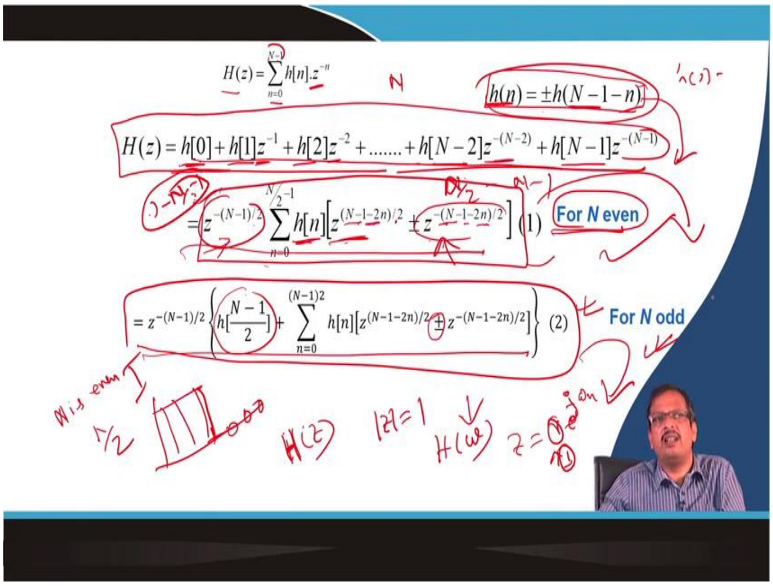
So, what is required if it is not a linear phase is that different frequency components will be shifted in time indefinitely, which means somebody is shifted in 10 samples somebody due to the phase change. So, I apply a filter and change the phase. Some frequency components start this way some frequency components start from here. So, that will create an output signal distortion.

So, if I want a linear phase, then what is required? The impulse response should support this condition: h[n] is equal to plus minus h[n-1] minus n. So, if it is plus, then it is a symmetric response; if it is minus, it is called an antisymmetric response. So, I can say both the phase and group delay in a linear phase filter are equal to N-1 by 2 samples for plane delay at every frequency.

So, N-1 by 2 is exactly half of the total filter delay. So, what is the requirement here? So, h[0] must be equal to someone. Let us see that the order of the filter is 10. So, N-1 is 9. So, if it is h[0], then this side will be h 9 n equal to 0. Now h[1] should be equal to h of 8.

So, n by 2, this side and that side of the impulse response are the same. If they are the same, then it is called symmetric filter symmetric response. If they are in opposite phases, if it is h[0] is equal to minus h 9 then it is called an antisymmetric filter. Now, I get the condition for which the filter will be linear phase. The impulse response must satisfy this criterion for the linear phase. Now, let us go for the design of a linear phase FIR filter.

(Refer Slide Time: 22:15)



So, how do I design what should what kind of frequency response we will get if I design a linear phase filter? So, what is H(z)

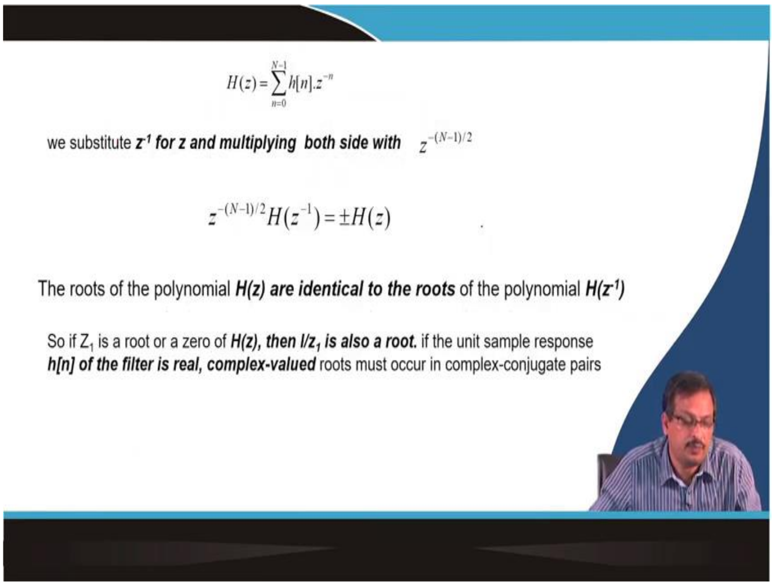
$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$$

This is the z transform H(z). So, I can say this is nothing but a h[0], h[1] z^{-1}, h 2 z^{-2} plus dot dot dot h[n-2] z^{N-2} h[n-1] z^{N-1}.

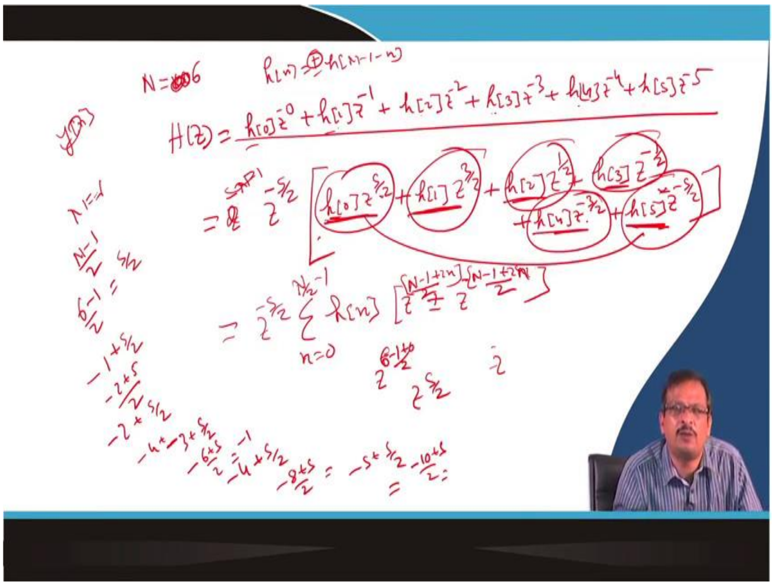
If the order of the filter is N, what is the condition? h[n] must be equal to N-1-n. Now, let us say that N is even, and N is even. So, the order is even. So, if the order is even, then N by 2, so this side N by 2 is equal to this side N by 2. So, h[0] to h[n] by 2; so, N by 2 number of from h[0] and N by 2 to N-1. So, 0 to N by 2 minus 1 response is equal to the response of the N by 2 to N-1; f N is even divisible by 2.

So, now, if I take this factor is common from all h minus z minus N-1 divided by 2. So, how many samples are there? They are similar to h[0], which is similar to what I said last. So, let us say forget about the n. Let us say take ok. Let us first compute the normal n value and then come to this equation.

(Refer Slide Time: 24:21)



(Refer Slide Time: 24:27)



Let us say I will take a slide here. So, let us say I said N is equal to 10. So, N is an even number. So, I can say the H of z H(z) is equal to h[0] into z^0 minus 0 plus h[1] z^{-1} plus h 2 z^{-2} plus let us say instead of 10. So, I have to write many things. Instead of 10, let us say it is 6. Let us say 6.

So, 0 1 2 3 is there, so, h 3 z^{-3} plus h 4 z^{-4} plus h 5 z^{-5}. So, if you see 1 2 3 4 5 6. So, 6 is the order now. I said if I take $z^{-(N-1)/2}$. So, what is N? N is equal to 6, so N-1 is divided by

2. So, it is nothing but a 6 minus 1 by 2 is equal to nothing but a 5 by 2. So, I take common minus $z^{-5/2}$ and take common from this part.

So, here it is, not there. So, I can say h[0] into $z^{5/2}$ plus h[1] into there is a minus 1. I have taken this one in common. So, this will be added up 5 by 2. So, it is nothing but a minus 2 plus 5 divided by 2. So, it is nothing but a 3 by 2 plus h 2.

So, it is nothing but a minus 2 plus 5 by 2. It is minus 4 plus 5 1 by 2 z^1 by 2 plus h 3. So, it is 3; so, 3 plus 5 by 2 minus 3 plus 5 by 2. So, it is nothing minus 6 plus 5 divided by 2, which is equal to minus 1 by 2. So, it is nothing but a Z to the power minus half plus h 4 into Z^{-4} plus 5 by 2. So, it is nothing but minus 8 plus 5 by 2.

So, is equal to it is nothing but a minus 3 by 2 plus h 5. So, minus 5 plus 5 by 2, so it is nothing but a minus 10 plus 5 divided by 2 minus 5 by 2 $z^{-5/2}$. If you see this component matching with this component component matching with this component, which one does match with this component?

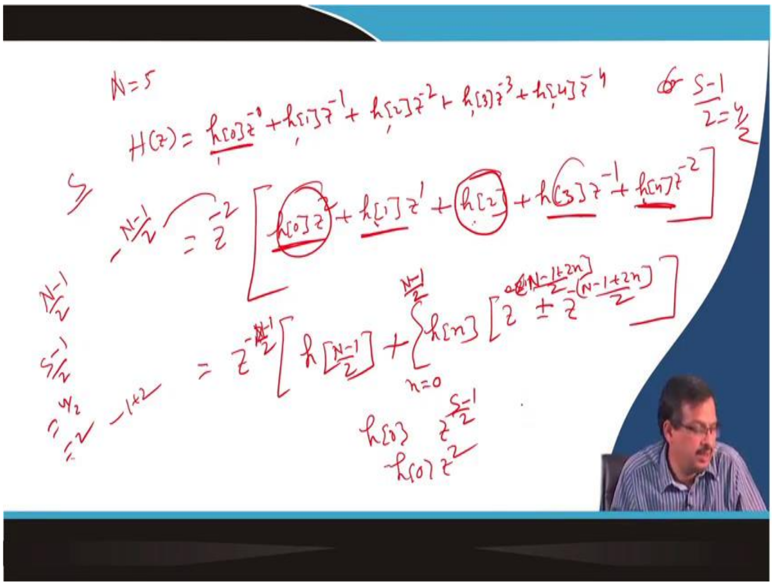
So, I can say that instead of writing n equal to 0 to N-1, I can say[0] to N by 2 minus 1. So, it varies from. So, those are clubbed together. So, I can group them into this component, and this component can be grouped together; the only difference is plus-minus. So, this component and this component can be grouped together.

So, instead of writing $z^{-5/2}$, here, m is equal to 0 to N-1. I can say N by 2 minus 1 h[n]. Now I said z to the power. This one is positive, and this one is negative. So, this one is N-1 plus 2 n plus minus z to the power. So, this can also be minus what I said: y[n] is equal to h[n] is equal to plus minus h[n-1] minus n.

So, if it is plus symmetry minus symmetry, so, h[0] $z^{5/2}$ can be equal to positive or plus minus 1 it can be negative if it is negative then it is anti symmetric, if it is positive then it is symmetric. So, it is nothing but a z^{-n}-1 plus 2 N. If a small n is equal to 0 see that it is z^n, n means your 6, 6 minus 1 plus 0 divided by 2 sorry divided by 2, which is nothing but a $z^{5/2}$ and this is minus 5 by 2. So, I can generalize if that.

So, it is nothing but a h[n] z^{N-1} minus 2 n by 2 plus minus z^{-n}-1 2 n by 2 clear. So, if n is sorry, if n is. So, N is even when the order is even. So, for order 6, I have shown you that order 6 is okay. So, when the N is even, the H(z) can be reduced to this one. Now, if the N is odd, what will happen? Let us take it.

(Refer Slide Time: 30:16)



So, now I said N is odd, let us take a small example: N is equal to 5, and N is odd. So, I can say H(z) is equal to h[0] z^{-0} plus h[1] z^{-1} plus h 2 z^{-2} plus h 3 z^{-3} plus h 4 z^{-4}. So, 1 2 3 4 5 order is mentioned 5 order n is equal to 5.

Now if I take N-1 divided by 2 what is N-1 divided by 2? 5 minus 1 divided by 2. So, it is nothing, but a 4 by 2 is equal to 2. So, if I take z^{-2} common, which is nothing but an N-1 divided by 2 minus ok, then if you see this one, there is a 0. So, I take minus 2. So, I can say h[0] into Z^{2} plus h[1] into z^{1} minus 1 plus 2 1 plus h 2 z is already taken out plus h 3 into z^{-1} plus h 4 into^{-2}.

Now, if you look at this coefficient, it matches this one. So, I can say z^{-2} h 2. So, what is the general form of 2? $z^{-(N-1)/2}$. So, if I want to write down the general form, what is the 2? It is nothing but an N-1 divided by 2. This no z is the rest are h[n]. So, how much sample is less than 1 sample is taken out?

So, out of 5 samples, 1 sample is taken out. How much is there? Only 4 are there? So, I can say n is equal to 0 to N-1 by 2. So, 4 samples will also occur in pairs. So, 1 pair and 2 pair, so, in the case of 6, it is nothing but a 5 n equal to 5, 5 minus 1 divided by 2; so, 4 by

2. So, N-1 divided by 2 h[n] into z to the power it is positive. So, 2 n, N-1 minus 2 plus 2 n plus minus z^{-n}-1 plus 2 n.

If you see n equal to 0, then h[0] z^n is 5 minus 1. So, z divided by 2 will be there divided by 2. So, it is nothing but a h[0] into z square, ok, understand or not? So, I can say that the generalized form for odd is nothing but this one. So, if I apply this condition, then if the order of the filter is n is even, then the z transform will be this 1. If the order is odd, then the z transform will be this one: clearly understand, do not remember it, clearly understand it, do not remember. Is it clear?

So, I employed the requirement of a linear phase filter. So, the linear phase requirement is this one plus-minus in terms of symmetry or antisymmetry if it is symmetry; that means the same side is negative if it is anti-symmetry. So, that is why once it is negative, it is anti-symmetry. Once it is positive, it is symmetric, but if N is odd, then one sample will be without z. If N is even, there will be 1. See that 1, no, none of the coefficients is without z.

So, for odd and even, I get 2 equations 1 and equation 2. In the next class, let us try to derive what the frequency characteristics of N are even and what the frequency characteristics are if N is odd because if I say H(z). So, how do I get the frequency characteristics from H(z), H(z) is z is nothing but a complex function $e^{j\omega}$.

So, if I if the amplitude is equal to 1. So, if the mod of z is equal to 1, then I get H ω. So, I put the mod of j is equal to 1 condition, and I derive the H ω. So, what is the expression for H[ω] ok? So, z is nothing but a r into $e^{-\theta n}$ or ω n ok.

So, z is a complex number j, there will be j, j ω n. So, r is 1; that means the amplitude is 1. All frequency component has a unit amplitude, and then I get the frequency response of H[ω] because I said that I want to pass band all 1 and stop band all 0. So, if the in-all frequency component unit amplitude is 1, the phase will be there. So, what will the frequency response for N order is even and odd will be derived in the next class.

Thank you.