Signal Processing Techniques and Its Applications
Dr. Shyamal Kumar Das Mandal
Advanced Technology Development Centre
Indian Institute of Technology, Kharagpur

Lecture - 27
Frequency analysis of long signals using DFT

So, in the last lecture, we talked about DCT, and then we talked about long data analysis.

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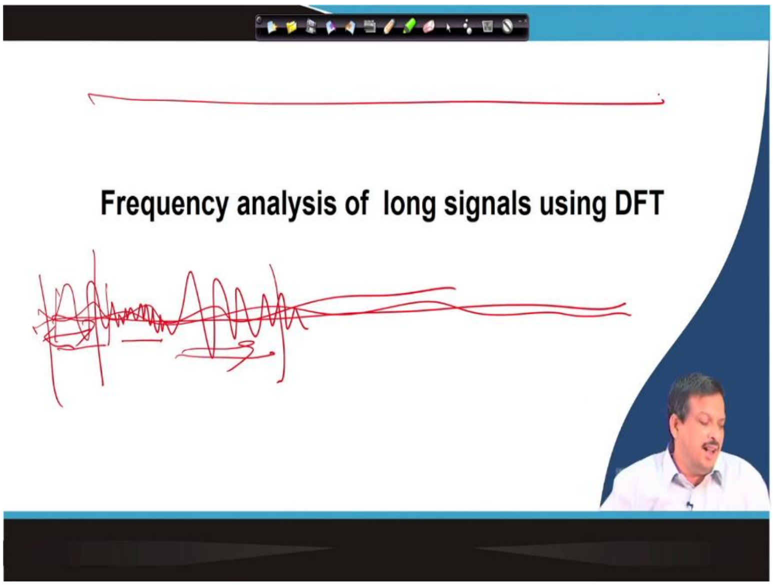

Today, we will talk about the Frequency analysis of a long signal using the Discrete Fourier Transform. So, what I want to discuss is how I can analyse my signal using a discrete Fourier transform if my signal is very long. Let us say I have a signal whose length is infinite, length is infinite. So, if I take the entire signal, the length of the DFT will be used.

So, if I want to analyze the frequency of the signal, I have to take a small portion of the signal. Another problem is that we have a signal that is not time-invariant; let us have a long signal along the timeline signal changes its property, a non-stationary signal. Then, if I take the entire signal at a time to analyse the discrete Fourier transform, what will I get? I get an average spectrum.

So, let us know that sometimes the signal is periodic, sometimes the signal is noisy, and sometimes again periodic. So, if I take them all together, I get spectra of the periodic noise and periodic average spectra. Let us say I do not want that one, I want to analyze the signal's small portion of the signal, and then again, I want to synthesize that signal.

So, that is called short-term Fourier transform, which I will do after this lecture. I will explain that also. So, initially, I thought that the signal was stationary, but the signal was very long. I have a long signal which is stationary.

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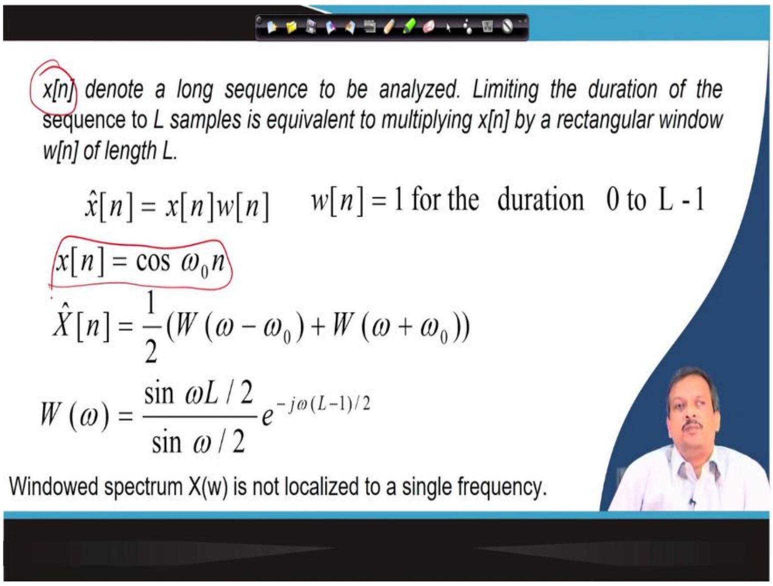

Let us say in the case of a x[n] is a sequence long sequence of a signal that has to be analyzed, and let us say x[n] is nothing but a cos(ω0n). So, I have a signal that is nothing but a x[n], which is cos(ω0n). So, what am I saying?

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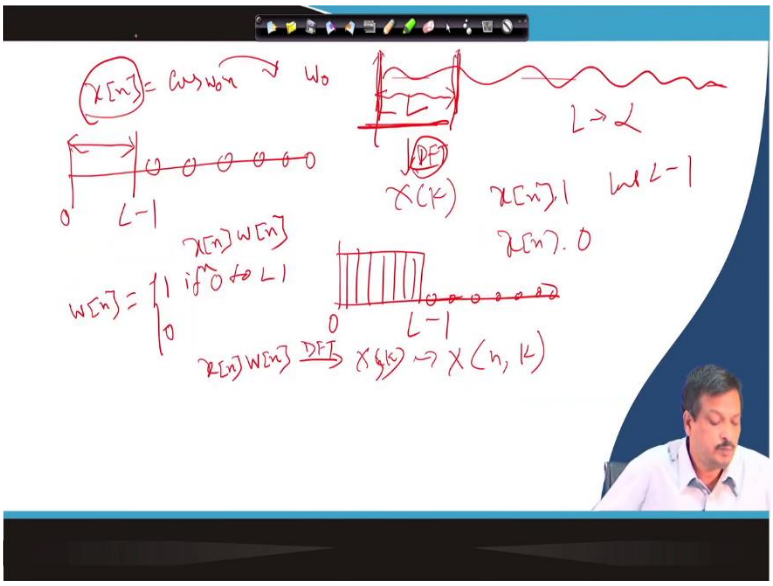

I am saying I have a signal x[n], which is, let us say, $cos(\omega0n)$. So, the frequency of the signal is $\omega0$, and that is $cos(\omega0n)$ has a long sequence; let us L tends to be an infinitely long sequence. So, I cannot take the whole sequence to analyze discrete Fourier transform at a time.

So, let us say I select some L number of samples of this long signal, and I do the discrete Fourier transform. So, I take a, let us say, up to out of long signal. I take the L number of samples, and then I do the discrete Fourier transform. So, if a long signal is represented by an x[n], DFT must be x(k), but not.

Why not? Because when I select some portion of the signal, what am I doing? I am saying the signal only this signal non-zero signal is now x[n] is non-zero only the interval of 0 to L minus 1; outside it, the signal is 0. So that means, basically, I am multiplying 1 of x with x[n], multiplying 1 up to L minus 1. After that, I multiply x[n] by 0. So, what are we doing? Basically, we are doing a signal multiplying and window function w[n].

So, w[n] is basically 1 if n varies from 0 to L minus 1, and n varies from 0 to L minus 1. Else, it is 0. So, what am I defining? I have an x[n], and I have a w[n], which is 1 0 to L minus 1; outside, it is 0. So, when I take the DFT, basically, I am not computing the DFT of x[n] alone. I am computing a DFT with w[n], and that DFT I am calculating. So, that DFT is not x(k), which is defined by which window? So, I am discretize the time also. So, here time also has an index; so, that is called X[n, k].

Now, what is X[n] k? So, if my signal is only cos, let us, I take another slide here.

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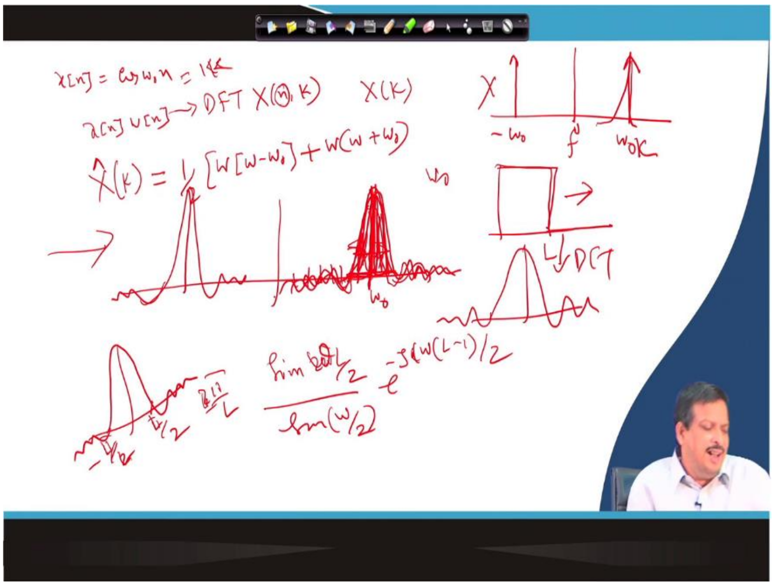

So, when I say my signal is only cos ω x[n] is equal to cos(ω_0n), then what is the frequency transform? What is the frequency representation of a cos function? If this scale is frequency, this is 0; then I said there would be one component in here, which is ω_0 plus, and one component will be here, which is ω_0 minus.

That is the frequency domain or Fourier transform of a cosine wave. If it is infinite, transform is infinite, that is the ideal case, but now what are we doing? Basically, I am multiplying x[n] with a w[n]. What is the w[n] signal? w[n] is nothing but a gate function. So, what is the frequency response of the gate function? It is nothing but a sin function. So, when I take the DFT of that, then my X[n], k is not X(k), so it is not like this.

So, what it will be? So, I can say my X(k). Let us say I have n. I have said the n is one first window. So, I can say X(k). Let us see that if X cap k it is not X k x cap k is equal to half of wω minus ω_0 plus wω plus ω_0; that means this sinc function. So, ideally, what will I get? I will get instead of an impulse at ω_0, I get a sinc function in here.

This is my ω_0, and I get a sinc function in here also. So, what am I getting? I am getting a sinc function at the. So, I cannot say this is a particular ω_0 frequency. So, the power of ω_0 will be distributed near frequency also. So, I get a frequency band. So, what is the bandwidth? Now, what is the representation of the sinc function? You know that if the length of the window is L, then the sinc function is sine. So,

$$\frac{\sin(\frac{wL}{2})}{\sin(w/2}e^{-\frac{j(w(L-1))}{2}}$$

is the DFT of this one. So, what does it mean? This means that instead of getting a single impulse at ω0, I get a bandwidth.

So, this is the main lobe, and those are called the side lobe. So, this is nothing but a π by; you can say π by L. So, this is L by 2, and there will be a L by 2 minus L by 2, or I can say this is twice π by L by π by L and this is π by L in the frequency discrete frequency range. So, I can say suppose this ω0 is 1 kilo hertz, so instead of getting an impulse at 1 kilohertz, if I take the DFT, I get a power distribution near 1 kilowatt.

So, not only a single 1 kilowatt but side-by-side frequency and impulse. So, that is called DSP leakage, and it also affects the side lobe. So, those components also get the power. So, this is a power spectrum if I take the power spectra. So, instead of getting an in the power spectra instead of getting an impulse on a particular frequency ω0, I get a main lobe width and then this kind of thing.

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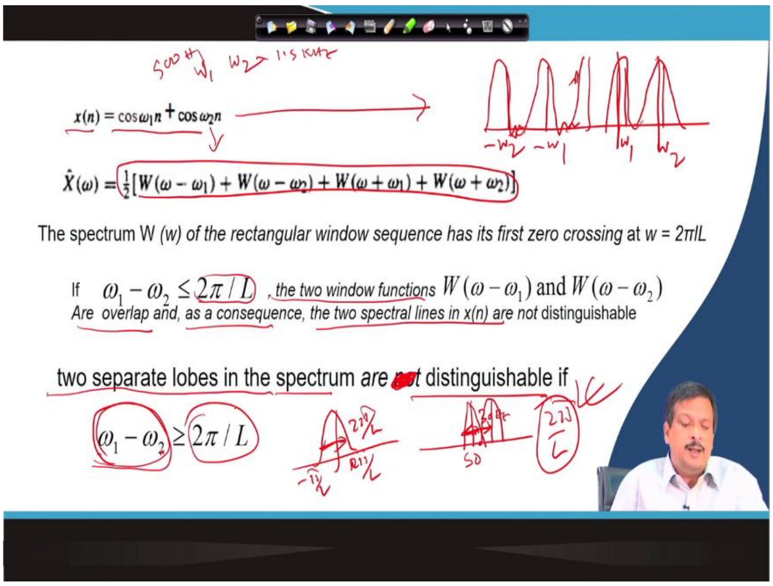

Now, make it a little bit more complex; let us go instead of a single frequency. Let us have a signal that has two frequencies. One is ω1, another is ω2. So, I have a sine wave or let us say this is 500 hertz and this is 1.5 kilo hertz; so two components 500 hertz and 1.2 kilo hertz. Again, if I take the DFT of a particular window, then I get this frequency response.

So, instead of saying if the ideal frequency response or spectrum of this signal is there will be a peak at ω1, there will be a peak at ω2 this side, and there will be a peak at minus ω1, and there will be a peak at minus ω2 that should be the ideal response, but since I am multiplying by the window. So, the window response will be convolved here. So, that is the main lobe, and there will be a side lobe.

Now, see if ω1 minus ω2 is less than 2πL, so you know the width of the main lobe is 2π by L. So, the π by L, this is minus π by L, so total width is nothing but a 2π by L. So, let us I have a peak.

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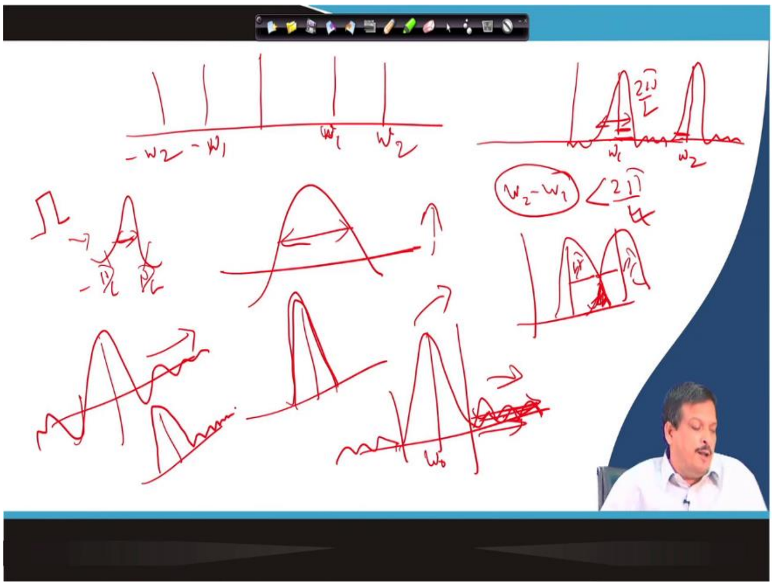

So, what is the ideal response? The ideal response is ω1 and ω2 minus ω1 and minus ω2. So, what will I get? Instead, I will get this kind of response. So, this is ω1, and this is ω2, and this width is nothing but a 2π by L. So, you know that if I use a rectangular window, which is a gate function, the sinc function has a main lobe at minus π by L to plus π by L. So, total width is 2π by L.

Now see that if ω2 minus ω1 is less than 2π by l, so this separation so this part is π by L, this part is π by L. So, if it is less than π by L, that means there will be overlap. So, this is π by L, and this is also π by L, so there will be overlap. So, the separation between the two frequencies, if it is less than 2π by L, I cannot get a distinct frequency component $\omega1$ and $\omega2$ at the DFT analysis.

So, what I said is that the two window functions overlap as a consequence of two spectral lines. I suppose getting one line here and one line here are not distinguishable because they are in overlap. So, what am I required? When I analyze DFT, I want there not to be an overlap. So, $\omega1$ minus $\omega2$ must be greater than 2π by L, so that is my resolution.

So, if it is this, if there will be no not-so-two separate lobes in the spectrum are distinguishable if $\omega1$ and ω difference between the two frequencies is greater than 2π by L. So, suppose I have a component here, let us say 50-hertz component here and let us say bandwidth in terms of hertz is let us say 30 hertz. So, I can say the maxim the resolution frequency resolution, which is defined by L, is 30 hertz to frequency must be separated by 30, so I can say twice π L is my resolution.

And that if I can only distinguish those frequencies, they are separable by $2\pi L$; if there exist two frequencies that are within $2\pi L$, I cannot separate that, so that is my frequency resolution. Is it clear? So, what is the limitation? Due to the multiplication of the window in the time domain signal in the frequency domain, I get a frequency response that is nothing but a combination of the signal frequency response and the window frequency response.

Now, see if I want to increase the resolution, so if my side lobe width if the side lobe width main lobe width is increased, then my resolution will be decreased; broad spectra frequency resolution will be decreased. Ok or not? So, if I want to increase the frequency liberation, then this lobe should be very sharp. There is another problem. What is the other problem? If I see if I suppose I have a sinc function like this, and if I plot the magnitude spectra, I will get like this.

So this is called the side lobe, and this is called the main lobe. Now, if the side lobe is the noise, the side lobe introduces noise for the other frequency. So, this is $\omega0$, and this side lobe introduces noise for the other frequency. So, there is a true trade-off. I have to reduce this side's low energy, and I have to suffer the main lobe.

So, I required a window whose main lobe is very sharp, and the side lobe is very small. That is my requirement for defining a window. So, I require a window whose main lobe is very sharp and very narrow and whose side lobe error is very low. So, that is my window requirement.

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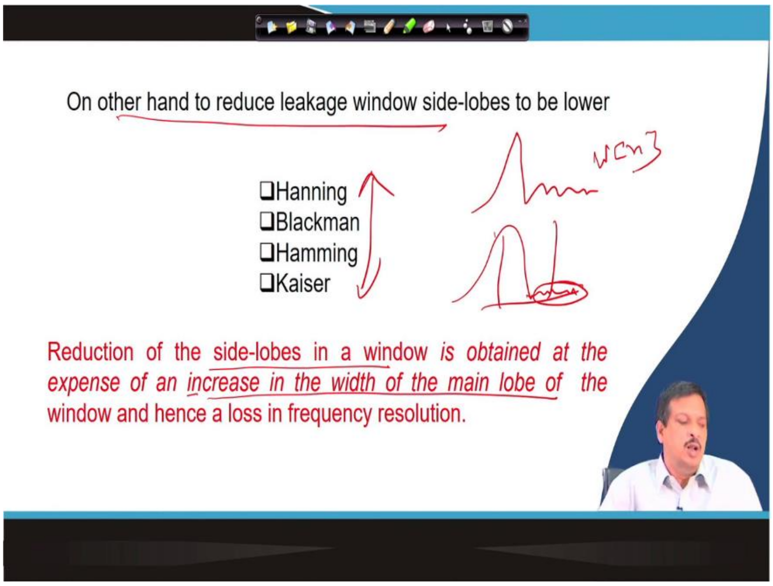

Now, if you see that window requirement, how do I define it? So, that side lobe produces a leakage. So, if I want to reduce the leakage, there are different kinds of windows: Hamming window, Hanning window, Blackman window, and Kaiser window, which reduce the side lobe at the expense of increasing the width of the main lobe.

Once the side lobe energy is reduced, the main lobe energy increases. So, in a rectangular window, I may say the main lobe is sharp, but the leakage is very high. If I want to go to the Hamming window or Hanning window, maybe the main lobe is very wide, and the side lobe leakage is much lower.

So, when I use the Hanning window, the frequency resolution is reduced because the main lobe width is high. So, it depends on which kind of analysis you want and which kind of analysis you want for your signal, which gives you the trade-off of choosing the window function.

So, w[n] by default, it is a rectangular window, but if I want to choose the Hanning window, Hamming window, Blackman window, or Kaiser window, there is all window functions are there.

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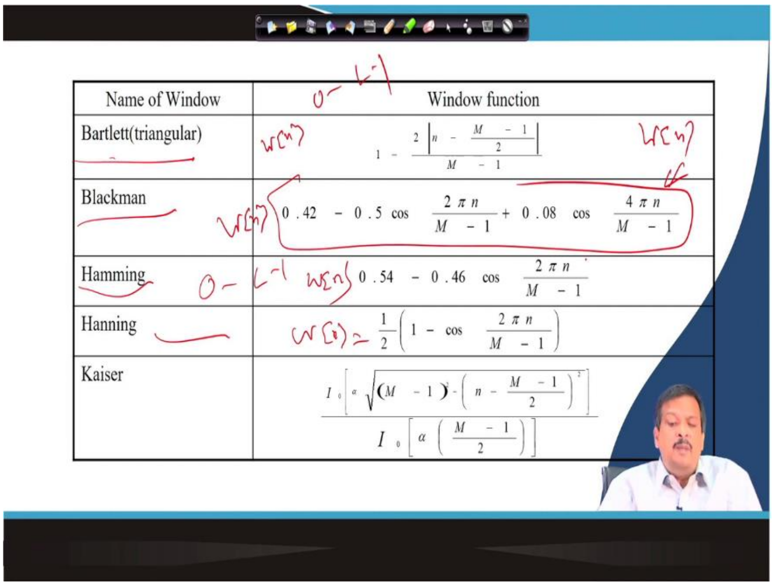

So, it is then Blackman window is this function. So, I can generate w[n] using this function. So, this is nothing but a w[n]. Let us say I generate w[n] up to 0 to L minus 1. I generate Hamming window 0 to L minus 1, which is w[n], and I Hanning window w[n]. So, this is the window function, which is a different window function I can use to generate w[n].

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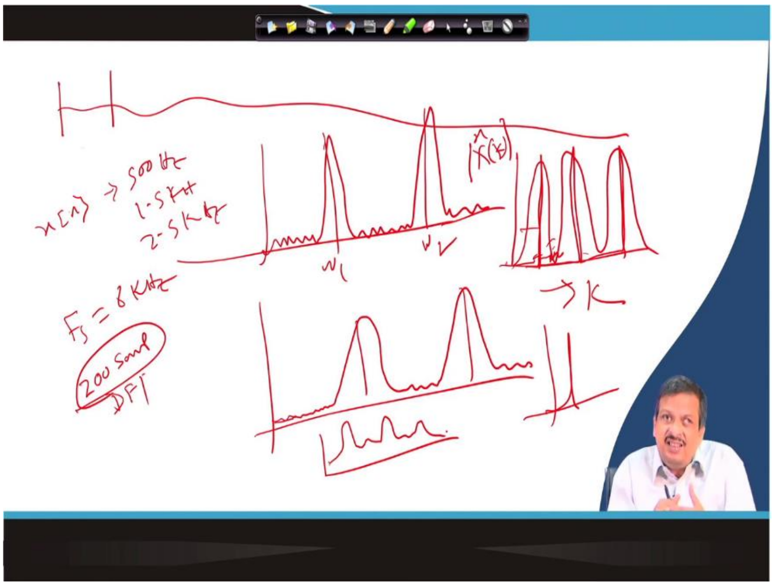

So, in summary, when I want to analyze a signal in the frequency domain of a long signal, I have to multiply the signal within the window. And the choice of the window depends on what I want. If I say I use a rectangular window, the leakage is much greater. So, if I say the rectangular window, my frequency and the spectrum may look like this: Let us say there is a noise, and then there will be a main lobe, there will be a noise, there will be a main lobe. So, this is my ω1, and this is my ω2.

Now, if I use the Hanning window, the noise may be reduced, but the main lobe width may be increased. So, this is Hanning's window. So, what can I do? It depends on my choice of what kind of frequency analysis I want. So, there are a lot of window functions available; you can use any one of the windows, ok? So, that is an important issue.

Now, again, for a practical example, you can do it. Let us say I told you to generate x[n] with a frequency component of 500 hertz, 1.5-kilo hertz and 2.5-kilo hertz and sample it F s equal to 8-kilo hertz then take a 200 sample and compute DFT and plot the spectrum of that DFT output.

So, I want the plot. This axis is the k, and this axis is the mod of the X k cap. Then you can see. So, what ideally do you expect? You expect there will be a spike single spike at 500 hertz, 1.5 kilohertz and 2.5 kilohertz depending on the amplitude you have multiplied. Now, when you say I am doing a DFT of 200 samples, you can see it will look like this due to the DFT leakage.

This is the main lobe, and there will be noise so that overlap creates a spectrum that is not a clear spike. There may be a spectrum that looks like this depending on the amplitude. So, I will, all of you can try it. Now, this limitation has come into the picture of signal analysis. A real example is the short-term Fourier transform. In speech processing, we use a short-term Fourier transform to analyze the signal in the frequency domain.

You may ask, sir, why do we require a frequency domain? We are very happy with the time domain, but we suppose you have to make some modifications at the spectral level. So, you have to go to the frequency domain modification and then take the inverse DFT. You get back the signal. So, DFT analysis and IDFT are the syntheses, so both are used in speech signal processing for short-term Fourier transform, which I will explain in the next lecture.

So, now you have an idea what will happen if my signal is long if I multiply by the I take a small portion of the signal; that means multiplying by a window function. So, in the next lecture, you can see how this affects synthesis and how this affects analysis.

Thank you.