Signal Processing Techniques and Its Applications
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Lecture - 27
Tutorial - 5

Ok. So, I have covered the properties of DFT this week and the linear filtering that overlaps the add and overlap save method. So, based on that, I will just solve a few problems so that you can understand whatever theory I have covered during the last 4 lectures.

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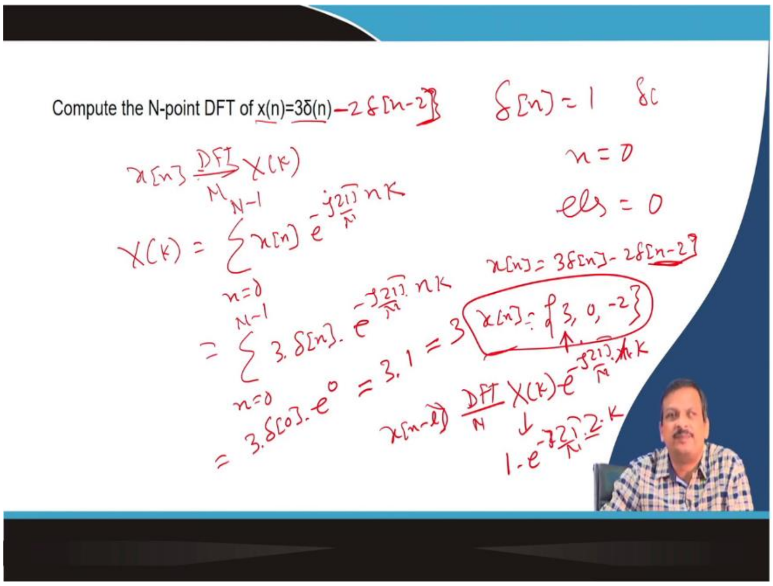

So, let us see the first problem is defined as computing N-point DFT of x[n]; let us see there is a simple first DFT, compute N-point DFT of x[n] 3δ[n], so, very simple. So, you know, ok. So, I know that what, let us say, is x N-point DFT of x[n] is equal to DFT and N-point is X(k). So, what do you know

$$X(k) = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi}{N}nk}$$

So, I can simply say n is equal to 0 to N minus 1 3δ[n] into $e^{-(j2\pi/N)nk}$. Now, you know the δ[n] is equal to 1 when n is equal to 0; elsewhere else, it is 0. So, I know this only exists when n equal to 0; so, 3δ[0] into e^0. So, I can say it is nothing, but a 3 into 1 is equal to 3.

Now, instead of δ[n], if I say 3δ[n] minus 2δ[n-2], 3δ[n] minus 2δ. So, my x[n] is equal to 3δ[n] minus 2δ[n] minus 2. So, what can I say? I can say my signal x[n] is nothing but a 3, 0, minus 2, where the 0th point is 3. So, what do you know?

You know x[n-l], what is the if I take the DFT N point DFT, what is the DFT X(k) into e to the power minus j2π by N into n l into k. So, if I say it is n minus 2, that means δ x[n] x δ is 1 at 2. So, it is nothing, but this is equal to 1 at $e^{-(j2\pi/N)2/2k}$. So, from the property, I can directly calculate, or I can calculate, take the signal and calculate the DFT ok.

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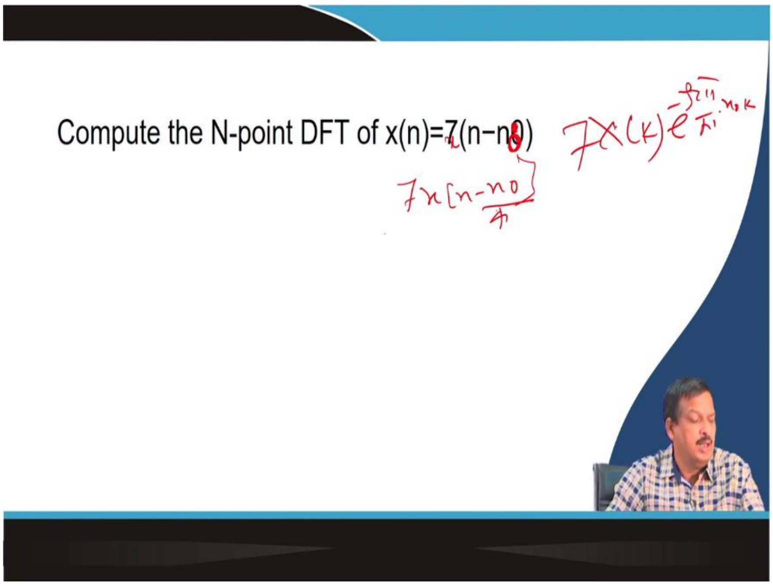

Let us say another compute that the N-point DFT of x[n] equals 7 into n minus n 0, n minus n 0. So, that will be x[n] minus n 7 into x[n-n_0]. Very simple, you can calculate n minus n0 as the shift x[n] is X(k), and n0 is the shift $e^{-(j2\pi/N)n0k}$, and 7 will multiply scalar multiplication ok.

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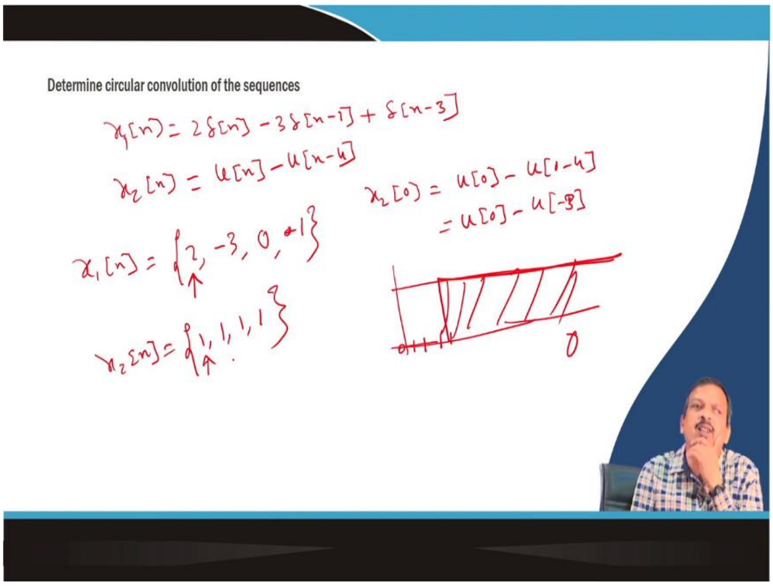

Let us say other, determine the circular convolution of the sequence, let us say first I do two things, I said x[n] is equal to let us say I said 2δ[n] minus 3δ[n-1] plus δ[n-3], x1[n].

And x2[n] is equal to, let us say, u[n] minus u[n-4]. From there, I first determine the signal, which is x1[n], and X[1] is nothing but a 2, minus 3, 0, 1, and here is the 0th point.

And what is x2[n]? So, let us say x2[0] is equal to u[0] minus u[0 – 4]. So, what is that? U[0] minus u[-4], so minus 3, which is nothing but a 0, or I can say the x2[n] is only there. So, if we see u[n], u[n] means all are there, and u[n-4] means 0, 1, 2, 3, 4; at 4, there will be a 1. So, this is subtracted. So, this part becomes 0. So, I can say x2[n] is nothing but a 1, 1, 1, 1 ok. So, I can do that. Now, what I said is to calculate the circular convolution of the sequence.

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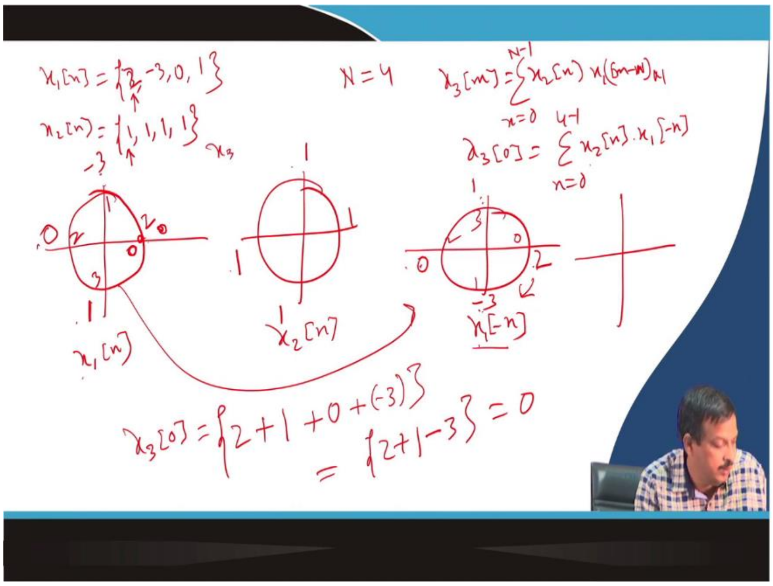

So, how do you calculate? So, I know x1[n]. So, I write down a top x1[n] is equal to 3, now what is 2, minus 3; 2, minus 3, 0, 1 and this is 0th sample and x2[n] is equal to 1, 1, 1, 1 and this is 0th sample. Now, I will calculate the circular convolution of N as equal to 4, and N as equal to 4.

So, 4 point circular convolution I have to calculate. So, what can I say? How do I represent circularly x1[n]? So, I draw that, seeing that you do not know anything, you just draw the circle. So, now, I get a 4 index; let us see this is index 0. So, inside, you write down the index; index 0 is this one. So, if I go clockwise, it is a positive index, so 0, 1, 2, 3. So, I know the 0th sample value is 2, the 1 sample value is minus 3, 0 is the second sample value, and 1 is in here. So, this is nothing but an x1[n].

Similarly, if I want to draw x2[n], so this is 0 1, this is 0 1, this is 0 1, 1. Let us they; let us do that circular convolution because 1 1 1, you cannot understand the shift. Let us say I want to calculate X[3] m, which is nothing but an x2[n] and X[1], you know, m minus N m minus N, circular convolution of N you know what is tell me, n equal to 0 to N minus 1 that is my circular convolution.

So, I am saying instead of X[2], I said X[1] I want to convert circularly. So, I have to calculate x3[0] is equal to n equal to 0 to 4 minus 1 x2[n] multiplied by X[1] minus n; so, X[1] minus n. So, if I want to draw the X[1] minus n. So, this is x1[n] X[1] minus n. So, minus n means flip the signal. So, if it is a 0 index, this is a positive index. So, this is a negative index, so 0, 1, 2, 3. So, 0 value is 2, 1 value is minus 3, 2 value is 0 and 3 value is 1.

So, x[n] 2 minus 3, 0, 1; X[1] minus n 2, 1, 0 minus 3; this side. Now, I know x3[0] is nothing but a sum of the product of x2[n]. So, x2[n] is 1. So, 1 multiplied by 2, 2 plus 1 multiplied by 1, 1 plus 0 multiplied by 1, 0 plus minus 3 multiplied by 1, minus 3. So, it is nothing, but a 2 plus 1 minus 3 is equal to 0.

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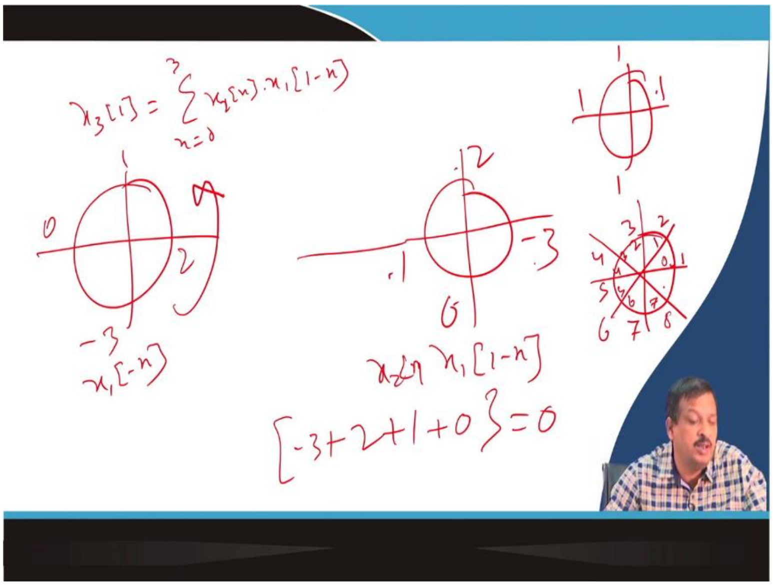

Similarly, when I say instead of this 1, if I say what is a x3[1], it is nothing but a sum of n equal to 0 to 3 x2[n] multiplied by x1[1] minus n. So, if I say this is my X[1], this is 0 2, 1, 0, minus 3; 2, 1, 0, minus, 3 is my X[1] minus n, I can say that x1[1] minus n should be the 1 positive shift. So, I have to shift this circle in 1.

So, 2 becomes; so I can say 2 goes to 1, 1 becomes here, 0 becomes 1 minus 3 comes here. Now, this has to be multiplied with x2[n]. So, I know x2[n] is nothing but an all-1. So, I can say minus 3, 1, minus 3 plus 2 1, 2 plus 1 1 1 plus 0 1 0. So, it is also 0. So I can calculate. So, instead of this kind of example, let us say I have taken another example; let us say I have an x1[n] calculate the 4-point circular convolution of x1[n].

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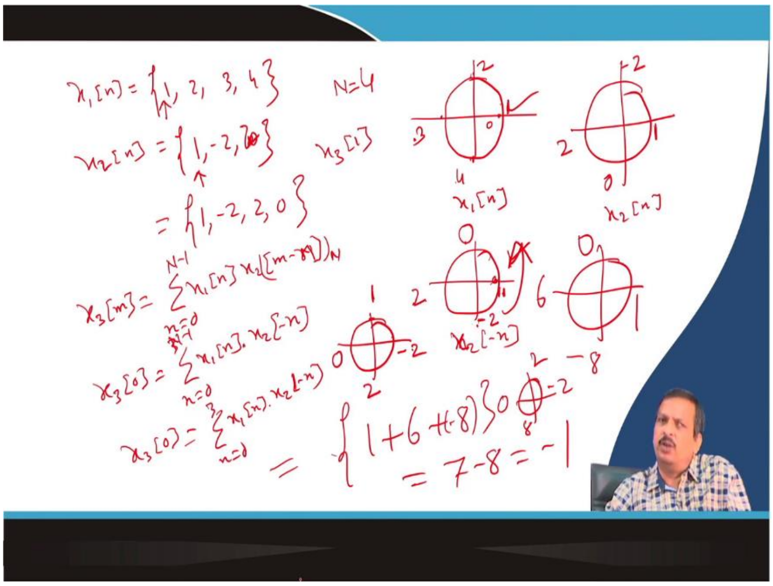

x1[n] is equal to, let us say, 1 2 3 4, and x2[n] is equal to x2[n] is equal to, let us say 1 minus 2 0. And I have to calculate whether 4 point N is equal to 4 or not 0; let us say this is minus 2 2 let us say. So, this length is 3. So, I have to make a (Refer Time: 12:47) 1 0 1, minus 2, 2, 0.

So, now I have to draw first you draw x1[n]. So, when you draw, think that this is your 0th sample and this is your 0th sample. So, an anticlockwise index is positive. So, if this index is 0, then this value is 1, and if this index is 1, the value is 2, this index is 2, the value is 3, this index is 3, and the value is 4, so this 1 is my x1[n]. How do I represent x2[n]? This index is 0th indeX[1], 1 index minus 2, 2nd indeX[2] and 4, 3rd index 0. So, this is my x2[n].

So, I know the circular convolution X[3] m is equal to n equal to 0 to N minus 1 x1[n] convolved with X[2] m minus N circularly. So, if I say the value of x3[0], So, I want to calculate x3[0], I know n equal to 0 2 3 x1[n] or x[n] minus 1 into X[2] minus n. So, if this is x2[n], then what is X[2] minus n X[2] minus n? So, X[2] minus n means holding a negative index. So, this index is 0, 0 is 1, minus 1 is minus 2, minus 2 is 2, and minus 3 is 0. So, this is X[2] minus n.

Now, I said x3[0] is nothing but a summation of n equal to 0 to 3 x1[n] into X[2] minus n. So, if it is 3, this is nothing but a product. So, 1 product will be, so what should be the product circle? This multiply with this 1, this multiply with this 0, these multiply with this 6; this minus 2 multiply with these minus 8. So, what is the sum? The sum is nothing but a 1 plus 6 plus 8 minus 8. So, which is nothing but a 7 minus 8 is equal to minus 1.

Now, when I say x2[1] and x3[1], this will be 1 minus n. So, 1 minus n means x of minus n has to be shifted this side 1 1 sample. So, if I want to shift, that means this is what? This value is 1, and this value is 1. So, 1 will go here and 0 will come here, 2 will be here and minus 2 will be here. Then the product will be minus 2 into 1, so the product circle will be minus 2, 1 into 2 2, 0 into 3 0, 2 into 4 8.

So, the sum of 2 minus 2 and 8 so it is nothing but an 8. That way, you can calculate the circular convolution for any sequence. I can also calculate the 8-point sequence. If it is 8 8-point sequence, you have to think this is my circle, so I get 4 points. So, this is 1, 2, 3, 4, 5, 6, 7, 8. So, this is the 0th sum 0th index, 1 index, 2 indexes, indeX[3], indeX[4], indeX[5], indeX[6], and indeX[7]. So, I can represent an 8 things. Now, again, you shift the same; convolution is the same, ok? So, that is circular convolution.

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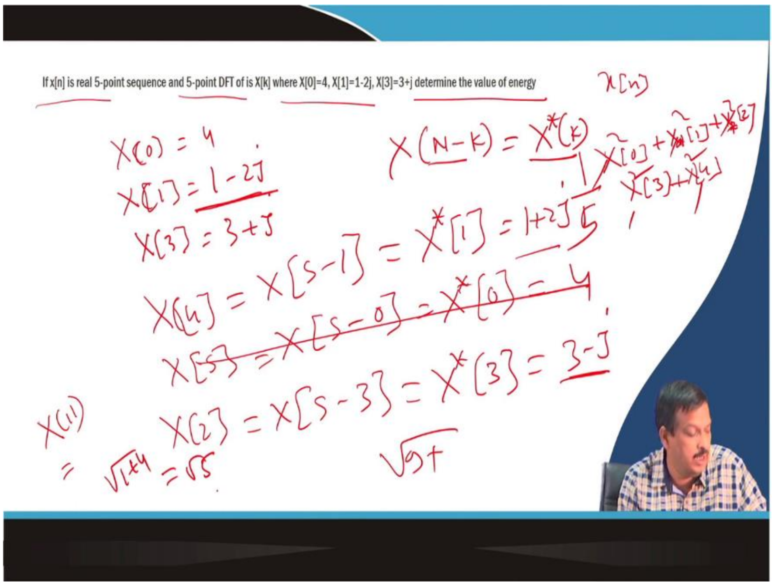

Next problem: Let us say if x[n] is a real 5-point sequence and 5-point DFT of X k is given, determine the energy value of the signal. So, I said, what is the energy of the signal? So what is given? X 0 equals 4, X(1) equals 1 minus 2 j, and X(3) equals 3 plus j; then what do you know? 5-point DFT X[n] minus k is equal to X star k.

So, X[n] minus k is the complex conjugate of X k. So, if I say X(4), it can be represented as X(5) minus 1, which is nothing but a complex conjugate of 1. So, what is the complex conjugate if it is 1 plus 2 j? 1 minus 2 j, so 1 plus 2 j. Now, what is the complex conjugate? X[5], X[5] is X(5) minus 0 again 4. So, I do not require X[5].

So, I have a 3 point is given. So, what is not given X[2] is not given; what is X[2]? X(5) minus 3 is nothing but a complex conjugate of X[3], which is nothing but a 3 minus j. Now, what is energy? Energy is nothing but an X 0 square plus X[1] square plus X[2] square plus 2 plus X[3] square plus X[4] square 5-point DFT. So, 1, 2, 3, 4, 5, and 5 components are there. So, since DFT is magnified by N, I have to divide by N; N is nothing but a 5.

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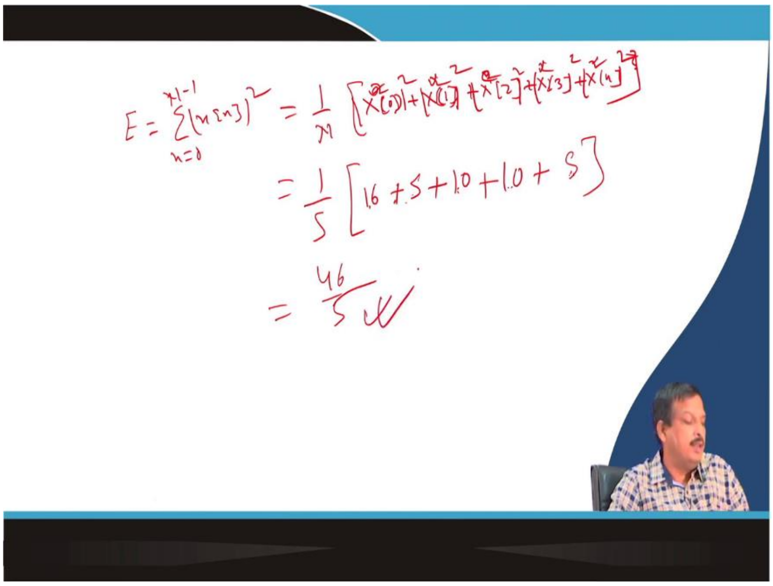

So, if I say the energy of the signal X[n] is nothing but an energy E

$$E = \frac{1}{N}\sum_{n=0}^{N-1}(x[n])^2$$

$$= \frac{1}{N}\left[x^2[0] + x^2[1] + x^2[2] + x^2[3] + x^2[4]\right]$$

$$= \frac{1}{5}[16 + 5 + 10 + 10 + 5]$$

$$= \frac{46}{5}$$

So, N is equal to 5. So, 1 by 5 into X[0] square, what is X of 0? If it is 4, the square means 16 plus X(1) and 1 minus 2 j. So, what is the mod of X(1) minus 1? What is the mod? So, this all will be mod. Sorry, all will be mod power; so, mod square mod X[2] square mod X[3] square mod X[4] square.

So, what is the power here? X[1]1 power is nothing but a root over a real square plus an imaginary square. So, it is the root of 5. So, the square of the root of 5 means 5 plus X[2], X[2] is 3 minus j root of 9 plus 1 10, root over of 10 square means 10 plus X[3] X[3] is 3 plus j again 10 plus X[4], X[4] is 1 plus 2 j. So, it is nothing but a 5; 1, 2, 3, 4, 5. So, I can say 5 plus 5. So, I can say 5 plus 5 10, so 10 10 20 30 30 46 by 5, okay? So, that is the energy of the signal.

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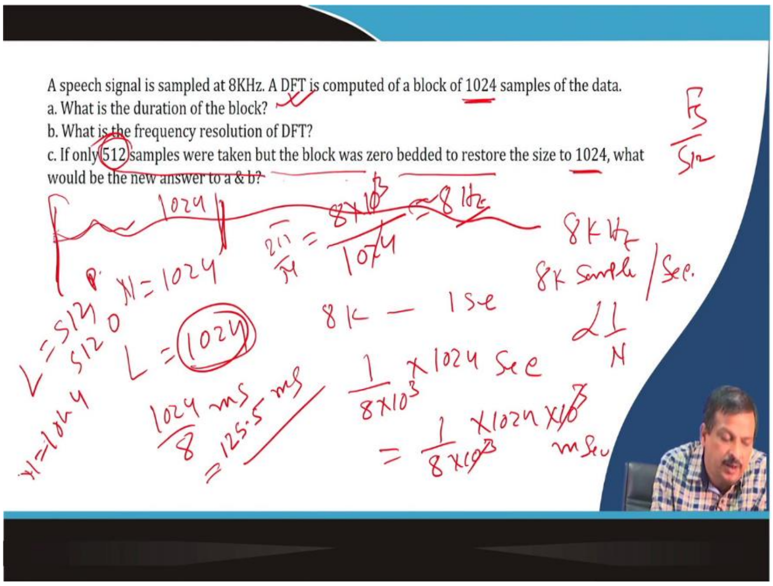

Similarly, let us say this: 1 A speech signal is sampled at 8 kilohertz. A DFT is computed of a block of 1024 samples of the data. What is the duration of the block? So, I have a speech signal sampled at 8 kilohertz; that means I can get 8 k samples in 1 second.

So, what is the length of the DFT? N is equal to the 1024-point DFT I have calculated. So, I have calculated a sample block, which has 1024 samples. So, L is equal to 1024; I know what the duration is; so, one 8 k sample at 1 second. So, this number of samples, how many seconds? So, I can say 1 by 8 into 10^3 into 1024 seconds, which is the duration.

If I convert it into milliseconds. So, 1 by 8 into 10^3 into 1024 into 10^3 millisecond, this is cancelled. So, 1024 by 8 millisecond ms. So, 1024 means you can check the block's duration 1 2 4 5 4 5, 125.5 milliseconds. What is the frequency resolution of the DFT? What is the frequency resolution of the DFT? What is the frequency resolution, you know? 2π by N, 2π is equal to F s, F s is 8 kilohertz 8 into 10^3 divided by N is 1024, approximately I can say 1000. So, it is approximately 8 hertz.

Now, if only 512 samples were taken, but the block was 0s added to do 1024-point DFT, then what would be the new answer of b? What is the resolution of the? Do you understand? I said instead of taking L, which is equal to 1024, I take L, which is equal to 512 points, 512 data points I have taken, and I padded up another 512 0 to make it 1024 point DFT. So, the resolution will never change; the resolution depends on the length of the DFT.

Now, if my length of the DFT is reduced to 512, then what should the new resolution? The new resolution will be again F s by 512, which is nothing but 16 hertz. So, you know that resolution is inversely proportional to 1 by N. So if we increase the length of the DFT, your frequency resolution increases, but what is happening?

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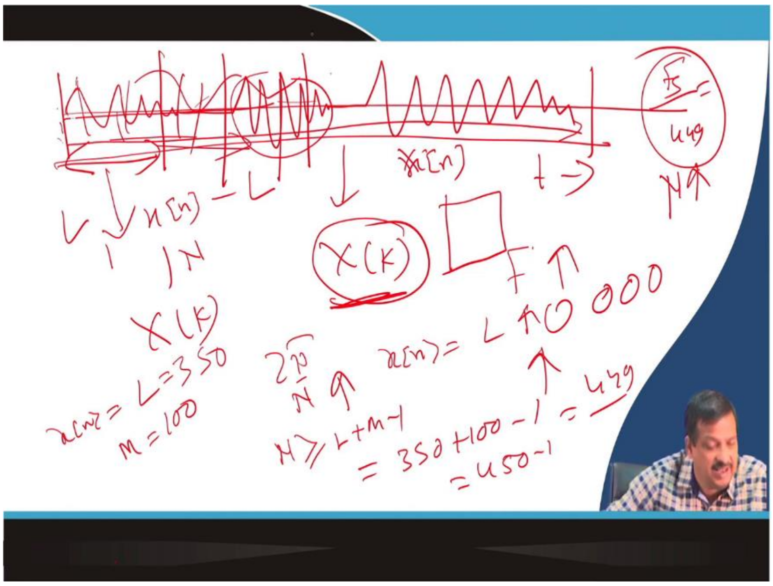

If you see, suppose I have a long signal. So, let us say it is a non-stationary signal; let us say speech signal. So, sometimes silence, then there is a signal, then silence. So, along the timeline, the signal is non-stationary. So, if I take an entire signal at a time, let us say this is my x[n] and do my X k. So, I get a frequency response, which is that I consider the entire time duration signal stationary; you know that this is the Fourier basis.

I am saying the entire time duration signal is stationary; that means a non-stationary signal, a stationary signal. Stationary means the frequency composition of the signal does not change along the timeline. So, it is an invariant signal, but the actual signal is a time-variant. So, if I take the frequency transform, I get an average frequency response of all the signals. So, it includes the silence response, including this portion.

So, now I said no since it is a non-stationary signal. If I make a small window, then small window I can say stationary. So, I am analyzing window by window of length L. Then I said I have taken a x[n] of length L, and I calculated X(k). Now, once I take n point DFT. So, the frequency resolution is 2π by N.

Now, if I increase N, I will have to take a larger sample. So, if I increase frequency resolution, time resolution is decreased, and I am not getting how the signal changes along the time line A time resolution increase means a small quotient of the signal and the length of the DFT is less.

Time resolution in decrease means a large portion of the signal I can take. So, if I increase the time resolution and frequency resolution, N will decrease less, and if I increase the frequency resolution, time resolution will decrease. Now, you may say no, sir, I will take x[n] very low L by padding up with 0s and making N very large. Then, one point is the computational complexity increases.

The second point is a very important point; the amplitude of the frequency response will get an average flat, and I cannot add so many 0. So, when I discuss the DCT, I will discuss this. So, this is the lacuna of the DFT, okay? So, that is that kind of thing.

So, use this in a practical scenario I may ask you that suppose I have a signal I want to sample frequency is given I want to compute the DFT length of this what should be the frequency resolution, what should be the time the amount of signal I have to taken what if that my filter response is these frequency response these, what is the length of the you can say that convolution, what are the minimum length of the DFT I should take?

Suppose I said I have a signal x[n] whose length is L, length is equal to 350, and I have a filter whose order is equal to, let us say, 100. What should be the minimum length of the DFT? So, you know the N should be greater than equal to L plus m minus 1, which is nothing but a 350 plus 100 minus 1. So, I can say it is nothing but a 450 minus 1, is 449.

How many 0 have to be added to the signal? m minus 1, 99 0; how many 0 will be added to the filter L minus 1. So, if the signal is sampled at 16 kilohertz, what should the frequency resolution be? You know the frequency resolution is nothing but an F s by N; N is equal to 490 449. So, that is nothing but the frequency resolution of my things, ok? So, that kind of answer you can give is okay. So, this we discuss about the 1 dimensional, discrete Fourier transform x[n]. Now, think about how I have an image.

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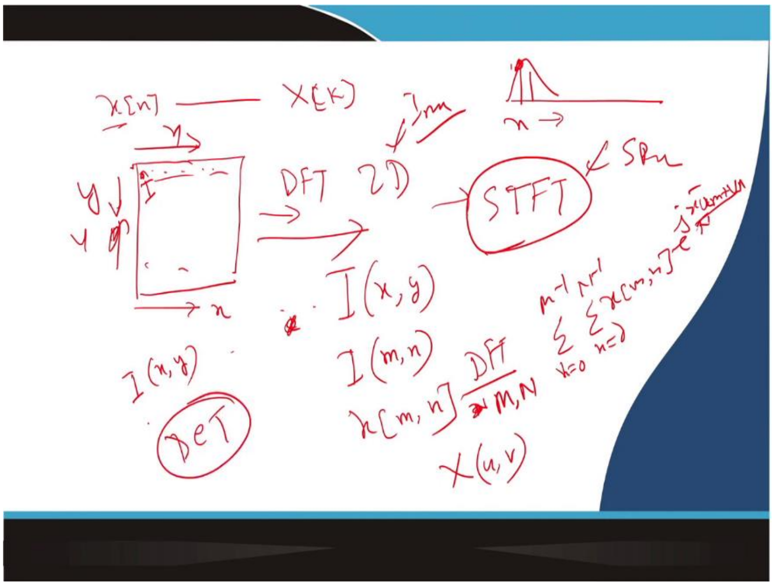

So, let us write taken one other slide; so, next week, I will start with 2 dimensional DFT and then go for STFT. So, what I say is if I have a x[n], I take the DFT X(k). So, only n index is varies. So, my signal is 1 dimensional, and x[n] varies on this side. Now, suppose I have an image. Now, if I can I do the DFT discrete Fourier transform? So, what is an image, what is an image signal? Is it nothing but a pixel?

So, if the pixel is represented by I. So, I is a function of x and y if this direction is my x and this direction is my y. So, it starts from here: this is x, and this is y. So, I is a function of x and y. So, now, like that x[n], the amplitude is a function of the n; I take that instant amplitude here; also, I digitize the image and get the pixel index. So, that is called spatial signal.

So, I my signal if my signal is x, or I can say the signal is image is I is a function of x and y, or I can say I have a digital index of both m and n instead of x[n], I have a I or signal or x which has a m and n both index, m 0 n0 first fix 0 0 pixel 0 1, 0 2, 0 3, 0 4, then 1 0, 1 like that way. So, those are the matrix representations; I have to do the DFT of this.

So, I have a 2 dimension, so 2 dimension I have to do DFT. So, I can take 1 dimension, and I can take DFT of 1 dimension and go to the other dimension. So, I have to take the m and n numbers let us say this is you can say the M and N number of DFT. So, when I say the M and N numbers of DFT.

$$\sum_{k=0}^{N-1}\sum_{n=0}^{M-1} x(n,m)e^{-j2\pi\left(\frac{kn}{N}+\frac{lm}{M}\right)}$$

So, when I say the DFT which is nothing but a X instead of k, there will be 2 indexes, u and v. So, I can say u with m plus v with n divided by N. So, details I will discuss in the next week that the; what is the 2 dimensional DFT, 2 dimensional discrete Fourier transform. Then, I discuss STFT Short Time Fourier Transform, STFT synthesis and STFT analysis. This is applicable to speech processing and image processing.

Next, I will discuss the DCT discrete cosine transform. So, next week, I will talk about speech processing, image processing, and discrete cosine transform related to the DFT, which is the difference between the DFT and DCT. I will discuss it the next week.

Thank you.