Signal Processing Techniques and Its Applications
Dr. Shyamal Kumar Das Mandal
Advanced Technology Development Centre
Indian Institute of Technology, Kharagpur

Lecture - 22
Properties of Discrete Fourier Transform (Contd.)

Ok. So, now, we say symmetry property is for a real-valued sequence.

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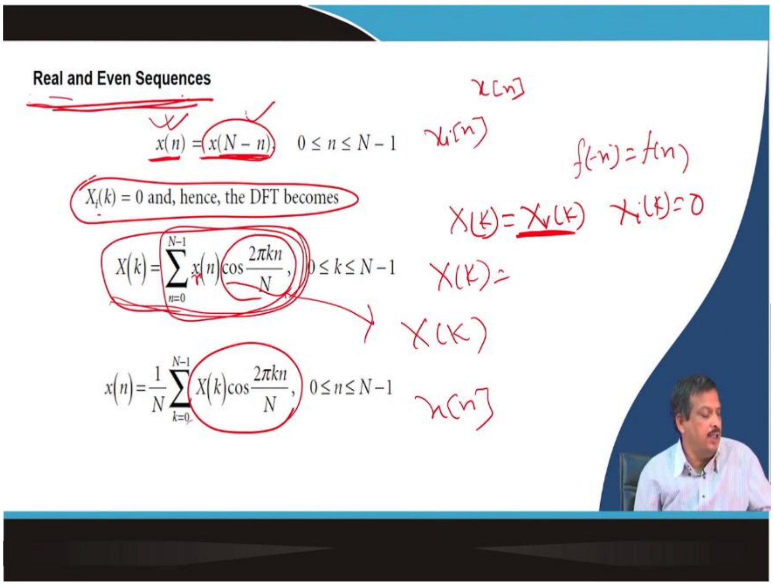

Now I said sequence is real and even sequence, real and even sequence. So, what do you mean by real and even sequence? Real and even; so, x[n] is real and even. So, a function is even if f[-n] is equal to f[n]. So, I can say in this case, the sequence is a symmetric sequence.

So, I can say x[n] is equal to x[N-n]. So, if I said x[n] is a real-valued even sequence, then I can say x[n] is equal to x[N-n] even sequence. So, when my sequence is even, that means my odd component of the DFT becomes 0, and the odd component of the DFT becomes 0. So, I can say X(k) is equal to only Xr(k). This Xi(k) will become 0 if it is an even sequence.

So, since it is real, then Xi n is also 0. So, I can say X(k) is nothing but an Xr(k), only the Xr portion. So, only it will be

$$X(k) = \sum_{n=0}^{N-1} x(n) \cos\left(\frac{2\pi kn}{N}\right)$$

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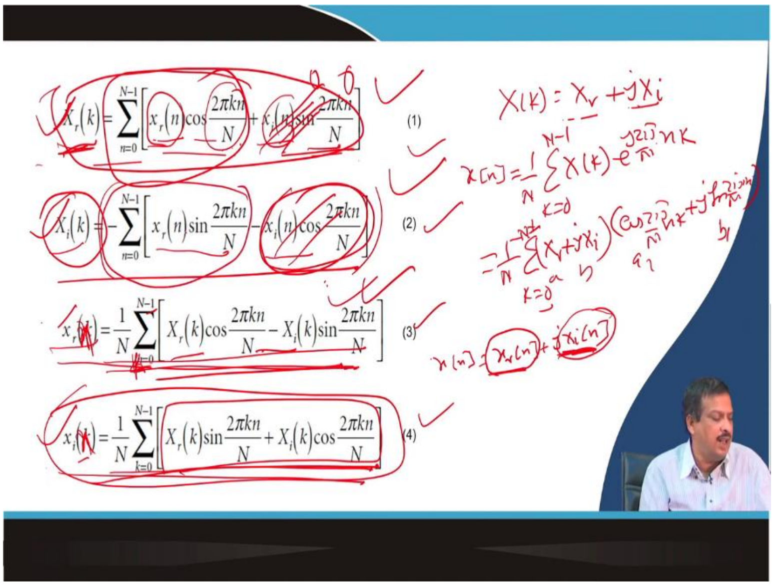

If you look at the expression, I will show Xr(k) x of r and x of i. So, this portion will be 0, and only this portion will be there. So, I can say X(k) is nothing but a; this is the DFT of the sequence. So, if my sequence is real and even sequence, then DFT is reduced to this one only. So, there is no imaginary part, only a real part, which is equivalent to a discrete cosine transform. Later on, we will come to that part, ok. So, the relation discrete cosine transform relationship will come to that part.

Similarly, if I take the inverse DFT of these X k, I will again get back the signal, which is nothing but a 1 by N X(k) cos ok. So, this is when the sequence is real and even sequence.

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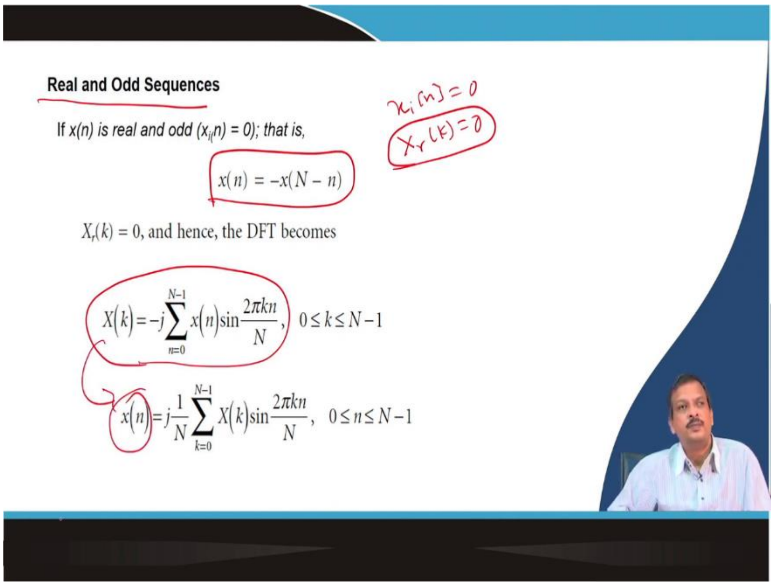

Now, the sequence is a real and odd sequence. So, an odd sequence means x[-n] is equal to -x[n]. So, this is the condition for an odd sequence if it is an odd sequence, real and odd. So, x small x of i n is equal to 0 and Xr(k) is equal to 0 because if it is an odd sequence, then Xr(k) will be 0. So, if Xr(k) is 0, then if this equation again goes to this equation, Xr(k) is 0. So, this part will not be there. Only X of only i k will be there, and then this part will not be there; only this part will be there.

So, minus x r n sin so ok. Similarly, if I take the IDFT, I get the x[n] sequence.

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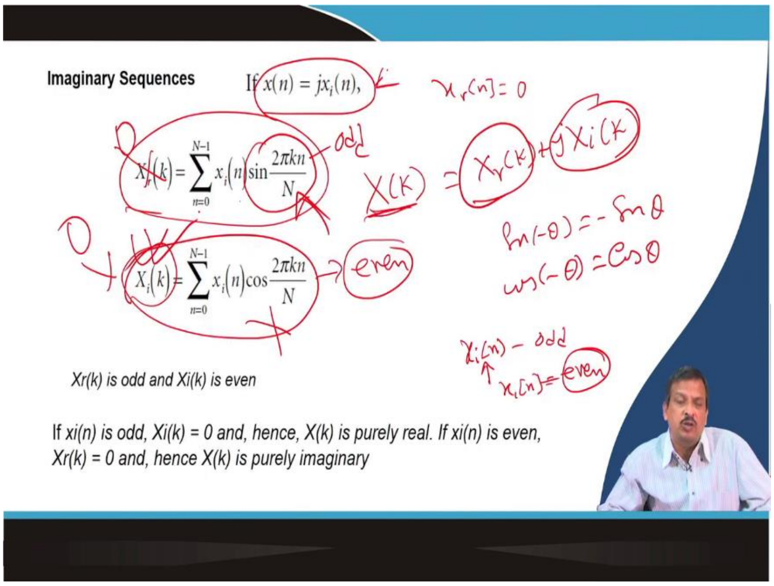

Similarly, if my sequence is purely imaginary, so, xr(n) is equal to 0 put that find out that expression. So, Xr(k) will be this one and Xi(k) will be this one. So, X(k) is equal to nothing but an Xr(k) + jXi(k).

Now, the interesting thing is that if the sequence is purely imaginary, then the real part becomes the sin component, which is odd. And imaginary parts, even components. So, sinθ is an odd function, and cosθ is an even function because sin minus θ is equal to minus sinθ, and cos minus θ is equal to cosθ.

So, cos is an even function, and sin is an odd function. So, now see that generally, the real part is an even function, and the imaginary part is an odd function. Now, here, if the sequence is purely imaginary, then I said the real part of the Fourier transform becomes an odd function and the imaginary part becomes an even function. Now, if I says my input I n is odd.

So, if x i n is odd, odd means the, if it is input is odd, then the frequency domain even function becomes 0 so; that means, X i k becomes 0, then I can say so, this part is 0. So, X k is purely the real part. Now, if my x i n is complex, but the function is even, if the signal is complex even, then I know the odd part becomes 0.

So, this part becomes 0 in the frequency domain, so this part is purely imaginary. So, for an imaginary sequence, if my sequence is odd imaginary, then the frequency transform is purely real. If my sequence is even imaginary, then my frequency transform is purely imaginary, is it clear? So, for imaginary sequence.

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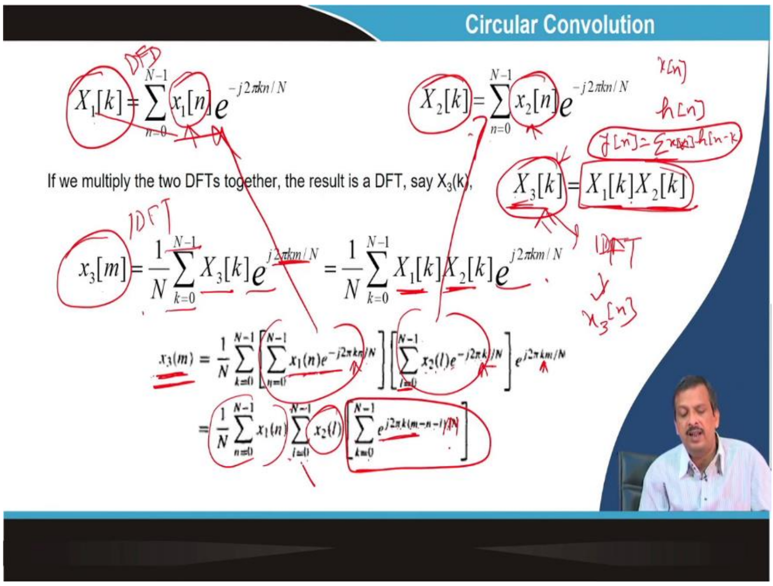

Now, if it is a so if for the imaginary sequence you know for the real sequence you know for a real even real odd sequence all I covered and I have also covered for a real sequence, the symmetry of a magnitude property ok. Now, I go to the circular convolution. So, you have heard about what is circular. So, how does this circular convolution come into the picture? Why is it used? What is the purpose? Why are you calling about circular convolution? You know the linear convolution.

Linear: This means that when a signal is input to a system, the output of the system is nothing but a linear convolution of signals and system response. So, systems impulse response h n. So, if I have a linear convolution, if x[n] is my signal and h[n] is my impulse response of a system, then the output of the system is nothing but a linear convolution of x[n] h[n-k] X(k) h[n-k].

So, you know that n is equal to k, equal to 0 to infinity or minus infinity to infinity. So, that is in real linear convolution. Now, I come to the circular convolution: why? So, interestingly, what you can observe is that suppose I have a sequence x1[n] and I have another signal x2[n]. So, I have a digital signal x1[n] and x2[n].

So, if I take the Fourier transform of x1[n] or DFT of n point DFT of x1[n], I get X 1 capital X1(k), and I get DFT of x2[n] is given in the capital X2(k). Now, if I say the product of these two, X 1 capital X1(k) multiplied by capital X2(k), the product is another signal, which is capital X3(k) that is it I multiply them, and I get another signal which is X3(k) capital X3(k).

Now, since all are in the frequency domain, if I take the inverse DFT IDFT of X3(k), I should get back a signal which is x3[n] time domain signal. So, let us say I take the IDFT of X3(k), and my time domain signal is x3[m] because here I am using n; that is why I said sequence length is m ok.

So,

$$m = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k) e^{j\frac{2\pi km}{N}}$$

That is the inverse Fourier transform inverse IDFT. Now, I put the value of X3(k) X1(k) multiplied by X2(k), and this will be e to the power remaining the same. So, x3[m] now I put the value of X1(k) and X2(k) X1(k) is this one and so, this one is this one and this one is this one.

Here I change the summation. Here, I take n, and here, I take instead of n l because of two different signals. So, this thing was m. I take l equal to 0 to N minus 1 x 2 l $e^{j2\pi kl/N}$. So, instead of l here I put l, and here I make n whatever n is there ok. So, it is k n by N. It is k l by N, ok. Now, if I see that if I do the product so, I take the summation side.

So, I can say n is equal to 0 to N minus 1 x1[n] and n equal to l equal to 0 to N minus 1 x 2 l and k is equal to 0 to N minus 1, and all e I put together because they do not have a k. So, I can k sum can be only e to the power. So, e to the power 2π k m is positive l is negative n is negative. So, minus n minus l divided by N.

So, this is my x3[m]. So, my x3[m] is this one, and k equals 0 to N minus 1 $e^{j2\pi(m-n-l)/N}$.

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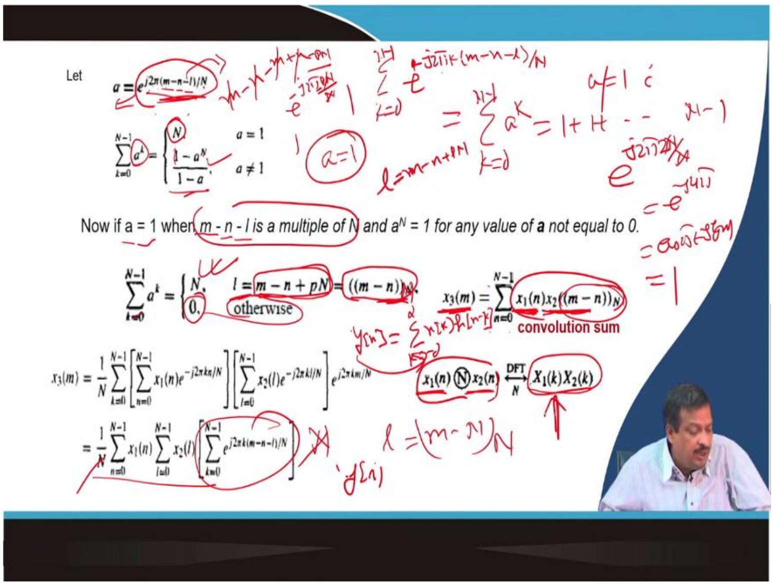

So, I can say the last part of the sum is k equal to 0 to N minus 1 $e^{-j2\pi(m-n-l)/N}$.

Let us say an equal to this one e to the power 2π 2 j 2π 2π m minus n minus l is equal to an except k. So, I can say it is nothing but a k equal to 0 to N minus 1 e to this part is a so, a to the power k is given because this expression does not have the k so, I said a to the power k is there. So, what is the summation of k equal to 0 to N minus 1 a to the power k?

So, if a is equal to 1, then it is nothing but 1 plus 1 plus 1 plus up to N minus 1. So, it is nothing but an n, but if a is not equal to 1, that is an N N-dimensional series. So, it is 1 minus a to the power N divided by 1 by a. If a N is equal to 1, it is purely 1 plus 1 plus 1 plus up to n minus 1 k equal to 0 to n minus 0.

So, n number of 1 is there so it is a n ok. Now if it is not 1, then it is 1 minus a to the power n divided by 1 by 1 minus a. Now, if I want to make this an equal to 1 when a will be 1, I want to make an equal to 1 when? If you see what is a, a is $e^{j2\pi(m-n-l)/N}$.

So, if I said m minus n minus l is an integer multiple of N. So, e to the power j 2π let us say it is 2 n by N is nothing but a $e^{j4\pi} = \cos(4\pi) + j\sin(4\pi)$. So, it is nothing but a 1. So, a will be 1 when this m minus n minus l is an integer multiple of N.

So, let us say I am writing down l is equal to l is equal to m minus n plus l is equal to m minus n plus p N. Now, if I put the value of l is here m minus n plus minus m plus n then this will be minus l so, minus p N so, I can say m m cancel n n cancel e to the power minus j 2π p N p is an integer divided by N.

So, N N cancels, so $e^{-j\theta}$ is also cosθ minus j sinθ. sinθ is 0, so cosθ is equal to 1. So, whether it is negative or positive does not matter. Now, if I share that so, what is the meaning of this? The meaning of this is that it is circularly shifting with a modulo of m m minus n, which is a modulo of n.

So, if m minus n is the modulo of n, then this sum becomes N. Elsewhere, it is 0 because if an equal to 1, this is 1 minus 1 divided by 1 is equal to 0, so, elsewhere, it is 0. So, the only consideration is that if l is an m minus n modulo n, then this sum this e to the power term becomes N. So, this N this is N will be cancelled.

Now, l is equal to m minus N mod N. So, I can say this is nothing but a n equal to 0 to N minus 1 x1[n] and x 2 l; l is nothing but a m minus n modulo N. This expression is also a convolution sum if I say linear convolution y[n] y[n] is sorry plane is not linear what is the linear convolution y[n] is equal to k equal to minus infinity to infinity X(k) h[n-k]; this is a linear convolution this is also x3[m] is nothing but a convolution of x 1 and x 2. But it is a circular convolution is a circular convolution.

So, what can I depict from this group? I can say that if I have two signals, x1[n] and x2[n], if I compute the circular convolution in the circular convolution time domain, the DFT of that is nothing but a frequency domain multiplication of the two frequency domain signals.

So, if X1(k) and X2(k) are in the frequency domain, I multiply if I take the inverse Fourier transform of that, and I get that x1[n] and x2[n] are circularly convolved. So, I can say the circular convolution between the two signals is nothing but an inverse DFT of multiplication of the frequency response of the two systems.

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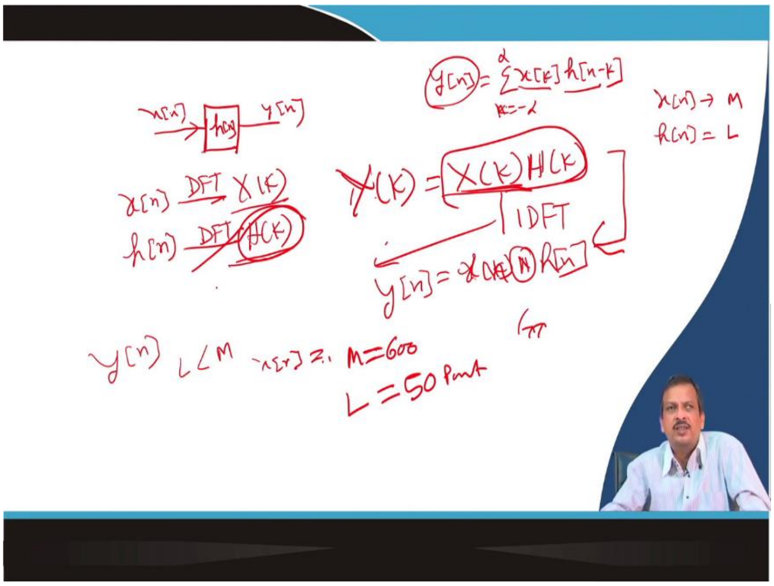

Let us give an example; let us say I have a system that has an impulse response h of n, and I have an applied signal x[n]. I get output y[n]. How do I calculate y[n]? The first procedure is y[n] in the time domain procedure is nothing but a linear convolution k equal to minus infinity to infinity X(k) into h[n-k].

So, this is linear convolution directly. I can calculate what y is. What is the other way? I can say that x[n] has a DFT Discrete Fourier Transform, which is X(k), and h of n has a discrete Fourier transform, which is nothing but an H(k). Now, if this is X(k) and this is H(k), then Y(k), Y(k) is nothing but a product of X(k) into H(k).

Then, if I say that the I DFT of this one is nothing but an X(k), the IDFT of this one is equivalent to X(k) circularly convolved with x[n] circularly convolved with h of n. So, the inverse Fourier transform of this product is equivalent to the circular convolution of X(k) and h of x[n] into h of n. So, what is the meaning of IDFT of this one? IDFT is nothing but a y[n], and the IDFT of the product is nothing but a y[n].

So, this is the use of circular convolution. So, in circular convolution, if I ask you what is the length of this y[n] let us say x[n] is a signal of length is M and h[n] is a signal which is length is L then what should be the length of output y n, if L is less than M, L is less than M, then what should be the y m? Think on it.

So, suppose I have one of x[n] input signals; let us say M is equal to 500 and let us say 600 points, and L filter response is, let us say 50 points. Then, what should the DFT length be in both cases? The DFT length of both cases must be more than 600. So, I will put 0 pad in here, make it 600, and then calculate.

So, I will come later on also that what should be the length of this convolution kind of thing or you also think on it what is the length of convolution of this thing ok.

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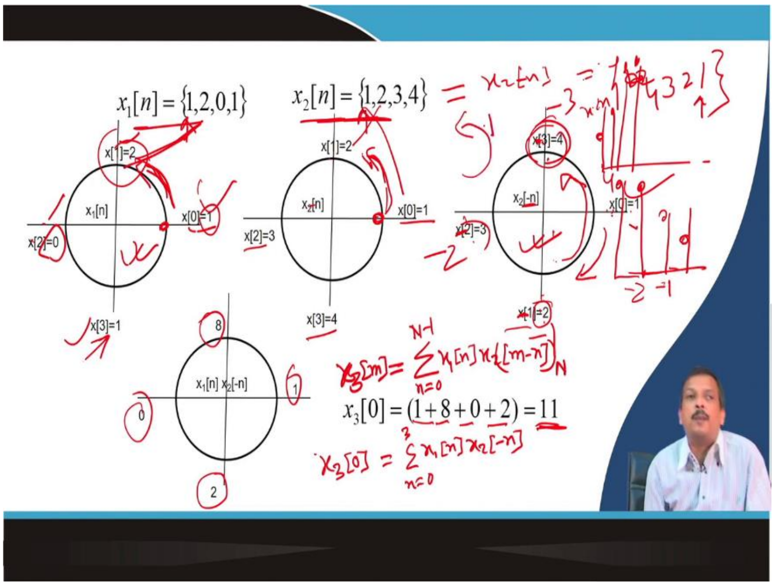

So, now give a circular convolution example; I will give an example, and I will end this class okay. So, what are the examples? Given what is, how do you compute the circular convolution?

So, circular convolution lets us say I have a signal x1[n], and I have another x2[n] ok signal so, if I want to represent the signal in x. So, x1[n] 0th sample is integration is this one, and here also 0th sample integration is this one so, if I want to represent in a circular sampling system. So, how do I represent? Let us say I start with this is my 0th index.

So, the index is positive if it is rotated anticlockwise. So, this is 0, this is 1, this is 2, this is 3. So, 0 value is 1 2 1 value is 1 1 sample is the first sample is 2, second sample is 0 and third sample is 1. Similarly, I can x 2. This index is 0, which is positive. So, 0 is equal to 1, 1 is equal to 2, the third second sample is 3, and the third sample is 4.

Now, what is x 2 minus n index is negative. So, if I said the clockwise index is positive, if the anticlockwise index is positive, then I can say the clockwise index is negative. So, if clockwise this is 0, this is minus 1, this is minus 2, this is minus 3. So, x2[n], what is the value of x 2 minus n? It is nothing but a 1, 2, so 1, 2, 3, 4. So, 1 2 3 4 1 will be here. You know what is x 2 minus n minus n. So, I have said that I have a signal x2[n], which has a 0th sample of 1, a first sample of 2, a third-second sample of 3 and a second, third sample of 4.

So, what is minus x2[n]? The 0th sample will be 1, the first sample will be minus 1, the sample will be 2, the minus 3 minus 2 sample will be 3, and the minus 3 sample will be 4. So, I can say x 0 is 1, minus 1 is 2, minus 2 is 3 and minus 3 is 4. So, you can put a minus 2 minus 3, ok.

So, this is x[-n]. Now, what I said? x of 3 m x of 3 m is equal to tell me n is equal to 0 to n minus 1 x1[n] into x 2 m minus n circular convolution circularly circular sequence of N ok. So, now, my length of DFT is 4. So, I can say x of 3 is m 0 0th sample. So, n is equal to 0 to 3 x1[n] multiplied by x 2 minus n.

So, this is my x 2 minus n, and this is x1[n], so it just multiplies so 1 multiply with 1 gets 1. This sample multiplied with this sample 2 into 4 8M 0 into 3 0, 1 into 2 2 then sum. So, it is nothing but a 1 plus 8 plus 0 plus 2 equal to 11 ok.

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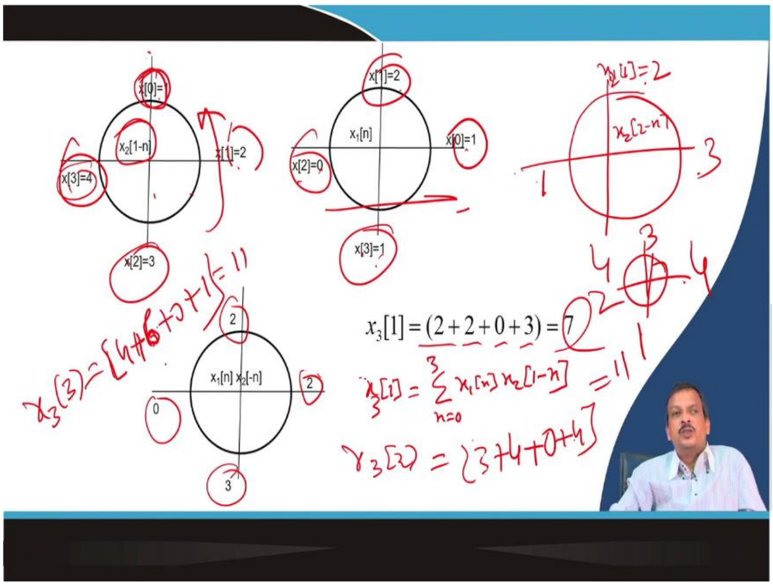

Now, said what is the value of x 3 1? So, x 3 1 is nothing but a n equal to 0 to 3 x1[n] into x 2 1 minus n. So, I want to make 1 minus n, so x 2 1 minus n. So, this is x 2 minus n. So, if I want to shift one sample to the positive side, then I get x 2 minus 1 and 1 minus n. So, the positive side means rotating this one anticlockwise in one sample.

So, if I rotate it, then 0 becomes here, so I can say x 0 will come to here, and x 1 will be the x of minus 1. This one will come to here. So, this one will come here, and this one will come here, and this one will go here. So, I rotate it rotate x[N-n] in anticlockwise 1 sample.

So, I got this one, and this one is my x1[n]. So, just product is nothing, but this multiply this, multiply this, and this multiply this. So, I get 1 into 2 2, 2 into 1 2, 4 into 0 0, 3 into 1 3. So, I get some 2 plus 2 plus 0 plus 2 is equal to 7. Now, what should we say? I want to draw x 2 2 minus n.

So, this will be shifted further one this direction. So, x so, this one will go here so, x 1 x x 2 1 will be 2 so, this will be 2, and this will be 1, and this will be 4, and this will be 3. Now, then, what should be the x 3 2 is nothing but this multiplied by this. So, 3 plus this multiplied by this 4 plus this multiplied by this 0 plus this multiplied by this 4.

So, 8 plus 3 11, then x3 minus n rotate it again. So, then 3 becomes here, 2 becomes here, 1 becomes here, 4 becomes here, then multiply again you get x 3. So, x 3 is nothing but a x 3 3 is nothing but a 4 multiplied by 1 4 plus 3 multiply by 2 6 2 2 multiply by 0 1 multiply by 1. So, that way, you can calculate the circular convolution one by one after another. So, this is the procedure for calculating circular convolution. Is it clear.

Thank you.