Signal Processing Techniques and Its Applications

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Lecture - 18
Discrete Fourier Transform Linear Transform View

Ok. So, we discussed the discrete Fourier transform. So, in the last class, we talked about the discrete Fourier transform frequency resolution. And today, we will talk about discrete Fourier transform as a linear transform view. So, before that, I just recap some portion of that previous lecture and then continue to the Discrete Fourier Transform as a Linear Transform View. So, let us know what the Fourier transform is.

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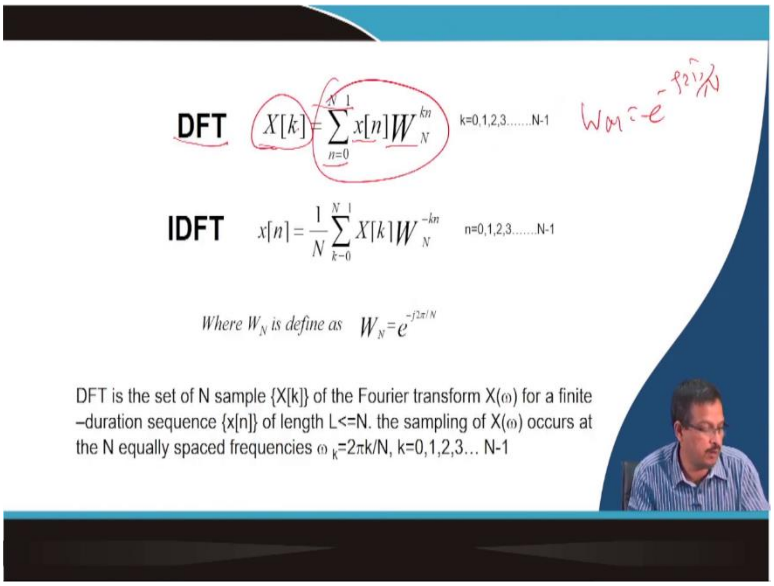

We have already said DFT is nothing but discrete Fourier transform

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$$
where $k = 0, 1, 2, \ldots, N-1$

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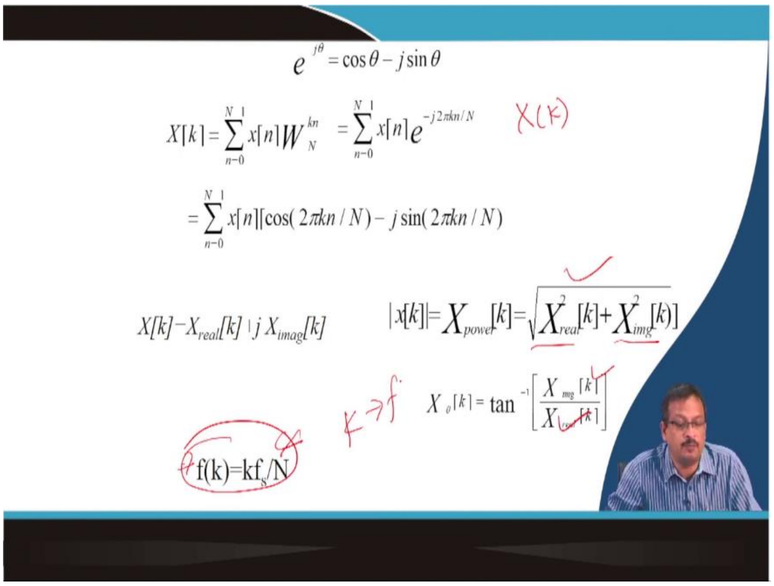

Then, what do we discuss? We discuss the properties, the different kinds of things, and the magnitude of the response. Because X[k] is a complex number. So, it has a magnitude and a phase. So, this is the magnitude, real square plus imaginary square, and the phase is tan inverse imaginary by real.

We also discussed the frequency resolution. So, we know that the relationship between analogue frequency and sampling, if the sampling frequency is given, N is given, then how the k is related to the analogue frequency we have discussed ok. So, let us say another thing. Let us say example that some of the examples we are talking about in DFT.

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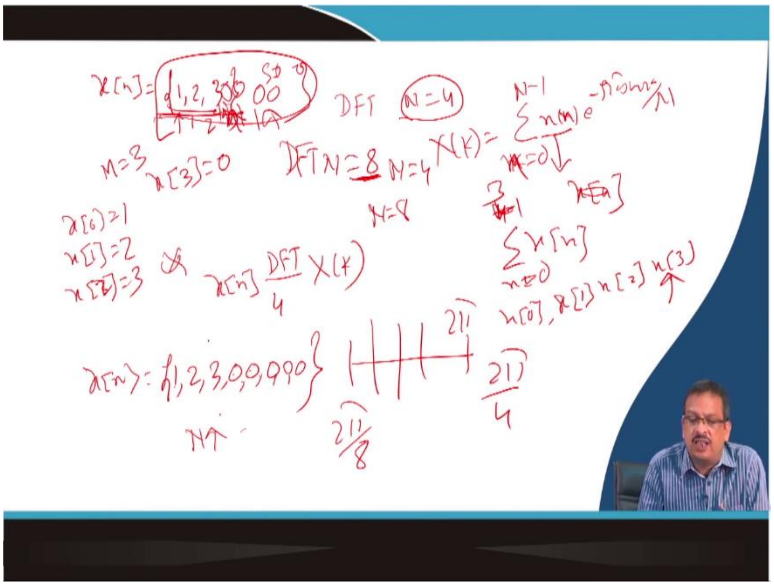

Let us say I have an x[n]; I have a signal which is, let us say, 1, 2, 3. Now, let us say I want to compute DFT of length 4, but the signal is that this is the 0th sample, this is the 1, and this is the 2. So, I have a signal whose length N lets us say M is equal to 3, but I want to compute DFT whose length N is equal to 4.

So, if I say, when I say k equal to 0 to N minus 1, X[n] e to the power minus j 2π by nk

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi}{N}nk}$$

So, if I want to do that, but I do not have x[4] because I have x[0] is equal to 1, x[1] is equal to 2, x[3] is x[2] is equal to 3, but I do not have, here what will come?

If n is equal to 0 to 4, 0 to 4 minus 1; So, x[n], this x[n] will come x[0], x[1], x[2], then x[3] because this is 0 to 3, 4 minus 1 is equal to 3. So, what is the value of x[3]? So, that means, I am assuming outside the length of the signal, the signal is 0. So, I can say x[3] is equal to 0. So, I padded a 0 here or inserted a 0 here, and then I said x[n] do the DFT of length 4 to compute X[k].

So, that is called zero padding; Even if I can say x[n] is equal to 1, 2, 3, I can compute the DFT of N equal to 8. Then, I have only 3 samples, but I am computing 8-point DFT. So, another 5 samples will be 0. So, that means, five 0, I will be padded in the length of the signal. So, my x[n] will be 1, 2, 3, 0, 0, 0 to the left will be 1, 2, 3, 4, 5, 6, 7, 8. So, all are 0. So, if I add 0, does it change anything?

Why do I add 0? Does it change anything? So, what do you mean by when I say N equal to 8, or what is the difference between N equal to 4? So, N equal to 4 means, I divided 2π frequency in four parts. So, if it is 2, the resolution is 2π by 4, but when I increase N, that means when it is N equal to 8, the resolution is 2π by 8.

So, I am increasing the frequency resolution, but signal-wise there is no change. Signal-wise, so, including the 0, once I inserted the 0, it did not change the signal, but it improved that frequency resolution, but the problem is that it also increases that computational complexity. As I said, suppose I have an X[k].

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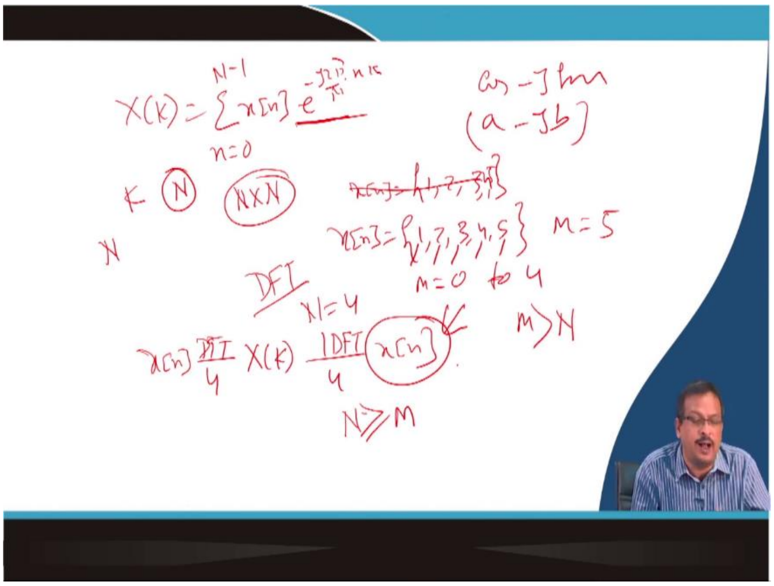

So,

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi}{N}nk}$$

Now, I said that computational complexity is a complex number. So, a complex number is nothing but a cos ± jsin. So, I can say it is a-jb. So, if it is a-jb, so for every k I require N number of complex multiplication.

So, for N number of k, I required N x N complex multiplication. So, when I increase the N using zero padding in the signal, I am increasing the computational complexity, but calculation-wise there is no change. I get the frequency response of the same signal of x[n] but with better frequency resolution. Can I make, suppose I have an x[n] and I have 1, 2, 3, let us say or 4, four number of let us say 4, 5 let us say x[n] 4 and 5. So, I have a signal x[n] that is equal to 1, 2, 3, 4, 5. So, I have a length. So, M, M is equal to 5. So, M equal to 0 to 4, M minus 1 is 4, 0, 1, 2, 3, 4.

Now, can I do N equal to 4-point DFT? Can it be allowed? Is it the signal? Suppose I apply a x[n] DFT with 4 points, I get X[k], and then I apply IDFT with 4 points. Can I get back x[n]? No, I cannot get back x[n]. Then, I have to consider part by part. So, there are two parts I can do. So, I have discussed this during the long signal DFT calculation. So, that is called STFT, Short Term Fourier Transform, which we will discuss later on.

So, if that M is greater than N, I cannot get back the signal x[n]; aliasing will happen. So, what are the requirements? My DFT must be greater than or equal to the length of the signal. If it is greater than that, then I can assume that all those signals that are outside the M are 0, which is called zero padding. Let us say do one example with zero padding.

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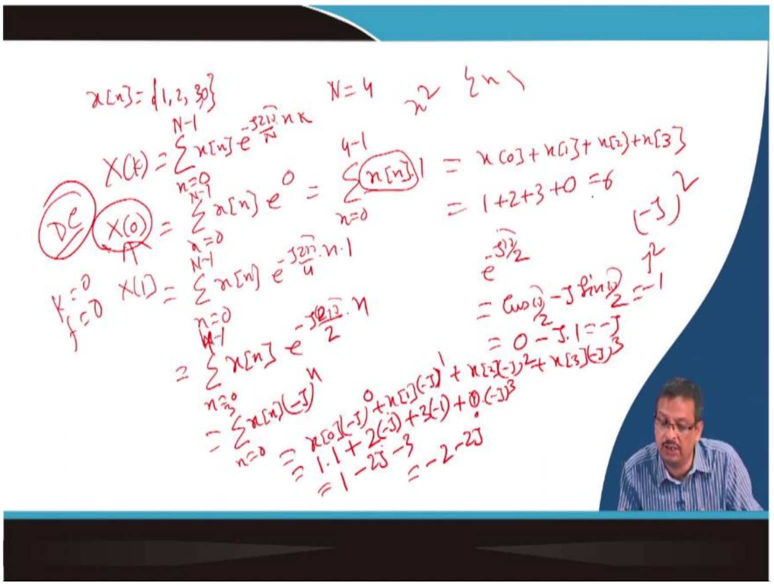

Let us say I have a signal x[n] that is equal to, let us say, 1, 2, 3. And I want to compute 4 point DFT, N equal to 4 ok. So,

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi}{N} nk}$$

Now, I said x[0]. So, I have to pad 0 here. What is the sample? 0. So, X[0] is equal to n equal to 0. So, I put n equal to k equal to 0.

So, k equal to 0, which means n equal to 0 to N minus 1, x[n] e^0, which is nothing but a n equal to 0 to 4 minus 1 x[n] into 1 equal to x[0] plus x[1] plus x[2] plus x[3]. So, 0 is 1 plus 2 plus 3 plus 0 is equal to 6. And, X[0], k equal to 0. So, frequency is equal to 0.

So, it is nothing but an average of the sample average value of the sample, which is why x[0] is called the DC component. It is not the energy. If I said energy, then x square has to happen because if it is negative, it will be cancelled. So, the average mean value of the signal, I am not doing 1 by n, is not there. So, I can say that the sum of the samples is X[0], which is frequency is equal to 0. That is why it is called DC value ok.

Similarly, what is X[1]? X[1], n equal to 0 to N minus 1, x[n] e to the power minus j 2π by 4 into n into 1. So, I can say n equals 0 to capital N minus 1 or 4 minus 1 x[n] e to the power minus j 2π by or π/2 into n. So, $e^{-j\pi/2}$ is nothing but a cos π/2 minus j sin π/2, cos Φ by 2. 0 minus j sin π/2 1. So, equal to minus j.

So, instead of that, I can write down n equal to 0 to 3, x[n] multiplied by minus j^n. So, I can say it is nothing but a n equal to 0, x[0] minus j^0 plus x[1] minus j^1 plus x[2] minus j^2 plus x[3] minus j^3.

So, if I say x[0] is equal to 1 into minus j^0 1 plus x[1] is 2 minus j^1 means minus j plus x[2] is 3 minus j to the power minus j square. So, j square j square is nothing but a minus 1 minus 1 plus x[2] is done. So, now, x 3, x[3] is 0. So, 0 into minus j^3 is equal to. So, it will be 0. So, it is 1 minus 2 j minus 3. So, I can say it is nothing but a minus 2 minus 2 j.

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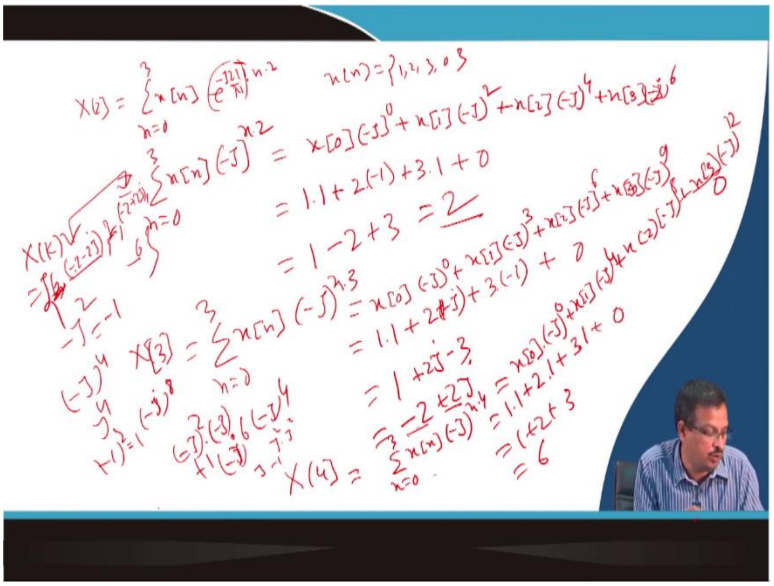

Similarly, I can calculate; let us say I take another slide, ok. So, now, X[2], X[1] I have calculated. So, k equal to 2 n equal to 0 to 3 x[n] $e^{-(j2\pi/N)nk}$ k equal to 2. So, I can say n equal to 0 to 3 x[n]. So, I already said that this part is nothing but a minus 2, so minus j^n into 2.

So, this is nothing, but I can say x[0] minus j^0 plus x[1] minus j^2 plus x[2] minus j^4 plus x[3] minus j to the power 6. So, I can say this is nothing but a 1 into 1 again plus 1, 2, 3. So, my x[n] is 1, 2, 3 and 0. So, 2 into minus j square, that means minus 1. So, I can say minus 1 plus x[2] is equal to 3 minus j^4.

So, it is nothing but a j^4, j^4 is minus 1 whole square equal to 1; so, 3 into 1 plus 0. So, I can say 1 minus 2 plus 3. So, it is nothing but a 2. Now, similarly, X[3] is equal to n equal to 0 to 3, x[n] minus j^n into 3, which is nothing but a x[0] minus j^0 plus x[1] minus j^3 plus x[2] minus j to the power I can say 3 into 2, 6 plus x[3] minus j^3 into 3, 9. So, I can say this is 1 into 1 plus 2 into cube minus j ok j square into minus j, so it is minus j.

So, minus j whole square into minus j, so minus j whole square means minus j. So, it is minus j, minus j is a plus j. So, it will be plus j plus x[2], x[2] is nothing but a 3, 3 into minus j whole cube whole 6. So, minus j cube means minus this is plus j. So, minus j is plus j. So, plus j into plus j; so, plus j into plus j minus 1 and plus 0. So, I can say this is 1 plus 2 j minus 3. So, I can say it is minus 2 plus 2j.

Similarly, I can calculate X[3] is done, X[4] also, X[4] you will see n equal to 0 to 3 again x[n] minus j^n into 4. So, this is nothing but again x[0] into minus j^0 plus x[1] into minus j^4, 4 plus x[2] into minus j^2 into 8 plus x[3] into minus j^3 into 4 12.

So, this part is 0. So, this part is 1 into 1 plus x[1] is equal to 2 into minus j^4. So, j square minus. So, that j square into j square. So, minus 1 into minus 1 plus 1, 1 plus x[2] is 3 minus j to the power 8. So, minus j to the power 8, again j to the power; so, it will be 1 only ok plus 0. So, I can say it is 1 plus 2 plus 3, which is nothing but a 6.

So, I get a sequence X[k], which is nothing but a first value. How much is it? The first value I have calculated is how much X[0] is equal to 6. So, X[0] is equal to 6, 1 is X[1] is minus 2 minus j. So, X[1] is minus 2 minus 2j, X[2] is 2, X[3] is minus 2 plus plus 2j minus 2 plus 2j, X[4] is 6. If you see that, all even components are real, and all odds components are imaginary. Even signal and odd signal ok. So, if I want to calculate the mod of X[k].

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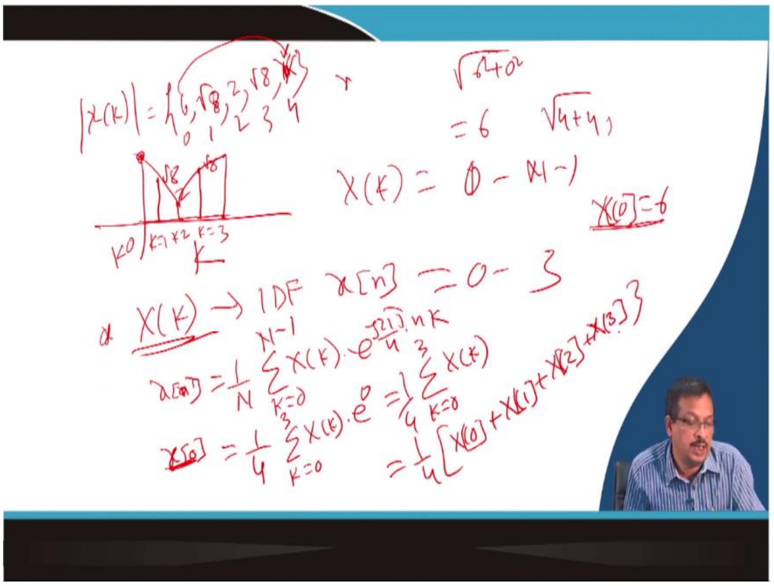

So, if I want to calculate the mod of X[k], then I know the first sample is only 6. So, the mod of X[k] is the root over the real square plus the imaginary square. So, 6 square is nothing but the 6, amplitude ok. Then, root over a real square plus an imaginary square; what is the value? 2 minus 2; suppose 2 squares means 4 plus 4. So, I can say this 6 root over of 8.

Then, again, I have a 2, the value is 2, and X[2] is 2. So, I know the value is 2, and X[3] is nothing but a minus 2 plus 2. So, 4 roots over 8; again, I have 6. So, this is a mod value. So, now, if I want to plot it, this axis is my k, k equal to 0, it is 6; k equal to 1, it is root 8, k equal to 2, it is 2; k is equal to 3, it is root 8 and k equal to 3 k equal to 3.

So, k is equal to 0, k is equal to 1, k is equal to 2, k is equal to 3 and k is equal to; so, 0, 0, 1, 2, 3, 4. So, if you see the X[k] will not be 4, X[k], k varies from 0 to N minus 1. So, it is 0 to 3. So, if you see the same component repeat back. So, I will not write this one understand? So, now, I will explain DFT symmetry later on in the properties of DFT. DFT has symmetry properties.

You can see the 6 is coming back. So, I can say it is a symmetry property n by 2 symmetry; so, I will come later on when we will talk about the properties of DFT. So, that way, you can calculate the DFT and see what you see. So, here x[n], now suppose I calculate now I X[k] is calculated. Now, if I take the IDFT of X[k], I should get back the signal of x[n]. So, what is IDFT?

So, I know x[n], x[n] is equal to 1 by N, k equal to 0 to N minus 1 X[k] multiplied by $e^{(j2\pi/4)nk}$. Let us say whether it is coming back or not, and I have to check. So, what is the value of X[0]? What is the value of X[0] is equal to 6? X[0] is 6, 6. So, I said X[0] is 6 ok. Let us say. So, x[0], this capital X[0] equals 6.

So, I say the 0th sample is equal to 1 by N, 1 by 4, k equal to 0 to 3 X[k] and e^2 j k n equal to 0; so, e^0. So, I can say it is nothing but a 1 by 4, k equal to 0 to 3 X[k]. So, now, you do that. So, 1 by 4, X[0] plus X[1] plus X[2] plus X[3]. Now, what do you know? My X[k] is nothing but a 6 minus 2 minus 2j. So, I just write down my X[k].

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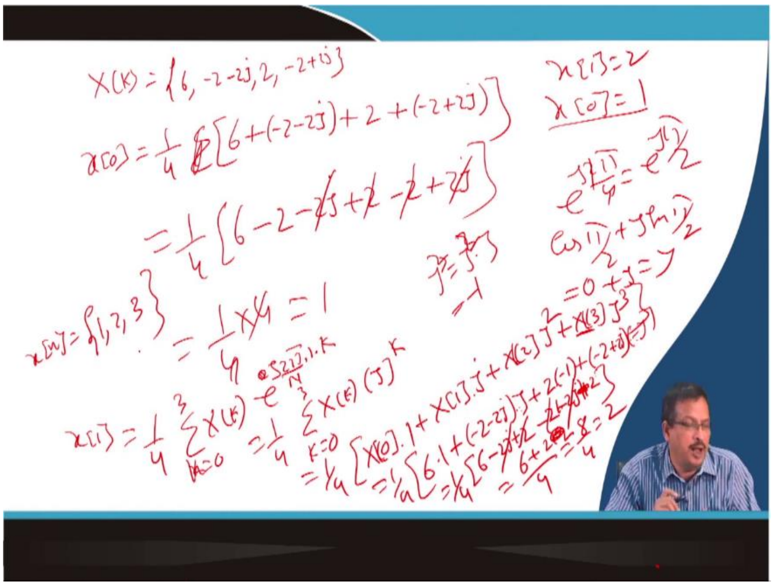

So, my X[k] is 6 minus 2 minus 2j, then, again, I think 2, 2, then again, I have a minus 2 plus 2j minus 2 plus 2j; that is my X[k]. So, what is x[0]? Signal first sample 1 by 4 summation I have already written. So, X[0] 6, X[1] minus 2 minus 2j, X[2] plus 2 plus minus 2 plus 2j. So, it is nothing but a 1 by 4 6 minus 2 minus 2j plus 2 minus 2 plus 2j.

So, 2j 2j cancel minus 2 plus 2 cancel. So, it is nothing but a 1 by 4 into 4 is equal to 1. So, x[0] is equal to 1, which is my first sample. I have considered signals 1, 2, 3 and 0. So, 1, 2, 3 and 0, I consider my x[0] to be equal to 1. Now, what is x[1]? Is nothing but a 1 by 4 n equal to k equal to 0 to 3 X[k] into e to the power minus j 2π plus 2π by N nk n equal to 1 k will be there.

So, $e^{j2\pi/4}$. So, it is nothing but $e^{j\pi/2}$, which is nothing but a cos π/2 + jsin π/2, which is nothing but a 0 plus j is equal to j. So, I can say 1 by 4, k equal to 0 to 3 X[k] into j^k, n equal to 1 j^k.

So, I can say 1 by 4, X[0] into 1 plus X[1] into j^1 into j plus X[2] into j to the power j square plus X[3] into j to the power cube plus see 1 by 4, X[0] 6 into 1 plus X[1] minus 2 minus 2j into j plus X[2] is 2, j square is minus 1 ok plus X[3], x of capital x[3] is minus 2 plus 2j into j cube, j cube means j square into j.

So, it is nothing but what j square is? j square is minus 1 ok; so, minus j. Now, if you calculate 1 by 4 6 j minus 2j minus 2j square minus 2j square minus 1, it is a plus 2 minus

2; here will be minus j. So, minus-minus plus 2j and minus j minus 2j square minus 1 plus 2 I think. So, if you see this will cancel plus 2 minus 2, this will be minus 2, and one will be minus 2.

So, minus 2, this will be no, and this will be 2 plus 2. So, this is nothing but a 6 plus 2 plus 2 divided by 4, I think so. one 2 will be cancelled, 6 plus 2 will be there; 6 plus 2 by 4. So, it is nothing, but an 8 by 4 is equal to 2. So, x[1] is equal to 2. So, I can get back the signal x[n], which is equal to 1, 2, and 3, if I do the inverse DFT.

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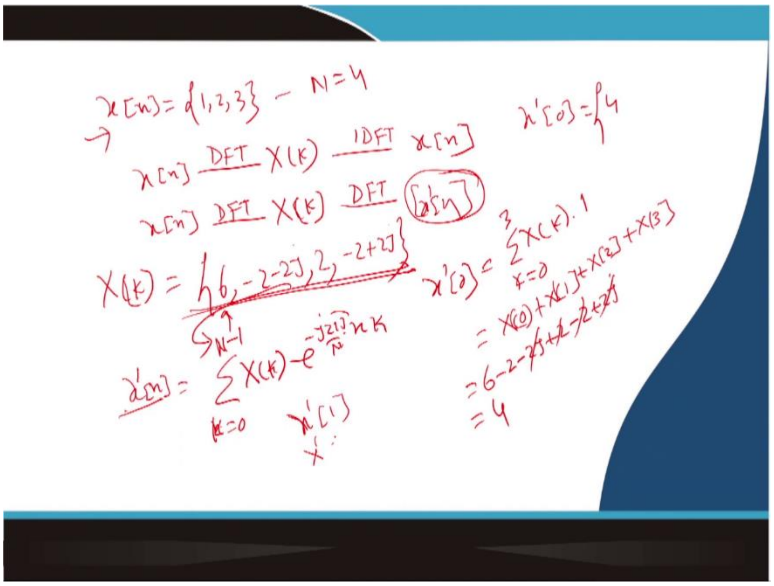

Now, if I told you, suppose I have a signal x[n], which is equal to doing it yourself 1, 2, 3, ok. So, x[n] if I take DFT, I get X[k]. Now, if I take IDFT, then I get x[n], which we have proved. Now, instead of that, suppose I have an x[n]. I take the DFT, and I get X[k] again. If I take DFT, what should I get? Should I get x[n], or what should I get? You do it, do it by yourself; you check, let us know this is the signal. Take the N equal to 4, 4 point DFT, calculate and see what is coming out.

So, I know DFT. So, I know X[k], X[k] is equal to I have already done it 6, minus 2, my 6, minus 2 minus 2j; second sample is 2; third is minus 2 plus 2j; minus 2 plus 2j ok. This is the 0th sample. Now, if I say instead of taking inverse DFT, I retake the DFT. So, what will I get? I will get x of it. Let us say this is x prime n. So, let us x dash n is equal to. So, n equals, let us say k equals 0 to I am considering this is a signal.

I am considering this as a signal, and this is another signal. So, I am letting this signal transfer to DFT. So, k equal to 0 to N minus 1 X[k] $e^{-(j2\pi/N)nk}$. So, if I say what is the value of x dash 0? I know it is only a k equal to 0 to 3, X[k]. So, it is 0 and 1.

So, it is nothing but a. I can say k is equal to 0 to 3. So, X[k]. So, it is x[0] plus x[1] plus x[2] plus X 3. Again, so, I can say that this is nothing but the 6 plus minus 2 minus 2j plus 2 minus 2 plus 2j, which is nothing but a 4. So, I can say x prime 0 is equal to 4 ok. Then you calculate x prime 1, x prime 2 and see what happens. So, now, let us. So, you can calculate DFT on your own, ok. So, in the next lecture, let us talk about the linear transform view of the discrete Fourier transform because I have already covered this example part in this lecture. So, in the next lecture, I go for the Linear Transform View.

Thank you.