Signal Processing Techniques and its Applications
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Lecture - 15
Inverse Z - Transform

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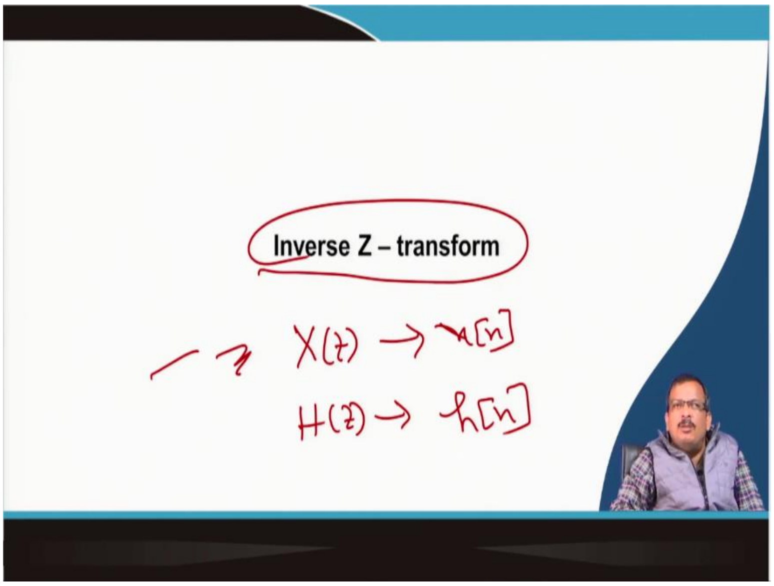

Ok. So, in the last class, we discussed about the pole-zero concept in the Z transform. Now, in this class, we will try to discuss the Inverse Z transform and what an inverse Z transform is. So, if you see the inverse, what do you mean by that? So, I have an X(z). Can I get back x[n]? So, X(z) is given to me, I want to find out the x[n] like the H(z) is given to me. The transfer function is given, and I have to find out the impulse response of the transfer function.

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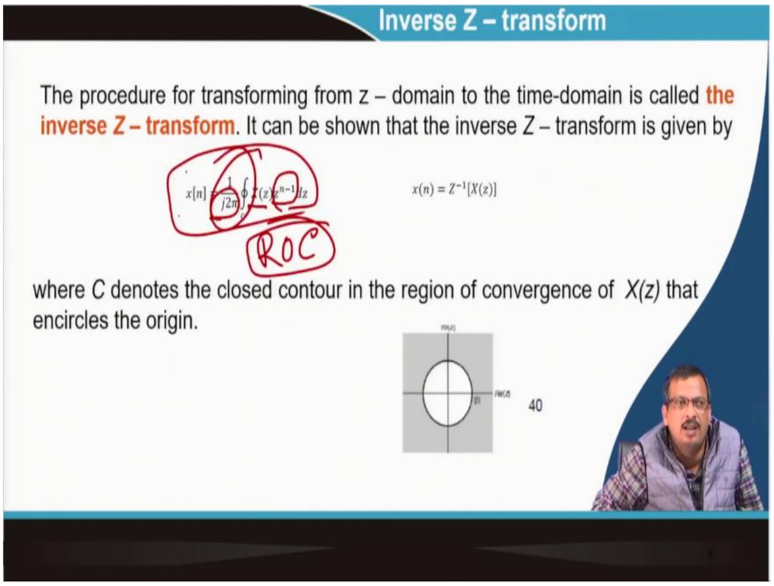

So, that is called inverse Z transform. What is its definition? The procedure for transforming from the Z domain to the time domain is called inverse Z transform. So, X(z) is given, I have to convert to time domain signal x[n].

$$x[n] = \frac{1}{2\pi j} \oint_C X(z) z^{n-1}\, dz$$

So, this is the integration why this C is given. C is called ROC within that ROC close contour of the region, and C denotes the close contour of the region. So, that means, within that ROC or close contour, I have to integrate X(z) to get back the x[n]. So, this is the definition.

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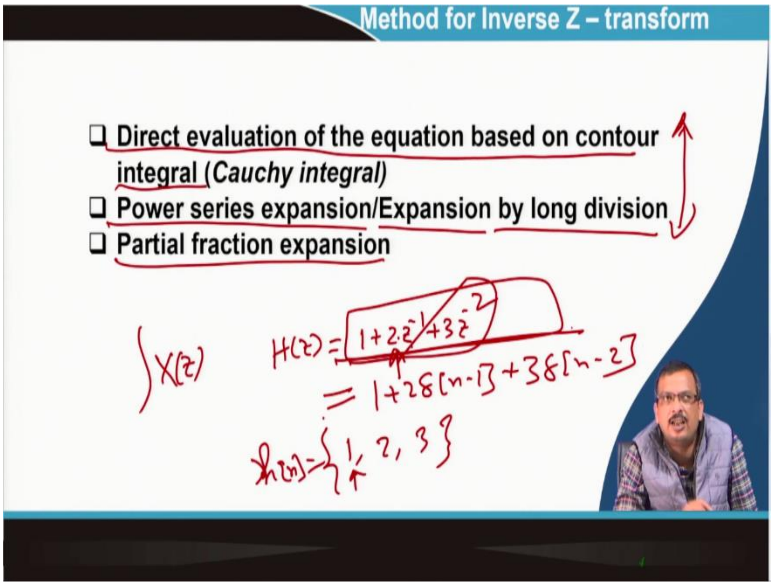

So, what are the procedures? There are three procedures of inverse Z transform. One is called the direct evaluation of the equation based on the contour integral. If you see this is the contour integral function, I am not going into details on the. So, you know that causing integral it is called.

We have already done it in mathematics score. So, using contour integral, if I know X(z) directly, I can apply contour integral to find out x[n]. The next method is power series expansion, expansion by long division, and partial fraction expansion, which are the three methods used for inverse Z transform.

Now, what is power series expansion? How do I do the power series expansion? So, as you know, I suppose I forgot about the word inverse. Suppose you know that I have a function H(z).

$$H(z) = 1 + 2z^{-1} + 3z^{-2}$$

Then what is the meaning? This means that if I say this sample is delayed by one sample and this one is delayed by a second sample. So, if I want to express the δ function of this transfer function, I can say 1 plus 2 into δn minus 1 plus 3 into δn minus 2, or I can say this is nothing but A1, 2, 3, and this is the origin of the signal. So, this is h[n]. So, whatever that is given, I have to express in this form that this is called power series expansion.

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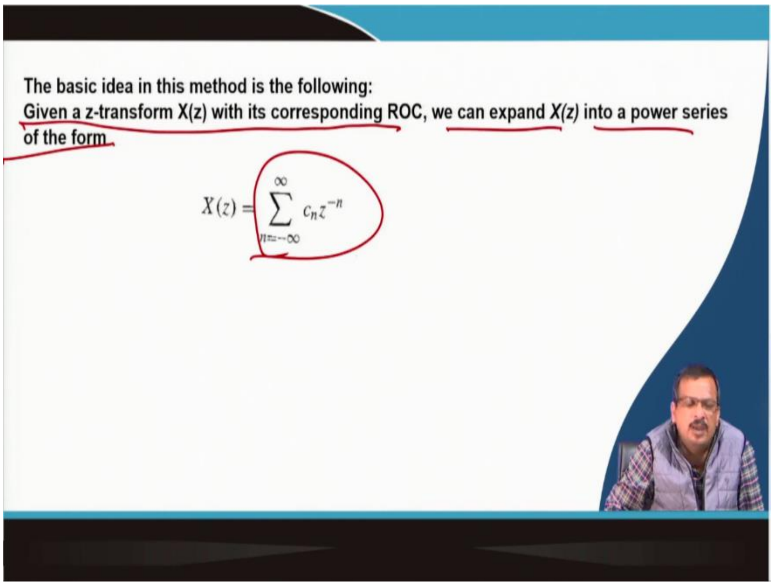

So, how do I do that? So, given A(z) transform with corresponding, we can expand X(z) into a power series form coefficient multiplied by z to the power something.

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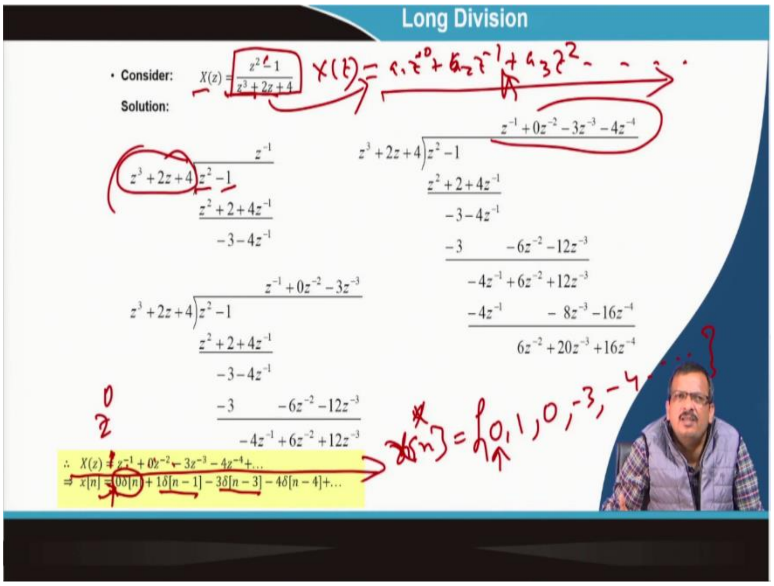

Let us see an example, let us say

$$X(z) = \frac{z^2 - 1}{z^3 + 2z + 4}$$

So, X(z) if I want to go for the power series expansion, I have to represent in some way that A1 into z^{-0} plus b A2 into z^{-1} plus a 3 into z^{-2} like that way I have to represent this one.

So, how do I do that? Simply in algebra long division. So, this is nothing but a this is divided by this. So, this is z^2 minus 1 divided by z^3 plus 2 z plus 4. I go for long division.

Once I go for long division I can find out this kind of expression. So, this is the expression. So, if this is an X(z) expression, then I know. So, what is there z^0 is not there; that means, 0 into δn, now 1 into z^1 sample delay δn minus 1 2 into my 0 into δn minus 2 3 z^{n-3}. So, I can say x[n] is nothing but a x[n] if the first sample is 0, the second sample is 1, the third sample is 0, then minus 3, then minus 4. That way, I get the time domain signal x[n].

So, that is called a power series. So, given the z domain transfer function, I can express it in a power series using long division, and then I can find out the representation in the time domain of the signal. So, whatever the transfer function is given, I have to express it in that way. So, how do we do that? This is simply a long division method; I can get the time domain signal.

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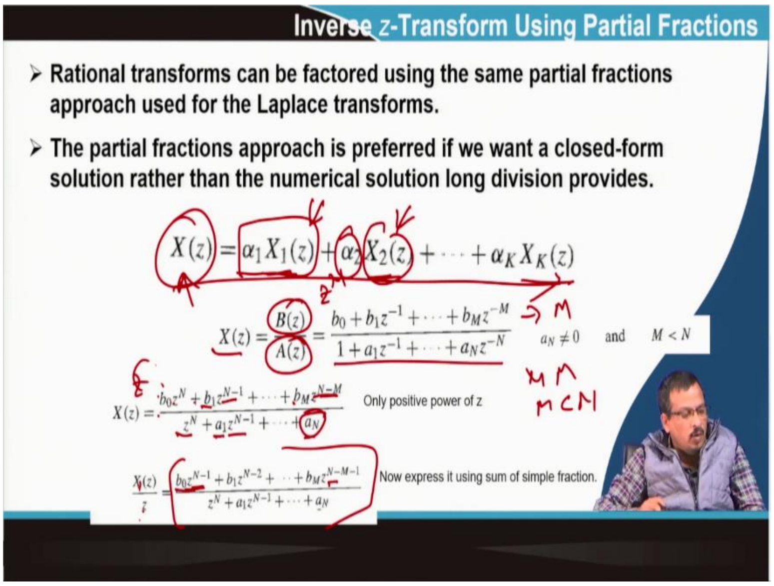

The next one is using partial fractions. So, rational transformation can be, you know that I am not reading the slide. You can read on the slide. So, what is what I do with partial fraction, what do you mean by that; that means, suppose I have a transfer function I have a transfer function X(z) somehow, I have to break it so that it consists of, let us say X1(z), X2(z), Xk(z) because the superposition principle is support this document linearity is supported.

So, I can say I have to express X(z) in terms of α1 X1(z), where I can find out the time domain representation of X1(z) very easily, and X2(z), I can find out the unknown representation multiply by α2 X3(z). So, if I am able to express it, then by partial fraction methods, I can calculate the value of α1, and if I know the time domain representation of X1(z), I can easily do that inverse Z transform.

So, how do I do that? Let us say X(z) is equal to B(z) by A(z). So, B(z) looks like this, and A(z) looks like this. So, there is an M number of 0 and an M number of poles. Now, if I make it positive power b0z to the power. So, if it is M is less than N I multiply by z^N. So, I can say $b0z^N$, $b1z^{n-1}$, and bMz^{N-M}, and the lower side also multiply z^N.

So, z^N z A1 into z^{n-1} and the last one is z to the power minus N plus z^N becomes 0. So, it is An. So, it is only the positive power of z. Now I said X(z) by z is equal to b0. So, I just take one z this side outside. So, this becomes $b0z^{n-1}$, and this is n minus 1 M minus N. So, X(z) by z, I get this one ok.

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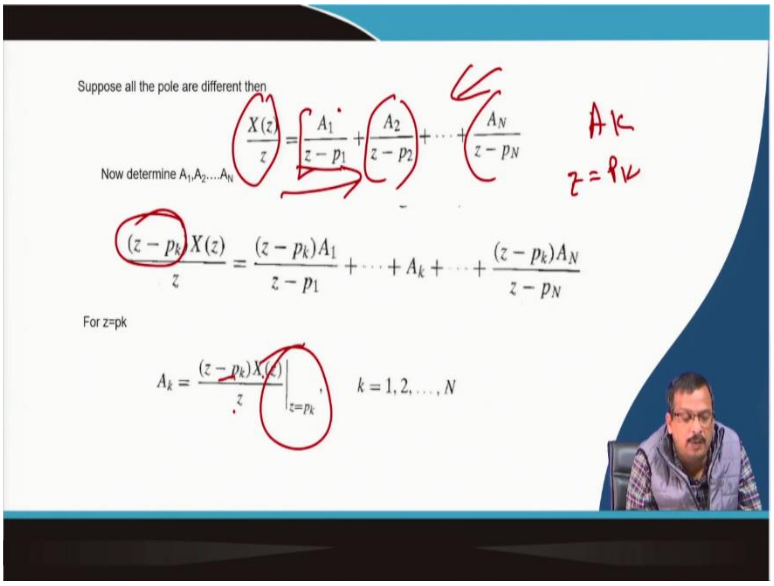

Now, I said I suppose this X(z) by z that can be a summation of A1 by z to the power z minus p1 A2 by z^{-p} that can be. So, I can factorize the lower portion, and I can express it in terms of this summation. Now I have to determine A1 A2, and An how do I determine A1 A2?

I will evaluate each of the X(z) by z by each of the pole pairs. So, let us z. I want to evaluate for z equal to pk or find out the value of Ak ok. So, Ak is equal to z minus pk π into X(z) by z at z equal to pk, I can find out the value of Ak. Let us see an example because if you have an understanding that is a little bit problematic, take an example.

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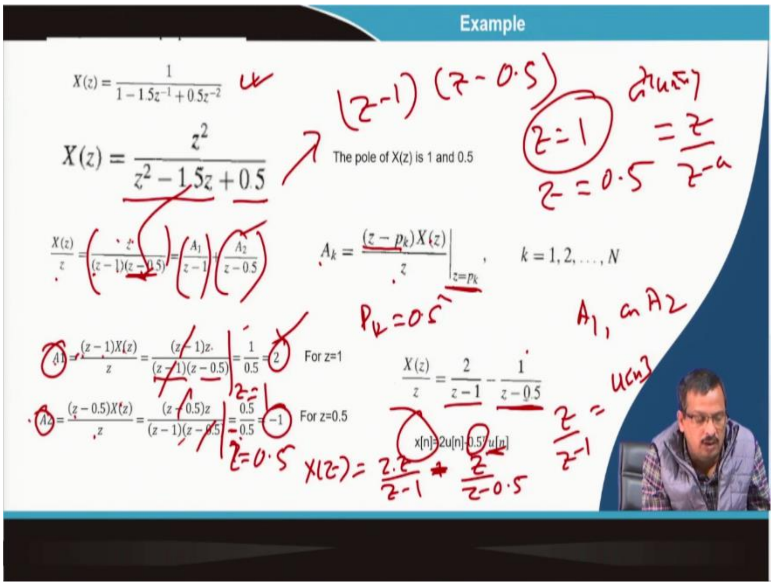

Let us say X(z) is given like this 1 by 0.5 into z^{-1} plus 0.5 will z^{-2}. So, if I simplify it. So, multiply z^2 by both sides. So, the upper side is z^2. So, z^2 minus 1 5 z plus 0.5. Now, if I factorize this. So, this will be z minus 1 into z minus 0.5. So, there are two poles: one is at z equal to 1, and the other is A(z) equal to 0.5. Two poles are there.

So, now I said X(z) by z is nothing but A(z) by a factor of this thing; let us say X(z) by z by this can be represented at A1 by z minus 1 plus A2 into A2 by z minus 0.5. So, I have to find out the value of A1 and A2. How do I do that? For the formula, Ak is z minus pk X(z) by z at z equal to pk.

So, I will multiply z minus 1 if the pole first pole is 1. So, z minus 1 X(z) by z, which is nothing but A(z) minus 1, divided into what is X(z)? X(z) by z is z by z minus 1 into 0.5. So, this, this cancel now I evaluate at for z equal to 1. So, this is nothing but A1 by 1 minus 0.5 0.5 2.

Similarly, what is the value of A2 again? I multiply. So, the pk value is equal to 0.5; I put it here: X(z) by z then, this, this cancel so, z by z minus 1 at z equal to 0.5 so, 0.5 by minus

0.5 which is nothing but a minus 1. Now, once I get that, then what is the equation? I know X(z) by z is equal to 2 by z minus 1. So, the A1 value is 2, and the A2 value is minus 1.

So, minus 1 by z minus 0.5, now then what is X(z)? I can say X(z) is equal to 2 into z by z minus 1 plus minus z by z minus 0.5. What is z by z minus 1 is nothing, but a u[n] as per the table I had done in the z transform in the table similarly. what is z my z by z minus 0.5? It is nothing but a 0.5 to the power. So, you know that a to the power An u[n] the z transform is equal to z by z minus a.

So, here, a is 0.5. I can say this is nothing but a 0.5^N into u n. So, this is my x n. So, I can calculate the inverse transform using fraction partial fraction, or I can calculate the inverse transform using power series expansion, or I can calculate the inverse transform using contour integral.

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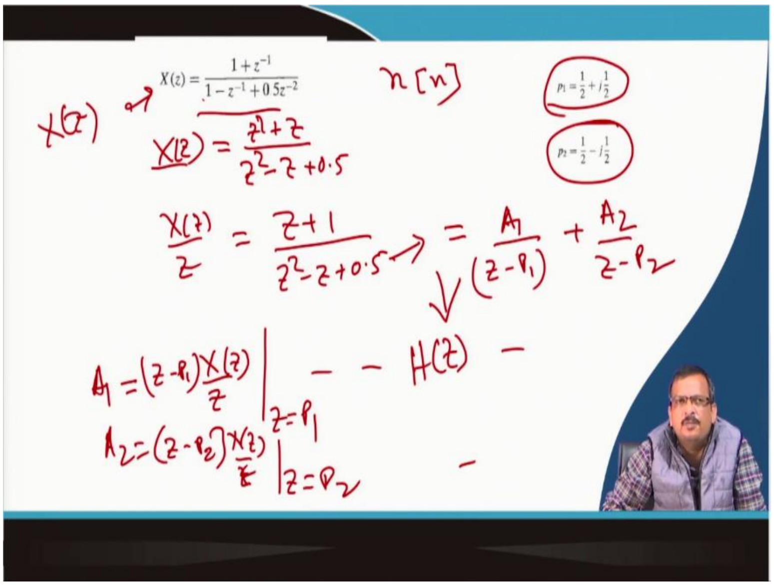

So, those are the procedures to compute the inverse Z transform. Let us give another example: let us say X(z), which is this one that finds out x[n]. So, if you see this, I can say that X(z) by z. So, what will be the X(z) by z or X(z) in power in positive? So, it is there 1 plus z^{-1}. So, I can say if it is positive, I want to make it positive. So, z^2, so, z^2 plus z divided by z^2 minus z plus 0.5.

So, I can say X(z) by z equals z plus 1 divided by z^2 minus z plus 0.5. Now, you factorize this portion. So, there is a p1 pole 1 and pole 2 p1 and p2 because they come in complex.

So, if it is complex, pole p1 and p 2. So, I can say this can be expanded as A1 divided by z minus p1 plus A2 divided by z minus p 2.

Now, I have to find out A1. So, when I find out A1. So, A1 is equal to z minus p1 into X(z) by z. I have to evaluate at z equal to p 1. Similarly, I can find out the value of A2 z minus p2 X(z) by z at the value of z equal to p2. You can determine the value, and then once you get the A1 and A2 values, you can write down the equation. Then, using the table, you can derive the time domain, which should be x[n] ok.

So, you can do your inverse Z transform. So, inverse Z transform very by you using MATLAB also can be implemented in MATLAB. you can do programming for that, and you can implement it in MATLAB, and MATLAB gives you the inverse Z transform. How do you estimate the initial value and final value of the transfer function X(z)? If I say that I find out the initial value of X(z), you know that I can apply the initial value theorem to find out the initial value.

Once you do it, you check whether the universal value is inverse transform and is satisfied or not, final value also you can say you can find out the initial value and initial value theorem final value theorem. So, what we have learned this whole week is the z transform, properties of the z transform, pole 0 concept and inverse Z transform.

So, those things will be used in signal processing. So, this z transform is purely meta. You can go into details on the Z transform. There are a lot of things are there in the Z transform you can use. So, Z transform, you have to remember only Z transform is nothing but a transform that represents the signal time domain signal to a Z domain. Z is a complex domain. So, I am representing a time domain signal in the Z domain, which is in the complex domain which is called frequency domain look like the frequency domain.

When I use Z t, when will we utilize Z transform? When we design either filter that time you can we can you can see that we use Z transform in most of the cases; z transform gives me the pole-zero concept of the system. So, once I get that A(z), I can determine how many poles there are and how many zeros there are, as well as goals related to the resonant frequency zeros.

So, zeros mean for which z value the systems become 0, I can design H(z) we can say that we can implement H(z) either in all pole models or all 0 models. When I use all 0 models, you can see that when you go for the filter design, either all pole filters or all 0 filters will go for the design.

I can implement H(z) in all pole types and all 0 types. So, those are the pole 0 implementations: 0 is related to the system function equal to 0, and poles are related to the resonance. Think about a real example. So, what do you connect with the resonance to Z transform? So, I can say that you have heard about the natural resonance frequency of any physical system.

So, I have a system, which is the second-order physical system. What is the meaning? Second order means I have a two-pole system. So, two poles mean there are two resonance frequencies: one is Z1, and the other is Z2. Two permanent frequencies can.

So, those are the physical concepts you have to remember what are the physical concept of Z transform Z transform is nothing but a complex domain transform. If I know my poles are outside the unit circle, that means the system is oscillatory. If the pole is inside the unit circle, the system is stable. If it is on the unit circle itself, it is self-oscillatory in nature. So, we can predict those things from the Z transform.

Thank you.