Signal Processing Techniques and Its Applications
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Lecture - 13
Tutorial - 02

Okay, so let us take a tutorial for this up to the second week, whatever we cover in the second week; that means signal properties of the system and then LTI system convolution and correlation.

So, just a few problems I will solve. You can solve any problem; if you have any doubts about many problems, you can email me. I can solve it for you and send it back to you, okay?

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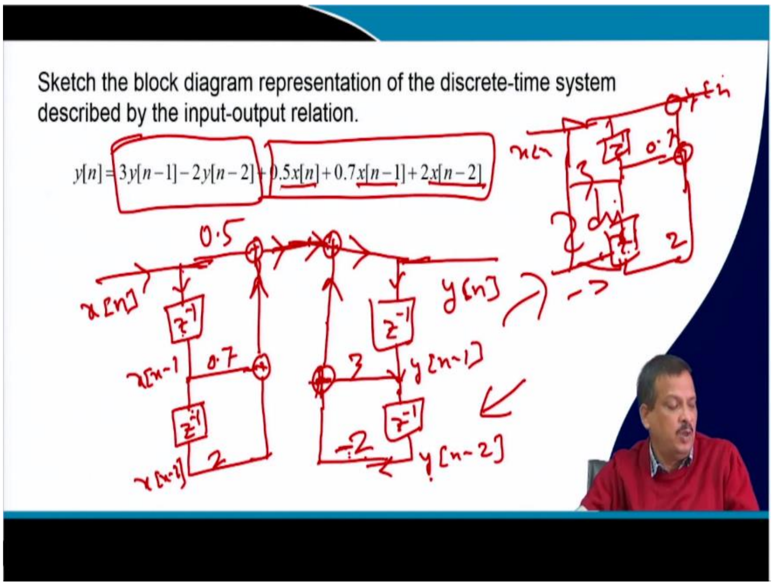

So, let us start with some problems. Sketch the block diagram representation of the discrete-time system described by the input-output relation. So, this is the input-output relationship.

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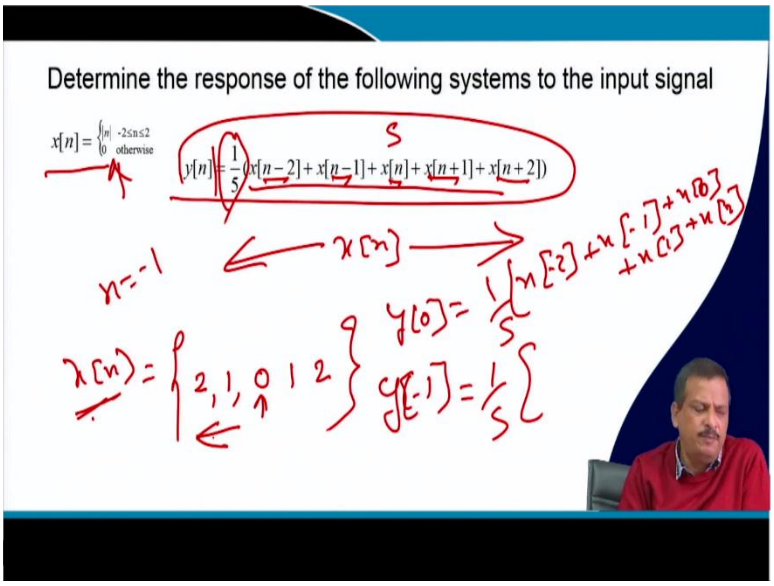

And I have a question: I have to sketch the block diagram. So, how do you do that? Very simple. So, initially, what do you do? You first define your input; then, you define your output. So, this is x[n], this is y[n]; I know that y[n] consists of x[n], x[n-1], x[n-2]. So, what is that meaning? I know that x[n] has to be delayed by$[z]^{-1}$. So, one sample delay. So, what do I get? I get x[n-1].

Then I have to derive another z on another sample delay, $z^{-1,\ \text{and}}$ I get another delay. So, I get x[n-2]. So, a constant multiplying factor x[n-1] is 0.7. Here, it is 2. So, I know the input is added up for these two, and this is 0.5. So, up to this point, I have made this question. Then I know y[n] is dependent on y, so y[n] has to be delayed by one sample, then I get y[n-1], then again has to be delayed by one sample.

So, put the arrow sign because the arrow sign is very important. Without the arrow sign, you get 0. So, this is the arrow sign. So, the signal will come from this side, and this side is y[n-2]. So, this portion multiplying factor is 3, and this portion multiplying factor is 2, and these two things have to be added up, so I did this one. So, this is a minus instead of an add. So, what will be minus, this will be minus.

So, 2 into y, or I can say it is minus 2 and put it plus here, ok and then both will be added up with the signal, and I get the output ok. So, what about the system discrete system given? I can draw a block diagram or signal flow diagram. So, if you see this one is a structure 1 realization director structure 1 realization, I can make it two realizations.

So, I can make it instead of 2 delay, 4 delay can I implement it using 2 delay so, direct structure 2. Yes, I can do that. You can do that. You just interchange the input with the output. So, this portion will be shifted in here and this portion of 2 will be shifted in here. So, ultimately, I will get x[n] and y[n].

So, this portion will be shifted here. So, the delay will come in the middle. Delay will come in the middle, so this will be 3, this will be minus 2 added up, and this will be 0.7, and this will be 2 added up and get the way. So, structure one implements structure 2. So, you can do that, ok.

Similarly, let us ask another question that determines the response of the following system to the input signal. So, this is my signal, and this is my system. y[n] is the output of the system. So, I have to find out the y[n] for a given input x[n] ok. So, if you see this, y[n] is nothing but a moving average of how many samples. So, it is nothing but a 5-point smoother. I take the x[n] sample and two samples from this side, and two samples from this side consist of 5 samples and take the average.

Now, x[n] varies, so what is my x[n] it is given like this way? For the sample value. So, when n is equal to minus 2, that is also 2; when n is equal to minus 1, that is also 1; when n is equal to 0, this is my 0th point. When n is equal to 1, this is 1; when n is equal to 2 this is 2. So, this is my signal; otherwise, it is 0.

Now, I have to find out y[0]. So, what is

$$y[0] = \frac{1}{5}\{x[-2] + x[-1] + x[0] + x[1] + x[2]$$

What is y[-1]

$$y[-1] = \frac{1}{5}\{x[-3] + x[-2] + x[-1] + x[0] + x[-1]$$

So, I can calculate because x[n] x[n] also exists on this side. So, then I have to calculate y[-1] also exists, y[-2] also exists, y[0], and y[1] I can calculate easily.

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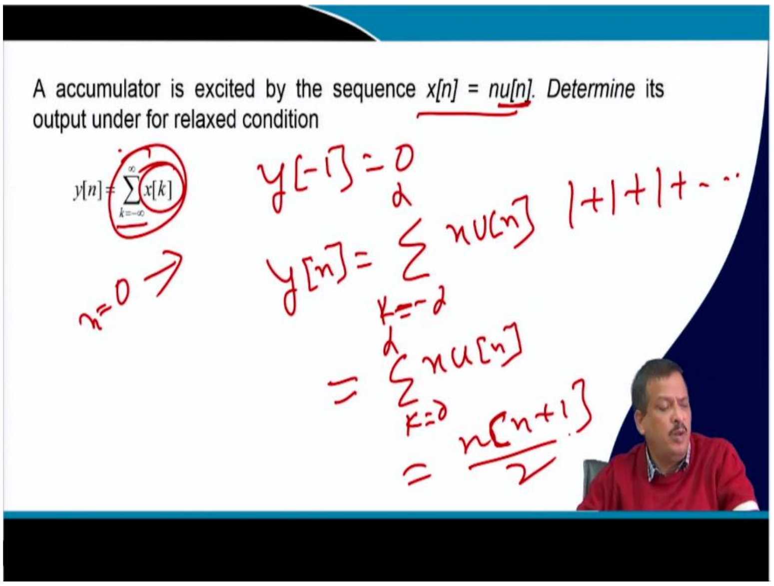

Then let us say an accumulator. This is the question: an accumulator is excited by a sequence x[n] equal to nu[n]. So, you know this is the unit step function; the discrete time unit step function determines its output under relaxed conditions. What do you mean by the relaxed condition? Relax condition means y[-1] is equal to 0; that means the y exists from n equal to 0 to this side, not y minus 1 minus 2 minus 3. That side does not exist. So, that is initially a relaxed system. So, initially, relax means t is equal to 0. You excited the system t equal to minus 1, and the system was relaxed ok.

So, what is the accumulator? An accumulator is nothing but a summation of the input. Accumulator means whatever the input sample you get, you make a sum that is the accumulator. So, I can say y[n] is equal to k, equal to minus infinity x[k], and a summation of x[k]. Now, you put the equation so y[n] is equal to k equal to minus infinity to infinity nu[n] summation of nu[n].

Now, I can say minus infinity to 0, it is not there. So, y of that side will not be there, and the n cannot be negative. So, I can say it is k equal to 0 to infinity nu[n]. Now, what is u[n] unit step function? 1 plus 1 plus 1 plus 1 up to infinite. So, I can say it is nothing but n into n plus 1 divided by 2, I think so. So, you can do it that way, ok.

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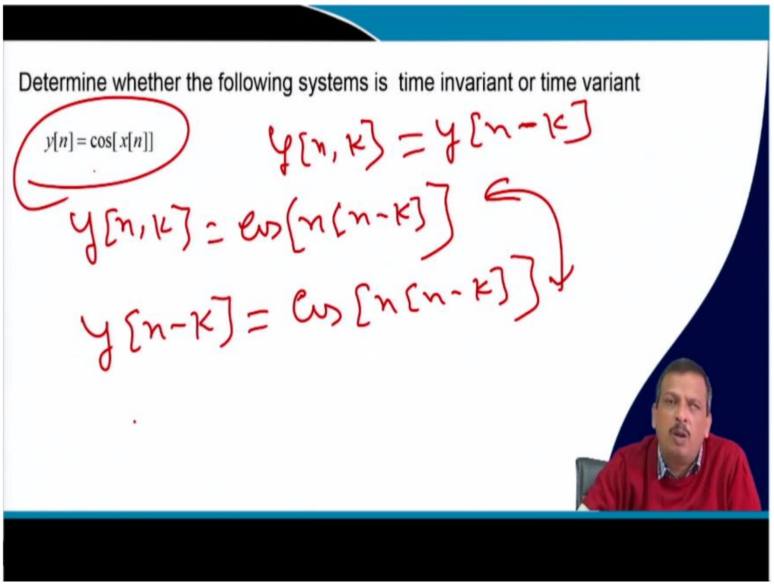

Another problem is determining whether the following system is a time-variant or time-invariant, time-invariant or time variant. y[n] equal to cos of x[n]. What is the procedure? The procedure is that y[n,k] must be equal to y[n-k]. So, let us say k time after k time, what should be the so, I know

$$y[n,k] = \cos [x[n-k]]$$

Similarly, I know

$$y[n-k] = \cos [x[n-k]]$$

y[n-k] just n index is replacing, cos of x[n-k] both are equal. So, I can say the system is time-invariant along the timeline system does not change its property ok.

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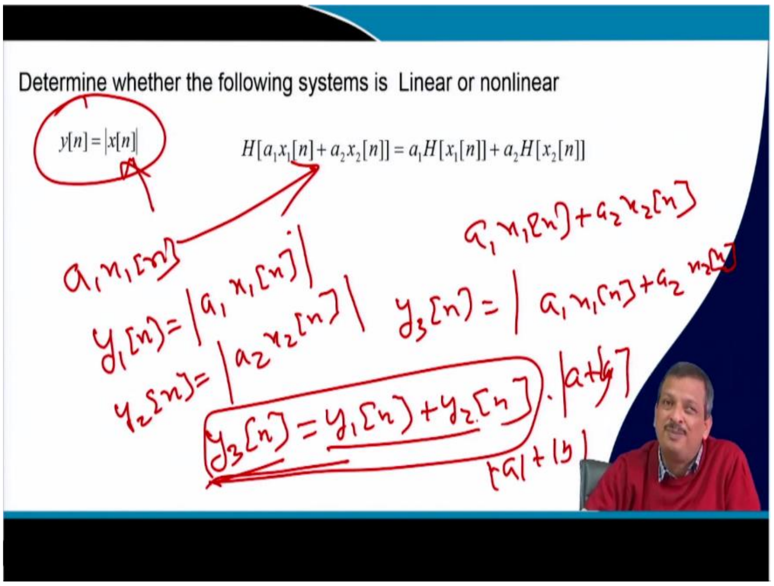

Similarly, we determine whether the following system is linear or non-linear. This is the system y[n] equal to the |x[n]|. So, if it is linear, then it satisfies the superposition principle. So, let us apply a1x1[n] as my input. So, I know that y1[n] is equal to the mod of a1x1[n] if I apply a2x2[n]. So, y2[n] is equal to the mod of a2x2[n]. Now, if I combine the two inputs, if I apply a1x1[n] plus a2x2[n] as my signal, then what should be the y3[n], y3[n] will be equal to the mod of that part a1x1[n] plus a2x2[n].

Now, see whether y3[n] is equal to y1[n] plus y2[n]. Is it okay or not? If it is satisfied, then I just say the system is linear. So, I can see the mod. So, the mod of a plus b is equal to the mod of a plus mod of b. If it is that, then I can say this y 3 is equal to y this 1. So, I can say the system is linear; if it is not, then I say it is non-linear.

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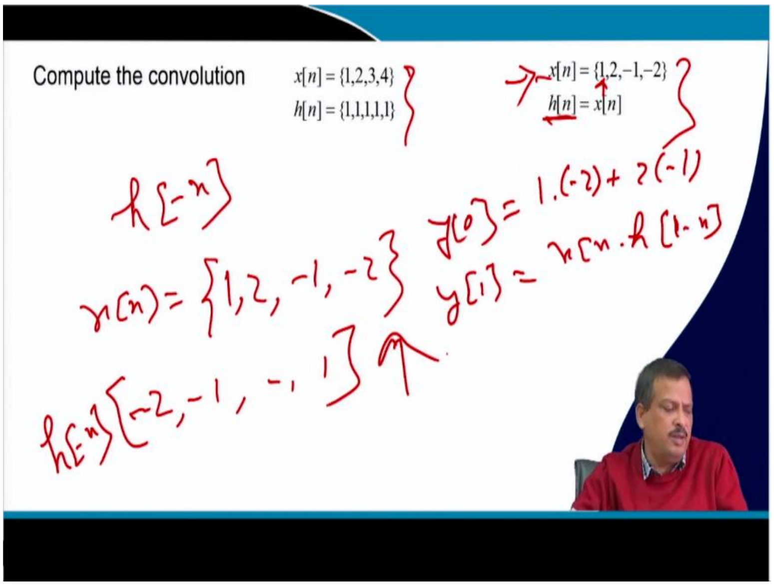

Compute the convolution. I have given two problems; this is one problem, and this is another problem: compute the convolution. So, if I do this one convolution. So, what is the formula for convolution? So, I know x[n] and h[n] must be the same. So, I can once I folded. So, h[n] I have to calculate h[-n] I folding it. So, if I have a x[n] this one 1, 2 let us say this one is the 0th sample; minus 1 minus 2 then if I fold it that means, reverse so, minus 2 minus 1, 2 then 1 h[-n].

Then I know multiply and sum y[0] is equal to multiply and sum. So, y[0] 1 into minus 2 plus 2 into minus 1 plus like that way y[1] I can get same way y[1] means delayed. So, I can say x[n] multiplied by h[n-1]. So, 1 minus folded is okay. So that way, you can calculate the convolution you do, and you can calculate you practice it.

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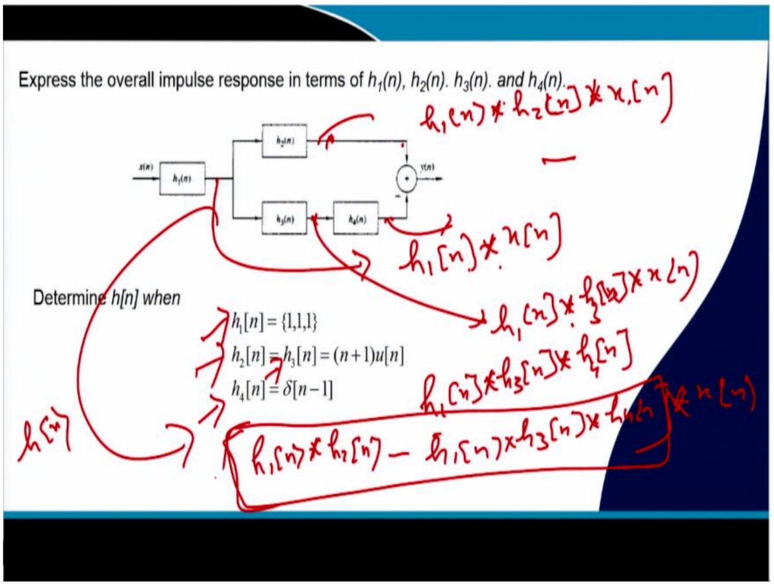

Then let us say this: express the overall impulse response in terms of h1, h2, h3, and h4 n. I have solved it; I have already done it in during my class also. So, this is x[n]. The output is nothing but a h1[n] convolved with x[n]. This output is the input. So, I can say this one is nothing but h1[n] convolved with h2[n] because it is associative with no problem convolved with x[n] this output I know. So, this is input. So, I can say h1[n] convolved with h3[n] convolved with x[n].

Now, I know this output this output is the input signal. So, h1[n] convolved with h3[n] convolved with h4[n] and this one will be this one, and this one will be minus; so, this minus this one. So, I can say that ultimately, it is nothing but a h1[n] convolved with h2[n] minus h1[n] convolved with h3[n] again convolved with h4[n] whole system will be convolved with x[n].

Now, what extra things have I given? I said that lets the h1[n] be given, h2[n] is given h4[n] given, and h3[n] is also given. So, now, I can easily compute the convolution at this point this point, this point, and I can finally find out what is my h[n]. So, I have to calculate this one, okay? So, this can be done this way.

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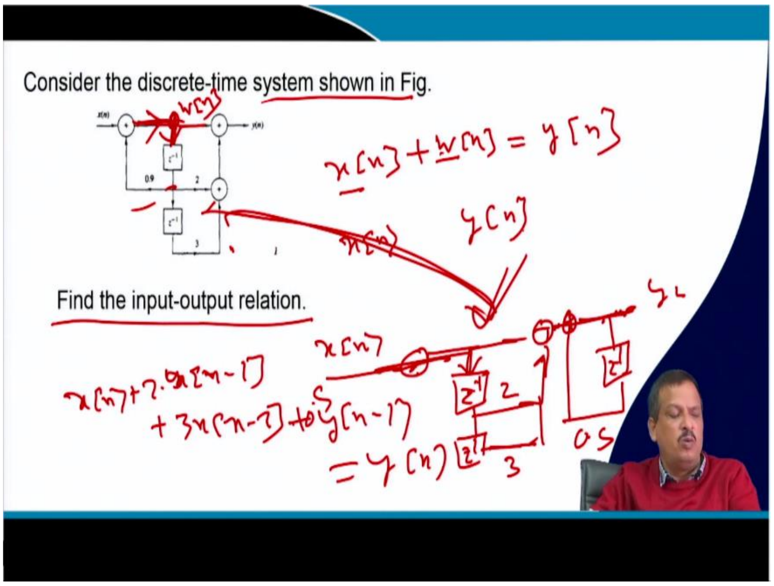

Similarly, let us consider the discrete-time system shown in the figure; this figure finds the output-input-output relationship. So, instead of writing the block diagram, I am saying you find out the relationship. So, you can do that. So, this is x[n], and this is going x[n]. So, now, if you look at x of this, it is a structure 2 implementation.

So, x[n] and y[n] so, this is the intermediate this is w[n] intermediate signal. So, w[n] is delayed by one sample. So, I can say x[n] plus w[n] is equal to y[n], or I can say us say the intermediate there is another signal intermediate signal there. So, let us say that this signal is w[n].

So, now you can calculate how the signal is flowing and what the relationship between x[n] and y[n] is. So, this is structure 2 implementation, nothing but the structure 2 implementation. If you want to do it, structure 1 is for simplification. So, you can initially easily do the structural implementation converted into structure 1 implementation and then do.

So, structure 1 is nothing, but this one will replace this side, and this side will replace this side. So, this side means that x[n] will be delayed by one sample. Then another sample and then that will be; that will be added up, that will be one is 3, one is 2 and then added up, then y[n] will be delayed by$[z]^{-1}$ and multiplied by 0.9 and added up. This is the structure 1.

If it is structure 2, this is the structure 1. Now, I can easily do I can say it is nothing, but this one is nothing but a x[n] plus 2 into x[n-1] plus 3 into x[n-2]. This one is nothing but a 0.5 into y[n-1] is equal to y[n]. You can do both ways either from there, creating a w[n] and then replacing the w[n] or from here, you can draw the structure 1, and then you can find out the relationship between the input and output.

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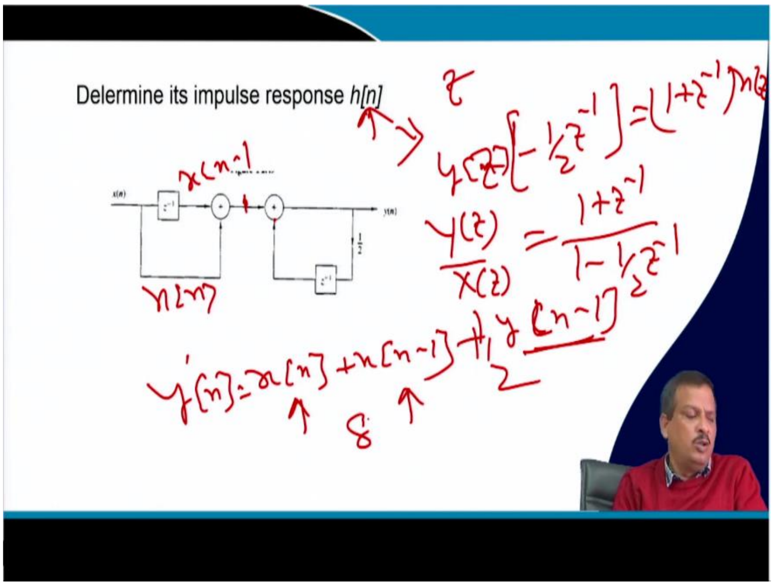

Determine the impulse response of h n I have given. So, what is the impulse response? I can say this is x[n], this is x[n-1], and this is x[n]. So, I can say this point is x[n] x[n] plus x[n-1] plus y[n-1] multiplied by half is equal to y[n]. Now, if I say that z to the power if we want to find out the h[n] in impulse response in terms of z domain or impulse response.

So, this is the 0th sample, and this is the minus 1. So, x[n] 1 sample delay, and this is y of. So, if I if it is in the z domain if I want to tell you, then you can easily set y[n] as equal to y[n] minus half z^{-1} 1 minus is equal to 1 plus z^{-1} into x[z], and this is y[z].

So, y[z] by x[z] is equal to you know that y[z] by x[z]. So, y[z] by x[z] equal to 1 plus z^{-1} divided by 1 minus half of z^{-1}. This is the z domain, the impulse domain also makes it in terms of delta response impulse response ok. So, you can do it.

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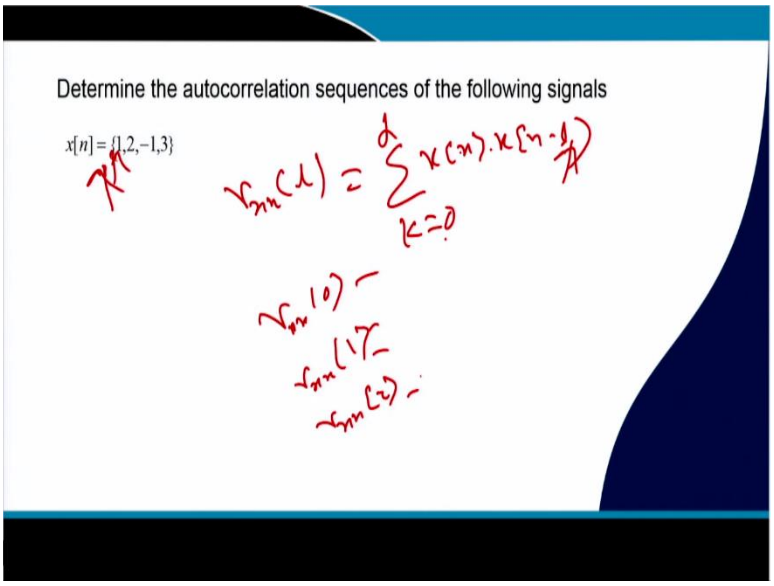

Next, determine the autocorrelation of the I am not doing it. So, if a signal is given, find out the autocorrelation $r_{xx}[l]$. So, I can say that $r_{xx}[l]$ is equal to k equal to 0 to let us say signal start from 0 point. So, I can say 0 to infinity x[n] multiplied by x[n] minus l. l equal to 0 I can get $r_{xx}[0]$ x, l equal to 1 I can get $r_{xx}[1]$ and if l equal to 2, I get $r_{xx}[2]$, I can put the value of l and I can just multiply and sum and get that all value ok.

So, this kind of problem will be there in the assignment. Also, you just try to solve it, and if you have any doubt, you can just you can send an email, that ok, sir, I cannot solve this problem. So, I try to solve it ok.

Thank you.