Communication Networks
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Module - 08
Media Access Control
Lecture - 39
Slotted Aloha- Stability Analysis contd.

So, in the last class, we started this Stability Analysis, we have defined a new for Slotted Aloha, we have defined a new variable called drift and then we tried to estimate the success probability, the drift amount, and the number of attempts on an average that will be happening within a slot. So, these are the things we have got. So, that is the basis for our stability analysis. Now, we will draw a graph to characterize this stability, let us try to see how we achieve that. So, this was our last analysis very good.

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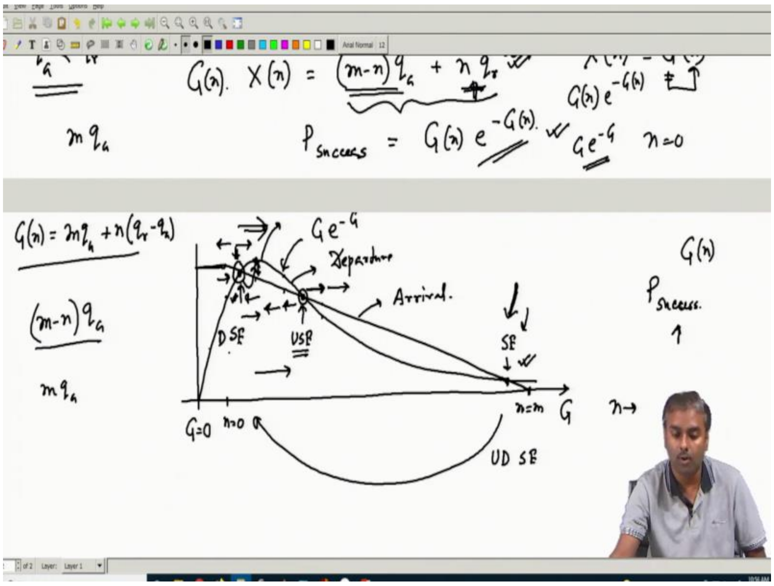

Now, let us try to see. So, this graph is now plotted with respect to G now of course, we have seen that G is a variable of n, in a way over here we also expect that n will be varying ok. So, this is a kind of hypothetical graph that we are trying to draw. So, as if over here n also will be increasing ok. So, at this point, G is 0, but this n increment I cannot capture over here that will be along with G increment as G increases probably n also increases.

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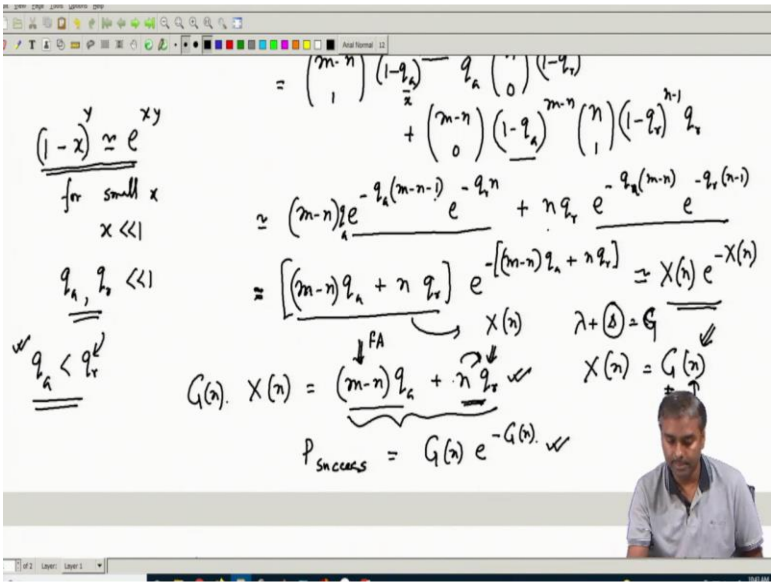

So, that is something we have seen that also means G is kind of as you can see G n is this one m minus n in to q a plus n into q r. So, as n increases if we have these particular things where q a is lesser than q r. So, this particular thing is where probably the stability analysis is most important, we will also talk about that. So, where means what we are trying to analyze over here is in a system where fresh attempts are means coming with lower probability.

So, our node will be attempting a fresh transmission that is not a very high probability, but what we will be characterizing is when they go into let us say back off or backlog. So, in the backlog, they will be attempting with a higher probability compared to new arrivals. So, a node in backlog will have having higher attempt of transmission compared to a node that is fresh and making an arrival.

So, over here over this condition only we will try to do the stability analysis because you will see also the other cases do not mean you do not have to do stability analysis. Because stability analysis is something where it has already gone deep into backlog, can it come back, can those backlog link?

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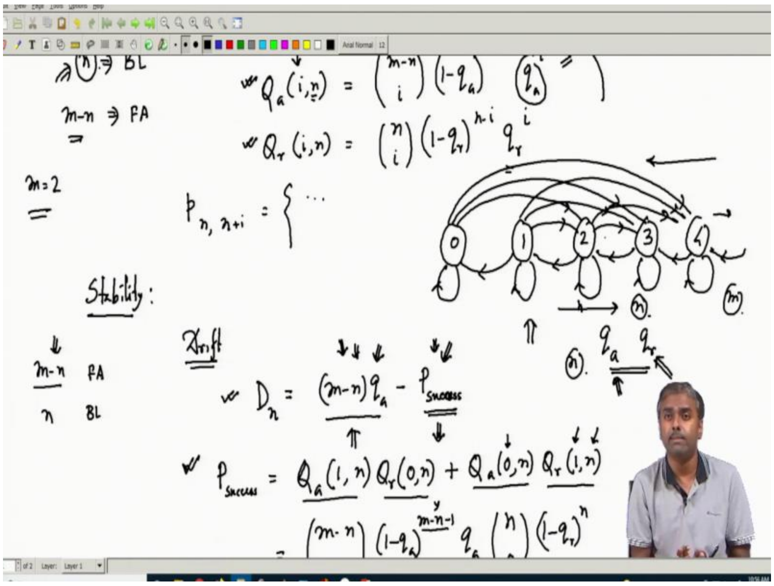

Stability means what? In a a Markovian system stability means can it actually come back, if it goes up in the backlog can it really come back? So, the most interesting stability analysis will be where m is a very large very, large number of nodes are there. , therefore, this is almost the number of nodes which means possible so, that goes towards infinity. So, this Markov chain also goes towards infinity.

Now, is there a possibility that once it is backlogged will it come back? So, that is actually stability analysis, what is the probability or what is the possibility that it will come back? If with a certain probability, I can say that it will come back then I have stability in the system, but if that is not certain; that means, there is a possibility that it will never revisit this system. So, the steady state probability that we are talking about that might not be of interest. So, this is something which we have to characterize.

, therefore, in stability analysis, the new arrivals are not the important thing, the important thing is if it is in backlog I need to give more probability; so, that they actually clear their packet and then come back ok. So, that is why we are taking this interesting scenario where q r is greater than q a and if q r is greater than q a from this equation G n which is m minus n q a plus n into q r as q r increases means n increases basically more emphasis will be in this term n into q r. So, that will be increased.

So, as we can see indirectly if G increases means there will be also an increment over this axis. So, this will be happening. Now, what will be initially happening? If you try to see G equal to 0 what does; that mean? That means probably it is in state 0 ok. So, n is equal to 0 because otherwise G cannot be 0 right? So, n is equal to 0, and at that point basically, nobody is making arrival ok. So, that is what is happening. So, at that point probably what will be happening, now try to see this try to do this analysis over here or try to plot means try to see what we are trying to plot over here.

So, let us try to see if we have something called P success, what is P success actually? In one slot how many packets will be put that is a kind of departure rate according to our analysis. So, if you see per slot how many packets we are putting in the system means those packets we are putting are successfully transmitted or they are successfully departing from the system. So, P success if we try to plot that is actually their departure rate ok.

So, this departure rate depends on this G n e to the power minus G n or we can say if G is the variable. So, that is G into e to the power minus G which is almost the approximate analysis we have done for slotted Aloha. So, that will look like we already know the pattern that is the pattern. So, this is actually the departure. Now, let us concentrate on the arrival ok? So, if you try to see the arrival what will be happening?

So, initially, you expect that n will remain 0 because initially, it's all fresh arrivals that are happening, fresh arrivals will be generally what they will be doing. So, whenever they make arrival so, as you can see it will be n will be 0. So, it will be m into q a that will be the arrival ok. So, many arrivals are happening because at that time there are no nodes that have gone into backup or backlog. So, all fresh arrival m nodes are there, and the average per slot arrival rates are this one only; so, m into q a. So, it will be m into q a for some amount of this one till n remains 0.

So, let us say this is up to this n is equal to 0, from there n will be increasing ok. So, from there onwards n will be increasing, now let us try to plot it with respect to n ok. So, what will be happening? So, this is actually m into q a. So, if you see this G n as a variable of n we are trying to plot now. So, this will be m into q a plus n into q r minus q a that is exactly what will be happening because q r is q a; so, as n increases ok. So, sorry n q r minus q a yes. So, this is our GN that we are trying to see ok?

So, now let us try to see if this will be the overall G n, let us try to see what will be the overall arrival just on that slot. So, arrivals are only this portion m minus n into q r q a because this portion is due to this collision right? So, not due to collision, I should say due to backlog, whoever has been collided and gone into backlog will be making these arrivals. So, if we just take out this arrival just for the system perspective, overall system perspective what is the fresh arrival that is happening?

So, this is the departure that we have plotted. Now, if we just try to plot the arrival then what will be happening? Just the arrival will be m min n into q a, till n is 0 this will be m into q a, but as n starts increasing this will be linearly decreasing with respect to n. So, there will be a linear decrement with respect to n and then this is where n equals to becomes m; whenever n becomes m; that means, more nodes are going into backlogged.

So, therefore, arrivals are getting reduced, actual arrival to the system. So, the system has a peculiar dynamics. So, what is happening? Initially, it will have all fresh arrivals, but slowly it is going into backlog. As it goes deep into the backlog the fresh arrival will be reduced and reduced because a backlog node cannot make any further arrivals, that is how according to our assumption. So, the buffer is only having 1 packet. So, if a node has been backlogged, he cannot make a fresh arrival till he transmits or retransmits the packet that is already there.

So, therefore, the actual arrival of the system will be depleted as it goes deeper and deeper into the backlog. Finally, we can see if it goes up to m; that means, all nodes are backlogged that is when the system cannot actually make any further arrivals. So, that arrival will be 0 ok. So, this is the system dynamics if you see, this is how it is departing. So, this is the departure and this is the arrival.

Now, at this point so, there are 3 crossover points as we can see, can we now characterize these 3 crossover points? So, if you try to see over this crossover point what will be happening? The difference between these two as you can see is actually the drift we have talked about, you can see the drift is m minus and q a which is the arrival and this is the departure. So, that is the drift ok?

So, as you can see this is actually the graph that characterizes all those parameters we have talked about. So, this is the success or the departure, this is the arrival and the difference between these two is the drift ok. Now, as you can see from this node onwards if he goes over there; so, what is happening? His departure is greater than his arrival. So, once he pushes in this direction because the departure rate is higher than the arrival; so, it will again take him back due to the system dynamics because the departure rate is higher.

So, it will reduce n and n reduction means it will push the system back, if he goes in this direction what will happen? So, over here his arrival is greater than his departure; so, that will again push him back in this direction. So, it will increase the n because arrivals are more; so, it will actually bring more systems sorry more systems towards backlog ok?

So, as you can see this particular point is where arrival is equal to departure and the point is such that whenever a left or right shift happens it brings it back, and the system dynamics bring it back. So, therefore, this particular point is characterized as a stable equilibria. So, this is an equilibria point because arrival rates are balanced with means departure rate. So, that is where we want to be, whatever is arriving that many are departing. So, the system is stable we have already seen that.

So, that is when the system becomes stable because as many will be arriving on average as many are departing. So, the system is capable of handling everything properly. So, it has an equilibria point of course, but it is also a stable equilibria because any drift from there brings it because of the system dynamics it brings it back. So, this is a stable equilibrium. What about this point? This is also another equilibrium because the arrival rate is equal to the departure rate, is this a stable equilibrium? Let us try to see if he departs over there.

Now, what is happening over here the arrival rate is higher than the departure, it has already increased. So, if it goes over here from the equilibria it has been disturbed the n has been increased, but there because the arrival is already more than the departure; so, therefore, it will go deeper into that. So, this is an unstable equilibrium, on the other side also same thing will also happen. So, other side n decreases ok so; that means, the number of backlogs decreases, but on this side, as you can see success is more than the arrival.

So, it will further reduce. So, this will further reduce. So, whenever he goes to this equilibria he has a chance of either going to this one point or another equilibria means he has a slight drift that will take him towards left or right. So; that means, this is an unstable equilibrium, it will never stay over here; by default, it's not means stable. What about this one? Same thing as this. So, this is also a stable equilibrium, we can already see that through the analysis. So, what we have got from here with the description that there are 2 stable equilibria and 1 unstable equilibria.

So, most of the time what will happen? He will jump around from this point to this point to those two stable equilibria. If he somehow goes out unfortunately from here due to probably a successive series of bad luck, let us say he is attempting multiple he is in backlogs, all backlogs are attempting multiple transmissions, and a series of times multiple probabilistic bad luck happens. So, every time he gets collided. So, basically, the backlog keeps on increasing.

So, basically, he even crosses this equilibrium, and then he will drift towards this; so, that might sometimes happen. So, even if it happens it will go to this equilibrium, and then again with successive good luck probably he will come back to this equilibrium. Now, among these two equilibria which one is my desired equilibria? As you can see I am more driven towards this departure than is the success. So, the success is higher that is where I want to be.

So, I want to be over here, ideally, I would like to be over here so; that means, I would like to push my equilibrium point towards this. So, I want to ideally shift my equilibria to this peak which should be my ideal behavior, but even if that does not happen due to the parameter settings ok. So, it depends on m n sorry m lambda q r q a and all those things. So, depending on this cut point, I do not know where that will be happening, but whatever happens, I can see that compared to this, this equilibrium is more desired.

But, I know that the system can occasionally go to this equilibrium, and because this is a stable equilibrium, once it goes to that equilibrium most of the time it might stay in that equilibrium, which is a danger. I do not want it to be like that, if it is it was an unstable equilibrium then I have a chance that it will again come back to me. But, because there is an at the farthest end where the throughput is really low, I have a stable equilibrium that is the problem for me.

This is the point that is problematic for me because it might happen occasionally it goes to that equilibrium and stays over there for a longer duration. Then my performance will be very bad because I do not want this to be a desired stable equilibrium. This is actually an undesired stable equilibrium, this is, of course, unstable equilibria and this is the desired stable equilibrium. So, I want to keep the system in the desired stable equilibria.

Now, even though there are equilibria there are stable equilibria, I know it has a steady state analysis. We already can do the steady state analysis by means of this particular Markov chain DTMC.

So, we can solve this DTMC, we can get stable equilibria not a problem. It does not have a stability problem, but it has now I can see this observation, that it has 2 equilibria unfortunately and one of the equilibria is not my desired equilibria. So, what can I do? And, the same system can switch between these two equilibria. So, sometimes my performance might be very bad and I have no control over here. Can I now introduce a control? That will be the next question and that will be the next design also.

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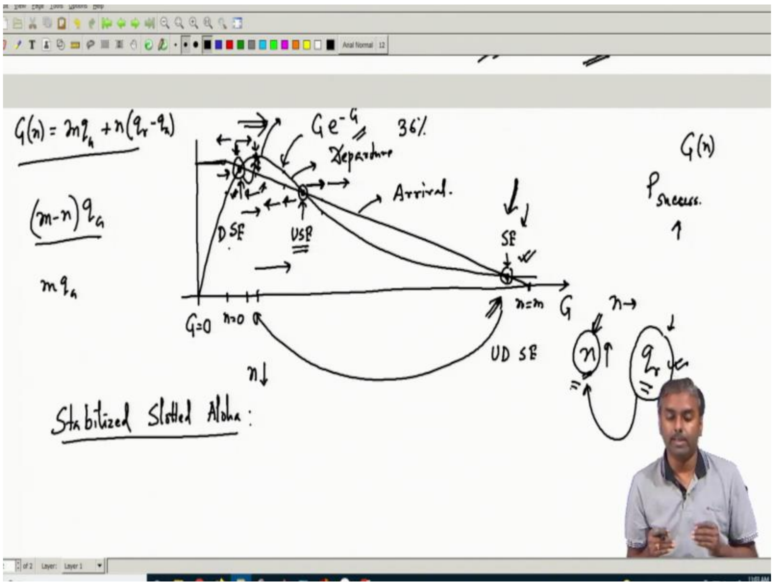

What is that extra control that I can put forward; so, that it does not hover around this, it most of the time it hovers around this. Not only that we also try to achieve; so, that this equilibria point goes into the peak, where I get maximum this one; G into e to the power minus G we have already seen 36 percent or around 36.8 percent throughput we can get, that is the maximum we can achieve.

So, can I reach towards that? How do I make sure that most of the time the system stays over there; that means, my stable equilibria are somehow dragged towards that desired stable equilibria, and that desired stable equilibria also is dragged towards the peak point of my success. How do I do that? So, now what we can see is what is happening over here. So, let us try to characterize these two equilibrium points.

So, this equilibria point is at this point what exactly is happening? The number of backlog nodes is very high. So, n is very high, over here number of backlog nodes is probably very low in this unstable equilibria. So, whenever these backlog nodes are high what will be happening? The corresponding collision probability will remain high because, if still I keep my q r value high then what will happen? They might collide quite a lot ok.

So, therefore, what I can see is that depending on this n, I should probably adaptively modify this QR. In such a manner in such a manner I modify these things; so, I can generally whenever it goes over there I can bring it back, I can resolve the collision quickly and come to this stable equilibrium point that is what we need to do. So, therefore, immediately a new thing comes into the picture which was not earlier there.

When we started doing this slotted Aloha ok, this is called non-stabilized slotted Aloha; we were not talking about stability over. There ok there was no stability analysis; so, everything was meant I have given a parameter, it is a parametric slotted Aloha we do not change anything, it is not adaptive, and it is not stabilized ok? So, we do not think about stability q r value is fixed, lambda is fixed and all those things ok.

I know that lambda and m that is a system parameter I cannot change that, that is something that will be coming to the system, and I cannot. But, this QR is my own thing, it is the protocol-related stuff. This is something I can adaptively vary. So, now, this stability means or stabilize slotted Aloha means where I will be actually additionally adaptively varying my q r.

Now, how do I vary this qr? As we have seen already q r should be varied seeing n, how many nodes are in backlog. So, therefore, immediately the question comes in can I estimate this value of n at any time instance? How do I estimate this value of n? Because, how many nodes are in backlog how do I know? Every node independently is doing something, how every node be able to estimate who else is in backlog, and how many nodes are in backlog?

So, can I do this estimation? That estimation is going to be a critical parameter or a critical part of the protocol. So, how do I estimate this n, and accordingly with respect to n how do I vary this q r value? That will be the next phase of slotted Aloha which is called the stabilized, the target is very simple whatever throughput maximum throughput we can get in slotted Aloha; now, we are devising at that time we told somehow that G has to be 1 ok. When is whenever this G is 1, I get the highest peak this one.

So, we have at that time we are told that it could achieve 36 percent throughput, but we never devised any mechanism to say how to always achieve that 36 percent maximum throughput, that we have never told. Now, we are trying to give a mechanism through which we can always take this slotted Aloha towards this highest throughput. So, that that G the composite arrival due to a retransmission attempt as well as fresh arrival, how do I keep that towards 1, that should be our next set of discussion. So, we will try to achieve this particular part.

So, let us try to see how we can do that in a perspective where n is known; that means, n can be directly gathered from the node information or n can be estimated or if I cannot estimate the n in a heuristic manner I can actually try to predict n. So, these are the things that we will be now covering. So, let us try to see what we can do with respect to n ok. So, in stabilized slotted Aloha a slight modification has been proposed by people. What was the modification?

So, what they told I no longer discriminate between these two nodes that are fresh arrival and backup. So, I do not discriminate between these two nodes. Actually, to make things all simplified, I will say that for the nodes that are immediately coming we also take them into backup or backlog.

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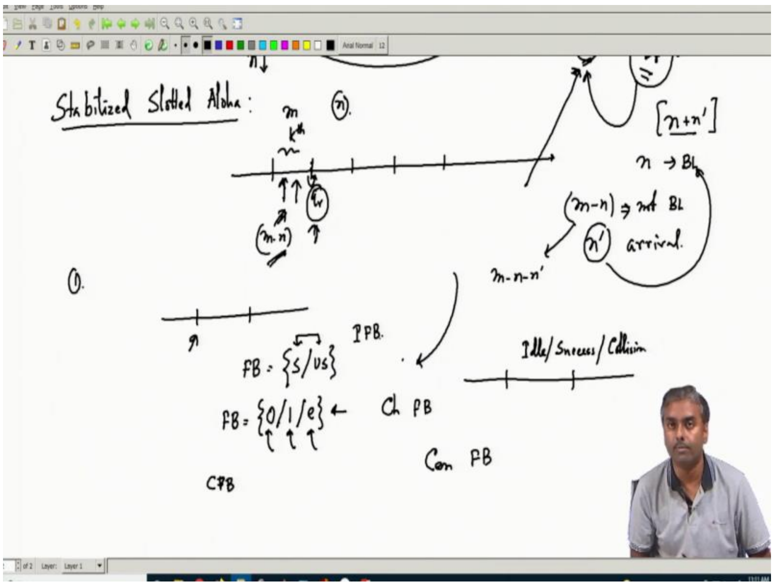

So, what would be happening then, what is the modification corresponding to modification? So, the corresponding modification will be something like this, I have slots where some arrival happens ok and of course, so, let us say if according to the earlier description we had m nodes out of that n are in backlog ok. So, m minus n is making these arrivals. So, m minus from m minus n nodes these arrivals are happening and there are n backlogs which will be with probability q r which will be also attempting trials over here.

Earlier what was our assumption? If they make an arrival immediately after the next slot they will be transmitted, now we do not do that. So, what we say is even this m minus n nodes whoever makes an arrival will be immediately taken into a backlog that is a new thing. So, basically what we will be trying to see that if some nodes make an arrival over here. So, out of m let us say some n dash number of nodes sorry out of m minus n which are not backlogged, n are backlogged.

So, n is backlogged, this m minus n is not backlogged. So, out of them, this m minus n n dash let us say n dash number of nodes make arrivals in a slot. So, we are talking about this slot kth slot making an arrival, and this n n dash node making an arrival. So, immediately at this point what we will do? We will take this and dash it backlog. So, my total backlog will be n plus n dash and this will be m minus n minus n dash so many will be not in the backlog. So, these n plus n dashs now will be attempting transmission with probability q r that is it, that is the modification that we are doing.

So, basically for fresh nodes earlier the only modification that has been done, earlier the fresh nodes were given priority. So, fresh nodes were always attempting transmission immediately in the next slot. Now, we are saying just that modification we are doing that nobody is fresh node considered to be fresh node. So, everybody will be taken as backlog and there will be attempting transmission with some probability q r and that will modify this q r ok.

So, the backlog is updated with fresh arrivals as well and they all will be now making attempts of transmission with probability q r, this q r will be modified which is exactly what we will be trying to do. Now, over here what we will try to see is that one attempt will be. So, we will be describing two protocols over here; one will be which will be attempting to estimate this in sitting at a node here. The situation is that sitting at a node, I do not know how many are backlogged.

I cannot keep track of that because, who is colliding, who has made an attempt, who is colliding how do I know ok? So, I will not be knowing that, if not means if we do not assume that there is a feedback mechanism centrally somebody is there who collects everybody is whether he has been somebody has successfully transmitted, somebody has collided.

So, all the information he takes, also he broadcasts all this information to everybody; then they will know who has attempted in in this particular slot, what was the status of his attempt and all those things; otherwise, generally, I will be getting my own feedback only ok.

So, what is my own feedback? So, my own feedback means in a particular slot if I attempt a transmission ok; so, that transmission whether it was successful or not; so, this feedback I can get ok. So, now, we are talking about this feedback that we can get. So, there will be the classification of feedback.

So, one feedback I am trying to see where it will be just my own feedback. If I attempt transmission, remember I am again saying condition that I have attempted transmission whether my transmission was successful or not, this is generally being done by an acknowledgment packet by the recipient.

So, basically, if I transmit some packet and then the receiver gives me an acknowledgment with that I will know whether this was successfully transmitted or not successfully transmitted. So, I can only know feedback; so, my feedback will be only successful or unsuccessfully transmitted.

This is the only feedback I can get, you will see later on that we will be describing generally the wireless axes have these mechanisms, only this feedback you can get, no other feedback you can get. There are some things where I can get a little better feedback. What is that better feedback? At least I will know what has happened to the channel. So, I can at least get the channel feedback.

So, in ethernet generally, that happens. So, wired media probably will probably be common wired media or ethernet LAN kind of thing or hub ethernet, we will discuss these things physical things. So, there this feedback is possible. So, I can not only my own things I can get feedback from the channel, whether the channel was idle or channel had a successful transmission or the channel had a collision. So, all these things I can get. So, this is a new feedback. So, over here this feedback is the channel feedback.

What is channel feedback? Channel feedback says 0 1 or e; what does that mean? The channel was idle, the channel was having 1 successful transmission or the channel had multiple attempts and that is why the packet collided. So, this is the channel feedback that I will be getting. So, as you can now see there are two feedbacks that a station can get; one is called only his feedback, and if he transmits he knows whether the transmission was successful or not. So, this is binary feedback whereas, he can also get channel feedback, for some media he can get that.

So, channel feedback means he will know additional things, he will know whether the channel was idle whether the channel was attempted and somebody has successfully transmitted or it was erroneous. So, this is the additional feedback he can get or he can get the overall feedback that who has attempted whose transmission, this is the centralized feedback. So, is the centralized feedback ok, or should I write centralized feedback and this is channel feedback and this is individual feedback?

So, we will later see that depending on this feedback different algorithms have to be devised. Some of the algorithms will be stronger in estimating n and correspondingly achieving this 36 percent throughput, but some of the feedback will not be that capable. But, they will have their own heuristic and we will try to see some of them if we can do some analysis of them.

So, in the next class, we will try to do these things ok.

Thank you.