Communication Networks
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Module - 08
Media Access Control
Lecture - 38
Slotted Aloha- Stability Analysis

So, far we have started discussing different kinds of Media Access Control protocols. So, basically what we have discussed is Slotted Aloha and AAloha and some of their analysis of applying DTMC. So, we will continue on that same thing.

So, basically, we are trying to analyze the whole thing and then trying to see how we can improve. So, we have told that at least in this particular section of the course we will be showing how a particular efficient media access control protocol can be designed through successive analysis. So, that is something we are still doing. So, we will do something more today a quick recap of what we have done so far.

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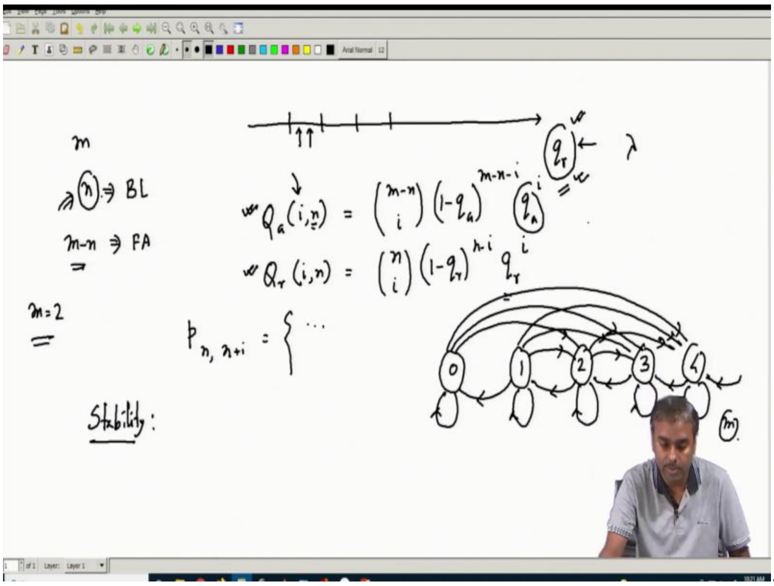

We have been told that we have some time that is slotted each slot is of one unit of time and then in slotted Aloha our algorithm was something like these packets will be arriving randomly, but whenever they arrive all these packets will be accumulated and at the end of the slot whichever packets arrive in the slot end of the slot they will be attempted ok.

But while attempting we will be trying to see what is the efficient way of attempting, because sometimes what might happen if we attempt and they collide then they get backlogged. So, we have also talked about there being a series of backlogs, and then backlogged also will be attempting, but what is that attempt rate that something we have decided that it will be with some probability q r they will be attempting ok.

So, this is q this q r is a kind of design parameter that is a very important parameter will means today we will try to give light or throw light into this design of q r, and then we will try to see is a possibility of designing q r in a different way ok all together; that means, do not fix some value of q r whether we can adaptively change the q r and we will see those things ok. So, this was our conjecture with this we have also devised a particular DTMC we have seen that, in that analysis.

So, in that if I just quickly recapitulate or just list out the equation. So, we had this Q a i n and we have another thing called Q r i n. So, basically, we had m nodes out of them and are in the backlog stage this is what we have already told. So, these are backlog nodes and m minus n nodes are making fresh arrivals ok.

So, this is something we have seen then this Q i n and this n we have been told that this should be sufficient if we take that as a state description, and correspondingly because the times are slotted we will be correspondingly devising a DTMC analysis. So, this is something we have done, and then these two probability matrix we have defined if you remember we had them defined like this m minus n c i ok 1 minus q a whole to the power m minus n minus i into q a to the power i. So, this was one thing.

And then the other one was n c i and then 1 minus q r whole to the power n minus i and q r whole to the power i. So, this was our definition this telling given that is in state n what is the probability that i nodes will be simultaneously attempting i fresh node among this m minus n node will be simultaneously attempting transmission corresponding single node transmission attempt was defined as q a ok. So, this was something we have already done.

And q r is already we have defined that backlog node they will be attempting. So, again it is in state n. So, out of n nodes, what is the probability that i node will be attempting this backlog thing right? So, this was our, these two probabilities with this we could give the entire transition probability matrix. So, that is something we have done and with that, we could also draw the corresponding Markov chain which if you remember we were having say 0 1.

So, these are the backlog stages 2 3 4 something like that and then there were probabilities like this, cell probabilities were there, everybody could come down to 1 stage, and then going up from 0 we have told that it is restricted 1, it cannot go and it can go to any other state, similarly from 1 also it can go to any other state and so on. So, from 2 also the same thing and so on.

So, this probability transition matrix we have drawn and then this p some if it is in state n to n plus I that entire transition probability we have actually given description with respect to this Q a and Q r. So, this is something we have done and of course, I will tell you as homework probably you should solve this.

So, let us say I have taken this goes up to m right. So, let us take I have taken m equals to 2 ok. So, it can have at most 2 number of nodes. So, for m equal to 2 can you solve this particular thing ok. So, is it possible to solve this Diamark of chain? So, this will be restricted to up to 2 and then corresponding equations you write down and then try to solve this whole thing. So, you will get the steady state analysis this is easier this is something we have already done.

So, pi p is equal to pi you will write only there are up to 2. So, it will be 0 1 2 3 state only pi 0, pi 1, pi 2 corresponding three equations will be given and you will also have a normalization equation pi 0 plus pi 1 plus pi 2 equals to 1. So, this means you just solve all those sets of linear equations, you solve them, you will get the corresponding pi 0, pi 1, and pi 2.

So, the steady state analysis is possible from there you will be able to also see for a given value of q r and lambda which is the arrival rate. So, you will be able to get the corresponding probability of which state it will be, and all those things ok. So, what are the backlog state and all those things, and from there you can do a parametric analysis. So, through analysis and all those things that something is possible, we will also try to see how you do that.

So, that part we will go on and analyze, but there is something which we need to discuss a little bit more, that was the heading of this particular class also that is the stability of these things. So, this stability analysis is something that cannot be completely captured from this steady state analysis.

So, that is something will try to do, we will try to see how we can actually explore these things and get our stable understanding and from that stable understanding we will be able to give a very important comment on Q and we will be also doing a very important observation. So, let us try to see how we do this stability analysis and for that what we will have to define.

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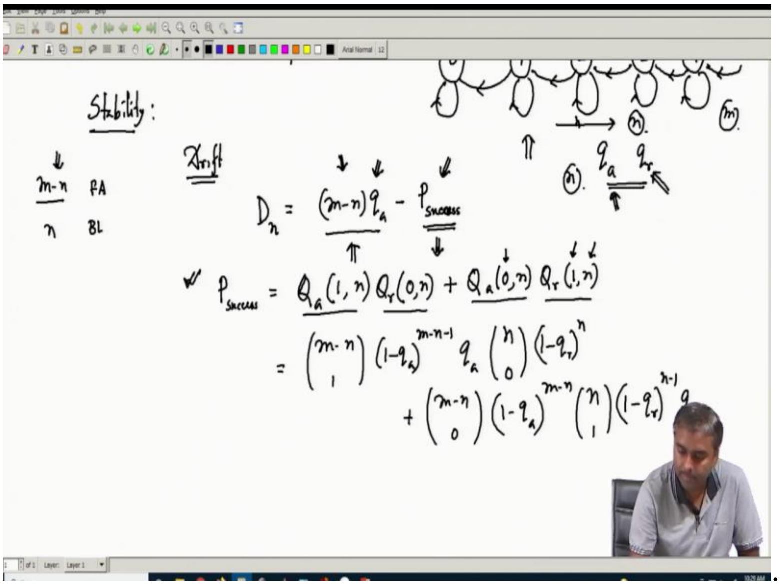

What we will do for stability analysis we will try to do something called drift which we will define what we mean by drift ok. So, drift it is like this, over here at difference means backlog stage as you can see at different backlog stages it has different numbers of fresh arrivals and different numbers of backlogged attempts ok retransmission attempt and depending on the probabilities corresponding probability q a and q r these things will be differing. So, if I am higher at n; that means, more stations are backlogged. So, therefore, this will be predominating.

So, more backlog stations will be trying to do for a given value of q r more stations will be trying to do retransmission. So, most of the time it will try to do retransmission compared to because m minus n will be reduced then because more number of stations are in the backlog so, less number of fresh attempts will be happening. So, as you can see at different stages of backlog there will be different kinds of phenomena that will be happening.

So, what is actually drift, drift is something that we will be defining now let me define that. So, at a given n value drift is defined by this m minus n into q a minus something called P, P success ok, let us try to understand what is this. So, P success is at this n what is the probability that it means there will be a successful transmission ok.

So; that means, given he is in nth stage m minus n new station will be making a fresh arrival in the previous slot and all the n stations will be trying to do retransmission with the probability q r, what is the probability that there will be a successful transmission that is called the P success ok. So, that is the P success; that means, our attempt has been successful means or it in that slot a successful transmission is being carried out.

So, this will be P success and what is m minus n into q a; that means, how many numbers of on an average how many numbers of or what is the average number of the sorry average number of new arrivals that is happening ok. So, m minus n number of fresh arrivals is happening with each one is doing with probability q a, what is the average number of fresh arrivals that will be happening this is this, why we are calling this as drift.

So, this will be the amount of backlog increased and this will be a kind of amount decrease. So, the difference between these two is the drift ok. So, you can talk about this as a drift-in n ok? So, basically, how over here when he is over there at a stage n ok. So, whatever that n is from there how it is drifting, is it going towards more back of or is it actually getting reduced, that back of number is getting reduced. So, that average drift when he is sitting at stage n we are trying to capture by this ok.

Let us try to understand this P success from our earlier description Q a and Q r description and corresponding to this probability transition description. So, we can characterize by this one you will see that I will explain it later let me first write the equation Q a 1 n, Q r 0 n plus Q a 0 n Q r 1 n ok. So, that is very nice that can be understood that this is the success probability because when the success will happen.

The success will be happening see we have two groups of users, one is this m minus n user who will be making fresh arrivals and the other is n arrival which is backlogged. So, when there will be a success if one of the backlog is attempting and no fresh arrival has happened this is a success because only one attempt has been made and that will be successfully transmitted. So, as you can see Q r 1 n means only out of n at state n; that means, n backlogs this one is there only one is attempting.

So, that is the probability that one is attempting, but at the same time, I have to make sure that none of the fresh arrival is happening. So, in Q a I only have 0 arrivals. So, these two together if they happen so, are independent because these two nodes whether they will have arrived or somebody will be attempting transmission are independent. So, their probability will be multiplied.

So, this is one case, the other case is that none of them from backlog will be attempting, but one arrival will be happening. So, that is why Q a 1 n multiplied by Q r 0 n. So, this is the whole probability that there will be a success ok? So, that is the success probability ok?

And if you say in one slot probability of success it tells you the average reduction ok in the n that is what we can see. And this is actually the average increase. So, that is the drift of n we are trying to capture. So, this is what we will be trying to capture. So, if I now try to understand what this P success is ok. So, that will come from these two formulas that we put over here. So, let us try to put that.

So, I can write m minus n. So, I equal to 1 now into 1 minus q a whole to the power m minus n minus 1 into q a whole to the power 1 multiplied by this will be n c 0 and 1 minus q r whole to the power n ok, that is the first term. The second term will be m minus n c 0 1 minus q a whole to the power m minus n multiplied n c 1 1 minus q r whole to the power n minus 1 into q r ok. So, that is the P success. Now we will try to manipulate this a little bit with some assumptions.

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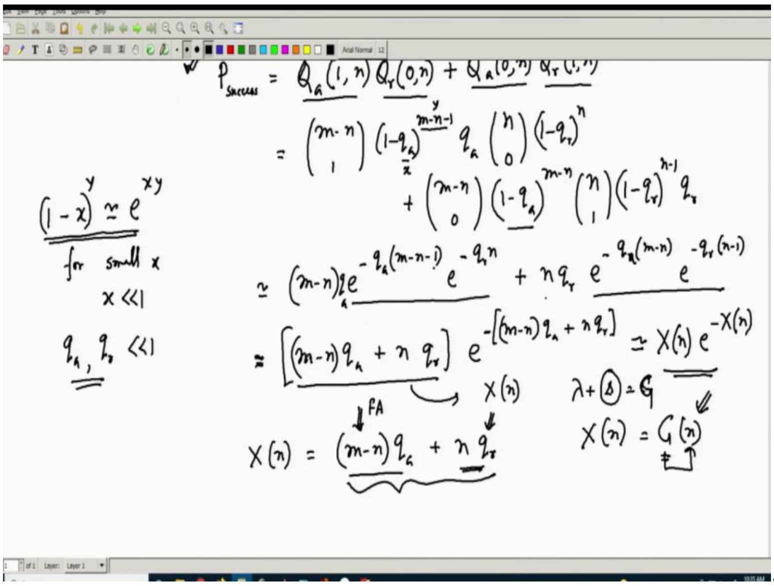

So, one of the approximate these things will take which is this one 1 minus x to the power y can be approximated as this can be proven e to the power x y for small x; that means, x less than 1 much much less than 1. So, this approximation or this approximation will be put over here as we can see, there is a means all these functions have this 1 minus x to the power y.

So, if this I take as y and this I take as x and generally this value of x means we can take that q a or q r much much lesser than 1, because if these are very close to 1 then that will be difficult always it will be in collision. So, that analysis does not make sense because then what is happening at the next slot everybody is attempting?

So, most of the time if everybody is attempting any n value higher than that, it means never the collision will be resolved. So, generally, this is a fair assumption that q a and q r value should be lower then only we can do meaningful analysis. So, if that is the case we can approximately say this to be a m minus n 1. So, that is m minus n ok.

So, we can write now e to the power minus this x into y ok. So, we can write it as q a m minus n minus 1 into this q a is remaining over here. So, q I can put it over here n c 0. So, that is nothing we can do that is one 1 minus q r to the power n again I can write e to the power minus q r into n, this is one this we can again write.

So, n c 1 is n q r goes over here and this happens to be e to the power minus q r m minus n sorry q a and this is e to the power minus q r n minus 1 ok. So, as you can see this e to the power same thing is happening, q a m minus n q r n and this is another q r. Now, if I take almost q a q r almost similar ok. So, if these two values are much much less than 1 and if we take that similar. So, this argument happens to be almost the same. So, what that is then? So, I can write this m minus n q a plus n q r this and approximately this can be written as e to the power minus m minus n q a.

So, this one q I am neglecting this because that is with respect to this one it will be much smaller plus n q r ok. So, as you can see I can now write this as a function of n I will write it as some X n. So, then, therefore, this can be approximately right written as X n e to the power minus X n ok. So, that is the value we get. Now let us try to characterize what is this X n. So, this X n that we have written is actually m minus n into q a plus n into q r, can you now try to see that this is actually the average arrival means average attempt because m minus n nodes are making fresh arrivals each one have a probability q a.

So, q a into m minus n is the average number of in one slot at stage n or at state n in one slot this many fresh arrivals on an average are happening and if you see n number of loads are stations are in backlog each one with probability q r attempt transmission. So, this is the average number of backlog transmissions happening.

So, this is actually the average number of overall transmission attempts that are happening over a particular slot. So, in a way, I can say this x n is nothing, but our G n, G you remember that variable we have given with respect to the total attempt that is happening now this is a variable over n. So, if the n varies accordingly we will be able to see depending on the value of q a and q r we will be seeing how many attempts are being made.

So, this is the average number of per-slot average number of attempts that are being made. So, if that is the thing that is almost like that lambda plus some retransmission attempt we were talking about some lambda plus delta and we were saying that to be G. So, this is that G now we have characterized it.

So, now as you can see this particular P success ok.

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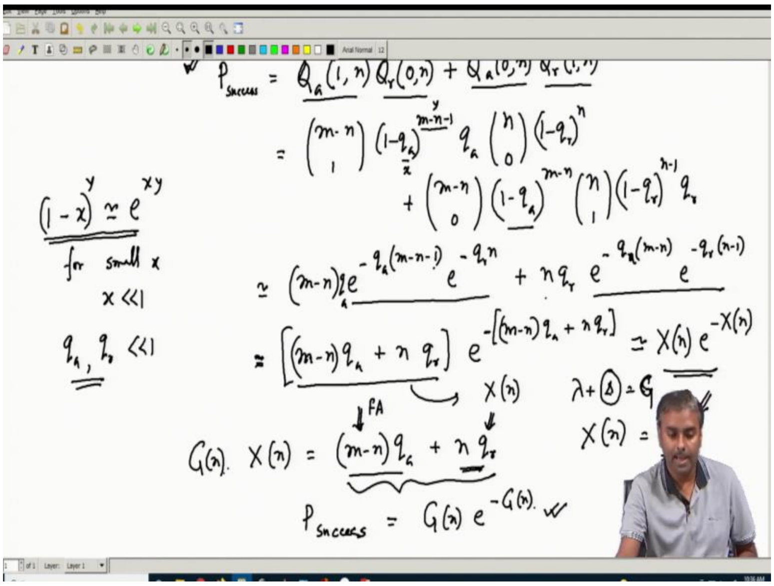

So, this particular P success has become according to our analysis, this G n e to the power minus G n ok. So, according to our analysis, this has happened. So, we could characterize P success, we could also see what our G n ok and from there, we have also got this drift equation. So, these three things we have got. Now with these three things in the next class what we will try to do, we will try to see if can we talk about the stability of this kind of system. So, that is something which we will be attempting.

So, seeing these three metrics analyzed from our actual understanding of the whole queueing system the DTMC we are trying to characterize means if we make a summary that we are trying to characterize at a particular value of n what is the amount of attempt that is being made, what is the average amount of or probability of success in that slot and what kind of drift in n that is happening.

So, these three things we are now characterizing what we will try to see taking all these things we will try to see whether we can analyze the stability of this system and then try to see what should be the guiding value of q r to make the system more stable that is what we will be analyzing next.

Thank you.