

Analog Circuits and Systems through SPICE Simulation
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Lecture – 58
Folded Cascode Noise (Contd.)

Welcome back. Let us resume our discussion on the noise analysis. And try to make certain other conclusion sort of it regarding the sizing of the devices and the conflict with other design optimization related to the bandwidth and gain.

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The whiteboard contains a circuit diagram of a folded cascode stage with three noise sources labeled 1, 2, and 3. Source 1 is the tail current source, source 2 is the input differential pair, and source 3 is the cascode stage. The equations written are:

$$V_{n1}^2 = V_{ng1}^2 + \frac{I_{nch1}}{g_{m1}^2} = \frac{V_{gs}^2}{4} \left(\frac{k}{\omega L C_{xf}} + \frac{4KY}{g_m} \right) g_{m1}^2 (g_{m3} r_{o3})^2$$

$$V_{n2}^2 = \left(\frac{1}{2} \right)^2 V_{gs}^2 (g_{m3} r_{o3} r_{o2})^2 g_{m5}^2$$

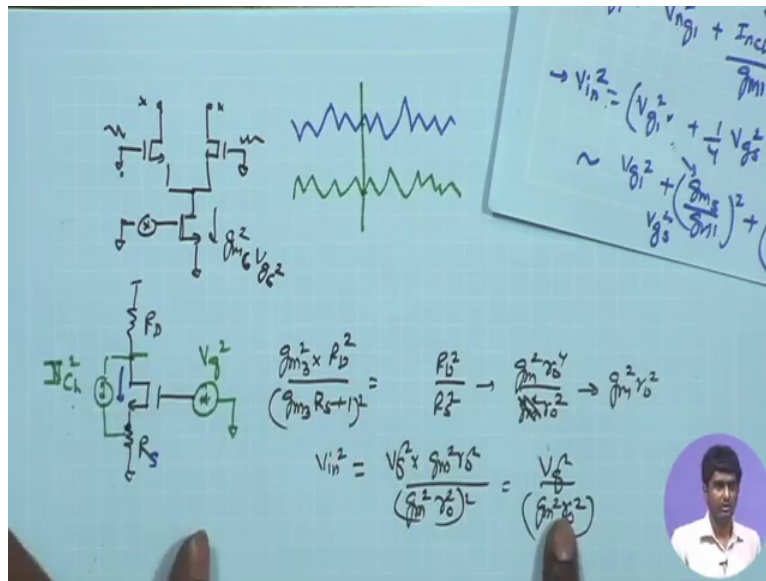
$$V_{n3}^2 = \left(\frac{1}{2} \right)^2 V_{gs}^2 (g_{m4} r_{o4} r_{o5})^2 g_{m5}^2$$

$$\rightarrow V_{in}^2 = \left(V_{g1}^2 + \frac{1}{4} V_{gs}^2 \frac{g_{m5}^2}{g_{m1}^2} + V_{g2}^2 \frac{g_{m2}^2}{g_{m1}^2} \right)$$

$$\sim V_{g1}^2 + \frac{1}{4} V_{gs}^2 + \frac{g_{m5}^2}{g_{m1}^2} \left(\frac{1}{2} \right)^2 V_{gs}^2$$

So, you look at the noise contribution of the 3 devices the m 1 m 2 m 2 and m 5. We have of course, not looked in to the noise contribution of the tail current source. And we have not looked in to the noise contribution of these 2. The analogy with the tail current source is that the noise contribution with the tail current source it is going to be divided equally into the 2 path. So, if we are looking at the tail current source.

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Whatever noise is produced by the tail current source that is going up the 2 paths. And is going to resulting equal and voltages at you know both the nodes. And in and the fully differential operation will be cancelled out therefore, we do not worry so much about the tail current noise contribution.

So, this is one point that we need to keep in mind. So, basically the 2 paths or the 2 noise signal that are going in up this channel and then getting folded and going out to the differential output. They are both these sources are correlated. So, if I assume that these are the 2 noise currents produced by this g_m 1 square whatever I guess it was 5, let me see the number. So, I guess 6, this is was 6. So, g_m 6 square V_g 6 square. That is getting equivalently divided with 2 branches and getting folded down therefore, these 2 sources are correlated and when we take the differential output it was getting cancelled. So, it is affect is not appearing significantly in the differential output.

So, we ignore the contribution because of this gate noise as it is the channel current choice noise other important factor is we are using the differential operation only to figure out the overall noise. And the main issue there is that when we look at the noise signals at the 2 gates they are uncorrelated they are independent. So, whatever input referred noise is calculated we are coming from the different 2 different branches each of the devices are having different noise contributions they are uncorrelated. So, the 2 noise signals over here are uncorrelated. If we magnify these 2 noises they are going to result

in some random noise signals at the 2 gates. And as I result if I want to define the overall signal overall noise signal at any instance.

So, at any instance we can find out what is the instantaneous noise voltage at the positive input, what is the instantaneous noise voltage of the negative input and the difference of that gives you the differential input just like we do normally for differential amplifier operation any arbitrary signal can be broken down in the common mode and differential signal common mode signals does not get significantly amplified only the differential part gets amplified and appears at the output with a much magnified response. Therefore, we are considering only the differential response and doing that we look at this as an AC ground and then figure out the total output referred noise and the output referred noise. So, this is another point we should be taking note of with respect to the overall analysis.

Now, let us try to as another issue which is regions with a noise contribution of these cascode device the cascode PMOS and cascode NMOS. So, I can look at that both the noise contributions the gate noise the flicker noise which is model as a series voltage in the gate and the second one channel current noise. So, I can just look at the NMOS condition and the result to be similar to the PMOS. So, for the NMOS I have large impedance R_D in the drain right. So, if I am considering the m_3 it is seen a large impedance into the drain which is $g_m r_o$ times r_o . So, I can call it R_D which is a large value. Likewise m_3 is also having a large resistance in the source which is r_s . So, I can call it r_s . So, of course, R_D is much greater than R_s in this condition. And then on the top of that we had the noise contribution because of this channel current if we can assume it and then you have an overall signal the noise signal at the gate I can call it V_g^2 I can call it V_{ch}^2 . And because of this I would like to find out what is the overall output voltage over here.

Now, if I here I cannot directly translate this V_{ch}^2 divided 2 equivalent gate voltage, because for this case the source is not ground not an AC ground, as a result I cannot directly divide this V_{ch} sorry V_{ch}^2 I cannot divide directly this V_{ch}^2 divided by g_m^2 and get a equivalent gate noise voltage, that can be done only the source grounded. And therefore, let us first consider the V_g^2 .

Now, if I consider the effect of V_g^2 and find out what is the output noise because of V_g^2 . This is our common source amplifier this source degeneration and

therefore, definitely the overall gain from this input to the output can be written as g_m of this device which is I guess $g_m \cdot 3$. $G_m \cdot 3$ square times the R_D upon $g_m \cdot 3 R_s$ plus 1. This is approximate expression this not exactly true when you are having very large impedance connected you get a slightly different expression over here, but approximately I am writing the same expression.

. So, you are having sorry $g_m \cdot 3 R_D$ square upon $g_m \cdot 3 R_s$ square, this is the overall gain. And once again if I assume that $g_m \cdot 3 R_s$ is much greater than 1 which is 2 here. So, I am getting R_D square upon R_s square. And therefore, we can see in this case you are reduced to g_m square r_o 4 upon g_m square r_o sorry upon r_o square. And therefore, you are having an overall expression g_m square r_o square. And now if I look at the input referred noise this needs to be divided by the overall gain which is $g_m r_o$ whole square. So, V_n in square because of this is going to be V_n^2 g square times g_m square r_o square divided by g_m square r_o square whole square. And therefore, this is going to V_n^2 g square upon g_m square r_o square.

Which is significantly smaller contribution much smaller contribution as compared to other devices. Even at the output if I look at it if you consider the for example, input device all the devices which are connected to the sources their V_n^2 g square is directly getting multiplied by the gain of the amplifier, which is g_m square r_o square. So, there you are having a multiplication factor which is $g_m r_o$ whole square. Whereas, the conclusion of this source is lower by a factor of $g_m r_o$ if I am considering V_n^2 g it is getting only multiplied by $g_m r_o$ whereas, if I am considering the gate noise voltages of the devices which are connected to the v_d and ground in our case the m_2 and m_5 . And m_1 their noises are getting multiplied by $g_m r_o$ whole square. And as a result the noise contribution of these 2 can be ignored as compared to the noise contribution of m_1 m_2 and m_5 . So, this is one effect likewise I can look at the I channel what is the result of I channels contribution. So, intuitively we can see what is going to happen, if I assume I channel to be increasing or decreasing in a certain direction right.

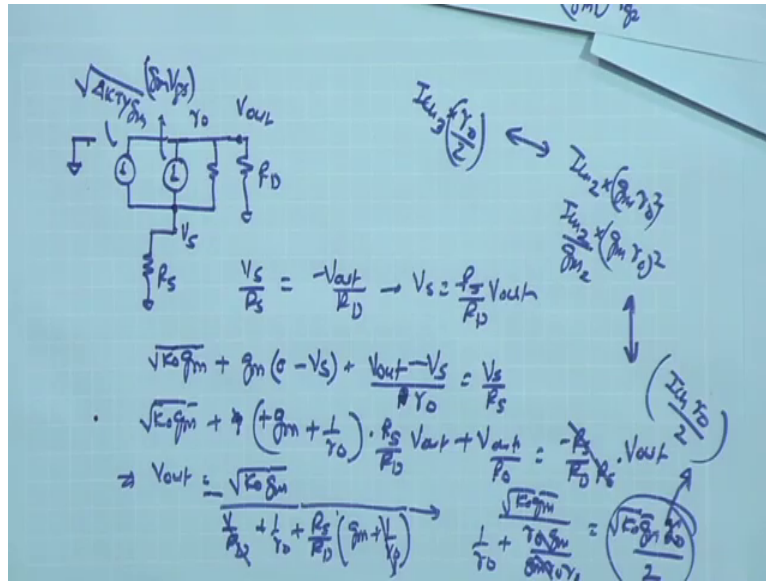
So, if I assume that I channel is a random current rate is go up or go down in a certain direction ultimately it is going to subtract to the channel of current. So, assume that you are having a fixed current I_{dc} and on the top of that you have this randomly fluctuating actioner it can go up and down. Suppose at a certain time instant it goes up; that means, the current is positive this is I channel is positive flowing from drain to source over here.

And therefore, it will increase the source potential right. So, if there is an increased channel current will try to increase the source potential over here because of this r coming in R_s coming in therefore, we did not mention r_s . So, here remember this was R_s and that is what I used. So, if I increase the I channel at a particular instance that will increase the r the v_s over here. V_g is a constant potential by analyzing the effect of this channel V_g is AC ground constant potential. As a result when you when your source potential over here goes up it is reducing the V_{gs} and therefore, will suppress the increase in the current.

So, it will whatever I assumption I took in the beginning that, because of the noise there is an increasing current. And that is causing those increase in source potential that will lead to reduction in V_{gs} of this MOSFET and that will suppress that current. So, we are having kind of a negative path is cancelling out any increase. So, that this kind of negative feedback kind of is going to suppress the effect of I channel in this case same thing is true for V_g also that can also be explained with the help of this negative feedback. So, ultimately when we are talking about a noisy generated common source amplifier that negative feedback action is present. So, V_g going up try to increase the channel current and that will try to increase the voltage at the source of this device. And that will reduce the again V_{gs} and hence try to suppress the increase in the channel current.

So, this negative feedback mechanism present in the cascode device it helping us in reducing the effect of noise contribution of these devices on the output and hence the input referred noise therefore, we are ignoring it. And analytically also we can try to in this circuit we can try to solve the k_{cl} and figure out what is the expression for I channel. You can do that very quickly if I look at it if I look at this model. So, assume the V_g is 0.

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So, MOSFET gate is 0 gate potential is AC ground and therefore, I do not I can look at the gm V gs term you have the R D which is between the drain and AC ground and you have the ro and also you have the Rs over here. We know that R D much greater than ro much greater than rs. And here you are having the gm V gs definitely and apart from that you also have the noise current which is for kt gamma gm it is not dependent upon the V gs (Refer Time: 11:27) just the constant term which is having some rms amplitude. So, you can call it some constant times gm. So, this is for kt gamma gm this one and this is your gm V gs. V g is anyway 0 so it is just gm times vs.

And then based on this we can figure out what is going to be the output potential over here, V out and that is going to give me the exact expression for the V out dependency on the current produced by the channel noise.

Now, if I assume that this is vs the unknown voltage and this is vd, at preconstance we can assume that there is some polarity. Of course, over all the noise current is random it is not having any polarity, but for a particular instance we can say that it is spiking up and as a result the current is in certain direction. And therefore, let us see that if the current is in this direction given by the magnitude for kt gamma gm peak amplitude, times some constant of core depending upon the frequency of integration. And these are this is the value which is you know gm V gs it is just gm times vs. And remember that this is the current over here supposed to be the root of this quantity. So, this is the current

density the mean square value I_{nch}^2 and when we add it we have to take the root under g_m value.

So, root under g_m times the bandwidth as we say. So, let us let us do it whether let us call it root under g_m . So, some constant and root under g_m this is $g_m v_{gf}$. And then we can figure out what is going to be the effect of this noise current $I_{channel}$ on the output. And let us look at v_s first of all. So, v_s upon R_s is the current flowing in that must be equal to the current going into the circuit I can call it minus V_{out} upon r_d . So, I can express v_s as R_s upon R_D minus times V_{out} . And then I can write down the overall expression of current over here which is root under k naught times g_m which is the current does dependent on g_m plus g_m times V_{gs} which is 0 minus v_s plus you have V_{out} minus v_s upon R_s sorry, so this should be equal to v_s upon r_s . This is expression we have finally, and I can solve for v_s you have k naught times g_m root under plus v_s I can basically replace by V_{out} and then so for it.

So, you have minus g_m minus 1 upon r_o times v_s which is if you get R_s upon R_D times V_{out} over here plus V_{out} upon r_o and this is equal to minus again R_s upon R_D upon R_s times V_{out} . And then you can cancel out. We are now I can take this on the other side and find out the expression for V_{out} . So, V_{out} is going to be given by root under k naught g_m divided by with the minus sign divided by 1 upon R_D , 1 upon r_o plus R_s upon R_D times g_m plus 1 upon r_o . This is the overall expression I am getting for the V_{out} . Magnitude point of view I will look at the mod of this quantity only.

And now if I look at these quantities in the denominator I can take some simplification based on whatever assumptions I had. So, I know that R_D much greater than R_s and therefore, this quantity is much smaller than 1 . And then you have the g_m and 1 upon r_o out of these again I can ignore the 1 upon r_o terms because in general I am assuming that it is much smaller than g_m . And among these 2 numbers 1 upon R_D and 1 upon r_o we know that R_D is much greater than r_o . So, I can ignore the 1 upon R_D term. And now if I look at R_s and g_m to the denominator side I have 1 upon r_o and $R_s g_m$. So, R_s is just r_o . So, you have $r_o g_m$ upon you have R_D which is once again large $g_m r_o$ times r_o . So, I am basically left with 2 upon r_o in the denominator and root under k naught g_m over here. So, root under k naught g_m and $2 r_o$ is are r_o by 2 is what we are getting the root under $g_m r_o$ by 2 and k naught is some constant number. So, the constant terms are going to determine my overall unit. So, we can see that the magnitude point of view it is this is

the noise term for $kT \gamma g_m r_o$. So, this voltage once again is proportional to the root under the v_n square because of I_{ch} square.

So, I can call it $I_{ch} r_o$ conclude. So, this is basically nothing else, but expression for I_{ch} . So, $I_{ch} r_o$ by 2. So, this is the overall voltage I am getting. So, I can call this I_{ch} this is a channel noise current magnitude times r_o by 2. And then if I want to refer it back to the input once again I will divide this quantity by the $g_m r_o$ whole square. So, here itself I at the output point I can compare what is happening.

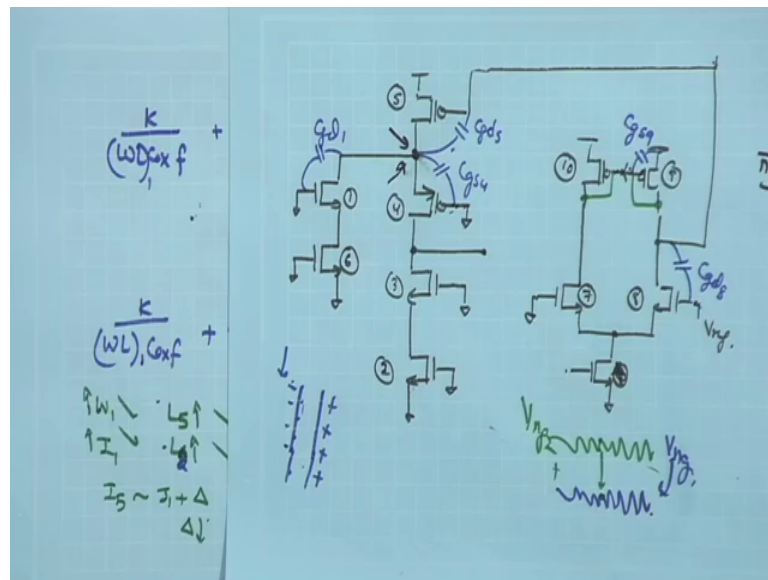
So, if I talk about the I_{ch} of the other MOSFET if I talk about the I_{ch} of say if I talk about the I_{ch} of the m_2 , or I talk about I_{ch} of and then getting multiplied by g_m square or $g_m r_o$ whole square. So, if I look at the magnitude I_{ch} times g_m times $g_m r_o$ square. So, if I compare this type of contrast this with the channel current effect of m_2 . So, I_{ch}^2 is getting divided by g_m^2 to give you the gate noise voltage of m_2 . And then getting multiplied by $g_m r_o$ square. So, basically I have I_{ch}^2 getting g_m times r_o square. Whereas, the I_{ch} of the cascode device is just having factor I_{ch} call it 3 times r_o by 2. So, here you have a r_o by 2 and you are having $g_m r_o$ times r_o right.

So, magnitude point of view once again the same conclusion that the overall noise voltage at the output, because of the cascode device is g_m out times smaller as compared to the devices which are connected to the V_{DD} ground that is the m_2 and m_5 . Therefore, as compared to the noise contribution of the I can see them I can call them common source devices because their sources are grounded and then noises are contributed in the form of a common source amplification. As compared to those devices the cascode devices noise are lower by 1 upon $g_m r_o$ factor therefore, I can ignore them this is the analysis just to conclude this result.

Now once we have concluded that and we look back at this expression over here we can again correlated back to the analysis that we did with it to frequency response and try to see how these 2 requirements are conflicting requirement of lower noise and requirement of higher bandwidth and good gain if at all there is any conflict. So, before we go further any discussion on whatever we did with respect to the noise contribution of cascode devices? All right.

So, let us go back and look at the expression that we had for the overall output referred and input referred noise because of the main contributing devices. And also look at the corresponding frequency response analysis that we did with respect to these capacitances.

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So, here once again we can see what is going to happen if I choose the device dimensions to minimize the noise. And one of the important factor that we mentioned in the beginning I need good gm for minimizing the noise and for that I need (Refer Time: 20:14) W by L or W 1 of m 1. Therefore, a large W 1 of m 1 definitely going to make the c_{gd} 1 dominant it is going to increase the c_{gd} 1 which is going to contribute this pole. Larger L 5 and L 6 as I said it is not going to increase c_{gd} 5 significantly because c_{gd} 5 depends upon the overlap capacitance. So, c_{gd} is dependent upon the L overlap it is the constant parameter for a given technology time W . Therefore, it does not suffer or does not increase when we are trying to apply larger L 5 or larger L 2 I should call this 2 it is not really as 5 6 it is should be 2.

Because L 5 or L 2 because this is the bottom device we have talking about. So, L 5 or L 2 full increasing the L 5 or L 2 does not really increase their c_{gd} decrease significantly as the result it does not have a significant fight with the compensation. And other another factor is that the cascode devices are not contributing significantly to the noise. And therefore, I can bring their W by L ratio down significantly without sacrificing my noise

figure. And as a result the cgs for effect can be minimized. So, that is not going to play a very important role. So, the main contributor is going to be cgd 1 at this node. And therefore, even for the differential response as well as common mode response the non dominant pole at this point is going to suffer because of the cgd 1. And other issue that we also figured out is the conflict between the swing and the gain and the noise.

So, if you are going for lower noise design then once again you are targeting larger L_4 m_5 and smaller W by L for m_4 . And that means, that you are going to have poorer or much significantly larger overdrive voltages for m_5 and m_4 . And that is going to reduce my available swing over here, same thing on the lower side I would like to have lower W by L_4 m_3 larger L_4 m_2 and both that is going to reduce the headroom on the negative side and going to impact my swing. So, lower noise is going to impact my swing in a negative fashion. So, that is another trade off we should be aware of. And for the point of view gain as I said the gain ultimately depends upon the g_m and the r_o products.

Now, r_o if we see is coming as $g_m r_o$ times r_o of this particular device g_m again depends upon the bias currents noise also depends upon the bias currents. So, increasing the current in the first device is going to help us in increasing the gain it is also you know going to help us in increasing the or reducing the overall input referred noise. Here if you increase the current over here of course, it also reduces the r_o of m_5 , but that can be compensated if you are having a larger channel length for m_5 significant additional length for m_5 .

So, at least for the noise definitely I would like to bet on the I_1 I need sufficient I_1 . So, that the noise input referred noise is reduced and at the same time what we discussed was if you are reducing the I in the in this particular stack, that is also in a way going to help us in reducing the input referred noise because we saw that the ratio I_2 upon I_1 or $g_m 2$ upon $g_m 1$ that was also helping us in reducing the noise. So, these are the factors we should be aware of while doing the design for low noise and make sure that while doing noise optimization going for lower noise their also meeting the required specifications for the gain and the bandwidth.

So, as next step you know since we have done this analysis for noise frequency response swing etcetera will take couple of examples, their the design steps will be shown algorithmically that given design spec specs at the higher level how do you come back to

the transistor level design, and how do you approach this sizing of these different transistors step by step, in a logical fashion, considering all these constraints considering all these you know design requirements in terms of specifications.

So, while addressing these specifications one by one starting from say bandwidth starting from the gain bandwidth product which is ultimately dictated by the higher application. Like we have discussed that the chopping frequency is going to mandate higher frequency that gives for the bandwidth requirement. And the position requirement gives us the gain requirement. So, gain valid product is coming from the higher point and from there we can see how the gain valid product is defined in this particular amplifier. So, we have seen for the 2 stage op amp how does the gain valid product get define. It is defined by the gm upon the cc. So, the gm upon cc the starting point if you need to setup particular gain bandwidth product for the amplifier where gain is said to be 10 the power of 4 for meeting a required accuracy bandwidth permitting required chopping frequency. We have this gain bandwidth product being determined by gm upon cc that becomes a starting point. And then you have the overall bandwidth or the cut off frequency means determined by the ro of the first stage basically the p_1 which is determined by the gm times gm ro time cc and the ro of the first stage.

So, step by step you know considering these constraints we can figure out the dependency of the sizing and resolve the designs resolve the sizing required for the different transistor in the first stage. In the second stage likewise in the cascode also we can start from given a specification and tracing back try to find out the dependency of each of these specification on the transistor sizes and try to set a compromise. So, generally analog then happens to be iterative. So, you once you set the transistors to meet a certain specs some other specs may get violated and again you can have you may have to go back and look at the design in iterative fashion fix them other specs again some third specs can get violated. So, it may take several iterations to arrive at the design which is meeting the given set of specifications within a reasonable margin.

So, we will take 2 such examples both with the 2 stage op amp that we have spent enough time on and also the folded cascode and that will facilitate that will help you in arriving at proper sizing. So, far in simulation assignments I have not ask you for going for appropriate sizing of the transistors. They have just given some starting sizing starting point for the sizes and have ask you to compensate those and get the loop stable.

So, another important factor is the sizing of the transistor that we will arrive at looking at this concept involving all these concepts and looking at the dependency or the sizing on all these analysis. Any question before we close?

Student: sir (Refer Time: 27:26) find out the noise we were taking only the differential period. So, one side of the differential (Refer Time: 27:31) input referred noise we have got with the differential input. So, this same input this same input referred noise with the negative sign of the (Refer Time: 27:46) this should be amplified

No input referred noise means you are having you know rms value of the noise, you are determining what is the rms value of the noise and what is the frequency content of the noise. So, that same magnitude of noise can appear at any of the terminals. So, what I discussed was ultimately if both the input devices are having certain noise and we are considering only the differential component of that noise. So, noise are uncorrelated at the both the gates and they are random and they are having a wide frequency range starting from low frequency to high frequency.

Student: (Refer Time: 28:16) must be exactly negative of the this (Refer Time: 28:20) you will get amplified?

Why?

Student: Because (Refer Time: 28:24).

Any arbitrary signal at the gate of the differential amplifier V_1 V_2 you can break it down into differential component common mode, component differential component gets amplified common mode component gets separates. So, not only in the last topic even previous topics, you can just go through very quickly. And if we have any issues we can discuss.