Analog Circuits and Systems through SPICE Simulation
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Lecture - 05
Basic Analog Design Part III

Welcome back. And a first point which is representation of noise.

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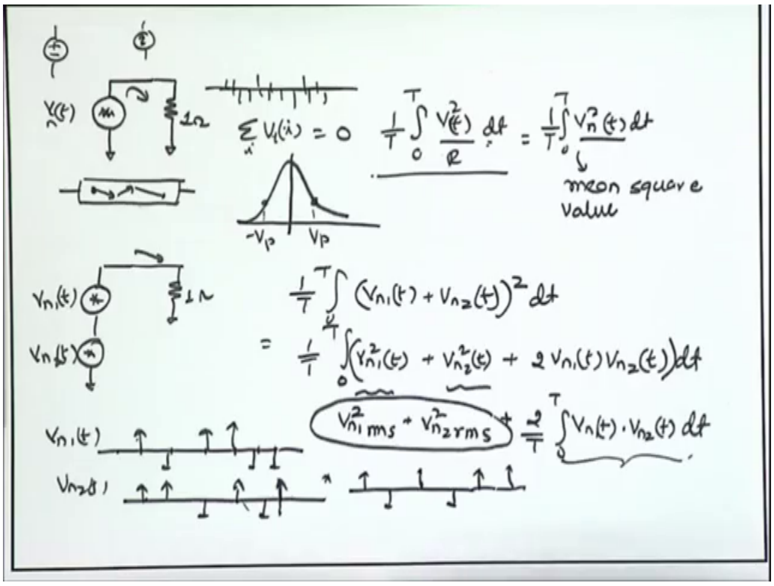

So, as I said noise can be treated as a random voltage or current being produced by certain random physical phenomena like the random fluctuation in the electron trajectory induced by thermal fluctuation. And we can represent this as a time domain signal V t right, although noise is not really a signal. It is just a random voltage fluctuation we can treated as a time domain signal V t and we can define the power delivered by this my signal to a 1 ohm resistor.

If we talk about instantaneous value of V t being random it is going to be almost symmetric across the zero point. So, if it sample V t for long enough duration sum of all this samples sigma V t i for all i's for a very long duration we will get something close to 0, because these positive and negative fluctuations when add up to 0s if we sum them up for long time that the characteristics of a truly random signal.

So, if I talk about the axial distribution it may be something like a Gaussian amplitude distribution where equal probability of finding an amplitude V p as minus V p. So, on and average the summed value of all the samples over long period will be close to 0 that is the characteristics of a random noise signal voltage or current noise. But a more meaningful quantity as I am defining the average or the mean which is 0 the more meaningful quantity is the power delivered to circuit component and we can define the or quantify the noise source as in terms of the power delivered to say a 1 ohm resistor. That is the more convenient way of defining a noise. So, what do we say how do we define out the power delivered by the signal my signal V t to a 1 ohm load?

So, we can write down V t square upon R which is in this case 1 ohm and integrate it over or sufficiently long duration divided by t and this gives us the definition of power. So, if I say call it V n square and define the R as just a 1 ohm resistor then this value is nothing but the mean square value of the signal v t; V n t. So, this is something like the mean square value of the noise signal V n t. So, we can represent the noise source in terms of its mean square, because it represents the power delivered to an effective 1 ohm load.

So, it is mean imaginary load we are assuming or we are defining either quantifying this V t or the noise source V n t in terms of the power it delivers to this 1 ohm load. This is just a way a convenient way of representing or defining noise.

Now, what happens if you have say multiple noise source. Suppose I have two noise sources in series V n 1 t and V n 2 t. And I want to find out what is the equivalent noise power delivered to this load. Recall that if you look at the symbol of the noise we did not put a plus or minus like we do with DC sources or an AC source, we put a star; that means it is not the directions is not significant polarity is not significant we are dealing with the mean square value or the power delivered to the load that is the convention we have defining noise so we do not worry about the polarity. So, we just put a star over there.

Likewise if we talk about the noise as a current source we put a double arrow; up as well as down. So, that we can signifies it is not really important to mention the polarity or the direction of the noise current flow. We are just concerned with the mean square value.

When we talk about the power deliver to this 1 ohm load once again we can use the same expression 1 upon t 0 t the total signal we will take the superposition of this to the time domain signals V n 1 t plus V n 2 t square by R R is by definition here 1 ohm. So, this is what we get. And we can write this as V n 1 square t plus V n 2 square t plus V n 1 t V n 2 t dt there 1 t .

Now, look at the first term and the second term that is just V n 1 square t dt 1 upon t 0 t that is just the mean square value of the first source, I can just replace this by V n 1 square rms. Likewise a second term V n 2 square t dt 1 upon t integrate over 0 to t. And once again V n 2 square rms. Now if we look at the third term this something like a correlation between the two noise sources V n 1 t times V n 2 t dt. And the third term over here as long as these two noise sources are independent they are going to be uncorrelated and as a result this product V n 1 t V n 2 t integrated over a long duration is going to lead to 0.

Simplest example is: suppose my noise is having only two levels; 1 and minus 1 equal probability it is a either it can take a value 1 or minus 1 just a simplistic case. So, I am sampling at different time instances and by with equal probability the two sources they can take 1 or minus 1 values. Overall over a long duration is 50 percent probability that the noise can take plus 1 or minus 1 value. This is V n 1 t and V n 2 t.

So, without going into detailed derivation and proof I can say like if I multiply these two they are uncorrelated, if I multiply this two and look at the overall signal again it will have almost 50 percent probability of plus 1 and minus 1 value. And therefore, being sum over sufficiently long duration once again it will result in 0. This is the simplest case. Of course, you can have any amplitude over here it is not limited to plus 1 minus 1 it can have any intermediate amplitude.

But the main point going to convey is that if we having two uncorrelated noise sources and you are multiplying them together summing it up for long duration, once again the resulting noise value, the signal over here summed over a long duration it will tend to 0. So, this correlation product if you take the mean of this there is something like taking the mean of V n 1 t times V n 2 t that once again you know tends to 0 if we integrate it for long duration.

So, this becomes 0 and we are left with only the first two terms which is the V n 1 square rms plus V n 2 square rms. That gives us the significance of defining the noise in terms of their mean square values. If we are dealing with two independent noise sources we can add their mean square values. We do not need to worry about their time domain amplitude, we just clear about the mean square noise produced by the first source, mean square noise produced by the second source and we add the mean square we add the power of the noise we do not add the time domain amplitude of the noise. This is one of the important concept let it be the representation of noise in the circuits.

Now if I talk about the frequency domain characteristics; how do we define the frequency domain characteristics of noise?

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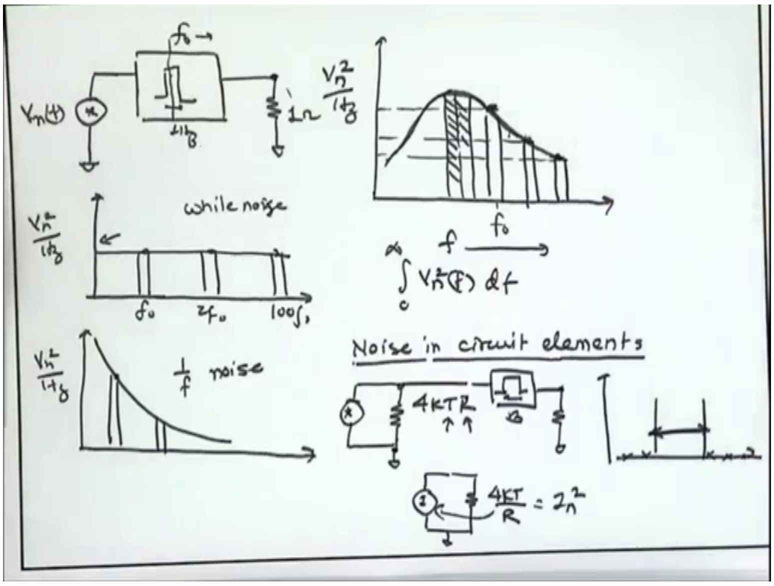

So, to do that once again we have a light source we want to figure out what is the frequency domain characteristics of this noise source V n t. To do that conceptually we can put a very sharp filter over here say of bandwidth 1 hertz, center frequency is f naught this is some center frequency f naught and bandwidth 1 hertz. So, it will pass the signal only within this 1 hertz bandwidth centered along f naught, it will block all other components of the v and t. And then we supply it to the load which is the 1 ohm load.

Now, if I once again plot the mean square value coming over here that denotes the power delivered to this 1 ohm load when I am centering my filter at f naught. So whatever amplitude V n square I get here it is going to give me the power delivered to the 1 ohm load when the center frequency of this filter is at f naught. Now I can keep changing this f naught, I can keep splitting this f naught and at different-different points I can calculate or I can find out what is the V n square delivered to the load. And if I take very fine sections ultimately it is going to give me smooth curve some characteristics depending upon what is the signal source.

So, here I am writing V n square because this is the V n square upon R that is V n square upon 1 which is essentially the power delivered to the 1 ohm load and on the x axis we have keeping the frequency. So, by joining these points you can smooth curve I get the noise power spectral density.

So, it is telling us how the mean square power of the noise delivered to the load varies with frequency, which frequency is having highest component. So, the peak over here we say that at this frequency you have very high component of the noise. In some cases as we said there are two dominant sums or two very commonly encountered spectral densities of noise: one of them is white noise where it is almost flat. So, this V n square that we get is almost flat- whatever frequency it take whether it is f naught or 2 f naught or 100 f naught ideally the noise power that you get it will be almost same at all these frequencies. So, it is a white noise.

And remember the way we have defined this V n square is by passing it through a 1 hertz filter. So, we are talking about power delivered in this 1 hertz bandwidth. Basically it is the rms or the mean square value per hertz. So, we define this as V n square per hertz. If we have to find out from a given noise power spectral density the total noise signal present in the particular source we all we need to do is integrate this curve.

So, I will just integrate this V n square f d f from 0 to infinity. So basically I need to integrate all these sections and get the total mean square noise represented by the signal. That is going to give me the amplitude or the total mean square value of the noise source. So, this is the corresponding frequency domain representation, this giving me the distribution of the mean square value of the noise across the frequency range of interest. And I have to find out total mean square value I just need to integrate all these stripe the stripes and get the total mean square value.

And a way or common way of representing noise may be taking the square root along the y axis rather than putting it at V square per hertz we can also represent it as volt per root hertz; we are just changing the unit along the y axis, but this is the main definition.

So, white noise is one commonly encounter spectrum another common spectrum is 1 upon f noise where the noise increases with frequency lower frequency. So, as we go towards lower and lower frequencies the noise goes on increasing. So, this is 1 upon f noise which is also very commonly encountered in circuits. Now let us see how do we represent noise in the circuits, so starting with the resistor.

What we have seen is at as so our discussion towards the end of the last class we had seen that the equivalent noise voltage produced by a resistor that can be modeled as a white noise source. So, the spectrum of the noise source produced by the resistor is almost is flat it is having equal magnitude at all frequencies. So, it is a white noise and the power spectral density can be just treated as 4 KTR it is proportional to R it is proportional to the temperature. And we just discussed the physical mechanism behind that briefly. Larger is the temperature larger is the thermal fluctuation of the carriers in the resistor. Likewise if larger is the R once again more scattering and more random fluctuation in the trajectory leading to more noise voltage. So, this is the magnitude of the noise spectrum density. What we can say is this level the magnitude mean square per hertz will be 4 KTR.

And if I have to find out that before a this resistor if you put say a filter; suppose I put an ideal filter of bandwidth B, this is the bandwidth B. And I want to see that here what is the noise power this you are going to pass. So, once again I have to look at the spectrum the bandwidth is located here, those of noise only in this region will be able to pass others are getting block. So, the out of band noise is not passed only the bandwidth which is accepted or passed by the circuit that is getting communicated to the output node.

We can also represent the noise of the resistor in terms of an equivalent, current source just by taking the Thevenin equivalent I can write down the equivalent noise current source produced by the resistor that is 4 KT upon R. So, this is the equivalent noise current source produced by this resistor of value R. (Refer Time: 16:08) you multiply this with R square if you remember this is I n square; this is I n square. So, this is the mean square value of the noise, likewise this is the mean square voltage. So, if I have to multiply this 4 KT upon R with R square to get the resulting voltage which is same as 4 KTR.

Another point to remember is that this is not the mean square value as it is the noise power spectral density that is it is denoting the magnitude of this curve means; what is the noise power spectral density you have to integrate over a desired frequency range to find out what is the total mean square power delivered by this source to a particular load.

So, once we are comfortable with the noise definition and representation of noise in a resistor we can draw an analogy and define the noise sources in a MOSFET.

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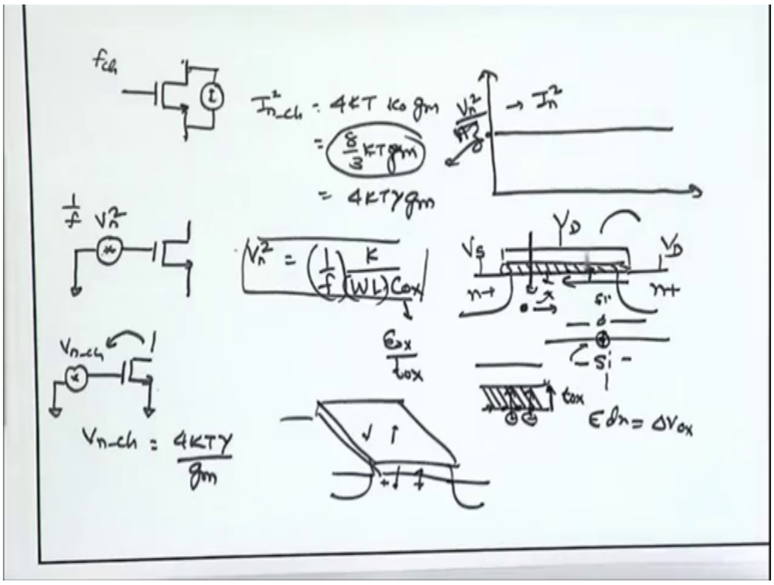

So, just like a resistor when the MOSFET is on it is in saturation region it is having a current flowing through the drain to source. And when we talk about the small signal current that is proportional to the gm. So, effectively the transistor is having some effective on resistance in the channel proportional to gm. So, we can say the effective noise current source I n square channel is going to be define in a similar way 4 KT some proportionality constant say k naught times gm.

Generally, this number for sufficiently long channel MOSFET is sum to be 2 by 3, so we get 8 by 3 KT gamma times gm where rest of the factors are constant and we get basically 4 KT times gamma gm, where gamma is around 2 by 3; so this is the equivalent noise factor. So, we can represent it as 4 KT gamma gm as the equivalent mean square noise for the MOSFET channel.

So, similar to the MOSFET similar to the resistance case where this channel noise current is coming because of the resistive conduction path between the drain and source of the MOSFET. And just like the resistor this is having the white spectrum. So, once again if I talk about this noise this is again having a white spectrum and the magnitude over here V n square per hertz is denoted by 4 KT gamma gm. So, this is basically I n square. So, I n square rms or mean square value.

Another important noise source in MOSFET comes because of so called 1 upon f noise which is slightly more involve to explain. So, that is represented as a voltage source in series with gate. Here the V n square or the mean square value as I said it is having a 1 upon f dependence on the frequency here proportionality constant K and then we have W L times Cox.

Here the main mechanism that is involved in the production of this 1 upon f noise deals with the carriers flow across the channel. So, whenever we have electrons accelerating from say source to drain and we have a current flow you have interaction between these accelerated carriers and the interface oxide. And there can be exchange of carrier between the oxide interface and the channel. So, because of electric field in the positive direction you are having for a NMOS for example; you are having a electric field from drain to the sorry from the gate to substrate, likewise you have a electric field from drain to the source. So, electron definitely they have a strong positive velocity from source to drain because of the positive electric field, likewise they do have some gradient of motion or the velocity in direction perpendicular to the gate.

And there is a continuous trapping detrapping phenomena that can happen at the interface. So, interface is not perfect you can have some dangling bond at the interface between the S i and the S i o 2. So, inside you know at the between the S i o 2 and S i you can have certain bonds which are broken they lead to dangling bond and this becomes the trap of these electrons which are accelerated in the channel. So, there is continuous trapping detrapping phenomena of these carriers at this interface.

And because of which we are having an effective change in the threshold voltage. Remember how did they define threshold; voltage threshold voltage concept came because of the accumulation of charge so we need certain maximum positive, certain minimum positive charge deposited on the metal gate to get sufficient number of negative charges inside the channel so that the continuous channel is formed.

So, suppose at a certain instant I already pump in some negative charges they get trapped in to the oxide over here, then you already have some trapped negative charge and then you may require smaller positive charges to balance that total negative charge. This is how threshold voltage can reduce. So, threshold voltage can fluctuate whenever you are having these negative charges getting trapped detrapped from this oxide surface.

So, for example, whenever you are trying to increase the gate voltage little bit you expect that the channel charge will go up by in a proportional amount corresponded to gm, but if you are ending up increasing the trap charges the electron charges over here. The increase in channel charge will not be so much, because some of the trapped negative charges they are compensating for the additional positive charge that you have put on the gate. Therefore, it will fluctuate from the desired value.

So, there is a basic mechanism we were just in a hand-waving argument the without going to the deeper physics or derivation we are just trying to state the basic mechanism that this is the main phenomena, which results in the 1 upon f noise. And this 1 upon f noise characteristics is mainly resulting from this trapping detrapping process. If you are going towards lower frequency this trapping detrapping process is able to trap the low frequency signal much more easily.

So, if you are changing the gate voltage slowly this trapping detrapping phenomena is much slower and therefore it is able to trap the change in gate voltage in a much better fashion and it is able to introduce more noise. Whereas, if the signal at the gate is changing much faster as compare to that this trapping detrapping from is slower, so that is not overlapping with the frequency or it is not really align with the frequency of the signal. So, this is shifting at lower frequency.

So, lower is the frequency larger is this trapping overall integrated trapping detrapping phenomena. So, if I want to find out total charge integrated because of these trapping detrapping phenomena, it will increase if we integrate it for a longer time or a longer duration. That means the corresponding frequency spectrum is lower. If I observe this noise for a longer duration amplitude will be integrated this trap detrap charge integrated over long time will give me larger change in the threshold voltage, larger change in the charge accumulated. And therefore, this 1 upon f noise comes in to picture.

Other term when you look at Cox; what is this Cox? This Cox is epsilon ox upon tox. So, this also comes because of this charge trapping detrapping. So, if you have thicker Cox and then you are trapping some additional charges over here, because of that the electric field that is produced and integrating the electric field produced because this negative charge is same it will produce the larger voltage drop if I increase this tox. So, the overall voltage drop produced across this oxide will be larger if the oxide thickness is larger, because we have just integrating the using gauss law you can get you know electric field produced by this additional trap charges if the tox is larger e times tox it is the delta vox. This is small change in electric field produced across the oxide it will also going to be larger.

Therefore, larger is the tox larger is the integrated noise or the 1 upon f noise. And the other factor that is W times L it denotes the total cross section area of the MOSFET. Basically what happens is because there is so much trapping detrapping phenomena happening all across this oxide interface, if you increase the area of the gate on an average these trapping detrapping phenomena will cancel out, because we have a large area. So, if you are integrating or summing up all this trapping detrapping phenomena over a large area it will become closer and closer to 0; just like you know the phenomena of forming noise at different instances.

So, here we are not summing the noise at different instances, but we are summing up this trapping detrapping across a large area. Larger is the area more is the probability of that total summation approaching 0, because in this entire gate area you are having some trapping detrapping going on. So, larger is the area across which you are integrating smaller is the mean value

So, just like in case of a random signal we integrated over a long duration and then this plus minus 1 will tend to 0 if I sum up for a long time. Just like flipping up coin if you are flipping it up or down if you do it for a large duration or large number of tries the mean value will be 50 percent up 50 percent down. Likewise here if I assume that there is trapping detrapping going on if you sum it up over large area the mean value tends to

0, because there is plus minus. Same phenomena, rather than averaging in time given we are averaging over area and therefore we have W L coming in to picture.

So, this is the expression for the noise spectral density coming because of the 1 upon f noise V n square which is represented at the gate. And once again we are getting some important information regarding the device. So, larger W and L is able to minimize the 1 upon f noise. And therefore it may be advantages in certain cases wherever 1 upon f noise is becoming critical we see that larger W and L can help us minimizing that. Likewise, here if you look at the channel noise it seems like an larger gm is giving me larger channel current, but it may not be the right picture or may not be the most effective way of observing this noise.

More effective way may be to represent or capture the effect of this channel noise current in to an effective gate voltage noise voltage. So, this is something similar to the gm vgs current that we get from drain to source. In the MOSFET model we any way had gm vgs flowing from drain to source. So, for a applied gate to source voltage vgs we have a gm vgs flowing between drain and source. And for a given vgs the small signal current flowing from drain to source happen to be gm times vgs. Now, I have this noise channel current defined in parallel with the channel conductance that is 4 KT gamma gm and, how can we represent it in form of an equivalent noise voltage at the gate so I can just divide this by gm square, because gm square times this noise equivalent noise voltage is going to give me the current.

So, I can look at it I can represent. So, this is the 1 upon f noise, this is the channel noise f channel. So, in order to represent the f channel provided my source is grounded, assume that the source is grounded I can represent it in the form of V n channel by dividing this mean square current value 4 KT gamma gm by gm square. Remember this is mean square value, so if I have to represent it in terms of a gate voltage it will be divided by gm square. So, V n ch will be 4 KT gamma upon gm. So, the equivalent noise will take refer to the gate can be represented as 4 KT gamma gm.

So, in circuits generally it may be important to look at the input referred noise; that if you are applying the signal at particular terminal say between the gate and source. So, as compare to the applied signal what is the mean square value of the noise over here. So, actually the noise will be present because of the channel current flow, but I can refer it to the input, I can represent it as an equivalent noise voltage at the input. And that is convenient to compare with the applied input signal. So, we call it input referred noise and this is coming to be 4 KT upon 4 KT gamma upon gm.

So, it seems that here if I look at this form increasing gm may help us in reducing the input referred noise. So, this is not giving me as a right picture, because when we talk about noise we always have to talk about noise in terms or in relation with the signal. So, we will see that a larger gm also has a role in amplifying signal, when we try to have a larger amplification we need larger gm. So, gm plays an important role in amplifying the input signal.

So, if we apply a input signal over here V n the small signal current that we get is gm's times V n. So, that talk about the mean square is gm square times V n square. So, the output signal square is proportional to gm square times V n square, whereas the noise is proportional to gm. Therefore, relative to the noise the signal is more amplified. So, gm is playing an important role in amplifying the signal whatever weak signal is present over here it is able to amplify that signal by the factor of gm square gm square times V n square.

Therefore, although it may look like that the channel current noise is 4 KT gamma gm and creating a larger gm may miss up with by noise, but in fact if I refer it back to the input if I compare it with the input signal then it seems like no not really in larger gm may help to reduce the input referred noise at the gate. There is another very important concept- while dealing with circuits more complicated circuits where you have bunch of transistors inside the circuits we try to derive what is the equivalent input referred noise of that circuit so that it can be compared with the input signal.

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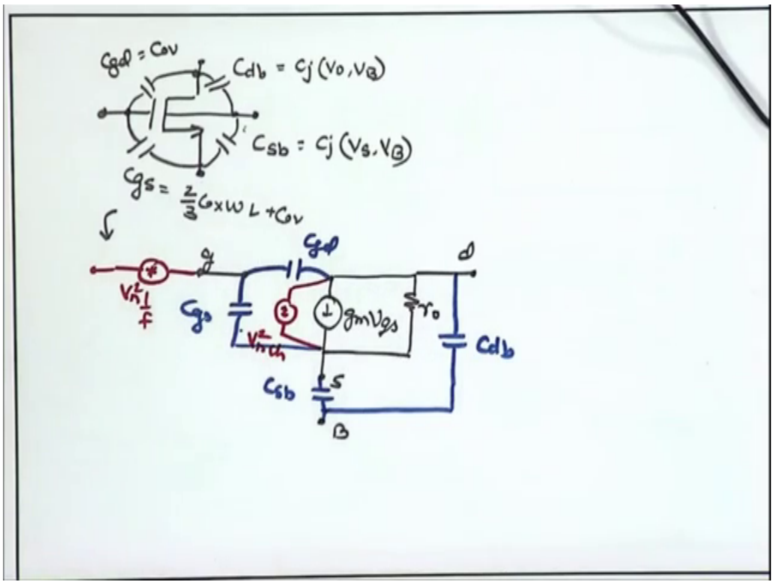

Now if I want to complete the small signal model that we earlier arrive that including the parasitic capacitances: I can add my noise sources over here, that is this is the V n square 1 upon f noise coming if series with the gate. And you also have a channel current noise which is we can call it V n square channel which is coming between drain and source. So, this basically completes our MOSFET model. We have incorporated the effect of two noise mechanisms in the MOSFET: that is 1 upon f noise and the channel noise.

And now we have some hint also that while analyzing the circuits what do we need to do the channel current noise: we can refer it to the input and then solve for input refer noise. That is the main step we are going to follow while solving some of our circuits using this concept.

Thanks a lot.