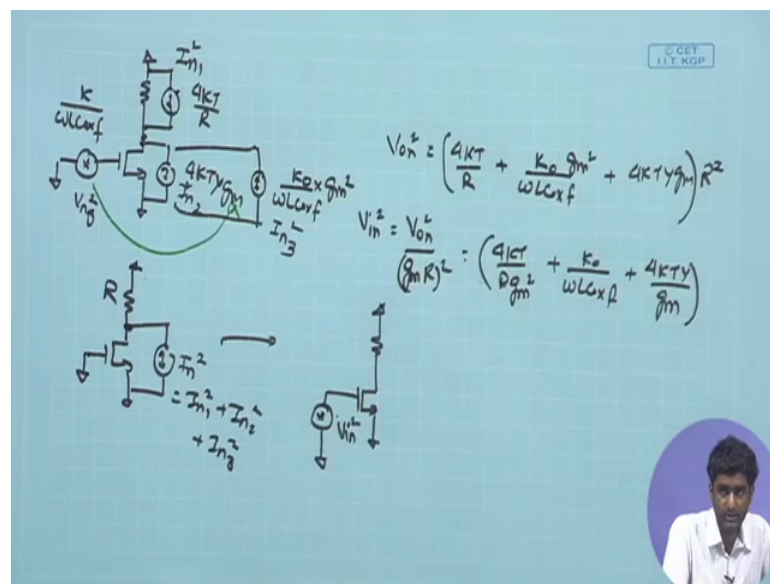


**Analog Circuits and Systems through SPICE Simulation**  
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**Lecture - 25**  
**Noise Analysis And Sizing**

Welcome back. So, let us start with our translate to be analysis of the fronted amplifier we are looking at.

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So, we can start the analysis with a simple common source amplifier with resistive load and then go towards active load amplifier which is ultimately going to link with our overall two stage amplifier with the help of which we have been building our fronted amplifier. So, here I have considered a simple common source amplifier and I can represent the overall noise constituent of these two resistors and the transistor over here.

So, I can draw the noise source of the resistor put it as  $4KT$  upon  $R$  likewise I can put the noise current source of the transistor put it as  $4KT$  gamma  $g_m$ , and I have the  $1$  upon  $s$  noise which is going to be given by  $KWLC\omega \times$  times  $f$  and since I have to get the effect of only the noise I can set the input signal to be  $0$  for not considering any applied input signal apart from this noise sources that you have in the circuit and therefore, I can find out the overall noise voltage at the output terminal.

So, if I take the superposition remember the small signal transconductance is not going to come into picture directly because the  $V_{gs}$  there is no additional input signal coming because of the  $V_{gs}$ , but only way that small signal transconductance is going to come is because of this noise signal at the gate. So, if I consider this as an input signal remember that is this is your  $V_n$  square. So, this is the mean square. So, in order to get the small signal current resulting from this noise board at the gate I need to multiply this with the  $g_m$  square. So, that will also ultimately add up as a small signal current in parallel. So, you have another small signal noise current which is basically going to come from this input source which is going to come from this input source which is given as  $K W L C_o \times f$  times  $f$  times  $g_m$  square.

So, there I can capture the effect of this noise in terms of a current another result if I look at the ac analysis the overall voltage over here, I need to take up the summation on the mean square values. As we have discussed earlier if you have uncorrelated noise sources like the noise source and the gate is uncorrelated with the channel current noise because the mechanisms are different their origins are different therefore, these two are uncorrelated noise sources, likewise the resistance noise is also uncorrelated with the other noise sources therefore, these are three independent uncorrelated noise sources and therefore, in order to get the final output voltage and need to just take the sum of their mean square that then automatically obtaining from this combination.

So, I have the  $V_o$  square given by the 3 noise sources  $4 K T$  upon  $R$  plus let me call it  $K_0$ . So, that does not get confused with the Bossman constant  $K_0 W L C_o \times f g_m$ . Square plus the  $4 K t$  gamma  $g_m$  times the overall small the overall resistance at this point. So, these are the total current summed up  $I_{n1}$  square,  $I_{n2}$  square,  $I_{n3}$  square that multiplied by the total small signal resistance. So, this is nothing is what you know similar to our case where you have overall resistance of the MOSFET and in total you have some current coming over here. So, this is set to 0. So, I have captured the all the noise sources ultimately their between the ac ground and the drain. So, this all these three noise sources are between the ac ground and the drain.

So, I have combined them into  $I_n$  square which is equal to  $I_{n1}$  square, plus  $I_{n2}$  square plus  $I_{n3}$  square,  $I_{n1}$  square, plus  $I_{n2}$  square, plus  $I_{n3}$  square. So, this is  $I_n$  I can call this  $I_{n1}$ , I can call this  $I_{n2}$ , and I can call this  $I_{n3}$  or other squares and as a result I can get the total  $I_n$  square and you have I have to find out the overall noise voltage

coming at this point is just going to be  $I_n^2$  times the total small signal resistance over here, and once again this is the passive resistance is much smaller than  $r_o$  I can just multiplied with the  $R^2$  that you have the passive resistance over here.

So, this multiplied by  $R^2$  gives me the  $V_o^2$  over here and this is the output noise. So, overall output noise. So, it can be termed as output referred noise, it means what is the overall noise voltage or the mean square value of the noise voltage in other words the rms of the noise voltage produced at the output node in case you do not have any signal applied. So, you can just trying to reach the noise voltage since the noise voltage absence of any signal, this is the mean square value that we are expected to obtain.

Now another important quantity in analyzing circuits is the input referred noise we want to find out the equivalent noise voltage at the input, which would produce the same noise voltage at the output provided this was an ideal circuit noise less circuit. So, for that I need to translate this  $V_o^2$  which is the output referred noise to an input referred noise by dividing this entire  $V_o^2$  by the gain of the circuit. (Refer Time: 05:39) do what is the small signal gain of this amplifier this is just  $g_m r$ . So, we are going to divide  $V_o^2$  by the gain of the circuit which is  $g_m R^2$  to obtain the input referred noise. So,  $R^2$  term gets cancelled and you are having the  $1$  upon  $g_m^2$  coming into picture  $4 K T R g_m^2$  plus  $K_{naught}$  upon  $W L C_{ox}$  plus  $4 K T \gamma$  upon  $g_m$ , this is the overall expression that we obtained for the input referred noise.

So, I can translate this into an ideal circuit. So, that the transistors and the resistors are non noise free, but I am having an equivalent input referred noise  $V_{in}^2$  which is capturing the noise contribution of all the transistors and resistors in the circuit and representing it as an equivalent input referred noise, which would produce the same  $V_o^2$  same output referred noise provided this as an ideal circuit and if you are having an input signal apply to this amplifier which is very minuet we would like to make sure that the means rms value of this input referred noise input it is much smaller as compared to my input signal. So, that it does not corrupt it for example, if I won 7 pre precision or 7 bit accuracy in the recorded data and the data coming over here is 1 millivolt peak to peak I would like to make sure that the  $v_{in}$  in the rms value of this noise that you are

having at the input point that is lower than 1 percent of that 1 millivolt. So, 10 make this is should be within 10 micro volt peak to peak.

So, the rms value should be within 10 microvolt approximately to ensured seven procession for the overall amplification. So, it is a good way to compare the input signal with the overall noise contribution of the amplifier you can be done at the output node also. So, you can look at the amplified input signal and compared with the output referred noise or you can look at the input signal magnitude and compare it with the input referred noise of the circuit and make sure that the input referred noise of the circuit is much smaller than the input signal that you are trying to process through this particular circuit.

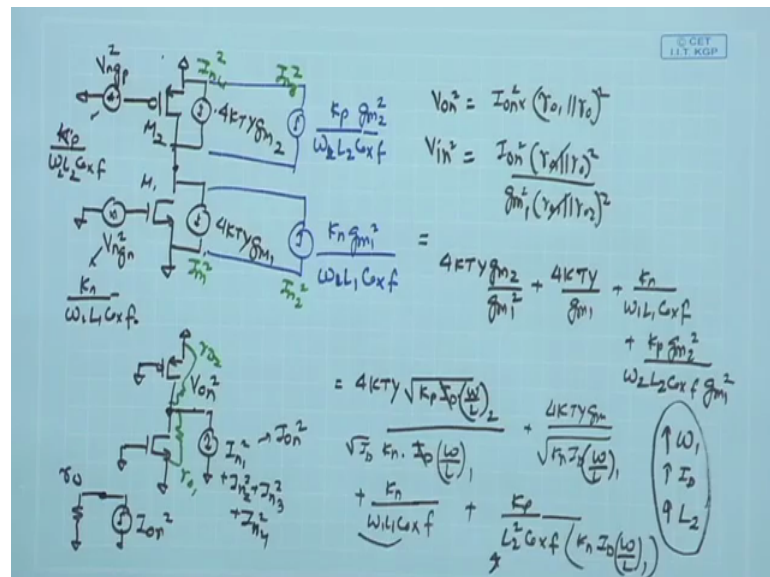
And, now if you look at the dependencies in this case we can look at the gm and the R dependencies we can see that all throughout we have 1 upon gm dependency over here 1 upon gm squared dependency over here. So, definitely having a larger gm for (Refer Time: 08:14) is going to help us in reducing the overall input referred noise. Likewise if you look at the R term over here it is coming under the denominator and therefore, increasing the R term is going to suppress the input referred noise, and the reason is that this R term although it is contributing to a rms value given by root under  $4ktr$  right. So, you have the mean square value given by  $4ktr$  rms value root under R, but the input signal is getting magnified by R.

So, the magnifications for the input signal proportional to R. So, the input signal gets magnified by the factor R was I am looking at the outputs voltage square with respect to the input voltage that is have a dependency R square wherever the noise is having just to dependency of R and therefore, the signal to noise ratio gets improved if I am looking at if I am using a larger value of R. So, respite this R contributing noise which has the mean square value proportional to R since it is amplifying the input signal by R, it is improving the signal to noise ratio and therefore, we can see that the R is coming in denominator larger R value is helping us in minimizing the input referred noise and likewise although the gm is contributing to this channel current source since the gm is also conducive in amplifying the input signal the output signal is directly proportional to the gm times the  $v_{in}$ .

So, it is also amplifying the input signal and as a result we can see that having a larger gm is helping us in getting a better or a lower input referred noise because it is the amplifier is able to amplify the input signal more strongly proportional to gm if the gm is larger.

Likewise we can look at an active load circuit where you are having a PMOS as an active load and then look at the concept of the input referred noise of the common source amplifier with active load. Any question on this part before we proceed towards active load.

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Yes any question on the passive load part for the active load once again I can put a PMOS and NMOS and then look at their combined voltage sources the noise voltage sources. So, once again I am going to have the Vn we can call that V ng p square and then you are arranged with channel current of the PMOSFET, likewise you have a channel current of the NMOSFET and the V n g n square I am just denoting this terms and the channel current over here we know that this is 4 KT gamma g m 2 if I call this m two and call this m one likewise here you have 4 KT gamma g m 1 and by doing the similar analysis you need to find out the output noise voltage over here or mean square voltage over here.

Now if you look at the V n g p square, you know that this value is 4 sorry K p upon WL of p. So, I can call it W2 L2 Co x times f likewise this is K n upon the W1 L1

times  $C_o \times f$  and then I can translate these two gate voltages also into the channel currents just like we did in the case of the passive load by multiplying with a  $g_m$  square. So, as long as the transistor source is grounded you can directly convert the gate voltage into a channel current because ultimately that noise voltage at the gate appears as  $V_{gs}$ .

So, it is easy to convert it directly into the  $g_m$  times  $V_{gs}$ . So, or  $g_m$  square times is  $V_{ngp}$  square this be the equivalent channel current likewise for the NMOS also the source is grounded. So, as long as the source is grounded and you are having the gate signal in this case the noise which is having mean square value  $V_n^2 g_n$ , that you can convert it into a equivalent channel current by multiplying with  $g_m$  because any gate signal multiplied by  $g_m$  square will give you the equivalent channel current.

So, as long as the source of the NMOS and PMOS are grounded you can do that, but we will see that if you have some other circuits where you have some transistors where the source is not grounded like a cascode device that we have studied their we cannot apply the same thing their we need to look into some other considerations while translating the gate voltage into channel current.

For these two it is clear to see that the gate voltage can be translated into the channel current. And therefore, I can write down the equivalent noise current coming because of the gate voltages translated into channel current, this becomes  $K_p W^2 L^2 C_o \times f$  times the  $g_m^2$  square, and likewise this becomes  $K_n g_n^2 W^2 L^2 C_o \times f$  and ultimately we have the same scenario as the previous case and what we are going to do is we are going to get the total output voltage  $V_o^2$  which is going to be the combination of all these 4 noise currents. So, I can call these currents as  $I_{n1}^2$  and  $I_{n2}^2$  likewise  $I_{n3}^2$  is our mean square values of the noise current  $I_{n4}^2$  square. So, ultimately these are the 4 noise currents which are a flowing from the ac ground towards the output node.

So, if I look at the equivalent circuit it is nothing is, but the two MOSFETs and an equivalent noise current source given by  $I_{n1}^2$  square, plus  $I_{n2}^2$  square, plus  $I_{n3}^2$  square, plus  $I_{n4}^2$  square; flowing from ac ground towards the drain and all we need to do is find out the output noise voltage  $V_o^2$  just like we did in the previous case only thing is in this case we have an overall small signal resistance over here given by  $r_{op}$  parallel  $r_{on}$ .

So, all this current  $I_n$  I can call this  $I_o$  noise that is will be multiplied by  $I_o$  noise square times the output impedance which is  $r_{o1}$  parallel  $r_{o2}$  square to give me  $V_o$  noise square right if you have a overall small signal current over here given by  $I_o$  noise square multiplied by the small signal resistance of these transistor  $r_{o1}$  and  $r_{o2}$  that is also between the drain and the overall ac ground. So, I can further simplify this and I can you know just represent this as  $I_o$  noise square and  $r_o$ , which is basically  $r_{o1}$  parallel  $r_{o2}$  and we are finding out the output noise voltage over here that is the equivalent circuit we have and I can find out the  $V_o$  noise square equal to  $I_o$  noise square multiplied by  $r_o$  square.

And then in the next stage is that we will need to look at the input referred noise so for that once again I will divide this entire  $V_o$  noise square by the gain of the circuit basically gain square of the circuit which is  $g_m$  square  $r_o$  square. So, if I have to get back to  $I_n$ ,  $V_i$  noise square there once again I will look at this quantity divided by  $g_m$  square times  $r_{o1}$  parallel  $r_{o2}$  square. So, basically in order to get the input referred noise I have to get the  $I_o$  noise square divided by  $g_m$  square that is the quantity you are interested in order to obtain the input referred noise.

So, let us find out expression for this total input referred noise like which is  $I_o$  noise square divided by  $g_m$  square. So, all you need to do is just sum of all these 4 components of the noise and divided by  $g_m$  square. So, if you look at the  $4KT$  term this is going to give me  $4KT \gamma g_m^2$  upon  $g_m$  plus  $4KT$  sorry  $g_m$  square  $4KT \gamma$  upon  $g_m$  and then the other two terms which is basically  $k_n$  upon  $W_1 L_1 C_{ox}$  and  $f$  plus  $K_p g_m^2$  square upon  $W_2 L_2 C_{ox}$ .

$F$  times  $g_m$  square. So, this is the overall expression that I get for the input referred noise and from here I can look at the I can ex find out the expression for  $g_m$  in terms of  $w_l$  and the parameters of the MOSFET the bias current to the MOSFET, and arrive at the dependency is how those overall input referred noise depends upon the sizing factors and the bias current of the circuit.

So, here we are seen that the  $g_m$  square if you increase the  $g_m$  square definitely the first term diminishes as will the second term diminishes. So, we can rely on larger  $g_m$  to suppress the first two term that is obvious, and lower  $g_m^2$  over here to suppress the first term the first two terms.  $G_m^2$  is also coming over here in the second term therefore, we need to look at the second the last term also in order to decide about the  $g$

$m_2$ , but for  $g_{m1}$  we are confident that having a larger  $g_{m1}$  suppresses the first as well as second term and the dependency point of view if you look at the dependency here you have  $g_{m1}^2$  dependency therefore, if you increase the  $g_{m1}$  as compared to  $g_{m2}$  definitely this term goes down its sup gets suppressed likewise the second term goes on linearly.

So, the first two terms are relatively straightforward to look at and remember the currents in both of the transistors are same. So, the way we have bias then they are common source amplifier low transistor input transistor. And therefore, the currents are same. So, the only factor that remains is the  $W$  by  $L$  ratio and the constant term was like the  $\mu_p$   $C_{ox}$  which are just constant factors. So, I can write in down for  $K_T$   $\gamma$  and here you are going to get let me write it down as root under the constant terms I am write. So, writing it as  $K_p$  and the then you have  $I_D$  which is going to be here times the  $W$  by  $L$  of this MOSFET 2 and likewise in the denominator side I have the  $K_n$  times  $I_D$  times  $W$  by  $L$  of MOSFET 1. Likewise a second term also same thing  $4 K_T \gamma g_m$  and root under  $K_n I_D W$  by  $L_1$  and the last second term once again its already simplified in terms of the  $W$  by  $L$   $W_1 L_1 C_{ox}$  times  $f$ .

And the last term we need to break down the  $g_{m2}$  which is going to have  $W$  by  $L_2$  component. So,  $w$  two over here gets cancelled  $W^2$  of  $g_{m2}$   $W^2$  over here will get cancelled. So, you are left with only  $L_2$  over here and other  $L_2$  coming from  $g_{m2}$ . So, it becomes  $L_2$  square. So, you have  $k_p$  upon  $L_2$  square coming over here  $C_{ox}$  and  $f$  and then you have the  $g_{m1}$  term coming again as  $K_n I_D W$  by  $L_1$

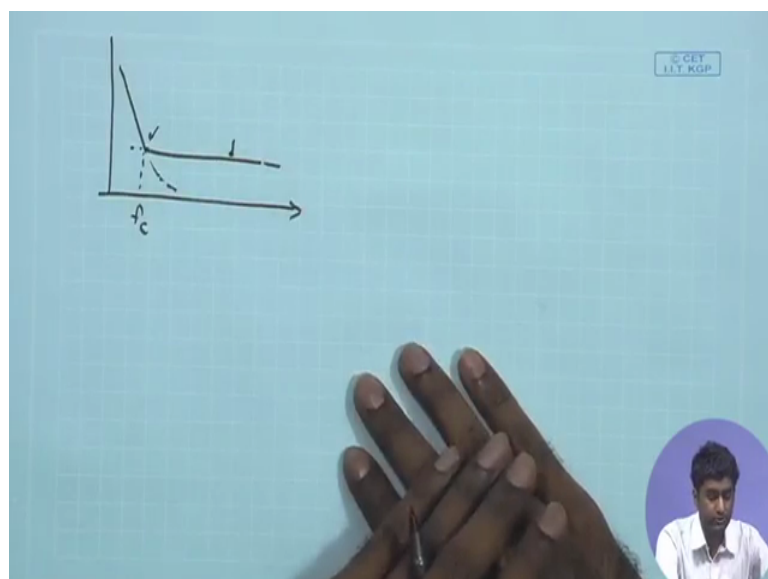
So, this is these are the 4 terms that we are getting ultimately and you know point of view your sizing we can see that for the load transistor or the PMOS device having a larger  $L_2$  is going to help us and suppressing the  $1$  upon  $f$  noise component likewise having a larger  $W$  and  $L$  having a larger  $W$  and  $L$  for the input device is going to help us and suppressing the  $1$  upon  $f$  noise component likewise the thermal noise or the white noise component can be suppressed significantly by increasing the  $I_D$  as well as the  $W$  by  $L$ . So, here also you have root under  $I_D$  dependency and everywhere  $I_D$  is coming the denominator. So, by increasing the  $I_D$  you can significantly suppress the thermal noise floor of the circuit



So, we can rely on relatively sufficient ID and a larger  $W$  by  $L$  to suppress the white noise floor and at the same time we can rely on along the channel length for the load device along with you know the ID and  $W$  by  $L$  of the first stage which is anyway suppressing the  $1$  upon  $f$  noise to reduce the overall  $1$  upon  $f$  noise component. So, if I look at the  $1$  upon  $f$  noise term increasing  $W$  by  $L$  or  $w$  as well as  $L$  of  $M$   $1$  is you know having similar effect on the on all the terms; however, is you looking at the last term over here  $L$   $1$  is coming in the numerator. So, therefore, once again it can degrade the noise if you are just increasing the  $L$ . So, for the input device it will be more efficient to increase only the  $w$ . So, that the  $1$  upon  $L$   $1$  term over here and  $1$  upon  $L$   $1$  term here that does not mix up with the noise. So, I can go for a large  $W$   $1$  I can go for a larger sufficiently large ID, I can go for sufficiently large  $L$   $2$  in order to reduce all these 4 terms.

So, a larger  $W$   $1$  and larger ID is going to suppress the first second as well as third term, large your  $L$   $2$  is going to help us in strongly suppressing the last term over here. So, we can see the line direction that we can obtain from this you know computation in order to minimize the input referred noise and another very important concern happens to be the  $1$  upon  $f$  noise corner that we have discussed. So, as we see the  $1$  upon  $f$  noise term over here will go on increasing as we go towards lower frequency whereas, the thermal noise floor that is constant, as a result if we want to look at the corner frequency where is the intersection of this  $1$  upon  $f$  curve intersection of this  $1$  upon  $f$  curve and the white noise curve.

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So, as we go towards higher frequencies the  $1/f$  goes down as a result its magnitude diminishes whereas, at and therefore, the white noise become dominate and this is the frequency  $f_c$  or the corner frequency below which the  $1/f$  noise term becomes very significant and if you are processing certain signal if we rely only on sizing then we would like to make sure that this  $f_c$  or the corner frequency is lower than the signal content, on and also we would like to make sure that this white noise floor is having much lower magnitude or much lower rms value as compared to the input signal.

So, but the white noise value as we see is depends upon the first two term. So, having a larger  $I_D$  and larger  $W$  by  $L$  ratio of the first transistor is this (Refer Time: 24:06) go to help us in reducing the white noise floor, likewise if I look at the  $1/f$  terms having a larger  $W$  by  $L$  ratio of the first transistor, and also having a larger  $L^2$  term is also going to help us in minimizing the contribution of the  $1/f$  noise term and push this frequency towards lower values.

So, these are the two these are the two considerations we need to make while allowing at the design minimizing the white noise floor and trying to suppress this corner frequencies to a lower value. And in general we will see that for signals which are having very low frequency content you may not be feasible or practical to push it towards much lower frequencies in designs it is convenient to push  $f_c$  to 100 hertz or even down to 10 hertz, but below that it becomes very challenging. So, few up to few tens of hertz you can get it down, but below 10 hertz is become very challenging or it will start burning a lot of you know it will require a lot of bias current or it will require in practically large sizes of this  $I_{nw}$  to push the  $f_c$  below say 10 hertz. Therefore, if your signal content is you know below 10 hertz then we need to take care of some other schemes to mitigate the effect of this  $1/f$  noise and look at the  $1/f$  noise contribution the chopping sequence the other schemes which can we employ to mitigate this.

So, we have looked at the sizing consideration this for a appointing of view of sizing you can minimize the thermal noise contribution and try to push the  $1/f$  noise towards lower frequency, but if you are stereo content lights you know at even lower frequencies done what are the steps to be taken. So, we need to address that and also when then we need to look at the differential amplifier configuration, then look at how the noise contribution of the transistors affect my response of the fully differential amplifier with which  $V_{i1}$  building our op amp.

So, at the starting point we can say that ultimately the differential amplifier for the differential response is having an ac ground over here. So, it boils down to the same common source amplifier. So, we should not create a lot of difference, but only a minor point need to be addressed like the tail current source. So, so that that is another small issue that we can address.

Student: (Refer Time: 26:32) noise is due to trap trapping a (Refer Time: 26:37), but coupling of is there any there is no gate voltage (Refer Time: 26:40).

No, but continuously there is always you know. So, it is not only signal dependent randomly also you have under equilibrium you have trapping, detrapping going on gate signal when you apply the gate signal it is changing that trapping detrapping process, but when you have a the two opposing phenomena going on trapping and detrapping it which you can equilibrium where the trapping and detrapping is equal in magnitude, that only means that the mean value of the noise that is produced at the 0, that does not mean that the amplitude is 0.

So, we said that the noise signal is having mean value 0; that means, even if there no signal is trapping detrapping is going on that is not dependent upon the signal. Only thing is and in your signal is having such a low frequency content it is overlapping with the you know frequency content of this trapping detrapping process it can get corrupted much more.

So, we can take a short break and after that we can get started with some of the remaining analysis in terms of differential amplifier operation and our operational amplifier circuit.