Analog Circuits and Systems through SPICE Simulation
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Lecture - 24
Noise In Circuits

Welcome back. We are going to start with our analysis of noise and looking the noise minimization of our differential stage as we discussed in the last example. So, before we arrive at the noise analysis we just have a brief recap of the concept associated with noise, the time domain representation of noise frequency domain representation of noise. And then finally, transistor noise models and then looking at the circuit analysis based on those models.

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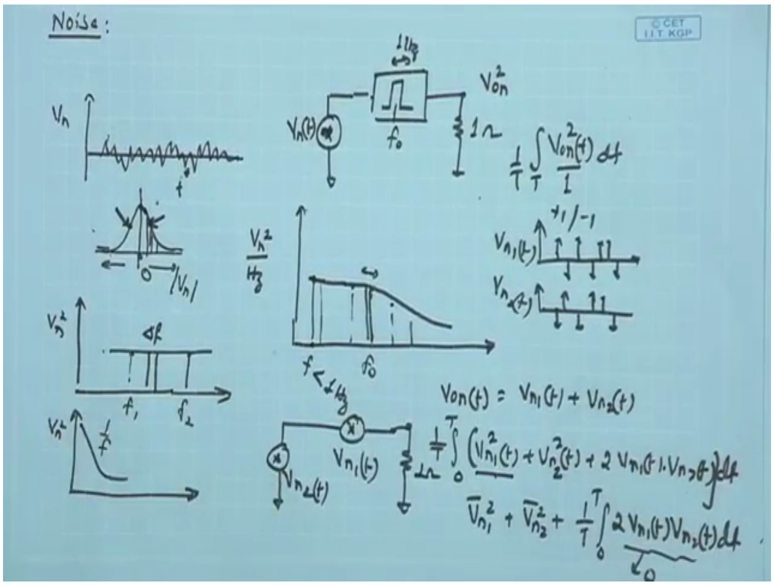

Let us get started. So, noise can be seen as a random signal which is having a close to 0 mean and a distribution of amplitude which can be relatively Gaussian. So, you can have a Gaussian white noise which is having an amplitude distribution given by a Gaussian characteristics; that means, the probability of the noise amplitude close to the 0 point will be maximum and as you go away from the 0 point it will be reducing approaching 0. So, this is the amplitude distribution. So, this is basically here and plotting the statistics of the amplitude. So, on the x axis I have V n magnitude of V n plotted or you can say V n plotted on both sides of 0s and on the y you have the statistics that what is the probability of the noise amplitude lying in a certain domain and on an average as you can see according to this distribution the mean will be 0.

So, if you add up the noise samples for a sufficiently long duration the mean will turn out to be 0 and if we look at the frequency spectrum. So, this is the time domain representation. So, I am talking about the amplitude how it is distributed in amplitude domain as we look at the samples. Likewise if you look at the frequency domain representation this is V n as a function of time frequency domain representation would determine the spectrum of this noise how it is spread in the frequency domain.

So, there are 2 major frequency spectrum of the noise that we encounter in our active and passive devices one of them is white noise that means, the noise component is uniformly distributed across the entire frequency range of interest; that means, whether we take low frequency f 1 or a significantly higher frequency f 2 the noise component or effective energy contribution of the noise will be similar at both these frequencies. And another dominant or bothers of noise component that we have in our circuit which comes from MOSFETs is 1 upon f noise where the noise spectrum increases with frequency. So, if I am plotting the magnitude of noise or magnitude of energy in the sudden frequency packet of the noise it will go on increasing with a certain frequency and that is termed as 1 upon f noise.

So, we will be looking at both these representation time domain and frequency domain representation of noise and the concept of noise spectral density and then look at the MOSFET models. So, if we look at the concept of noise spectral density it basically gives us the energy component of noise in a certain bandwidth delta f or delta omega and to do that basically we need to visualize a noise source V n t, we can call it you can denote it by a star mark because there is not having any fixed polarity this is a random noise signals. So, noise voltage and we generate represented by a voltage source with star mark and I can call it V n t.

And I have a band pass filter over here a very narrow band pass filter assuming that you know you are having a band pass of 1 hertz bandwidth and after that you have a register load of 1 ohm and whatever we enter you apply you just passed through this filter for just with a certain noise frequency, f naught and you are analyzing the voltage achieved at this particular point and you are taking the mean square of this voltage. So, you are taking output V o n square over time.

What is the increment of mean square? So, if we want to obtain the power delivered by this noise source to this 1 ohm resistor I need to look at the mean square value. So, for a large time duration if I just find out V on t square and V on t square divided by r which is you know 1 ohm that gives me the power delivered instantaneously at a particular time instance and the dt of that for a particular time divided by that time duration will give me the average power delivered during that time t. That is why we can look at the mean square value V on square delivered to this 1 ohm resistance and we can quantify the power delivered by that noise source or in other words the energy content of that noise source in a certain frequency range in a certain, one hertz frequency range across a center frequency f naught.

So, now, if I keep on sweeping this frequency from very low values going all the way to high frequencies. So, at each frequency value I will get certain magnitude of V on square which basically tells us that what is the energy content of this noise signal V n t at that particular frequency and only I join all these curves to obtain the V n square per hertz which is the power spectral density of the noise or simply noise spectral density that we obtain from that source. So, this is basically telling us that at a particular frequency f naught if I am taking a 1 hertz bandwidth within that what is the energy content of that noise signal or if I connect it to a bundle ohm load what will be the power deliver to that 1 ohm load because of this noise signal.

So, this is the general frequency domain representation of noise it can also be represented in terms of V n per a root hertz. So, I can take a square root and I can represent this unit as a V n per root hertz and in that case we have the overall noise shape the spectral shape remains almost same only thing is the magnitude becomes square root.

Now if we look at the, if you look at the bandwidth definition over here I have chosen it to be 1 hertz and definitely that would mean that you are having the power delivered in this 1 hertz bandwidth and then you are denoting it as the energy or power delivered per unit hertz. But definitely this definition also has to be valid at much lower frequency as very much lower than 1 hertz. So, if I have to get the spectral density at frequencies f lower than 1 hertz then only thing that you have to envision is that the bandwidth of this filter that you are having is reduced it is much lower than that particular frequency. So, if you are going save for a 0.1 hertz frequency and you want to get this spectral density value at 0.1 hertz then of course, in order to get the frequencies at 0.1 hertz, at 0.11 hertz and 0.12 hertz you will need to use a bandwidth which is further smaller much smaller than 0.1 hertz. So, probably I will use a 0.01 hertz bandwidth ideally just conceptually.

So, for obtaining the spectral density for frequencies lower than 1 hertz what you need to see is that this conceptual filter will be having the bandwidth much lower 0.01 hertz and then I will keep shifting it in small steps say 0.01 hertz and within that I will get the power delivered and divided that power delivered by that bandwidth delta double delta b of 0.01 hertz to get the power delivered per unit hertz or V n square per hertz. So, that is how I can take care of load frequencies.

So, this is how conceptually we can arrive at the noise spectral density of the particular noise. Now the other issue which time domain representation. So, if you are having 2 independent noise sources say V n 1 t and you are having another 1 say V n 2 t and then you are putting a resistor over here V n 1 t and V n 2 t, suppose you are having 2 independent noise sources and you are looking at the final value coming over here at this load resistors may be 1 ohm in that case if I look at the time domain value I was simply write V o n t is equal to V n 1 t plus V n 2 t.

And once again in order to find out the power delivered by this noise source the combination of these noise sources I will take the mean square value and therefore, I will write mean square value has V n 1 t square plus V n 2 t square plus 2 times V n 1 t times V n 2 t and then integrate this over a sufficiently long duration, integrated over a sufficiently long duration dt.

Over t and divide the whole thing by t. So, this is what I will do for obtaining the mean square value for the combined sources and this can be seen as the mean square value of V n 1 t because if I just take this integral and take only the first term V n 1 square t dt V n 1 square t dt integrated over 0 to t divided by 1 upon t. That is nothing else, but mean square value of the first noise source. So, I can call it V n 1 square with a bar to represent the mean square value likewise I can look at the sorry V n 2 t. So, if this second term V n 2 t square dt integrated over 0 to t divided by 1 upon t once again gives me the mean square value of V n 2.

So, I can call it V n 2 square and we have the remaining term as 1 upon t (Refer Time: 10:58) t 2 V n 1 t V n 2 t dt. Now this term is something like a correlation between the 2 random noise sources V n 1 t and V n 2 t and if the noise sources are independent they are coming from independent sources independent transistors or different resistors they are going to be uncorrelated and therefore, this term may altogether vanish. Therefore, when we want to get the overall noise power because of the 2 independent sources coming together I need to add their mean square values.

So, I can obtain the mean square noise spectral densities of the first source, means square noise spectral density of the second source and those mean square that you need to be added to get the overall effect we cannot add the time domain value, we do not add the V n 1 t and V n 2 t other we add the V n 1 square the mean square value of the first source plus the mean square value of the second square. So, basically this is going to give me the overall power delivered by the effect or by the combined noise sources V n 1 t and V n 2 t. Just to visualize you know how these noise sources will be uncorrelated and getting cancelled.

So, if these 2 are you know the simplest case is you are having only 2 levels in the noise equal probability each noise source can apply can you know take 1 or minus 1 values . Likewise the other source can also take by equal probability 1 or you know minus 1 value at different time instances. So, this is your V n 1 t and this is your V n 2 t. So, for as the sufficiently long duration each sample I am assuming that it can take only 2 values either it can go minus 1 or plus 1. Assuming this Gaussian distribution for all the amplitudes the same will be true, so I am picking up only 2 certain amplitude this 1 and this 1 for that the distribution is almost equal. So, the probability of the noise acquiring this positive amplitude and this negative amplitude is equal.

Therefore if I just pick 2 amplitudes plus 1 minus 1 arbitrary values their probabilities are going to be almost equal and then if we multiply these samples together the probability of getting 1 and minus 1 at individual point is also going to be all equal.

So, if I multiply these individual noise samples at these locations I am going to get plus 1 or minus 1 with equal probability and then if I sum all these together the value V n t times V n 2 t integrated over this entire period will again vanish. So, this is how we can demonstrate combining this amplitude distribution and time domain distribution we can demonstrate that any 2 uncorrelated noise sources their overall you know correlation product will be 0 integrate over a particular time period.

And therefore, in order to add the effect of 2 independent uncorrelated noise sources we need to look at their mean square values. So, anyhow holds true with respect to spectral density. So, if we have spectral density of 2 noise sources available one of them having a certain shape another one having a certain shape we need to add their spectral densities to look at the overall effect. So, these are the 2 concepts we need to take keep in minds in order to analyze the circuits.

Any question before you proceed towards MOSFET and registers look at their noise model and then go for the circuit analysis.

Now the 2 major noise sources that we have in the circuits is going to be the resistor noise.

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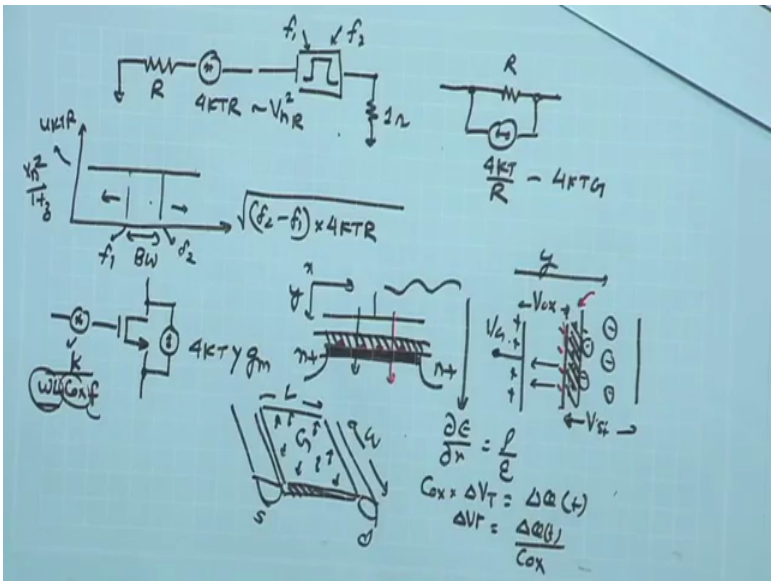

So, if you are having a resistor or value R is equivalent noise source can be modeled as 4 KTR means this is your V n the mean square noise spectral density of the resistor; that means, for a particular this is basically presenting at white noise because it does not have any frequency dependence. So, it will basically give you a flat spectrum value for KTR and here I am plotting V n square per hertz. So, if you want to get the equivalent noise voltage contributed by the resistor in a certain frequency domain I just need to multiply this with that bandwidth with this, you can call this bandwidth. So, 4 KTR multiplied by this bandwidth say call it f 2 and this is your f 1. So, f 2 minus f 1 times 4 KTR root under will give me the overall root mean square value or rms value of the noise voltage contributed by this R. So, suppose I am having this R and after that you have put a you know ideal band pass filter which is having a pass band from f 1 to f 2 and then you are trying to analyze its effect on a ideal resistor 1 ohm.

So, this is a noisy resistor and this is an ideal resistor 1 ohm I want to see; what is the noise voltage delivered by this resistor over here on to the component here. So, for that I need to look at this overall you know noise produced by the resistor look at the component between this f 1 and f 2 bandwidth which is passed by this filter and then look at the overall noise voltage in terms of the root mean square value. We have to go back to the noise spectral density from there we look at the bandwidth in which the noise is passed.

So, if it is a good filter and an ideal filters will block the noise component out of the band and it will pass only the noise within the band and then that gives me the mean square noise voltage over here integrated over this band f 1 minus f 2 root under that. And then once again here of course, we have to look at the exact value we have to take the voltage division between this 1 and R. So, the division factors also come into picture.

So, here just you are trying to show is that apart from the noise computed by the component, the other factor that controls the total noise signal at a particular point is the bandwidth through which you are allowing that noise to pass. Now if you look at the other major noise component that comes into our circuit is because of the MOSFETs noise. So, for the MOSFETs we have 2 major kinds of noise sources - one is your 1 upon f noise which is modeled as a voltage source in series with the gate and another is a channel noise which is modeled in parallel with the channel and here the channel current noise is having a similar concept as the resistor noise.

So, if you have a resistor just like you can represent the resistor noise in terms of a noise voltage you can also convert it into its terminal equivalent noise currents. So, basically the resistor noise can also be represented by its terminal equivalent noise current which is going to be 4 KT upon R, you will divide this 4 KT upon a square remember this is the V n square. So, this divided by R square will be give you the equivalent noise current that you can connect in parallel with the resistor and that becomes 4 KT upon R or you know 4 KT times G where G is the conductance of this R. Likewise for the MOSFET also you have this channel which is conducting the current and it has some equivalent small signal resistance across which the current is flowing.

So, that is going to ultimately complete control the channel conductance and hence the noise. So, for the MOSFET noise we have an overall noise channel current presented as 4 KT gamma which is a constant factor times gm where gm is the transconductance of this MOSFET and the gamma is a constant term it can be you know within 1 the normally used number is 2 by 3, but it can depend upon the process chosen. So, it can vary a lot composites to process and it needs to be specified. So, if I assume 2 by 3 then you get an overall expression 8 by 3 KT gm as the channel current noise of the MOSFET.

And remember a noise current we represented by double arrows because it is not a it is not a having any defined polarity it is we are going to deal with the noise current mean square value and hence the polarity in order of a significant importance. We just denote it by double able to show that the polarity is not of any consequence. Likewise the voltage source is represented by the star. So, that is polarity does not matter in the representation.

Now if you talk about the 1 upon f minus in the gate that depends upon the W L apart from that you have the cox and the frequency of the noise which is f and then you have the constant term k which is coming and it is also depending upon the process parameter. So, here we have the 1 upon f noise coming at the gate terminal because of the device physics and it depends upon the area of the device W L the overall capacitance and it is having inverse frequency dependence.

So, if I look at the noise spectral density we have the 1 upon f characteristics; that means, if you are looking at the frequency response of the noise signal produced by it is 1 upon f noise it is going to increase at higher frequency. So, we can briefly discuss; what is the mechanism of this 1 upon f noise, so that basically originates from the surface states and then interaction of couriers flowing in the channel with the surface states of the MOSFET.

So, you have this channel in a n MOSFETs which is being denoted by that this dark region where you have a lot of electrons shooting from the drain to source and the interface between the oxide and the channel is imperfect it has dangling bonds where some of these electrons can get trapped. So, basically there is a continuous trapping detrapping of electrons going on this interface.

Moreover you are having a gate voltage which you may be varying you are applying some signal because of which the electric field from the gate to the channel is also varying as a result the trapping detrapping process which is also going to depend upon this electric field is also going to vary. So, when the electric field over here increases you are attracting more electrons and therefore, the rate of trapping of these electrons will go on increasing.

So, whenever you know at these interfaces you are having you know trapping size for example, where you are able to capture some of the fast moving electrons when the gate voltage over here goes up the capturing rate increases whereas, the gate voltage goes down again it can go down. As a result you can see these phenomena as a effective reaction between the defect states that you have at the interface and the entities which are getting captured that is electrons. So, there are 2 entities that is the defect states which are capturing the electrons and the field moving electrons in the channel.

And there are reaction constants, the reaction constant as we can see it will depend upon the applied conditions also if the signal is on the gate of increasing electric field increasing, direction increasing you will end up attracting more electrons at these you know defect sides and they will start getting trap most strongly. And on the other side when the signal goes down it will be trap. Another result you have a continuous change of a fixed amount of charges that as stored at this interface.

So, you are having some spurious charges which are coming at the interface of the oxide and the channels which are getting changed as per the signal applied or these are fluctuating over time randomly and that leads to the 1 upon f noise characteristics that we are observing over here. Now what are these dependencies that we have W L cox. So, let us look at the cox which is basically giving us the integration of the electric field produced by this charge across this oxide. So, we know that the for in order to obtain the threshold voltage if you look at go back to the device physics and look at the origin of the threshold voltage that basically is coming from 2 components that is the oxide the voltage drop across the oxide V ox and the voltage drop across the silicon V si. So, when we apply a gate potential on the silicon V g we have a overall voltage drop across the oxide and some voltage drop across the silicon and here if we look at the voltage drop across the oxide it depends upon the total charge produced in the silicon.

So, for example, when you have a certain gate voltage you have a depletion region created in the silicon p type silicon and you have some invergent charge created over here close to the channel and as a result this total depletion charge in the p type silicon and the inversion layer charge combined together supposed to balance the total positive charge that we put on the gate. So, in the silicon when you are applying the positive potential at the gate for the NMOS we know that the P type substrate of the NMOS it will have a lot of holes the holes will get pushed down as a result of producer depletion regions. So, I am drawing the vertical view over here, this is the vertical view looking down into the MOSFET in the vertical direction this is the vertical view.

So, this is if I call this the x direction this is the y direction this is the y direction that I have drawn. So, the y direction when you are applying a certain gate voltage is producing electric field across the y direction and this P substrate that you have power NMOS is having a lot of holes and that will be pushing these holes away from the interface and exposing these negative charges and we know that beyond a certain threshold voltage V t you get a lot of negative charges exposed present in the channel region and you have an inversion taking place and you have moving electrons coming over here that is the case when you have the channel inverted.

So, these 2 charges together the inversion charge and the electron charge basically combined together managed or balance this total positive charge that you have dumped on the gate. And as a result when you are trying to increase or decrease the gate voltage the total charge on the gate is increasing or decreasing that is what we do right. So, while understanding the capacitance of the surface also we have discussed that how do the parasitic charectorances arise. So, ultimately it is because of the charge balancing between the gate and the substrate.

We are putting more charges in the gate to balance that you need more negative charges in the silicon. Likewise here also when you having positive more positive charges getting accumulated on the gate because of increasing gate voltage you will have or you will attract more negative charges over here to balance the electric field produced. Now if because of this signals you are also having some additional or spurious charge which are random in nature which are coming at the interface of the oxide and the silicon which are not under yours control these are random in nature because of that you will have an effective you know an effective you know imbalance or change in the channel charge.

So, basically now if you are having total tap some net tap charge over here combination with this tap charge along with the mobile charge plus the fixed charge will be balancing the positive charge over here. And since this tap charge is kind of random in nature it, is randomly fluctuating over time the total charge over here the mobile charge required to balance these additional positive charge that also fluctuate over time. That basically means that the effective threshold voltage that you need to turn on the channel that is also varying over time because the total charge required over here to balance this deterministic positive charge that you have put on the gate that is varying over time.

So that means, the overall threshold voltage that you have for the MOSFET that is fluctuating over time as you have more and more negative charges coming over here. And in order to look at the cox dependence you can see that if your negative charges are close to the interface whatever random charges you have trapped that interface they are lying close to the interface, we have an effective electric field produced by these charges which are going to be balanced by the positive charges over here.

And as a result we have the gauss law telling us is rho upon epsilon and if I look at in this region you do not have any fixed charges the oxide region you do not have any fixed charges and therefore, in this region whatever charges you are having that integrated over this entire distance gives us the overall delta V that is required and therefore, if I just look at the capacitive behavior the additional charges coming over here c times the total delta V that is produced across the oxide that should be equal to the delta Q. So, basically cox times the delta V t that should be equal to the delta Q that you are producing over time which is you know random which is varying randomly over time because of this noise characteristics.

And as a result the noise voltage which can be modeled as a manute noise in the threshold voltage that is dependent on 1 upon cox that is how this is 1 upon cox terms comes into picture. So, the bottom line is because of this random interaction of the electrons during the channel with the oxide interface you have continuous trapping detrapping of these electrons at the surface and if I look at the effective you know voltage drop to produce across the oxide delta V. So, this is the random change in delta V that is happening because of the random trapping detrapping of electrons. So, that is proportional to cox, the cox times the delta V proportional to the random dividing charge and therefore, this delta V that is happening across the oxide it is inversely proportional to cox and we can model this random change in cox as an effective noise term this is the effect of noise voltage which is appearing as a noise component in your threshold voltage. So, as a result you are having this 1 upon cos dependency.

This is the origin of the 1 upon the f noise. And then we have other terms coming over here the 1 upon x term W and L terms, y 1 upon f term because this trapping detrapping process which is in general of slow process and if your signal over the gate is changing much more slowly the trapping detrapping process will be able to follow that signal more closely. As a result the frequency component of the noise is more and more towards higher frequencies sorry lower frequencies as you go towards lower frequency the frequency content of the 1 upon f noise increases.

Effectively it also means that if you are integrating this random exchange of this charges and because of that you are exchange the you are integrating the delta V produced by this random transfer of charges over a long time it will end up having a overall larger magnitude. So, that basically means that at lower frequencies you are having a higher overall contribution because of this trapping detrapping process of the electrons so that bring in the 1 upon f factor. And then we have the last 2 term which is the W and L this is which is this the total area of the MOSFET.

So, that basically comes from the spatial averaging of the trapping detrapping process over the entire MOSFET. So, if you are having the source and drain region and this is the you know top view of the MOSFET you are having the channel over here and this is the gate region. So, inside this entire gate region there is a source and drain. So, inside below this entire gate region you have this trapping detraping process going on all throughout. So, you have this is the channel lengths and this is the W. So, I have drawn the top view looking from the top this is the view of the MOSFET.

And if I look at the spatial averaging, so this just like you have a time domain averaging that we talked about right. So, here we talk about the time domain averaging of the noise signal likewise you have this tapping detrapping process going on all throughout this area. And if I go for larger and larger area this overall summation of this trapping detrapping process average out and it will lead to cancellation of this large number of trapping detrapping process which are equal in probability and as a result you will get lower and lower overall noise signal. So, therefore, the effective noise signal coming because of these interfacial effects that reduces with the increasing area because of this averaging effect the spatial averaging effect. So, this basically counts for the spatial averaging effect.

The cox counts for the integration of this electric field produce across the oxide and that leading to the delta V change and then finally, the f corresponds to the following of the trapping detrapping process with in low frequency signal and also you know integration over a longer period leading to a larger overall noise.

So, that is the origin of this 1 upon f noise contribution coming at the gate of the MOSFET and this basically completes our; the noise model that we are going to use for MOSFETs. So, I have just given you a qualitative argument explaining these 2 noise components in the MOSFET - one of them arising from the channel current which is basically analogous with the noise current of a resistor and second one because the 1 upon f noise which has something to do with the interface effects in the channel of the MOSFET. So, we will be using these two for our circuit analysis and start our circuit analysis.