Digital Speech Processing
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Lecture – 09
Uniform Tube Modeling Of Speech Processing Part – 1

So, let us the in the first weeks we are study about that place of particular cells, speech acoustics and over view of signal processing.

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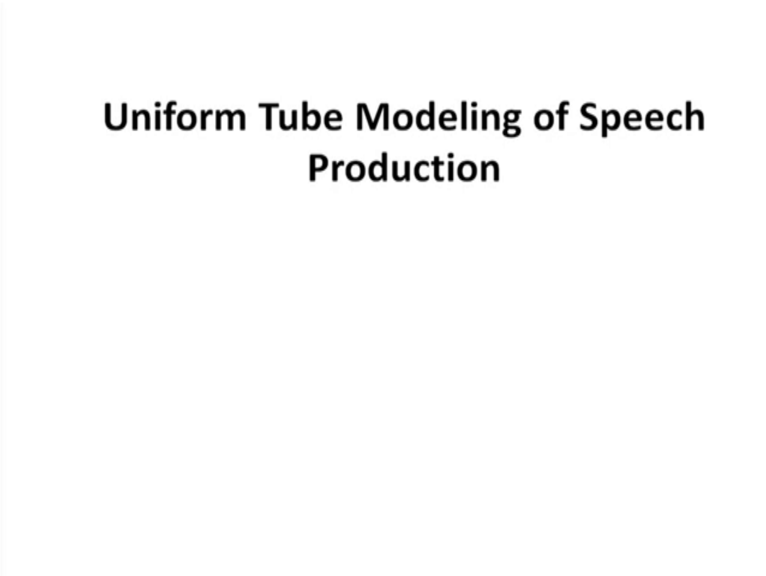

Let us start the new topics that uniform tube modeling of speech production. What was the purpose of that whole course were a speech processing is that we should know, how the human speech is produced. After knowing that can I able to mathematically model that human speech production, that is our motto. If I know the mathematical modeling speech production system, then studying the system property; we can know what kind of speech property you should look for in the acoustic signal. So, our motto is to developed a mathematical model of human speech production systems.

Now, if you see that we have said that this human speech production system has a 2 part. One is the glottal vibration that glottist the source of vibration. And that sound or vibration passed through the vocal cavity or you can that oral cavity and nasal cavity and produce the different kind of sound depending on the tube structure or the cavity structure. Now we want to developed if the glottis is produced sound, how can I model this tube model.

So that I can say; I can if I able to design the mathematical model of the human vocal cord system, then I changing the parameters I can produce the speech. Or what I know that if I know the exact model or approximate model of the human speech production system, I know the model property. So, I deriving the model property I can understand what kind of signal processing I should apply on human speech So that I can know exactly what is happening in here.

Now if I want to model, it the complex is that, human speech production model which is not a simple single tube. If I have a if you see in high secondary also you have studied, that there is a single tube if this end is closed, and a sound is generated here.

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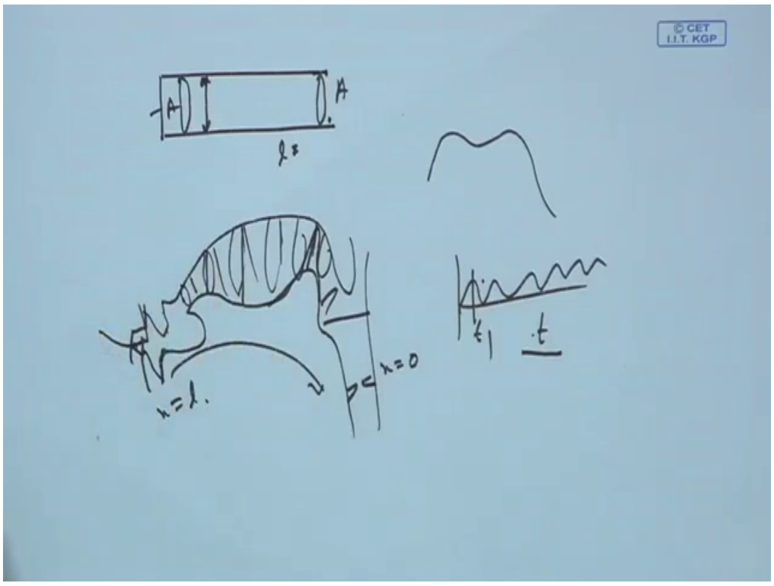

You know what kind of frequency you accept and what kind of sound we expect from the sound that depends on the l of the length of the tube, but in human vocal tract the this area of the tube in here it is fixed for a rigid tube, but human this area is not same throughout the tube. So, that area may be look like this; sorry, area maybe this, is the upper cavity; let us lips, then started this is the upper cavity which is almost fixed, and if you see the lower cavity the tongue of the tip and then back tongue of the tip and like this way. Now if you see depending on the different sound production system the structure the cross-sectional area of the different portion of the cavity for different sound is different. So, you can say there are 2 kind of variation.

Where I produce the speech, speech signal is changing along the time; that means, that constriction of the tube or structure of the tube is changing along the time. And also for a particular time let us the produce for this time t one that structure that the cross sectional area of the tube in all section is not same. So, 2 complexity one is that cross sectional area of the tube is not equal in every length. So, if I say this is the glottis or this is a vocal cords if this is the x equal to 0, and this is at the lip if you see this is the if this is the lip or either if the dental and then there is a lip. If I say that is a lip. So, and at lip let us x equal to l. So, length of the tube is l, but throughout the length here cross-sectional area is same if it is a and here it is also a. Now in here it is different, here it is different, here it is different, let us here it is different, here it is different, here it is different. So, different kind of cross-sectional area is there.

Now, once I produce ka maybe the back of the tongue touches the upper cavity. When I produce po lip lip is touches close together main cavity structure will be different. So, I can say for different sound or when it is continuously produce the different kind of sound, the cavity structures is changing with respect to time also. Now we not consider that let us we considering that at if the particular sound with fix cavity structure is there, that is another simplification will be done.

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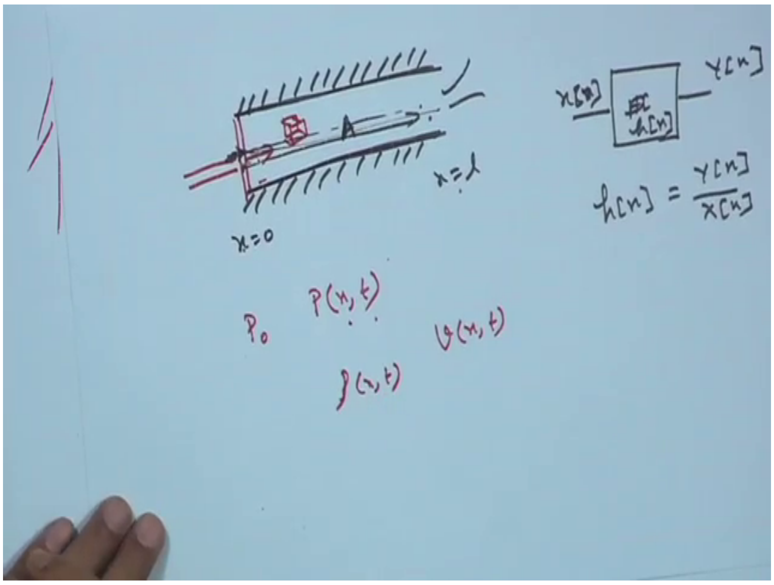

Let us think the whole human vocal system is nothing but a single cavity single tube or single tube which cross-sectional area is fixed a. So, I say throughout the what is glottist or vocal cords to mouth, throughout the whole system the cavity is uniform or even the tube is uniform and it has a cross-sectional area A. So, this is called uniform tube modeling.

Now let us I consider it is uniform, then we say cross sectional area is A. Even it is uniform, then also the walls of the cavity is not rigid this walls are not rigid if it is not rigid then if the air pressure increases the area cross-sectional area may change. So, this is less vocal cord x equal to 0, and this is lip site x equal to l. So, if I if the if the walls are not rigid, if this tube is not rigid wall then when the air pressure is passing through the tube walls will a the cross sectional area of the tube will be change because a wall will be deform. Now neglect that part also they do not consider that part. So, I am not considering we said neglect that part, let us the wall consider the walls are rigid.

Next one is that there may be a thermal loss there maybe (Refer Time: 07:03) loss there may be a friction loss, let us consider that loss also not there. So, we simplify that human vocal tract is nothing but a single tube, whose cross sectional area is constant a. There is no frictional loss there is no viscous loss there is no thermal conduction. Then try to derive the equation of this human vocal contract. So, uniform tube model. So, how do we derive that vocal tract equation? So, if it is a system, if you if you studied that electrical system signals. And system if I have a system if I apply a input let input is x n, let us I consider x n. And output is y n, then transfer function if it is h n h n, then we say h n is nothing but a y n divided by x n. So, we know how the sound is propagated input sound in here which is input is provided by the glottis or vocal cords. After vibrating it inject the partcular velocity, and that particular particules will be propagate along here, and radiated from the mouth, radiated from the mouth.

So, I have to know how the wave is propagated sound wave is propagated along this tube, along this tube. Now you have studied already studied that wave equation you know the wave propagation equation. So, what is still that if I produce a sound can I mathematically express the, So if I produce a sounds sound is propagated in the medium, who is a condensation and relaxation condensation and relaxation way. Can I develop a mathematical equation? How that tracer wave is propagated from one point to another point? That is the wave equation.

Now considering that amplitude of the pressure if I say the wave is propagated linearly then you see a linear wave equation. Here I am saying the amplitude of the sound is not bad enough that propagation become non-linear. So, I said amplitude is less with the limit within the limit and sound is propagated linearly in the medium and medium is also homogeneous. Then we can derive the linear wave equation.

Similarly here also we will try to derive the linear wave equation, how the wave is propagated along the tube let us try to do that. So, how do you do it? Let consider a single cube in here. So, pressure is. So, I can say this is a piston kind of things which produce the pressure in the air and that pressure is propagated along this cube. So, if I consider a single cubicle let us I draw the picture of neatly, let us I consider a single cubicle inside that tube whose dimension, Let us consider the dimension r.

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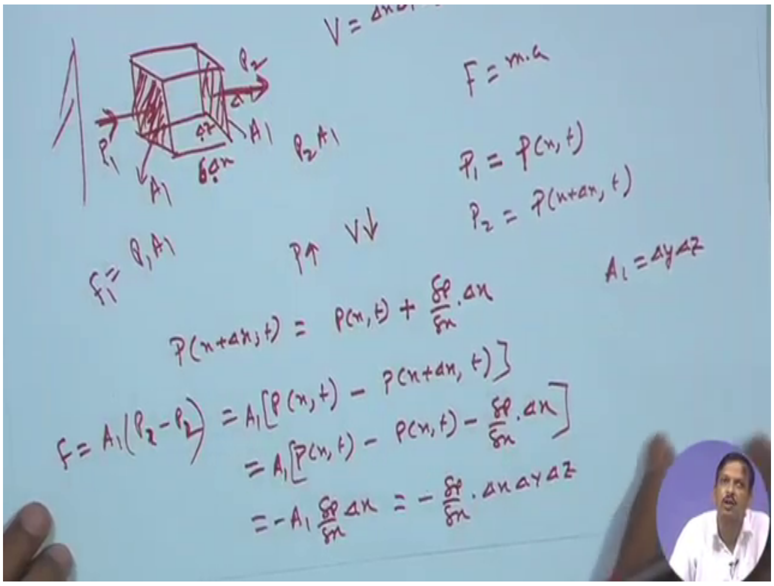

This is less del x, del y; let us tell it del x, del y, del z. So, let us volume is v is equal to del x del y del z. So, this is the volume and this is the cube this. Small cube inside the cube I am considering.

Now if I see there is a there is a pressure wall in here. So, the pressure is applied in here which is p let us I am saying. So, if it is if the t if the no pressure is applied by the piston, then inside the tube it is atmospheric pressure. Atmospheric pressure means atmospheric pressure is what? Let us the what is the average pressure or atmospheric pressure that is p 0. Once the piston apply a force a pressure then this that is p. So, change pressure is a function of the piston may apply a force in here, how the pressure will propagate. It that depends on the position where I am taking the position that x equal to 0 to l. So, this depend on the position and time.

So, change of pressure; the pressure change is a function of position and time. Now if you see; if I change; if I apply a pressure, then what will happen? The particle will start moving inside the tube, but average velocity of the particle is 0, because medium is homogeneous when sound wave is propagated medium does not change. So that means, the particle velocity is exist, but the average particle velocity is 0. So, let the particle velocity is b, small b and that also function of x and t. Now if it is a pressure wave condensation and relaxation, condensation and relaxation. So, density these also change. So, density is also function of x and t. So, in equilibrium I say medium is homogeneous.

So, I have already said some assumption I have taken, that amplitude of the pressure wave is within the limit where the sound is transmitted linearly that is why you call this linear wave equation. Now if it is linear wave equation then how do I derive that wave equation?

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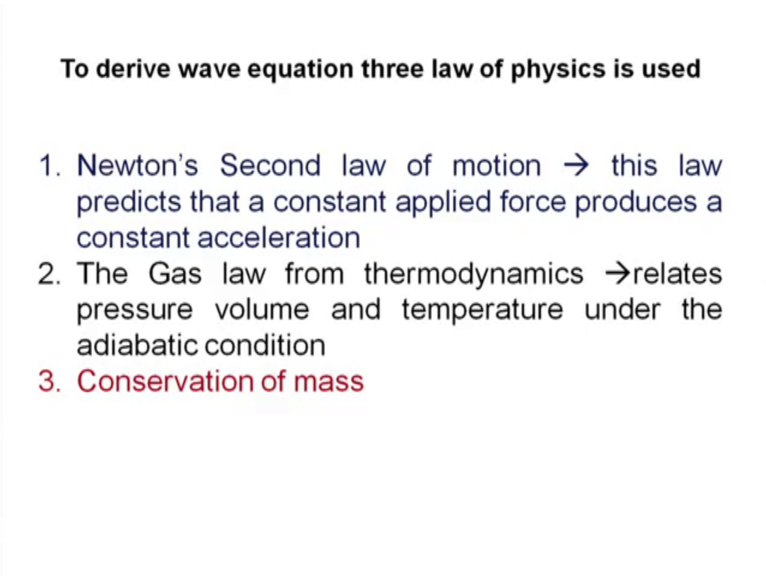

So, what do you understand my wave equation; that means, I want to know how the pressure form x equal to 0 is propagated along the tube, let us at x equal to l I want to know what is the pressure. So, I want to know the pressure movement equation, mathematical equation of the change of pressure with respect to position and time. And also mathematical equation for particle velocity with respect to position and time, and also the relation between the pressure wave pressure with respect to position time with the particle velocity and (Refer Time: 14:26) position and time.

So, that I want to know that is I want to know the derive those equation. So, for deriving those equation you already studied, it may be in acoustic or in wave equation derivation in first year physics. We will consider 3 principles of physics, one is called Newton's second law of motion. Second is called gas law or thermodynamics and another one is converse conservation of mass, think about it.

If I apply a pressure in this plane. So, change of pressure and the pressure is exist in this plane is let us p 2 this is p 1, then I can say the pressure difference. So, pressure if p 1 is the pressure then I know the pressure into area let us A 1 is the area and this is also A 1. So, pressure into A 1 give me the force let us this is f 1 which is acting on this plane. And force which is acting from this n is nothing but p 2 A 1. So, difference of force is the cause of the motion of this particular in this block. So, I can say Newton's second law of motion, the force equal to mass into acceleration. So, the force resultant force cause an acceleration of that cube which is nothing but a force equal to mass into acceleration, mass of the cube into acceleration.

Now second law said the gas law hydroxylated pressure volume and temperature under the adiabatic condition. So, what is the meaning? If I apply a pressure on this tube this is there this block suppose a block I apply 1 plane one pressure is applied. So, every day I apply a pressure the volume will be changed. If the volume will change mass would be change, but mass will not change it is increase the density due to the conservation of mass will remain same density will be change. Volume will change because that cube will be deform due to the applied pressure. So, although the if the pressure is increases the volume is decreased. But mass of the cube will be remain same. And this is an adiabatic condition. So, I say the region when the sound om is propagated there is no conduction of heat.

Now if that is the theory. So, using this 3 law we will have to derive the wave equation. So, again I can if you see the slides the assumption I have said media means homogeneous pressure exchange across the small distance, there is no friction in air particle air particle velocity is small sound is adiabatic.

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Assumption

1. The medium is homogeneous
2. The pressure change across a small distance can be linearized
3. There is no friction of air particle
4. The air particle velocity is small
5. Sound is adiabatic

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$$p(x+\Delta x,t) = p(x,t) + \frac{\partial p}{\partial x}\Delta x$$

Net force on the chunk is

$$F = A[p - (p + \frac{\partial p}{\partial x}\Delta x)] = -A\frac{\partial p}{\partial x}\Delta x = (-\frac{\partial p}{\partial x}\Delta x)\Delta y \Delta z$$

Assumed density in the cube is constant $m = \rho \Delta x \Delta y \Delta z$

From Newton's Second Law of Motion $F = ma$

Acceleration of the cube air is $a = \frac{dv}{dt}$

$$-\frac{\partial p}{\partial x} = \rho \frac{dv}{dt} \quad .....1$$

Now, if I want to derive that equation, what I will do let us this is p 1. So, I can say p 1 is nothing but A, p 1 acting on the pressure. So, it is p of x t, not capital p small p. What is p 2? Is nothing but a since del x is the length along the x direction. So, it is nothing but a small p x plus del x t.

What is p of x plus del x? T nothing but a p of x t minus p of x t and plus change of pressure to for the d x distance. So, it is nothing but a del p by del x into del x. I said change of pressure is linear, I am not considering the non-linear term. This is p 1 is p a p

2 x. So, if I say p 2 minus p 1 which is nothing but a p of x or I can say p 1 minus p 2 acting of pressure in p 1 here it is p 2. So, p 1 minus p 2 is nothing but it p of x t minus p of x plus del x t. Which is nothing but a p of x t minus p of x t minus del p by del x into del x.

Now if I say what is the force acting on this plane multiplied by the area. So, it is A 1 multiplied by A 1. So, I can say the one is multiplied. So, it is A 1 is multiplied. So, it is nothing but A 1 del p by del x into del x. So, what is A 1? Is a cross-sectional area of this area. So, this is nothing but the del z into del y. So, I can say it is nothing but a minus del p by del x into del x del y del z, because a is A 1 is equal to del y into del z. So, I get the force. So, it is nothing but a force acting on the cube. So, force is equivalent to mass into excellence Newton's second law of motion.

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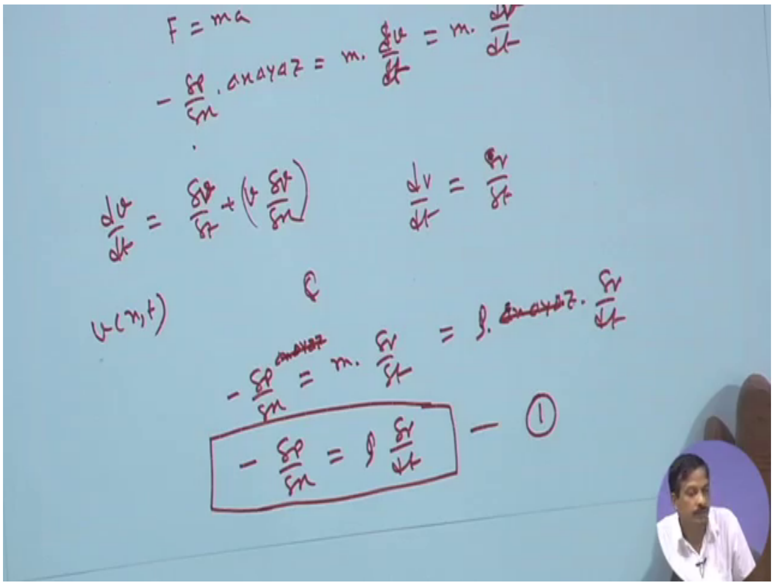

So, force is equal to mass into acceleration. So, what is force minus del p by del x into del x del y del z is equal to mass into acceleration, what is the acceleration? Del v by dv by dt rate of change of particle velocity is the acceleration. So, I can say it is nothing but a into dv d t, this v is the particular velocity.

Now if I say that then what is dv d v dt is a total derivative that and v is a function of x and t v is a function of x and t. So, I can say dv dt is nothing but a del v by del t plus v into del v by del x or not del v by del x. Now if I consider this term this is a non-linear term, if I drop this term that this term is common to be the particle velocity is very small, particle velocity is less negligible, in that case I can say that del dv by dt is nothing but a del v by del t.

So in that case I can write minus del p by del x, minus del p by del x is equal to m into del v by del t. What is m? M is nothing but a density into volume, which is sorry this that will be del x del y del z rho into volume. So, volume means del x del y del z into del v by dt, this will cancel. So, I can write minus del p by del x is equal to rho del v by del t, this is the equation number wave equation number 1. Minus del p by del x is equal to rho into del v by del t, the equation number 1. So, Newton's second law of motion. Now I come to the gas law. What is gas law said? P v is equal to nRT.

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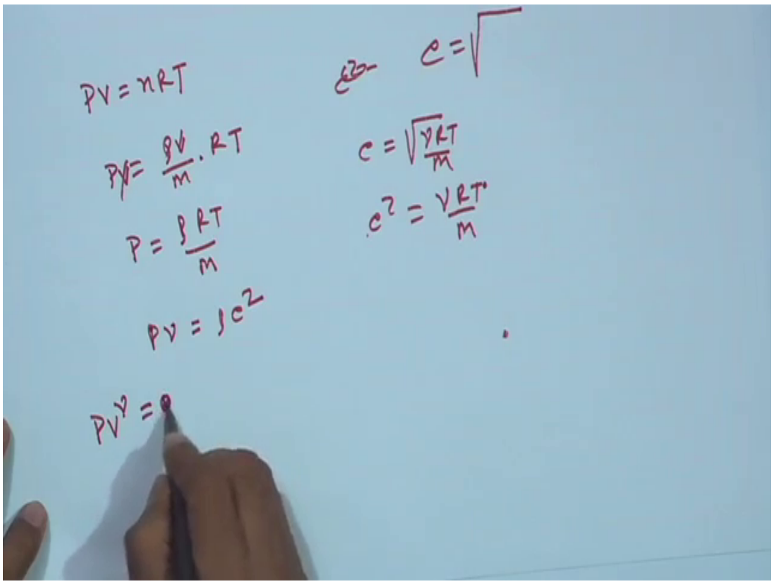

You know n is moles of air, r is gas constant, t is the temperature in Kelvin. That is a gas law adiabatic gas law I can say. Or I can say P is equal to rho v divided by m into R T. Rho is the density, v is the volume sorry, the P v, v is the volume, m is the molecular weight of the air and again R and T is there. So, if you see here it is there. So, P v is equal to n naught T and P v is equal to rho v by m into R T.

Now, if I say then p is equal to I can say rho, v cancel rho r t divided by m rho r t into m. Now you know the c square or c is equal to bulk modulus divided by that is you know. So, bulk modulus divided by density. So, instead of that I can write c is equal to root over of gamma R T by m. Where I derived the I am not deriving that one c is equal to. So, that that you can derive from the pressure the adiabatic condition that if you require you can see the book and that is available and I can next day I can also that do that also if we required. You send me the mail if it is required, then I can derive that things also or you can see that my another lecture in audio system engineering derivation of linear wave equation.

Now if c is equal to then I can say c square is equal to rho R T by gamma R T by m; what is gamma is nothing but specific heat ratio.

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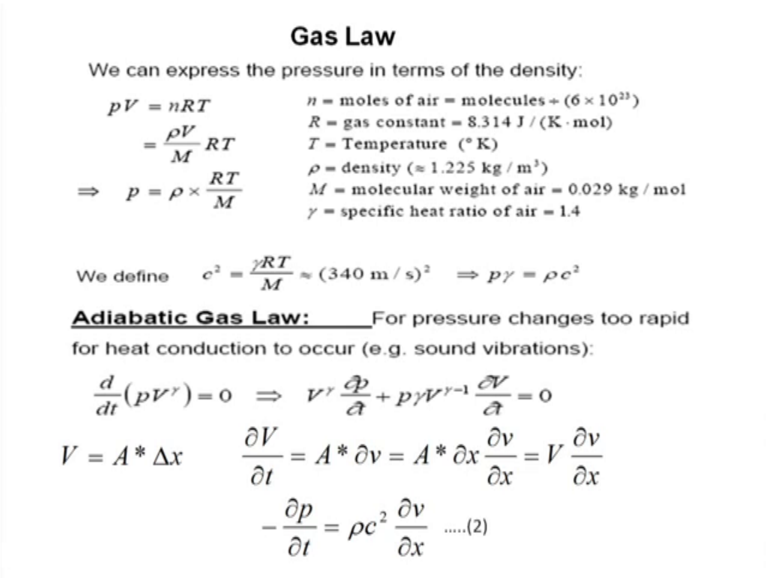

So, I can say from here I can say that p into gamma is equal to rho into c square p into gamma is equal to rho into c square. Another way I can say p V to the power gamma is equal to; sorry. So, another way I can say p V to the power gamma is equal to constant.

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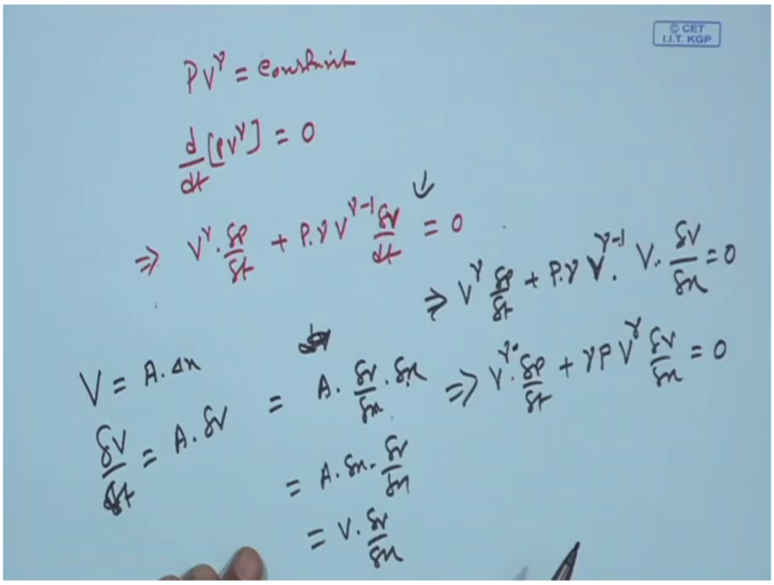

If I take the derivative with respect to t d dt of P v to the power gamma is nothing but 0. So, what is this? I can write v to the power gamma into del P by del t plus p gamma b to the power gamma minus 1 del v by del t, this v is volume. This V is capital V which is volume, minus del v into del t is equal to 0 I can do that that is equal to 0.

Now, if I say what is V? Volume is nothing but a, a into del x cross sectional area into del x. So, cube if you see the cube. Volume of the cube is nothing but a y z line cross sectional area del y into del z into del x. Now if I say dv or I if I say del v by del t is nothing but a, a into del x by del t which is del v if it is that then I can write a into del v by del x into del x I can write. Now I put this one in here. So, what I get V to the power gamma del p by del t plus p gamma v to the power v to the power gamma minus 1 into a into del x, what is a into del x? Into del v by del x.

So, it is nothing but a v into del v by del x. So, I can write v into del v by del x. So, it is nothing but a v to the equal to 0. So, v to the power gamma minus gamma into del p by del t plus rho gamma into p into v minus 1 into v to the power gamma del v by del x is equal to 0.

So this v is volume velocity and this capital v is the volume. Because here I have said del x by del t is nothing but a particular velocity. So, so particular velocity sorry particular velocity. So, this v is the particular velocity. So, I can say v to the power gamma is cancel. So, I can say del p by del t is nothing but a rho into p del v by del x is equal to 0.

I can say del p by del t is equal to minus rho P gamma P sorry, gamma p into del v by del x, which is nothing but a I said pressure into gamma is equal to rho c square. So, I can say it is nothing but a minus rho c square del v by del x.

So I can write another wave equation. So, first wave equation was del p by del p by del x is equal to minus rho into del v by del t.

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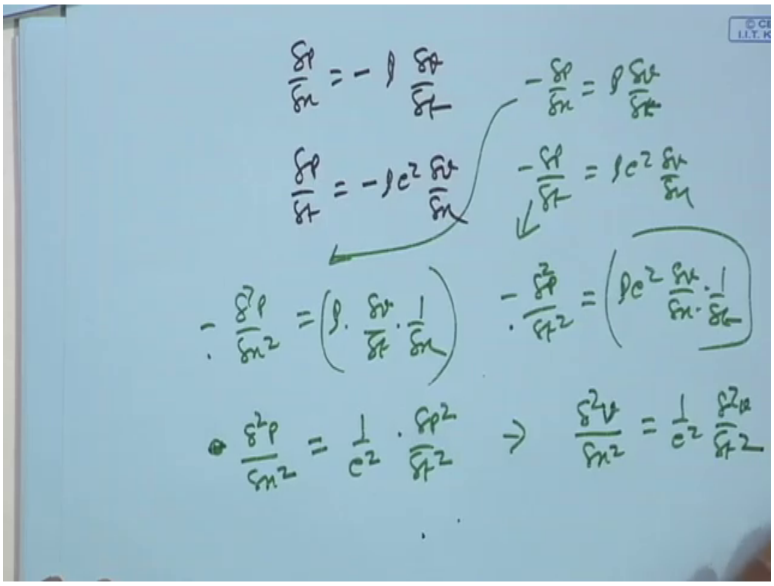

And here I get del p by del t is equal to minus rho c square del v by del x. Or another way I can write this one same way, that I can write minus or minus del p by del x is equal to rho del v by del t and del p minus del p by del t is equal to rho c square del v by del x. Now if I want to know the pressure equation. So, what I will do let us this equation derive respect to del t x. So, del square p by del x square let us minus is equal to rho into del v by del t into 1 by del x.

If I again do it in the respect to t minus del square p by del t square is equal to rho c square del v by del x into 1 by del t. So, if you see this side, and this side, what is the difference? 1 by c square only 1 by c square if I multiply. So, I can write minus del square p or minus this side minus this side minus. So, I can write del square p by del x square is nothing but a 1 by c square into del p square by del t square or not. Similarly I can derive the same equation which is nothing but a del square v volume particular velocity by del x square is nothing but a 1 by c square del square v by del t square. So, I can say this is the wave equation. So, this pressure wave equation this is the particular velocity wave equation.

So I can say this is the wave equation with a along when the wave is propagated along the x axis, I am not considered that wave would the any particular velocity with the x y and y and z direction. So, let us I say the wave is propagated along the x axis and there is a not other directional all particular velocity. So, friction force is not there viscous viscosity loss is not there. So, nothing is there. Then this is my velocity pressure equation this is my velocity equation. Now I what I know I know in the tube how the pressure wave is propagated, how the pressure wave is propagated I know in the tube. Now I have to derive. So, pressure a mathematical equation I know, but I do not know what is the solution of the p.

So, I to a second order differential equation, I can solve that second order differential equation, and find out the equation for p solution of that degree second order differential equation. And then try to find out how what is do the transfer function of this vocal tract tube. So, next class we try to derive the transfer function of single tube.

Thank you.