Digital Speech Processing

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Lecture – 30
Lattice Formulations Of Linear Prediction (Contd.)

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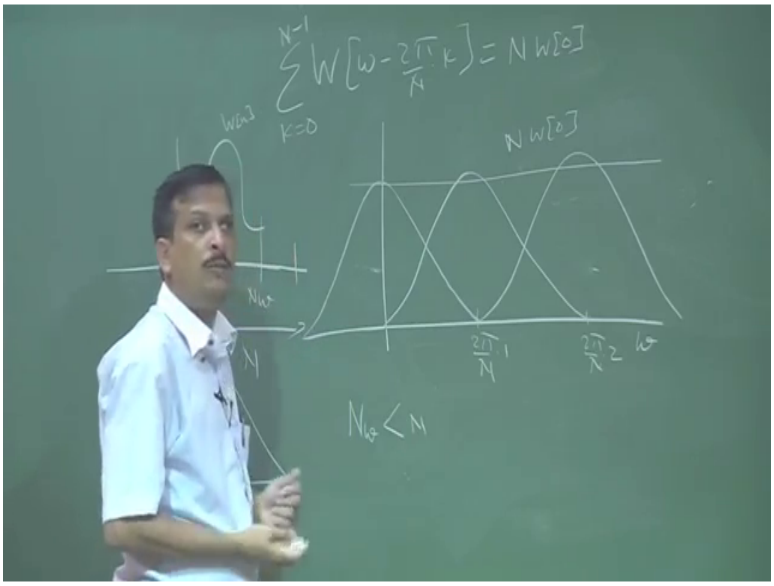

So, now what the FBS constant said that at k equal to 0 N minus 1 W omega minus 2 pi by n into k must be equal to a constant which is N omega 0. So, it express that suppose this is my, you can say this is my filter is this is W this is W N, so this is N w. And I have taken a analysis window which is N. So, N w is less than analysis window, so it is satisfied FBS constant, and it is said that the summation of filter response, ok. So if this is the frequency scale. So, this is omega this is my filter frequency response, this is my filter frequency response.

If this is the frequency response then I can say frequency response, this is 2 pi by N, so let us there is another filter, this is 2 pi by N into 2 into 1. So, summation of all summation frequency response of the analysis filter should sum to a constant, if I sum it, it would give me a constant which is nothing but a N into W 0, so what is the FBS constant that analysis window must be less than the length of the DFT or number of channel ok.

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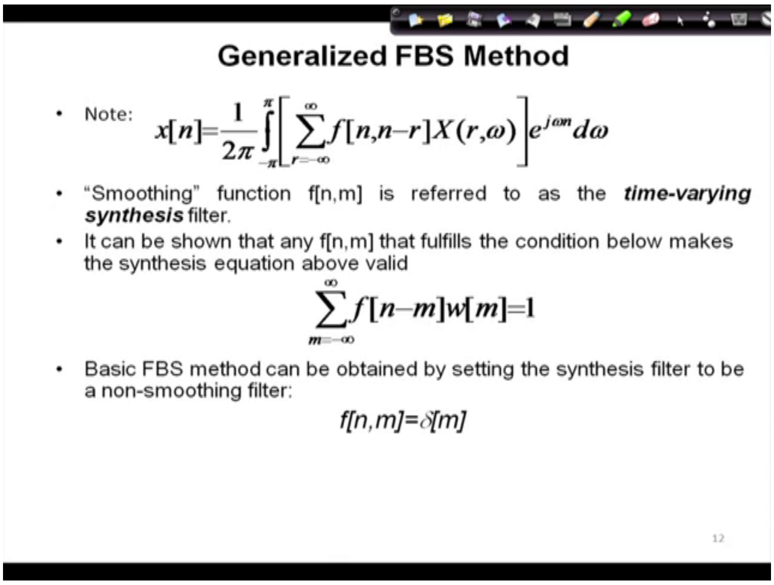

Then go for the generalized FBS method, now I do it generalize the FBS method. This, I have analyze with the FBS method with filtering view. Now let us try to generalize this FBS method. So, what is generalization set?

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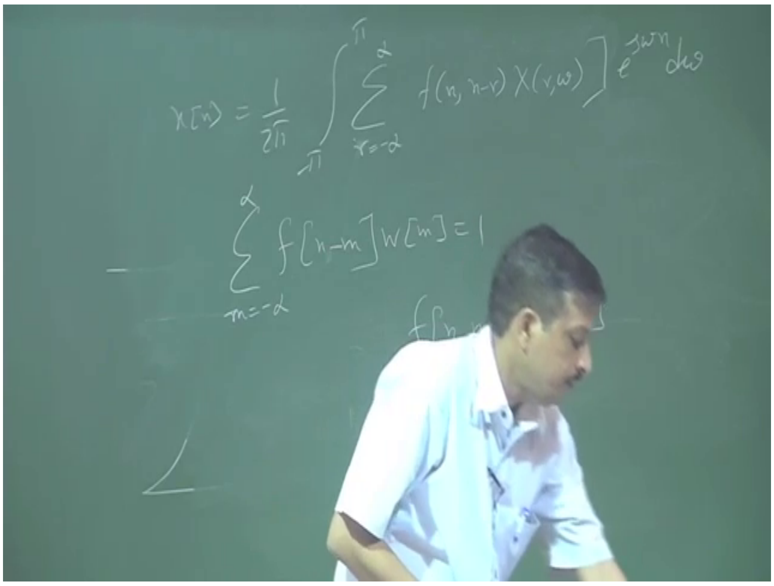

So, I can say x n is (Refer Time: 03:11) variable inverse Fourier transform, so this is 2 pi inverse Fourier transform, I have not taken DFT. So, it same analogy is same, so minus pi to pi, then frequency response, I can say m or r equal to minus infinity to infinity I define that f of n n minus r x of r omega e to the power j omega n d omega.

So, where r is the decimation in time r is the shifting of the window so first shifting, second shifting, third shifting so this is r, this is 2 r, this is three r, this is the decimation in time r. So, f of n minus r, I can say smoothing function f of n m n m is the smoothing function, time varying synthesis filter this is called time varying synthesis filter, it can be shown that any f of n m that fulfill the condition below, make the synthesis equation above valid. This is x n if this multiply by the analysis window f of n minus m into w m is equal to 1, where m equal to minus infinity to infinity ok.

So, in FBS basic FBS method the method can be obtained by setting the synthesis filter to a f of n m higher state it is equal to delta function delta n.

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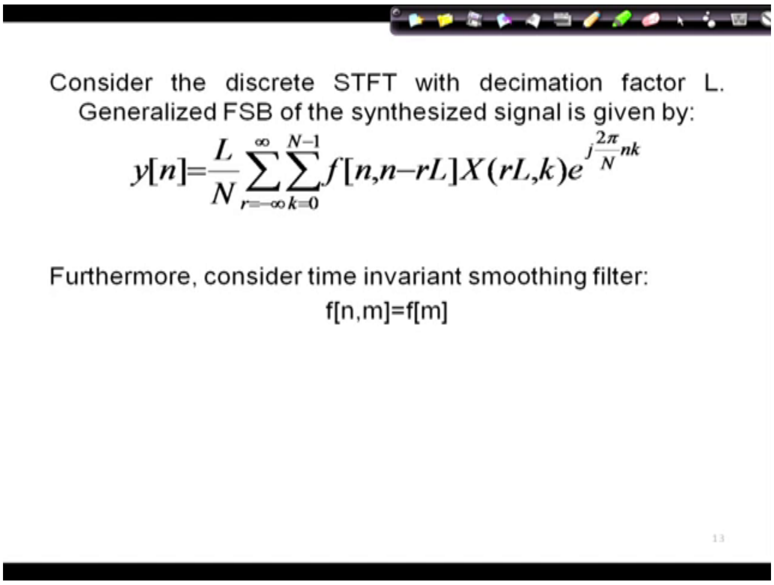

Now consider a discrete STFT of decimation factor L forget about this part, let us consider I have a signal and I shifted the frame with factor L.

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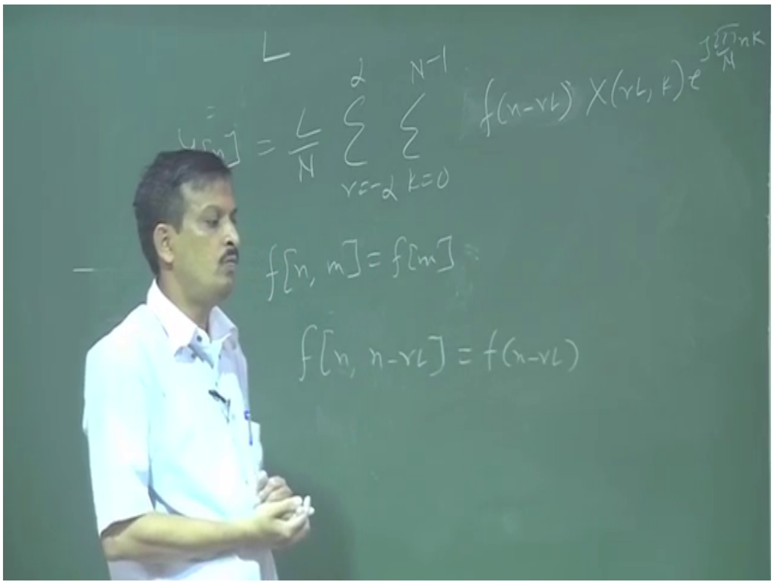

So, decimation time decimation factor is L, so I can say y by n after I take the IDFT is nothing but a L by N into r equal to minus infinity to infinity an k equal to 0 to n minus 1 n is the DFT length f of n r n n minus r L into x of r l k e to the power j 2 pi by n into n k. So, I said L is the decimation in time, so if it is n equal to nothing but a r into L, so r equal to 1 first shifting r equal to 2 second analysis window n equal to 3 third fourth fifth or that is why I will get, I replace that r L, ok. So, time varying smoothing function f of n m is nothing but a f n f of n nth time m number of m is the number of sample is equal to f m, then I can say f of n n minus r L is nothing but a f of n minus rn that time instant shifting, ok.

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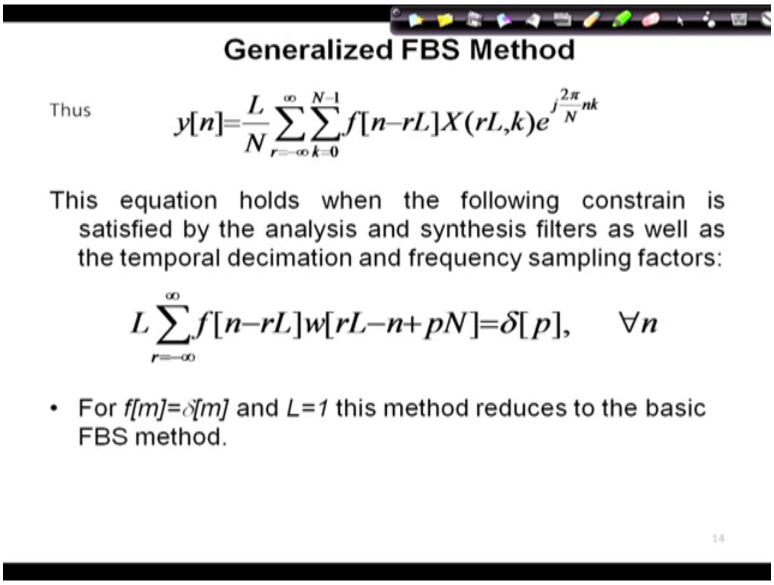

Then I can write down this equation instead of this one, I can write f of n minus r L. So, this equation hold only following constant is satisfy by the synthesis analysis and synthesis filter as well as the temporal decimation frequency sampling factor, when L into this function.

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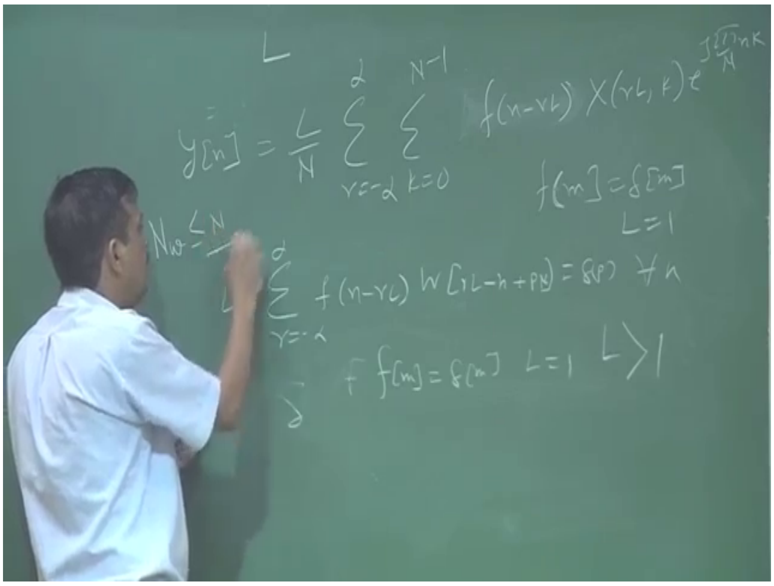

So I can say this is satisfy that y n is equal to x n when L into infinite of r equal to minus infinity to infinity f of n minus r L multiply by w r L minus n plus p n is equal to delta for all n.

So, f m is equal to delta if the f m is equal to f of m is equal to delta n and L is equal to 1 this satisfy, if f m is equal to delta m and L is equal to, so sample by sample recovery satisfy, so, this is since FBS method.

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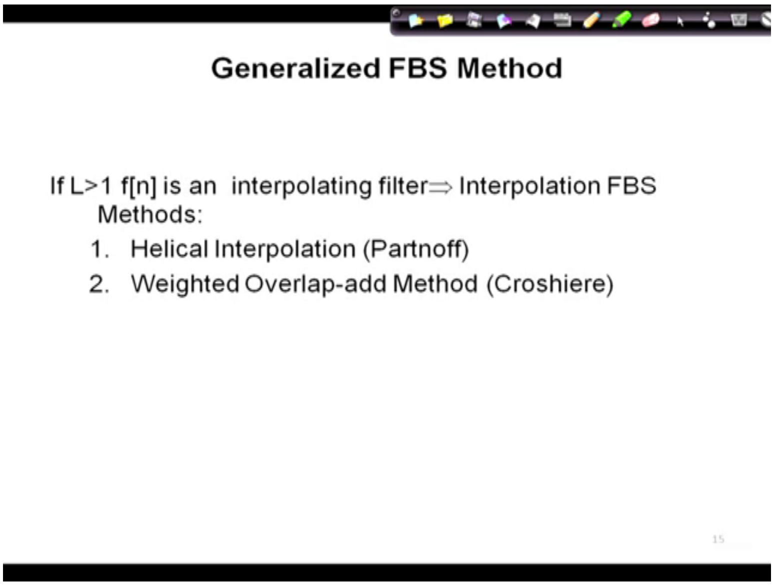

So, generalized FBS method, if L is greater than 1, then f m is an interpolating filter, so f m is f m is delta m only basic FBS method, but if it is f m is delta m, then the consideration is the time decimation L should be 1 decimation factor in time L should be 1, if I want L is greater than 1 then f m is an interpolating filter.

So, FBS method if I consider the synthesis function or smoothing filter is delta function and L equal to 1 then FBS method I can recover the signal, but if I want L greater than 1 in FBS method then f m is an interpolation filter. So, helical interpolation and weight it is overlap at method base interpolation that is there, so FBS constant is that FBS says that that analysis window size N w must be less than equal to length of the DFT. So, if I use 20 millisecond window which is 320 sample, in that case my the DFT length should be more than 320. So, 512 if I take n equal to 512 and window length is 20 millisecond that is fine no problem, I can completely recover is possible, ok.

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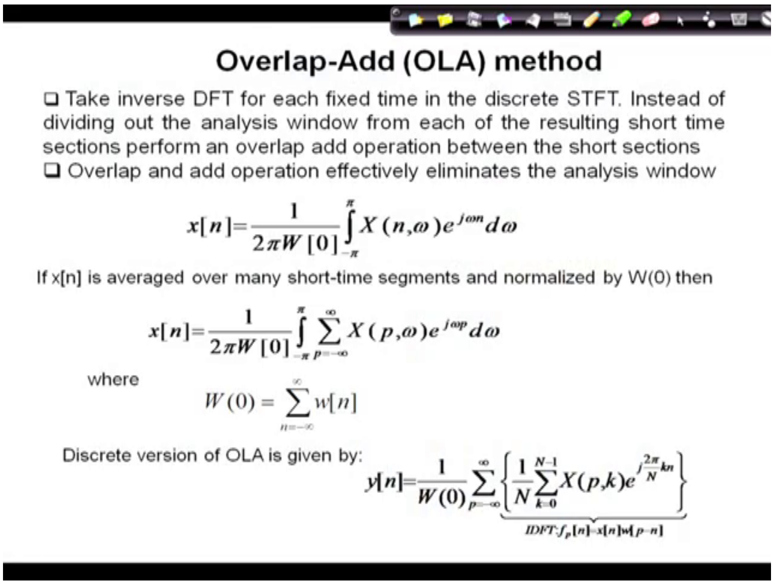

Another method of be since in synthesis is that, overlap add method, so this is another method of STFT synthesis this is call overlap add method. So, in overlap add method, so take inverse DFT for each fixed time of the discrete STFT instead of dividing out the analysis window from each of the resulting short time section I can do a add operation.

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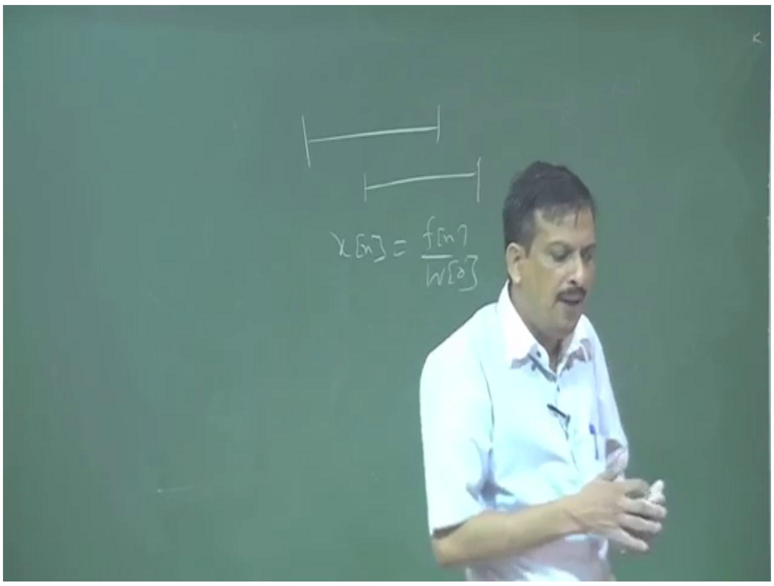

So, what I said that take the window analyze it and take the inverse transform, then what you get; you get, the signal divided by if I want to get exactly get back the signal, then I what I required that x n has to be f n divided by W 0, but instead of division what I do take 1 and take another 1 in here and overlap and add them. So, overlap add operation effectively eliminate the analysis, so what I am doing instead of dividing by the W 0, I take the overlap analysis window, and these overlap if I add them. So, due to these overlap addition, this effect of analysis window will be removed.

So, if x n is the average over many short time segment instead of normalized and normalized by w 0, then x n is nothing but a this one, I am mathematics you can see from thus this slides, so where W 0 is equal to w n give by this 1 ok.

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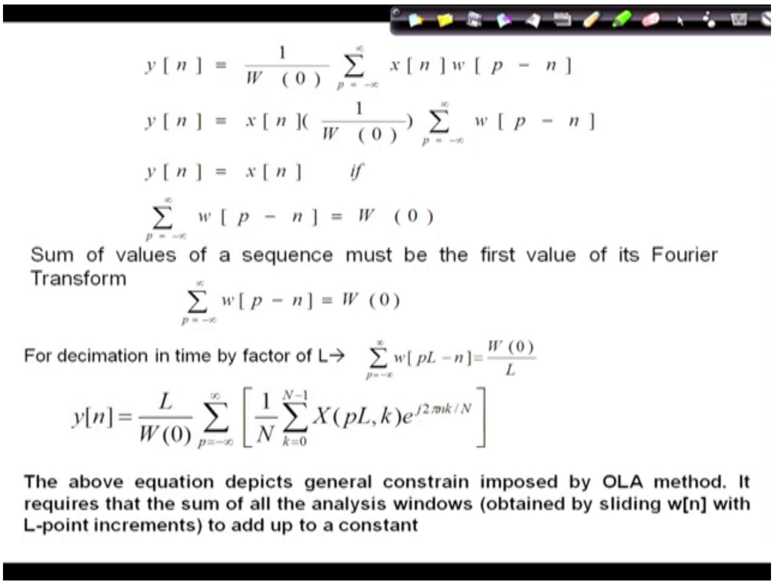

So, I can say next y n what is the y n in overlap add method y n is equal to 1 by W 0 into p equal to minus infinity to infinity so, all segments I sum up x of n into w p minus n. So, I can say it is nothing but a x of n divided by W 0 into p of minus infinity to infinity w p minus n.

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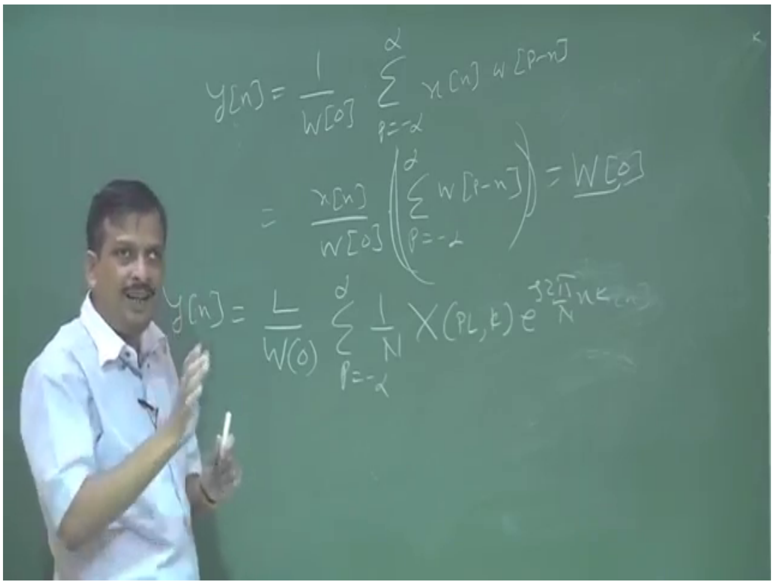

Now if I say y n is equal to x n only if this part is equal to w 0 or I can write capital W here, W 0 is not the 0th sample of the window say W 0 is nothing but a n equal to in minus infinity to infinity w n normalized, so this is the average or you can say the passed component of the Fourier transform, at k equal to 0, so, DC component which is nothing but the average of all sample give me the DC component.

So, here this is should be equal to W 0, now if it is W 0, so sum of value of square sum of sequences, so this is obviously sum of sequence is nothing but the first Fourier transform ok.

So, let us decimation time is L instead of shifting sample by sample I shifted L sample. It is nothing but a p equal to minus infinity to infinity W p into L I shifted L length minus n which is nothing but a W 0 capital W 0 divided by l. So, instead of shifting 1 sample take the sum, then I can say y n y n is equal to L by W 0 into p equal to minus infinity to infinity, 1 by n x of p L k e to the power j 2 pi by N into n k this is the synthesis equation. So, above equation what is the constant impose by the OLA method it require that the sum of all synthesis window, sum of all synthesis window, obtain by sliding w n with L point increment to added up a constant. So, I can say this 1, sum of all analysis window which is shifted by a L sample must be give a constant what is that; that means, that I can say sum up all analysis window.

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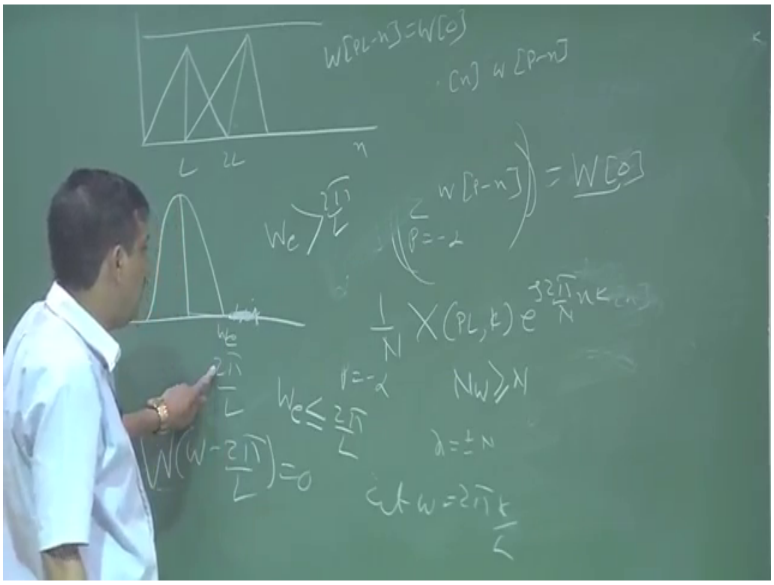

So, suppose this is my n number of sample so, this is my first window it is shifted by L, the second window will be here 2 L. So, sum of all analysis window shifted by time distribution factor must give me a constant which is nothing but the W p L minus n should be equal to some constant ok, if it is that what it is define, if it is W p L minus n is equal to constant. That means, this finite bandwidth, so OLA method it is the shown that that these constant is satisfy for all finite bandwidth analysis windows whose maximum frequency is 2 pi by L this is possible, when if I take the this is my analysis window, and this is omega c; this omega c, frequency whose maximum frequency is less than 2 pi by L and this is my this is 2 pi by L. So, omega c must be less than equal to 2 pi by L.

So, FBS method what I said N w must be greater than or equal to at least N, sorry greater than equal to N, if in that case there is a special constant, that n must be plus minus N like that way because that is also the relaxation is possible, here also relaxation is possible, but in that case this W omega minus 2 pi by L must be 0 here, all should be 0 it cannot be any value at omega equal to 2 pi by k by L must be 0, so I does not it matter if m omega c is greater than 2 pi by [laughter], then I have to ensure at every omega 2 pi by k 2 pi by L it should be 0 ok.

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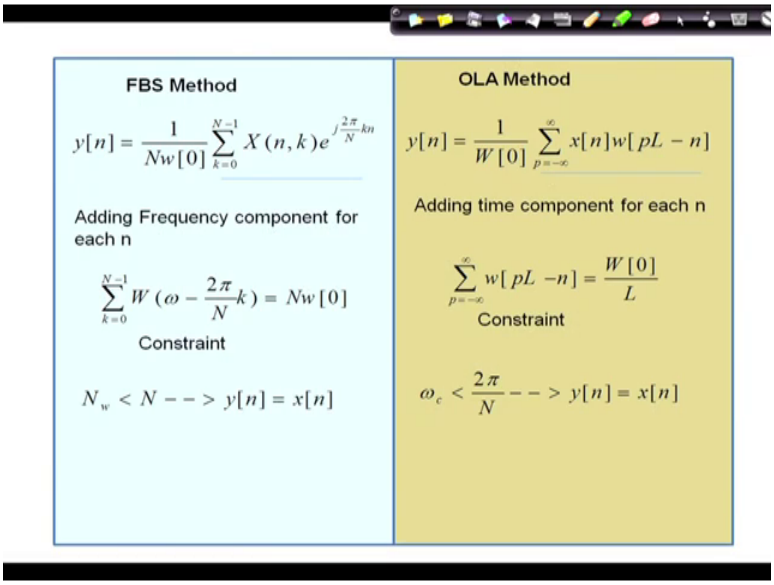

So, if I summarize FBS method the constant is this 1, which define me the size length of the analysis window must be less than the number of analysis channel, and OLA method define a constant which omega c the bandwidth of the analysis window must be less than 2 pi by N 2 ok.

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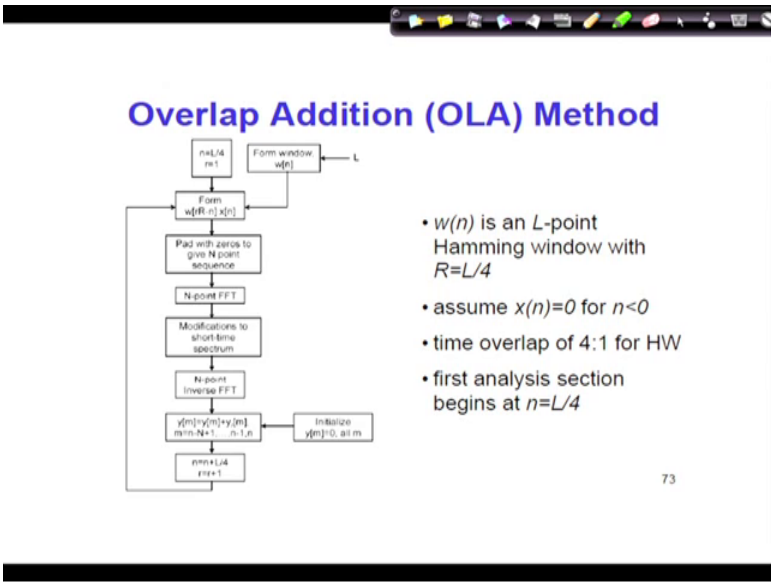

This I will come later on.

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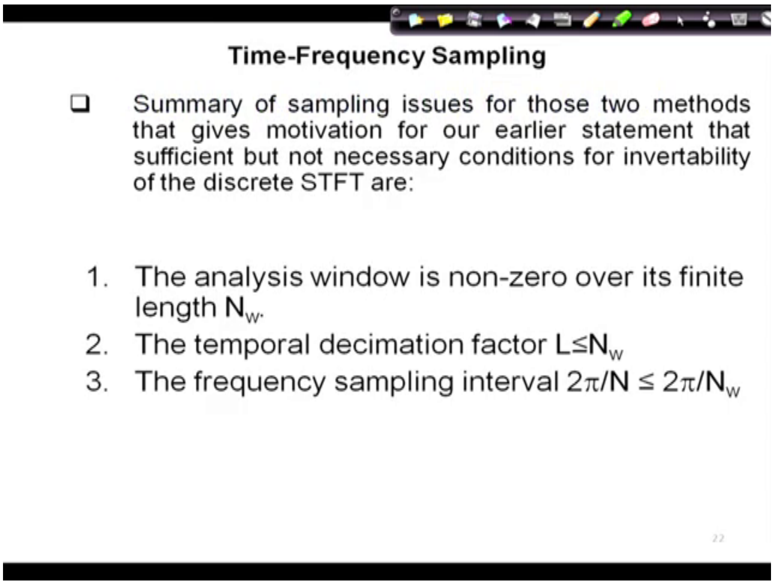

So, time frequency sampling which is very important, why we are doing that FBS method and OLA method, that I have to know I do not want that, I want to suppose this is my requirement I have a signal I cannot take whole signal at a time, and analyze its frequency domain and do some modification in frequency domain, and take the inverse to get the time domain signal this is not possible, because the signal is non stationary some part is modified.

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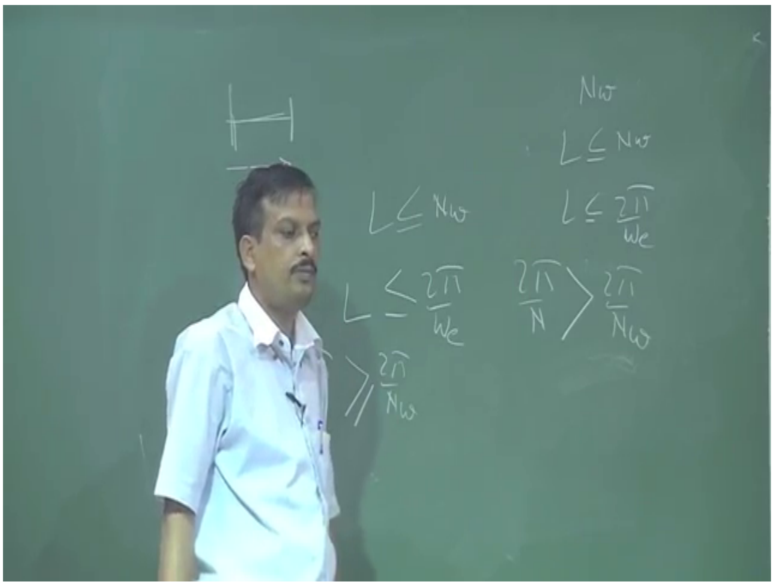

So, what I say I want to take part of the signal, and analyze it in frequency domain and do some modification take the inverse transform to get the signal back next take the next portion next portion. So, I want to know how much amount of shifting of this sliding is possible, it is generally it is possible, if I slide the analysis window once every sample, but which is very time consuming, and also time complexity because if I have a 4800 sample then 400 800 times I have to analyze the STFT. So, I want maximum allowable shifting for that I can recover the signal from the inverse transform, and also what kind of window I should use for that I know x of n omega for every omega.

So, what will be the bandwidth of the analysis window, and what will be the shifting decimation in time. So, analysis window is non 0 it is a finite length N W, so temporal decimation factor is L is must be less than the length of the analysis window that we have already said if it is greater than, then I know I miss the signal, and number of that 2 pi by N w must be greater less than the number of channel bandwidth, this is we know.

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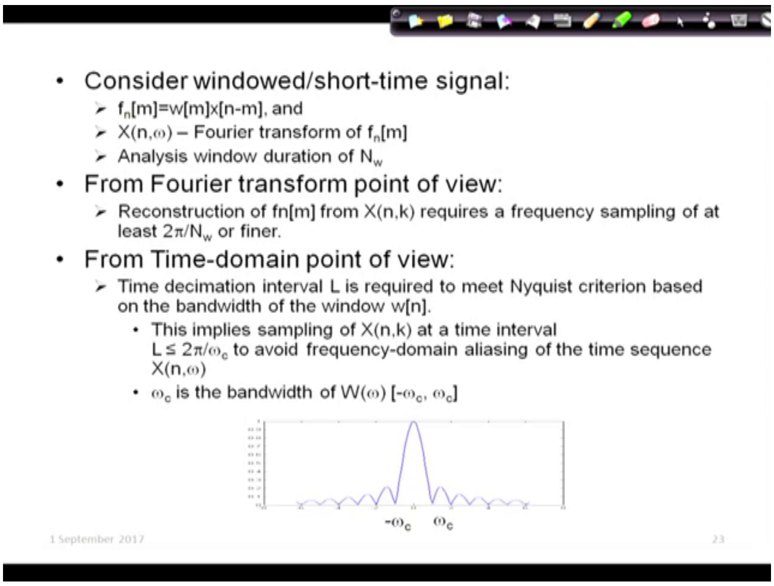

So, considering window for time signal f n omega is equal to omega w n into x n and x of n omega Fourier transform of x n analysis window is n w, so Fourier transform point of view reconstruction of f n omega f n m from x n k require the frequency sampling at least 2 pi by N w.

So, length of the analysis frequency sampling, so length of the DFT which define 2 pi by N must be at least 2 pi by N w, and for time domain view point the decimation factor L require to meet the nyquist criteria, which means that omega c the cut off frequency or you can say the bandwidth omega c the bandwidth L must be less than equal to 2 pi by omega c.

So I can say L is must be equal to 2 pi by omega c, and I have to chose N, so that 2 pi by N 2 pi by a frequency sampling at least must be greater than or equal to 2 pi by N w; N w, is the length of the window.

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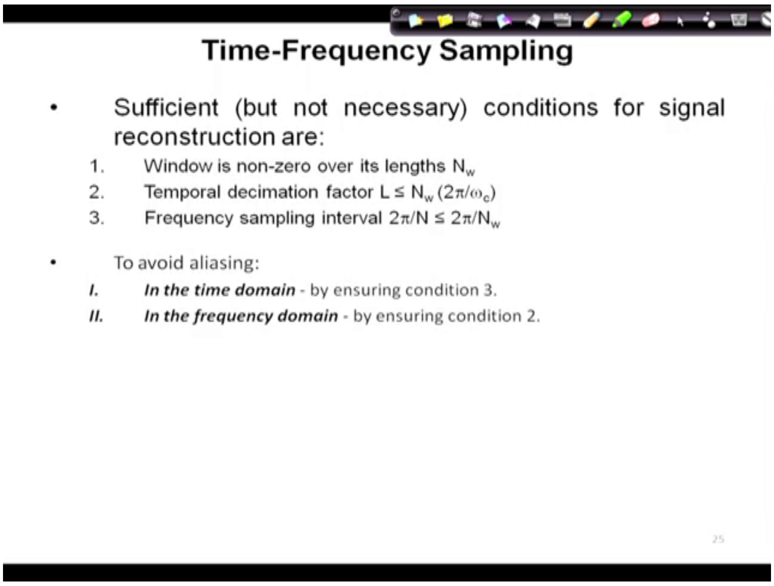

So, now I take a example so, window length is N w, take window length is N w, so temporal decimation factor L must be equal to n w less than n w and L must be equal to 2 pi by omega c and 2 pi by N must be greater than 2 pi by N w.

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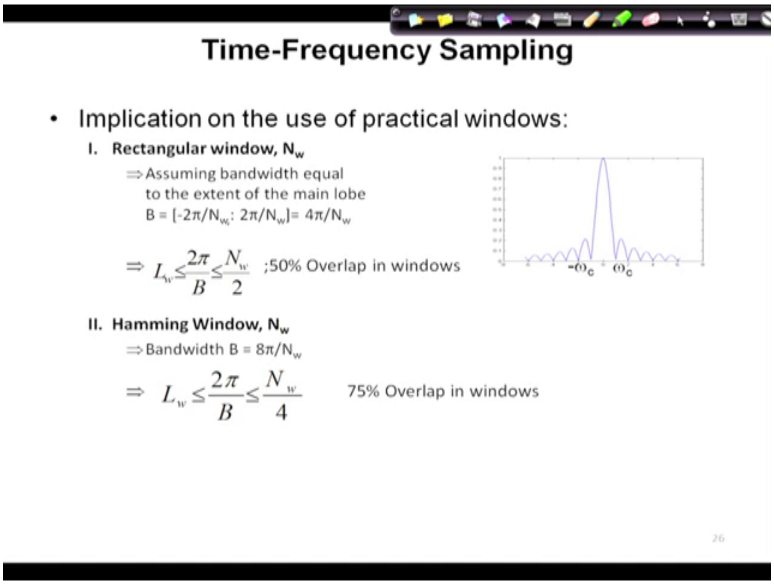

So, suppose take the example there is a call rectangular window length of the rectangular window is N w ok.

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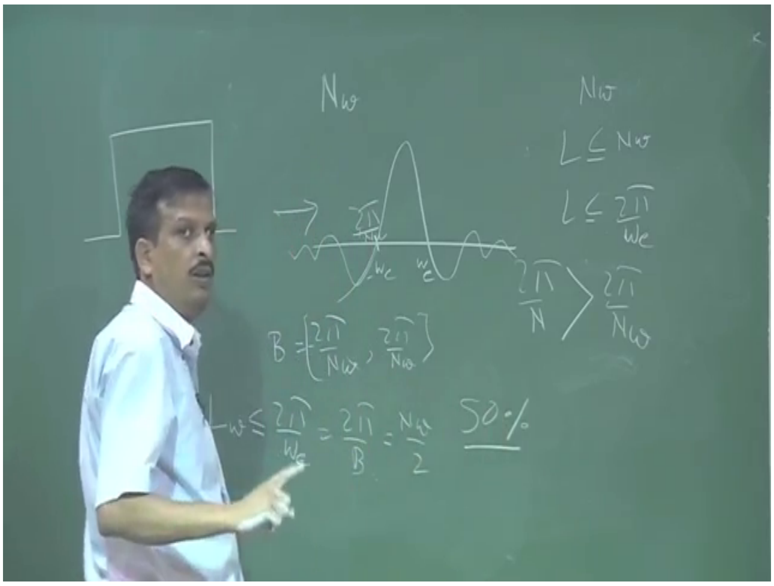

You know if it is a rectangular window, then what is the frequency domain representation of the rectangular window is nothing but a sink function, so a sink function this is omega c, this is minus omega c, this is plus omega c. So now, what is bandwidth is nothing but a plus minus omega c to plus omega c which is nothing but a 2 pi by L 2 pi by N w, so I can say 2 pi by N w minus 2 plus 2 pi by N w, this is the bandwidth so, L w shifting decimation time must be equal to 2 pi by omega c.

So, it is nothing but a 2 pi by bandwidth B, so it is nothing but a N w divided by, I can say 50 percent overlap is possible L maximum 50 percent shifting is possible, if it is rectangular window.

We will discuss next class we discuss others window, ok.

Thank you.