

Digital Speech Processing
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Lecture – 03
Review Of DSP Concepts

So, before we start digital speech processing, some signal processing concept has to be reviewed because those will be used frequently in many places. So, you may find difficulty that to conceptualise DSP problem. So, what I do that I will just review some DSP concept, but details I will not go because details; this is a separate subject which is called digital signal processing because this is a base point of knowing the digital speech processing.

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So, whatever the acoustic phonetics, speech production you know that is different because that does not require any DSP concept that much, but once I explain start that explaining the digital speech processing and modelling; discrete modelling, discrete time modelling. So, in that case some DSP concept is required.

So, I just quickly go through the some DSP concept. So, what I will discuss I have not discussed the details mathematics, I only discussed concept behind that DSP algorithm. So, let us start with that continuous signal.

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Concept of frequency in continuous-time and Discrete-time signal

Continuous sinusoidal Time Signal $x_a(t) = A \cos(\Omega t + \theta)$

- For every fixed value of the frequency F $x(t)$ is periodic
- Continuous time sinusoidal signal with distinct frequencies are themselves distinct.
- Increase the frequency result in increase in the rate of oscillation of the signal \rightarrow more period are included.

Complex exponent from $x_a(t) = A e^{j(\Omega t + \theta)}$

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Ω

$x_a(t) = A \cos(\Omega t + \theta)$

$F = 500$

$x[n] = A \cos(\omega_n n + \theta)$

$\omega = 2\pi f n T$

$\omega_n = 2\pi F \cdot n \cdot T$

$= \frac{2\pi F \cdot n}{F_s}$

$\Omega = 2\pi f$

$t = nT$

$f = \frac{F}{F_s}$

So, if I; if you know that this omega always represent that continuous frequency this omega. So, if I say sinusoidal signal let A is the amplitude cos omega t plus theta; theta is the range. So, this omega is continuous frequency. Now what are the property if you change this the frequency are distinct, if it is 500 hertz sinusoidal when is the frequency is 500 hertz if you increase the frequency in time domain suppose if it is like this. So, what you say this has an oscillation. So, the number of period in per second give me the frequency.


So, this there is a oscillation is there. Now if I increase the frequency. what will happen? The number of oscillation or number of period per second will be increases. So, high frequency means oscillations will be increases. So, that is the property. So, more period are included in one second more period are included in time. So, this is the low frequency and this is the high frequency signal. So, this is the concept of analogue signal. Now once I make the digital signal, let this $\cos \omega t$, I sampled and stored it in the digital signal. So, if it is a continuous if $X_a(t)$ is a continuous signal, then what should be the X_n ; X_n is a digital signal. So, x_n is nothing, but $A \cos$ instead of continuous ω we write this ωn plus θ this is called discrete frequency. So, this is continuous radian per second and this is that this frequency is per sample; I will come details I will come.

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Discrete time Sinusoidal

$$x(n) = A \cos(\omega n + \theta)$$

- A discrete time signal is periodic if its frequency f is a rational number
- Discrete time sinusoidal whose frequency are separated by an integer multiple of 2π are identical
- The highest rate of oscillation in a discrete time sinusoidal is attained when $\omega = \pi$ or $(-\pi)$.



So, this is written like this way and what is the property. Now a discrete time signal is periodic if its frequency f is rational number what is understand what is why the small f has come; now what is ω ? ω is nothing, but a $2\pi f$. Now if it is digital signal what I have done; I have replaced this t by n into capital T . So, this ω continuously becomes ω is nothing, but the $2\pi f$.

So, this frequency is continuous frequency, now once I make it discrete frequency then ω is equal to $2\pi f$, but this t is replaced by n into capital T . So, this ωn becomes ωn is nothing, but the $2\pi f$ into n into t . So, it is nothing, but the $2\pi f$ into n divided by f s;

f s is this. So, if the frequency content let I write this. So, on the frequency of this cos this digital signal is 500 hertz. So, let this is denoted by f. So, this denoted by capital f. So, f by f s; small f is nothing, but a frequency of the signal that is analogue frequency divided by f s.

So, I cannot say that this is a if it is periodic if this is a rational number then only it will be a periodic I am not discussing number system does not know what is rational number ok and then discrete time sinusoidal whose frequency are separated by integer multiple of 2π are identical very simple, if the cos is a distinct sinusoidal if it is integer multiple of 2π separated then they are identical. So, 2π is the maximum period. So, the highest rate of oscillation the highest value of small f what will the highest value of small f.

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Handwritten mathematical derivations on a grid background:

$$f = \frac{F}{F_s}$$

$$F_s = 2F$$

$$= \frac{F}{2F} = \frac{1}{2}$$

$$-\infty \leq \Omega \leq \infty$$

$$-\frac{1}{2} \leq f \leq \frac{1}{2}$$

$$\omega = 2\pi f$$

$$-\pi \leq \omega \leq \pi$$

On the left side, there are two circled terms: Ω and ω , each with an arrow pointing to the right.

So, f is equal to F by F s.

So, what is F s? As per sampling theorem, F s should be at least twice F. So, I can say F by 2 F. So, it is half. So, in continuous frequency, omega can varies from minus infinity to plus infinity, but if you see this small f can varies from minus half to plus half minus half to plus half. So, if is F s; if I say that 2π capital F is the analogue frequency; so, is continuous signal; omega is radian per second and this omega is radian per sample. So, this can varies from minus infinity to plus infinity. So, this frequency can also varies from minus infinity to plus infinity, but if you see, small f can varies only minus half to plus and was small omega what is the value of omega; omega is nothing, but a $2\pi f$.

So, f is nothing, but a minus F minus half to plus half. So, I can say ω can vary only from minus π to plus π where F_s is the sampling frequency.

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Continuous time signal	Discrete time signal
$\Omega = 2\pi F$ Ω is in Radians/sec and F is in Hz $-\infty < \Omega < \infty$ $-\infty < F < \infty$	$\omega = 2\pi f$ ω is in Radians/sample and f is in cycles/sample $-\pi < \omega < \pi$ $-1/2 < f < 1/2$
$\Omega = \omega/T_s$ $F = f \cdot F_s$	$\omega = \Omega T_s$ $f = F/F_s$
Where F_s is the sampling frequency and $T_s = 1/F_s$	

So, ω ; small ω represent the discrete frequency and capital ω represent analogue frequency, this is radian per second this is radian per sample radian per sample and this is radian per second. So, if I say this sometime this ω is called also normalised frequency.

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$F = 60 \text{ kHz}$ $F_s = 100 \text{ kHz}$

$$\omega = 2\pi f = 2\pi \frac{F}{F_s} = 2\pi \frac{60}{100} = 1.2\pi$$

$$2\pi \frac{F \cdot b}{F_s} = 2\pi \frac{60}{100} = 1.2\pi$$

So, if I say I have the signal whose capital F is 50 kilo hertz and less sampling frequency is 100 kilo hertz. So, what is the value of omega? Omega is nothing, but the 2 pi F where it is 2 pi F by F s. So, it nothing, but a 2 pi 50 divided by 100. So, it is nothing, but a pi; it is nothing, but a pi.

Now, if it is 40 kilo hertz, then it is 2 pi F into F by F s. So, t is nothing, but a 2 pi 40 by 100. So, it is 4 by 5 pi normalised discrete frequency. So, this omega is called discrete frequency, this concept is very important, this will be used in many where. So, this omega continuous this is discrete this is radian per second this is radian per sample is clear next concept is complex signal a complex; it is know you know everybody is know that is complex signal either, it is a vector everybody knows a complex vector and the complex signal has an amplitude and has an phase.

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Complex Signal

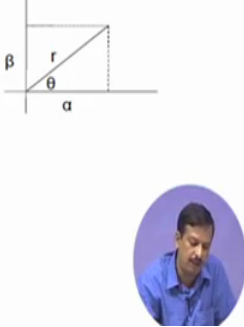
$$x[n] = (\alpha + j\beta)^n u[n] = (re^{j\theta})^n u[n]$$

$$r = \sqrt{\alpha^2 + \beta^2}$$

$$\theta = \tan^{-1}(\beta / \alpha)$$

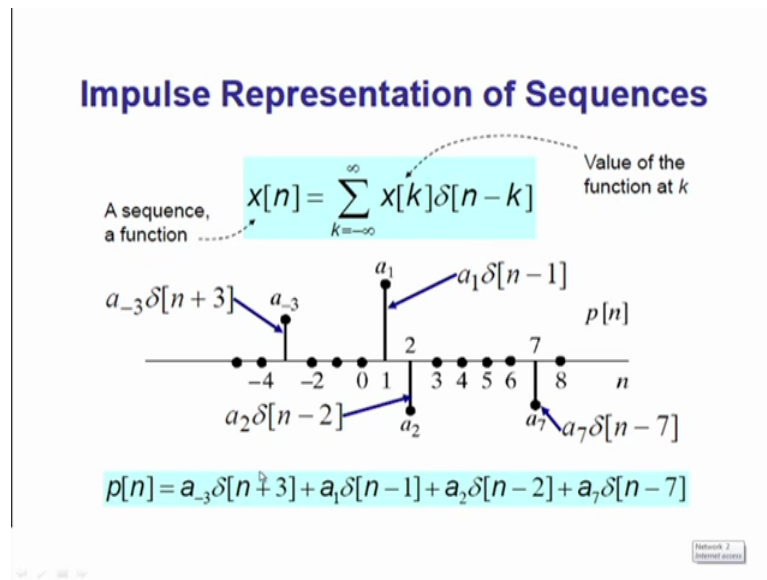
$$x[n] = r^n e^{j\theta n} u[n]$$

r^n is a dying exponential
 $e^{j\theta n}$ is a linear phase term



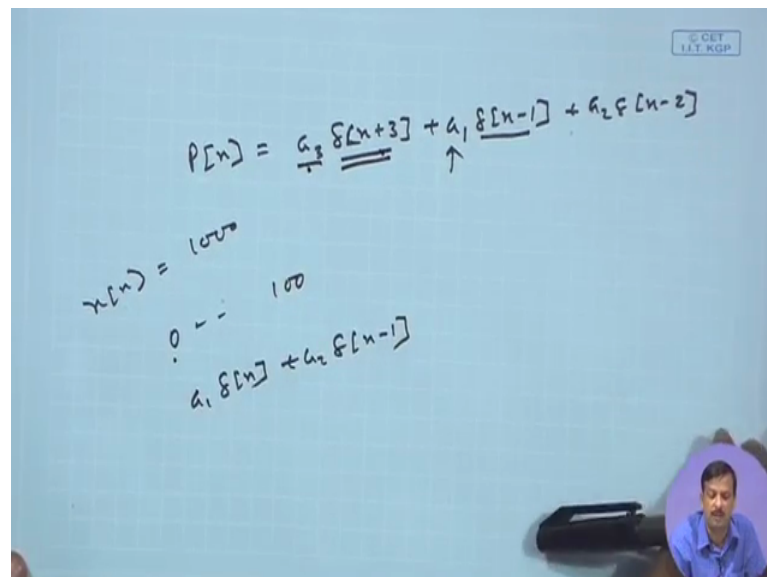
So, I can say if a plus j B is a complex number, it can written as r e to the power j theta where r is nothing, but a root over a square plus B square and theta is nothing, but a tan inverse d by a same things complex signal.

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Now, I come to the impulse representation. So, suppose this is given $x[n]$ is represented by $x[k]$ and delta function. So, $n - k$. So, if I say this, my $p[n]$ is like this; let $p[n]$ is my digital signal $p[n]$ is $a_3\delta[n+3] + a_1\delta[n-1] + a_2\delta[n-2] + a_7\delta[n-7]$.

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So, this a_3 is a coefficient and those are the delta position. So, at that position the signal has amplitude a_3 sample has an amplitude 3 at that position signal has amplitude a_1 . So, that can be represented like this way because it is plus 3. So, minus 3

point; there will be a sample whose completion amplitude is minus a 3, there is a sample cos sample position 1 2 like that way it will represent.

So, this is the impulse representation of a sequence. So, if I say have speech signal $x[n]$ who has suppose that is a 100 sample. Now if I say the sample start from 0 1 to 100, then first sample is nothing, but a the amplitude less first sample amplitude is a one with delta $n - n - 0 = 0$. Next a 2 delta $n - 1$ like that I can write. So, that will be used in different time one you process the digital signal.

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Classification of Discrete time signal

Energy signal and power signal: If E is the energy of a signal $x[n]$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

If E is finite then $x[n]$ is called an energy signal

Many signal possesses infinite energy, have a finite average power P . The average power define as:

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2$$

If P is finite and nonzero then the signal is called a power signal

Periodic and aperiodic signal: a signal $x[n]$ is periodic with period N if and only if $x[n+N] = x[n]$ for all n

The smallest value of N for which holds the above equation is called fundamental period.

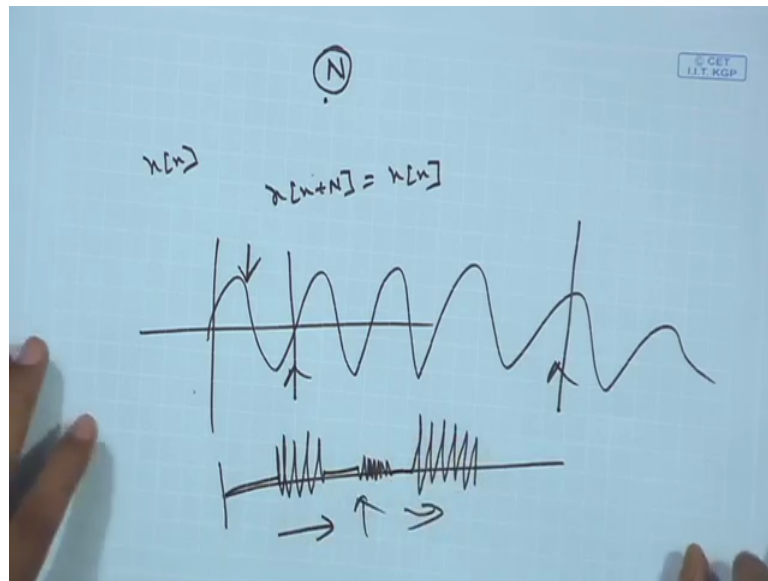
If there is no value of N that satisfies the above equation then the signal is called aperiodic signal.

Symmetric and antisymmetric signal: a real-valued signal $x[n]$ is called symmetric if $x[-n] = x[n]$

On the other hand a signal $x[n]$ is called antisymmetric if $x[-n] = -x[n]$

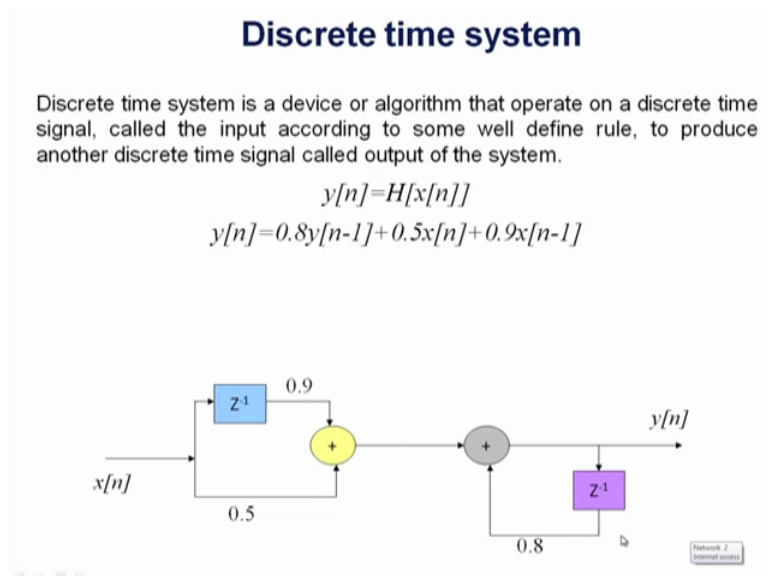
Now, next concept is classification of signal different type of signal; energy signal, power signal, I have not describing you know that if it is energy is finite then called energy signal. If power is finite, then it is called power signal periodic signal a periodic signal; a signal is periodic, if it is repeat its nature after certain time interval in time domain if it is repeat in nature lets after n sample, then I can say the n ; n sample is the period of the signal. So, $x[n]$ is periodic with period n if and only if $x[n + N] = x[n]$.

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So, finite amplitude; it is if this point and next repeating point is same, then I can say the signal is repeat itself. So, it is a period. So, digital signal x if n is the period, then x of n plus small n plus capital N is equal to x n , then you call signal is periodic, then symmetric and anti symmetric signal; a real valued signal x n is called symmetric if x of minus n is equal to x n ; on the other hand, if it is anti symmetric, then x of minus n will be minus x n .

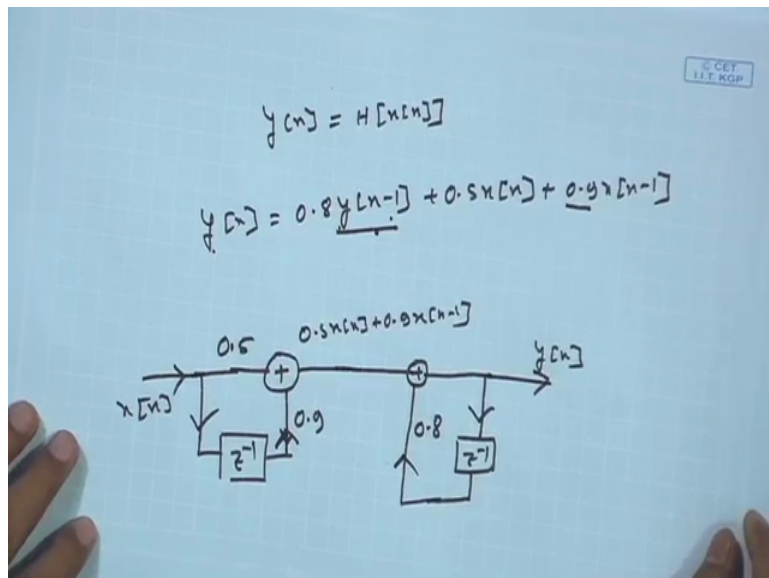
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So, this is a signal property; next discrete time system. So, signal periodic; a periodic signal symmetric anti symmetric; I have not discussed if n is called stationary and non stationary signal is also. So, what do you mean by stationary signal if the signal does not change its property over the time, then I can say it is a stationary; suppose I have a sinusoidal of 500 hertz; let this is a 500 hertz, if I take this time; if I take this time all the time it is 500 hertz sinusoidal. So, I can say the signal is a stationary signal does not change its property along the time.

Now, let us say the speech signal; if I say speech signal, if I say a sentence along the time the signal properties varies, sometimes it is silence, sometimes it is voice, sometimes it is silent, sometimes it is fiction when may be silence, then may be voice. So, I can say along the time signal changes its property. So, it is a time varying signal; later on you learn that signal processing algorithms are applied on a stationary signal. So, I have to consider; how I make the speech signal is non stationary. So, those kind of consideration we take. So, this is stationary and non stationary signal, then I come to the systems if you come system; this is called discrete systems, I am to going to the analogue systems you know that signals and systems that class.

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So, discrete system; let us output of a discrete system is $y[n]$ is nothing, but a H of input $x[n]$. So, H is the transfer function of the systems lets the system is like this $y[n]$ is equal to $0.8 y[n-1] + 0.5 x[n] + 0.9 x[n-1]$.

So, if I told you; draw the system diagram digital system diagram. So, I can say what is $y[n-1]$ means the $y[n]$ is delayed by sample number; 1 sample $n-1$ means 1 sample delay $x[n-1]$ means input is 1 sample delay. So, if this is my $x[n]$; let this is my $x[n]$. So, $x[n]$ has to be multiplied with amplitude 0.5 and has to be added with 0.9 multiplied $x[n-1]$. So, I can say let I take $1 \times x[n]$ in here input and apply a Z to the power minus 1 ; delay Z to the power minus one represent one sample delay and added with this signal with a amplitude of 0.9 . So, output is nothing, but a $0.5 \times$ of n plus $0.9 \times$ of $n-1$.

Now let this is my $y[n]$. So, $y[n]$ is nothing, but a this plus there is a $y[n-1]$. So, I can take one input from here and then Z to the power minus 1 0 apply and added with it with a multiplication factor. So, this is my system diagram. So, this is $y[n]$ and this is $x[n]$. So, signal flow is this way sorry this way this way or not. So, discrete system can be implemented like this way. So, Z to the power minus one is nothing, but a single sample display.

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Classification of discrete system

- **Static and Dynamic system**
 A discrete time system is called static or memory less if its output at any instant n depends at most on the input sample at the same time, but not on past or future samples of the input. In any other case the system is said to be dynamic or to have memory.
- **Time invariant and time variant system**
 A system is called time invariant if its input-output characteristics do not change with time.
 A relaxed system H is time invariant or shift invariant if and only if
 $x[n] \xrightarrow{H} y[n]$ implies that $x[n-k] \xrightarrow{H} y[n-k]$
 If the output $y[n-k] \neq y[n-k]$ even for one value of k the system is time variant

Now, a system consists of certain properties; I will come implementation later on also.

A system has certain property what are the property it is may be statics or it may be dynamic system a discrete system is called statics or memory less; if its output at any instant n depends at most on the input sample at the same time, but not past or future sample of the input.

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$$y[n] = x[n] \quad n < n-1$$
$$x[n+1]$$
$$x^2 \cdot$$
$$x[n] \xrightarrow{H} y[n]$$
$$\underline{x[n-k] \xrightarrow{H} y[n-k]}$$

So, suppose my y_n is only depends on x_n and x_{n-1} , but not x_{n+1} ; then I can say my system is static. So, or memory less system and if it is future sample then I can say it is with memory system. So, memory less system static system with memory system is dynamic system then time invariant and time variant system time invariant and time variant signal that is called stationary and non stationary signal, but if it is time variant and time invariant system.

So, if the system does not change its property over the time then I call it is a time invariant system; if it is change along with the time then I can say it is a time variant system if I consider this vocal track is a system along the time, it change its construction. So, it is a time variant system suppose access the suppose operator square always it will be square whatever the input will come it will square. So, property does not change along with the time. So, it is a time invariant system.

So, how do you mathematically said that if it is x_n is a input of a system H and output is y_n if I apply x_{n-k} I should get the y_{n-k} then it is called time invariant system along the time system does not change its property.

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•Linear and non-linear system: A linear system is one that satisfies the superposition principle. The principle of superposition requires that the response of the system to a weighted sum of signal is equal to the corresponding weighted sum of the response of the system to each of the individual input signal.

• Causal and non-causal system: A system is said to be causal if the output of the system at any time n depends only on present and past input, but not depend on future inputs.

$y[n]=F[x[n],x[n-1],x[n-2],\dots\dots\dots x[n-k]]$ where F is any function

If a system does not satisfy the above condition then the system is called Non-causal system.

•Stable and unstable system: An arbitrary relaxed system is said to be bounded input-bounded output(BIBO) stable if and only if every bounded input produces a bounded output. $x[n]$, $y[n]$ are bounded is simply translated mathematically to mean that there exist some finite numbers say M_x , M_y such that

$$|x[n]| \leq M_x \leq \infty \quad |y[n]| \leq M_y \leq \infty$$

for all n

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Now, linear and non-linear system. So, how to test the linearity of the system is linear if it supports the superposition principle. So, what is superposition principle if you know that if I apply 2 input $x_1[n]$ plus $x_2[n]$ to a system H whatever output, I will get this will be same?

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$H[a_1x_1[n] + a_2x_2[n]] = H[a_1x_1[n]] + H[a_2x_2[n]]$

LTI

$y[n]$

$x_1[n]$ $x_2[n+1]$

BIBO

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If I apply 2 input individually let us put a factor a_1 and a_2 ; this $a_1x_1[n]$ plus H of $a_2x_2[n]$. So, if it supports the superposition principle then I say the system is linear system. So, if the system is linear and time invariant, then we call LTI system linear time

invariant system that is called LTI system; that means, the system is linear system does not change its property along the time, then you know causal and non causal system. So, if it is not depends on the future, then I call system is a causal system means $y[n]$ is only depends on either $x[n]$ or $x[n-1]$, but not the future input future input will be $x[n+1]$ one if it not depend on $x[n+1]$ then I call it is a causal system; if it is depend, then it is a non causal system.

Then stable system unstable system what is stability if I apply a bounded input to input to the system it should produce bounded output if I apply bounded input if the system produce bounded output then I call system is stable system BIBO bounded input bounded output in bounded input should produce bounded output then stable and unstable system.

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Recursive and Non-recursive discrete system.

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Cumulative average of signal $x(n)$

$$y[n] = \frac{1}{n+1} \sum_{k=0}^n x[k]$$

$$(n+1)y[n] = \sum_{k=0}^{n-1} x[k] + x[n]$$

$$= ny[n-1] + x[n]$$

A system whose output $y[n]$ at time n depends on any number of past output values $y[n-1]$, $y[n-2]$ Is called recursive system

Then there is recursive and non recursive system that you know that this; this slide is self explanatory that explaining the recursive if the output depends on the previous output, then we call it is a recursive system, I am not describing details then very much or you can say the most commonly used signal processing algorithm in speech is called convolution. So, convolution is a most frequently used if you see that if I want to find out a output for a input in a system.

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Convolution

Convolution is one of the most frequently used operations in DSP. Specially in digital filtering applications where two finite and causal sequences $x[n]$ and $h[n]$ of lengths N_1 and N_2 are convolved

$$y[n] = h[n] \otimes x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = \sum_{k=0}^{\infty} h[k]x[n-k]$$

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Then the system transfer function is convolved with a input to produce the output of the system. So, convolution is most commonly used algorithm in DSP.

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Handwritten mathematical derivation of convolution on a grid background. The equations are:

$$y[n] = h[n] \otimes x[n]$$
$$y[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot x[n-k]$$

Below the equations, there are two terms: $h[k]$ and $x[n-k]$ with a multiplication sign between them. To the right, there is a vertical line with $x[k]$ above it and $x[k]$ below it, possibly representing a signal or a step in the derivation.

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So, a suppose y_n is produced using a system property H_n convolved with input x_n . So, how the mathematically it is written it is k equal to minus infinity to infinity H of k into x of n minus k . So, y_n is equal to H of k into x n minus k of y_n . So, convolution means that suppose this is my system and I apply a signal here. So, output is the property of the system must be convolved or modified the input signal produce the output. So, each

property of the; you can say that whole property of the system should modify the each input of the. So, that is why whole H k will modify the each input. So, this is convolution.

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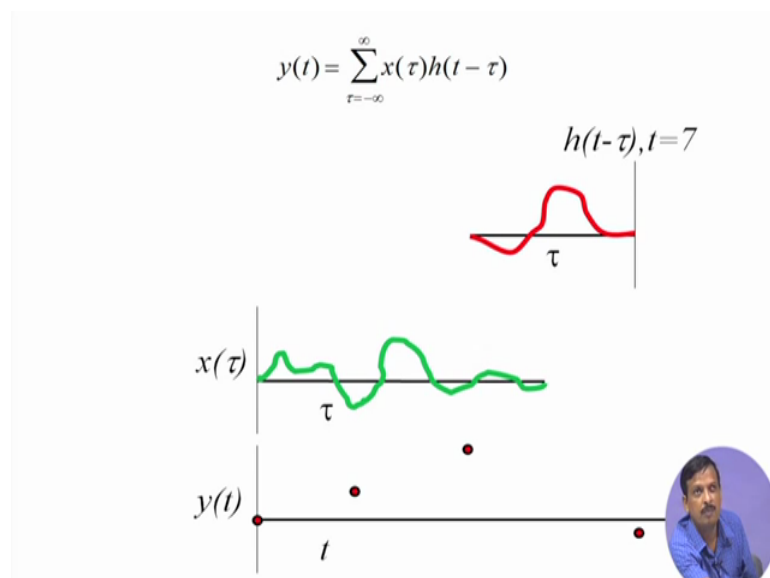
Operations involved

- **Folding**
- **Shifting**
- **Multiplication**
- **Summation**



How it is done it has a operation; how many operation is involved; it is folding shifting multiplication and summation if you look at this equation it is nothing, but a folding. So, if I say x_n equal to y equal to 0, then it is nothing, but a H k multiply x minus k. So, I can say the x k is folded to produce x minus k.

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So, I can give an example. So, let us this is my signal and this has to be convolved with this. So, green is the lets the system. So, sorry there is a wrong interpretation you can say that this is H and this is x lets I same thing is considered this is H and this is x. So, I can say this H is convolved or you can say that the x property of x is convolved with H t. So, what I said that the H has to be shifted folded. So, y t is means H has to be folded first folding then shifting t equal to 0 then product and sum this sample product to this sample produce sample then shift the signal y t equal to one then again product and sum then again t equal to 2 product and sum then again one sample shift product and sum. So, that way output will be produced.

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Convolution Example

$x[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$
 $h[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$

What is $y[n]$ for this system?

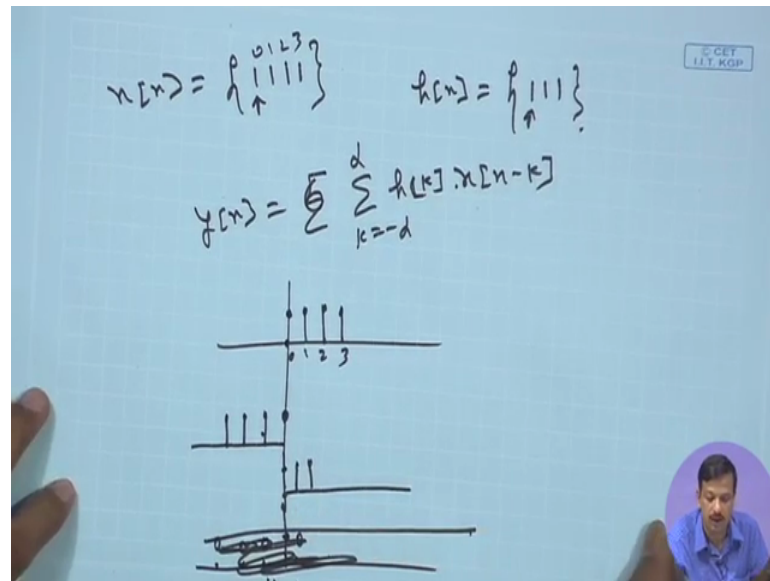
Solution :

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} h[m] x[n-m]$$

$$= \begin{cases} \sum_{m=0}^n 1 \cdot 1 = (n+1) & 0 \leq n \leq 3 \\ \sum_{m=n-3}^3 1 \cdot 1 = (7-n) & 4 \leq n \leq 6 \\ 0 & n \leq 0, n \geq 7 \end{cases}$$

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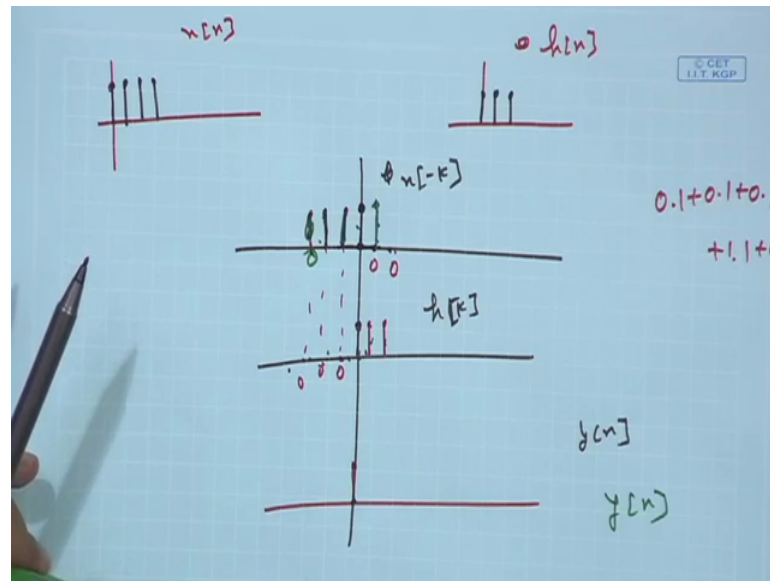
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Let us give an example let's $x[n]$ is nothing, but instead of writing that way let 1 1 1 and here let's this is 0 0 1 2 3 and let's $h[n]$ is nothing, but a 1 1 1 and here I am not giving the same example which is in the slide this is 2 signal. So, what is there $h[n]$. So, $x[n]$ let I want to produce $y[n]$ which is nothing, but a summation sorry summation of k equal to minus infinity to infinity $h[k]$ multiply by x of n minus k . So, which signal has to be folded $x[n]$. So, I can fold the $x[n]$. So, if I draw in pictogram. So, this is $x[n]$ first sample second sample third sample fourth sample.

So, 0 1 2 3; 0 1 2 3. So, if I want to fold it. So, it will be this side one 2 3 and if I want to plot $h[n]$; it is nothing, but a 1 2 3. So, my $y[n]$ first I fold it multiply by 0. So, $y[0]$ is nothing, but a multiply by this or this with this. So, 1 multiplied by 1 and this with multiply by this 0. So, this is 0, this is 0, this is also 0, this is 0, sorry, sorry, sorry, sorry, sorry, this is $y[n]$ I will draw nicely. So, it will be best; let draw it neatly. So, use this pen.

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So, this is my x of n sample number one sample number 1, 2, 3 and 4 and this is my H , H is nothing, but a sample number 1 sample number 2 sample number 3; 3 sample is there. Now what I said to produce the convolution lets I draw the first. So, produce the convolution x n has to be folded. So, I folded x n , I will draw here to here this is number 1. So, folding is this side 2, 3, let 4 I folded it and so this is x of minus k ; now I use the H k H k is nothing, but a 1 1 second sample third sample. So, this is H k ; now what is my y . So, this is lets y n . So, at y 0 y 0 this multiply; this and if I say if I projected; this side it is 0, here it is also 0, here it is also 0, here it is 0. So, here also it is 0.

So, product this multiply with this some with this multiply with this some with this multiply with this. So, 0 into 1 plus 0 into 1 plus 0 into 1 plus 1 into 1 plus 0 1 into 0 like that way; it will come this will multiply with this add and so, I can get first sample y ; to get the second y n , what I will do instead of doing here I shifted this signal one sample. So, what I can say lets shifted this. So, this will be one first sample this is the second sample this is the third sample 1, 2, 3, 4. So, this sample again become 0. Now I product this with this, this with this, this with this, this with this, this with this, this with this and add together and get the y .

So, that way I get whole y n . So, that is why folding shifting product and sum. So, convolution then there is a term called circular convolution.


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Circular convolution

• Circular convolution of $x(n)$ and $h(n)$ is defined as the convolution of $h(n)$ with a periodic signal $x_p(n)$:

$$y_p[n] = x_p[n] * h[n]$$
$$y_p[n] = \sum_{n=0}^{N-1} h[n] x_p[m-n]_N$$
$$m = 0, 1, \dots, N-1$$

where

$$x_p(n) = x(n \bmod N), \quad -\infty < n < \infty$$



Circular convolution of $x[n]$ and $h[n]$ is defined as the convolution of $h[n]$ with a periodic signal $x_p[n]$ if the signal is periodic then can do a circular convolution. So, this is the mathematics I am not going details of the circular convolution.

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Correlation

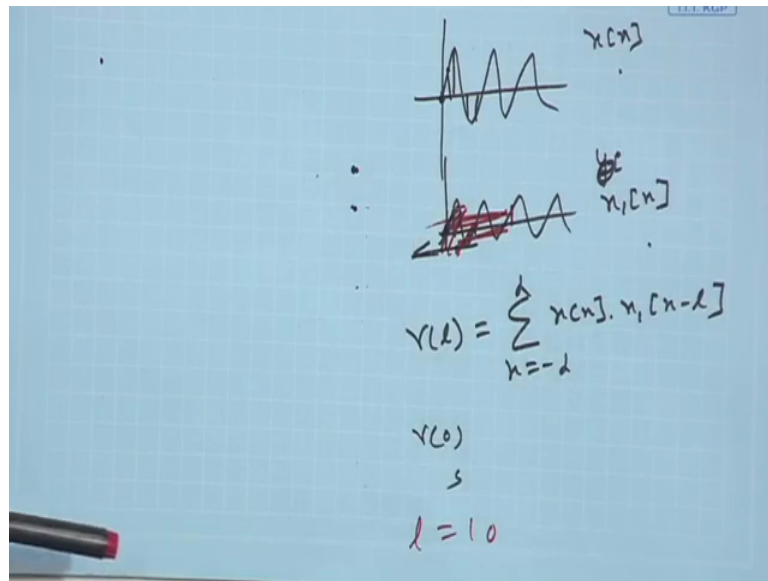
Correlation is a mathematical operation that is very similar to convolution. Just as with convolution, correlation uses two signals to produce a third signal. This third signal is called the **cross-correlation** of the two input signals.

If a signal is correlated with *itself*, the resulting signal is instead called the **autocorrelation**.



Next correlation; correlation is similar mathematics with a convolution what I want suppose I want to find out there is a signal like this.

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And there is a signal like this and to find out whether they are similar or not that is why it is correlation relation between let us this x_n and this is y_n or you can let us say this x_{n-1} . So, correlation between x_n and x_{n-1} is nothing, but a similarity between these 2 signal how they find out I just take the. So, it is a digital signal. So, I take the sample by sample I take the sample by sample. So, I can say that I can multiply the 2 sample and take then the sum product and sum is the correlation.

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Correlation

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)y(n-l) \quad l = 0, \pm 1, \pm 2, \dots$$
$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+l)y(n) \quad l = 0, \pm 1, \pm 2, \dots$$

Where, $r_{xy}(l)$ is the correlation coefficients

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So, I can say the first correlation coefficient or I can say l th correlation coefficient r l is nothing, but a product and sum n equal to minus infinity to infinity x n with x 1 n minus l or x n with n plus l x 1 n plus l. So, it does not matter whether I shifted the signal this way or this way both will give me the correlation. So, this is the l th correlation if it is 0th coefficient then I start from 0 sample to 0 sample if it is fifth sample then I shifted this signal this side fifth sample. So, start with this, this portion of the signal will be not used. So, will compare this portion signal with this whole signal understand or not then if l equal to 10, then lets this is the 10th sample. So, this portion of the signal is deducted and I compare this portion with whole signal. So, that is the correlation.

So, details that is may be a detailed discussion in signal processing books you can refer to the any signal processing book the algorithm also I have written down you can go through the slides and find out that things.

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Computation of correlation

$$r_{xy}(l) = \begin{cases} \sum_{n=l}^{M-1+l} x(n)y(n-l) & 0 \leq l \leq N-M \\ \sum_{n=l}^{N-1} x(n)y(n-l) & N-M \leq l \leq N-1 \end{cases}$$

```

FOR l=1 to lmax
{
  NL=M+1-l
  IF(NL>N-1) NL=N-1
  R(l)=0.0
  FOR(K=l TO NL)
  {
    R(l)=R(l)+X(K)*Y(K-l)
  }
}

```




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LTI Discrete-Time Systems

```
graph LR; X[x[n]] --> S[LTI System]; D[delta[n]] --> S; S --> Y[y[n]]; H[h[n]]
```

- Linearity (superposition):
$$T\{ax_1[n] + bx_2[n]\} = aT\{x_1[n]\} + bT\{x_2[n]\}$$
- Time-Invariance (shift-invariance):
$$x_1[n] = x[n - n_d] \Rightarrow y_1[n] = y[n - n_d]$$
- LTI implies discrete convolution:
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n] = h[n] * x[n]$$



Then the convolution correlation; so, then LTI discrete time system which has you have to consider because in signal processing or digital speech processing we consider the this production system is LTI system discrete time linear time invariant discrete system LTI discrete system linear time invariant linear system is linear system time invariant and a discrete system. So, linearity you know time invariant you know and LTI impulse response is direct convolution, I am not going details you know why I take that LTI system.

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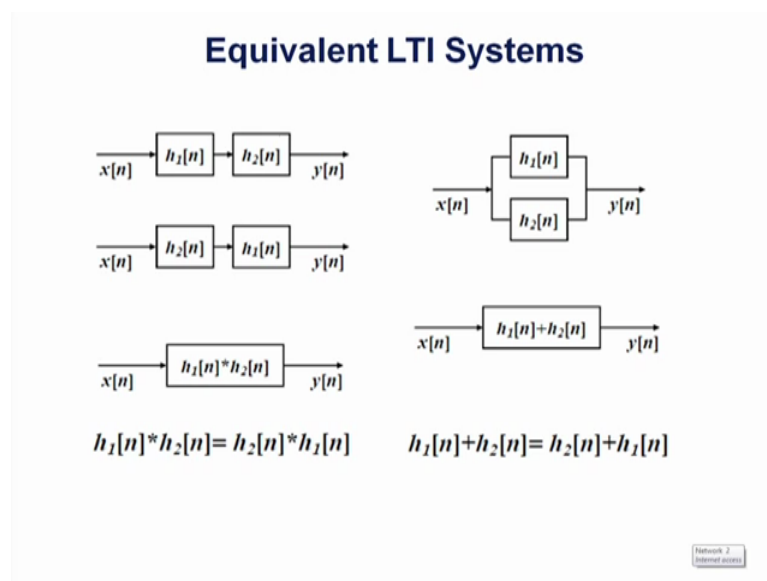
Linear Time-Invariant Systems

- Easiest to **understand**
- Easiest to **manipulate**
- Powerful processing** capabilities
- Characterized completely by their response to unit sample, $h(n)$, via **convolution relationship**
- Basis for **linear filtering**
- Used as **models for speech production**

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Because it is easy to understand, it is easy to multiply manipulate powerful processing capability characterised completely by their response to unit sample via convolution relationship basis for linear filtering used model for speech production used as a model for speech production. So, always we try to model, this one in a LTI system because it is easy to understand signal processing respect it is easy to understand it is easy to manipulate it powerful processing capability. So, that is why we always use LTI system linear time invariant system. So, you want to make this model as a LTI system linear time invariant systems.

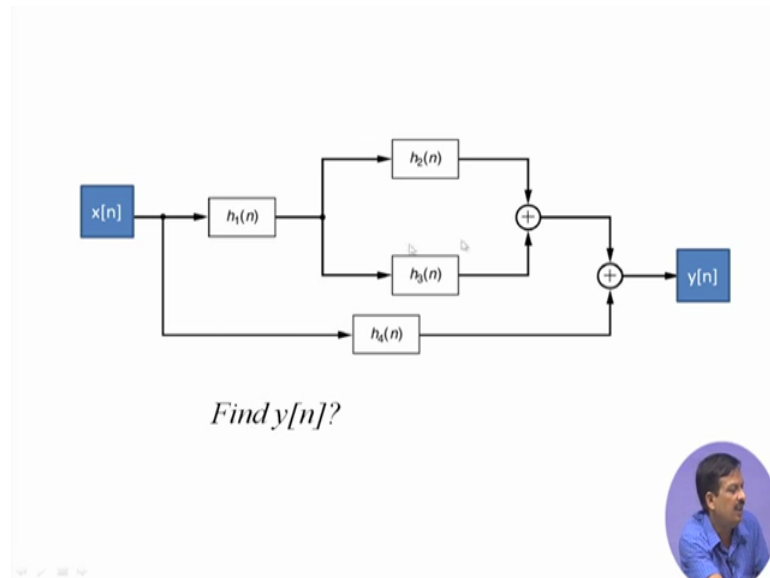
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Now, if you this is the same signal system you already you have did it if the $x[n]$ is input h_1 and h_2 are the 2 system function the transfer function and $y[n]$ is the output then it is a it is computable h_2 can be fast and h_1 can be the second also all those property you know that this is probably I can shift it here and here. So, I can plus it or I can star it.

So, this is a review of the system property.

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Then I can act you the draw the transfer function of this one. So, these 2 are parallel this is series with these 2 system and then again this; this whole transform summary will be parallel.

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Direct Form I

$$y[n] = -\sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

- Transfer function of recursive LTI system

$$v[n] = \sum_{k=0}^M b_k x[n-k]$$

$$y[n] = -\sum_{k=1}^N a_k y[n-k] + v[n]$$

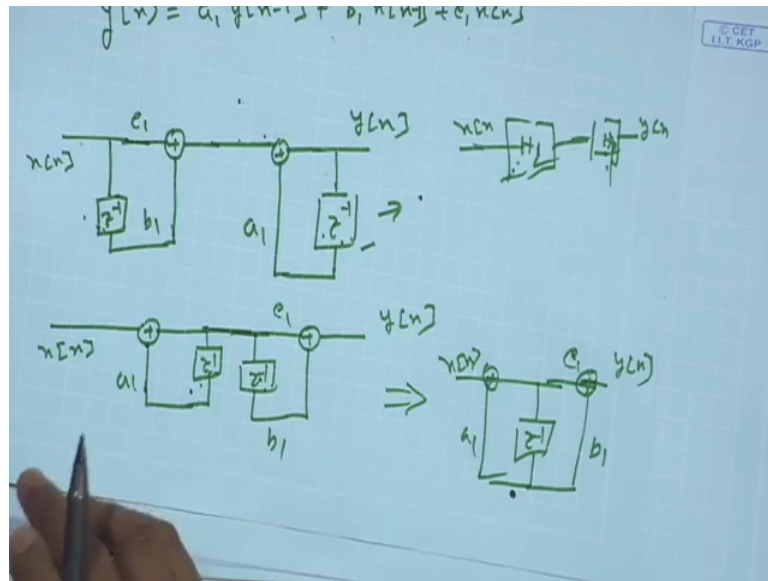
$$w[n] = -\sum_{k=1}^N a_k w[n-k] + x[n]$$

$$y[n] = \sum_{k=0}^M b_k w[n-k]$$

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You can practice it; now implementation is important. So, how do we implement a discrete system there is a 2 kinds of implementation one is called direct form one and one is called direct form 2. So, one is called structure one implementation another is called structure 2 implementation.

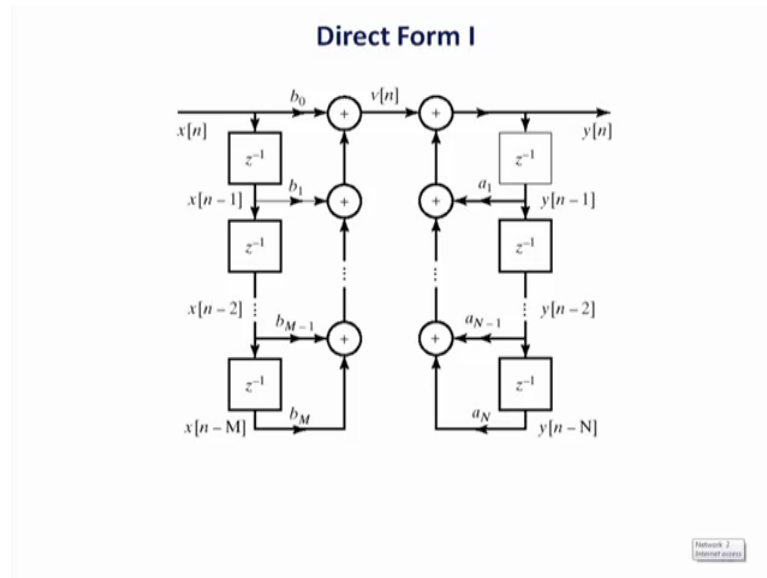
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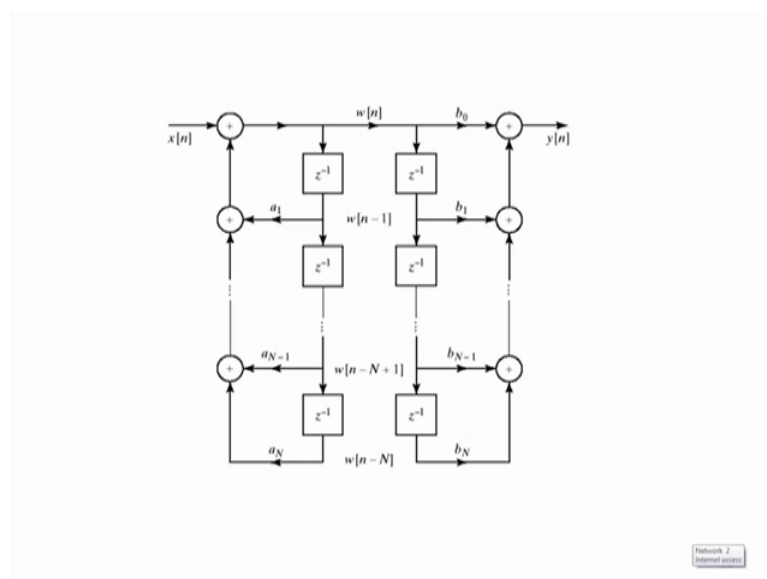
So, what I said suppose $y[n]$ is nothing, but $a_1 y[n-1] + b_1 x[n] + c_1 x[n-1]$; this is my system; it is a convenient system. So, I can write down the system; this is my $x[n]$ input and I can say that $x[n-1]$; let it is $y[n-1]$ plus lets c_1 into $x[n]$. So, I can say this is multiplied by c_1 added with a delay sample Z to the power minus one and multiplied by b_1 then output is $y[n]$ is nothing, but a delay sample which is Z to the power minus one by $n-1$ multiplied by a_1 ; this is multiplied by a_1 added with this input. So, this is the implementation of that system.

Now, I said that if a system has lets this is my input $x[n]$ and a system had H_1 then I have another system H_2 let us say this is H_2 and this is $y[n]$, then I can interchange these 2 system H_2 can be here and H_1 can be here does not effect that output. So, I can say I can change this side to this side and this side to this side. So, I invert that one. So, I can say $x[n]$ inverting it. So, I can say $x[n]$ plus this one is coming this side plus $a_1 Z$ to the power minus one, then this side coming this side Z to the power minus one plus b_1 . So, this is c_1 will be there $y[n]$. So, this is called direct form one because there is a required 2 memory 2 delay are separated input delay and output delay are separate if I implement this way then this signal is same. So, I can say instead of 2 delay I can replicated it by a single delay Z to the power minus one multiply a_1 and Z to the power minus one multiplied by b_1 and this is c_1 will be there $x[n]$ input $y[n]$ output. So, this is the advantage; this is our direct form one; this is called direct form 2 implementation. So, DSP book it is there.

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This is I said that I long this is one block and this is block once to interchange it will come same side. So, instead of 2 Z to the power delay I can club together and put the single delay. So, it is called delay from 2 implementation.

We used this thing when we implement this is our vocal systems using discrete time system. So, next class I will just complete this DSP things.

Thank you.