Digital Speech Processing
Prof. S. K. Das Mandal
Centre for Educational Technology
Indian Institute of Technology, Kharagpur

Lecture – 29
Lattice Formulations Of Linear Prediction

So, we have already discuss about the STFT analysis ok.

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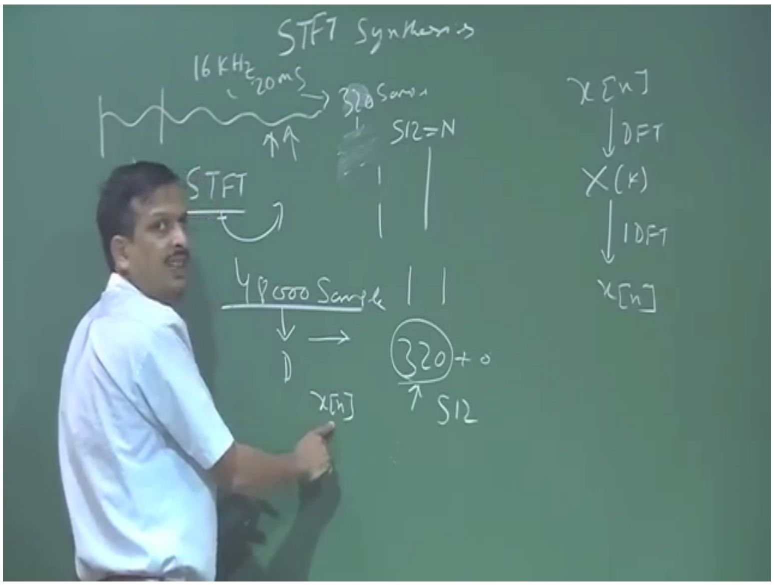

So, STFT now we discuss about the STFT synthesis. So, in STFT analysis what we have done that we have a, I have long speech signal we have taken a one segment of the speech signal and for that segment we have done that STFT of that speech segment of speech signal. Now if you see that in normal Fourier transform if xl is my signal, if I apply discrete Fourier transform on xn I will get xk once I apply the IDFT in here I should get back xn. So, this is the normal Fourier transform property ok.

Now STFT also follow the normal Fourier transform property. So, I can say I have done STFT can I get back the signal again, is it possible. So, what we want to hear the suppose something analysis I want to do in you can say the frequency domain and then again I want to get back the time domain signal. So, what I want suppose I have a speech signal and I want some processing in frequency domain some enhancement let some or if the enhance some frequency band of this signal, and then again I want to get back the speech signal. Since we have not taken the whole speech signal at a time we have taken part of the speech signal, then what we have done we have taken a that if you if you remember the example suppose I have a 16 kilohertz sample signal, I have recorded 3 second of my speech then I have 4800 sample in 3 second. So, I am not taken all sample at time and done that STFT I have done the Fourier transform then it is very easy, I take whole signal at a time and done the Fourier transform and get back the signal. Since the speech is non stationary signal this is true for stationary signal. Speech is a non stationary signal; that means, the along the time speech signal changes its property ok.

So, what we do we instead of analyze whole signal at a time, we take a small segment. So, if I take a 20 millisecond window then I get 160 sample sorry 320 sample. So, if I do that STFT analysis using Fourier transform. So, nearest 2 to the power something which is 256 no 512 point DFT I have applied n equal to 512 point DFT. So, I have (Refer Time: 3:37) up a 0, 320 sample plus some 0 to get 512 sample. So, once I done the reverse transform I will get a signal which is not the exactly 320 sample, it is something 212 sample, and also since it is not the you can say I cannot get back the x ns itself because that I have multiplied that signal with a window function. So, it is a multiplication of window function and the signal. So, my intension is that if I want to do STFT synthesis, I have to get back the xn exactly. So, in mathematics how do we get back, let us do the STFT synthesis in mathematics. So, what I have done for a given value of I have done STFT for time instant n this is my STFT output signal ok.

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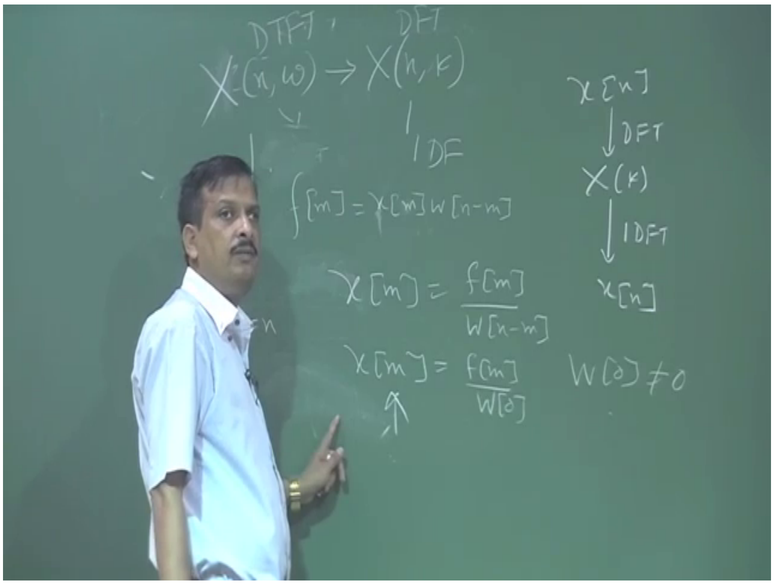

So, is has the property of Fourier transform. So, inverse transform is possible. So, if I this I have get frequency transform of what, signal plus window see if I take the IDFT in here or so this is DTFT IDFT in here in digital domain or if it is in DTFT, DTFT discrete time Fourier transform, if it is in DFT then it is n k this is DFT. So, if I take the IDFT I will get back xn multiply by omega n minus m because I have taken the frequency transform of this signal only xn multiply by the omega n minus 1.

So, I can say what I obtained fm which is nothing, but a xn or you can say if it is I write n then it is m this is n minus m. So, I have get back the signal which is multiplication of x m and omega n minus m not xn directly. So, if I want to get back the xn. So, xn, small xn is nothing, but a fm whatever I get back divided by omega n minus m, now if I evaluate fm or for every m equal to n. So, if m equal to n then x of m is nothing, but a fm divided by w 0. So, if w 0 is not 0 then only I can say I can get back the signal. So, if w 0 is non 0 then only I get back the signal ok.

So, which is already explained if you see here this slide so I have taken a this is my f n m is my frame signal.

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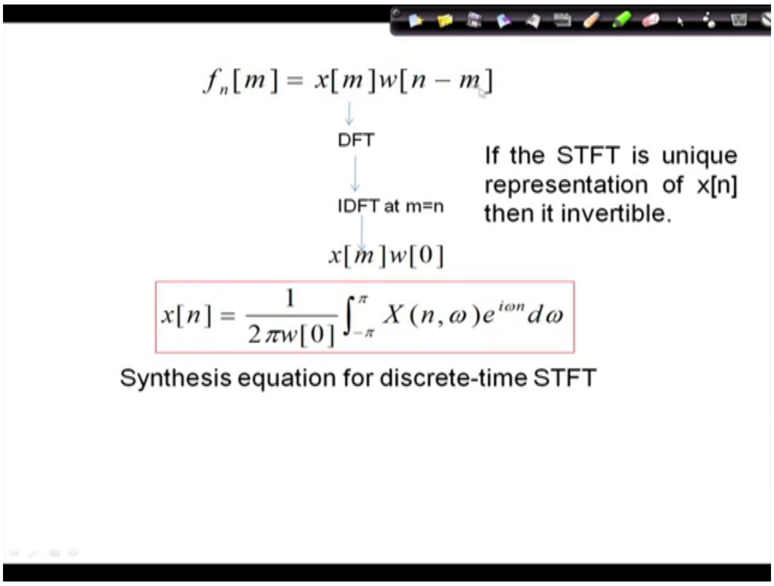

Which is nothing, but a xm multiply by the window function I take the DFT and then I take the IDFT and evaluated at m equal n I get x m multiply by omega 0. So, xn is nothing, but a inverse Fourier transform of frequency domain representation of STFT with 1 by 2 pi into w 0. So, this is a synthesis equation of STFT analysis, now what is the problem in here so for every m is equal to n I have to evaluate it. So, suppose so there is a 2 case is there suppose I have a signal.

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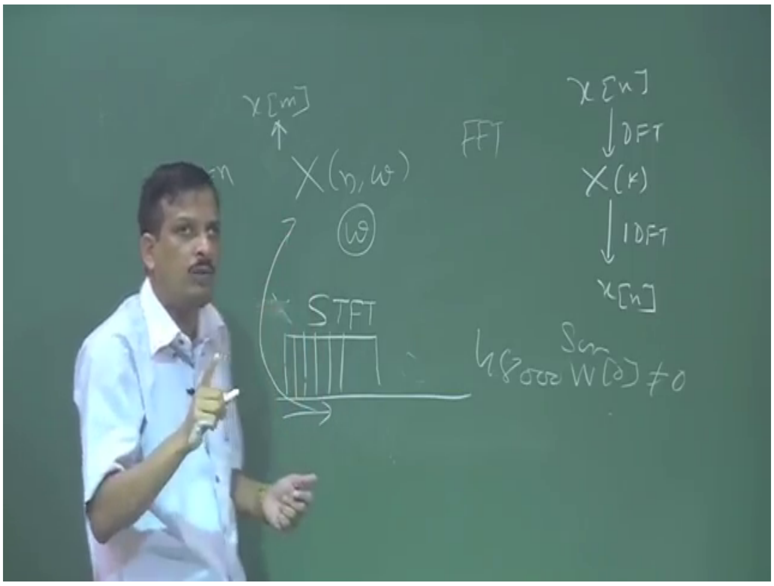

So, I can say the xm is recovery is possible a 2 condition, one is that I have to know sample by sample recovery process at for every m equal to n I have to do the recovery process. So, for every sample by sample recovery process I have to done and I have to know x of n omega for every omega value. So, I can say that complete recovery is possible if sample by sample recovery process is done and x of n omega must be known for all omega.

So, suppose I have a long signal and I do STFT, STFT then what is required that for every sample so I have taken a some portion of the signal and for every sample I have to recover it. So, suppose I have a 408 4800 sample point then synthesis is possible if I shifted the window analysis window for every sample which is computationally not feasible because FT FFT calculation is a time consuming matter lot of complexity is there. So, if I want to sample by sample recovery that is not a possible solution, second one is that I have to know x n omega for very omega. So, what kind of things I should allow or what limitation is what kind of shifting is allowed for that yes it is complete recovery is also possible and I have not process sample by sample. So, there is a 2 things we have to analyze. So, let us case one what is the case one lets case one is that.

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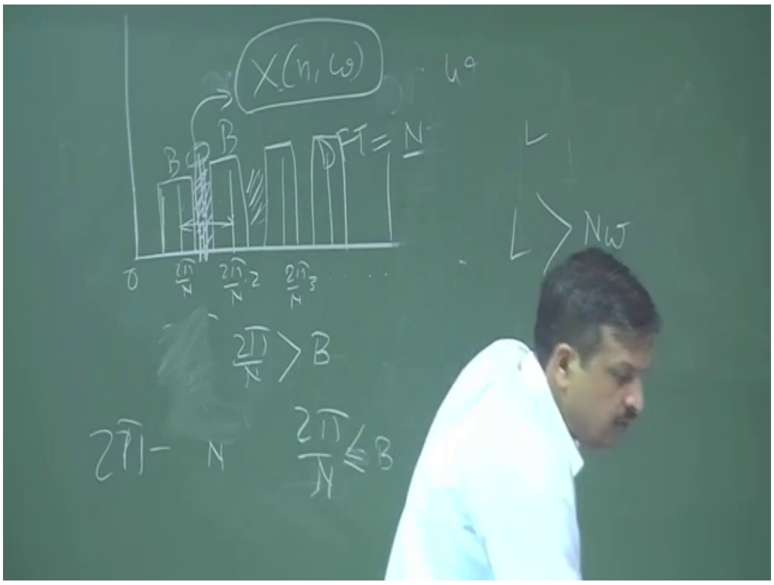

Suppose I have a signal I have done that STFT of a signal of nth window which is nothing, but a xn omega. So, let us I have taken the DFT length of the DFT n is equal to their length of DFT, DFT length is equal to N. So, what we have done that whole frequency scale 0 to 2 pi is the highest frequency I have divided 2 pi by N N resolution.

So, I can say whole frequency scale can be think about a some filter this is 0, this is 2 pi by N, this is 2 pi by N N into 2 this is 2 pi by N N into 3. So, that way now if I say this is the number of DFT, DFT length is equal to N then I can say whole frequency scale is divided into N number of channel or N number of filter. N number of filter or N number of channel now I have when I do the STFT I multiply the signal with a window function.

So, let us the bandwidth of the window is B bandwidth of the window means I take the frequency response of the window and find out the bandwidth of the widow isB. Now in STFT analysis what is happened the frequency response of the window function is convolve with the original frequency response of the signal. So, I can say the frequency response of the window function if the bandwidth is b. So, there will be a b at every 2 pi by N modulation as we already explains in DFT filter view, now if I say if this is my bandwidth this is my bandwidth. So, this distance between this these 2 is pi 2 pi by N if the bandwidth is less than the 2 pi by N. So, if the 2 pi by N is greater than B so; that means, that there is a gap between the 2 filter this is the gap if there is a gap. So, this portion of frequency is not pass through the filter. So, if it is this portion is not pass through the filter, then I cannot say the X of n omega is known for all omega.

So, since there is a spectral loss. So, complete recovery of x m is not possible, due to the spectral loss. So, what will be the optimum condition at least 2 by N must be equal to the bandwidth 2 pi by N must be equal to the bandwidth or I can say B must be greater than equal to 2 pi by N if it is there then I can say none of the omega is left out all omega is pass. So, if it is B equal to 2 pi by N then the picture will be what will be the pictures. So, pictures will be this is my 2 pi. So, this is b again there will be a b. So, this is b again there will be a 2 pi there will be a b in here. So, I can say if the bandwidth is b then all frequency. So, this gap is not there that is why I know X n of omega for all omega. So, that is the one limitation, second limitation is that I have said the sample by sample recovery. So, I have shifted the window one sample which may not be necessary only thing is that that consider xn band decimation factor is L. So, STFT is applied for every L sample. So, x of w of n is non zero over the n w n w is the length of the window then if L is greater than N w. So, there are L is greater than n w what is the case let consider this slides.

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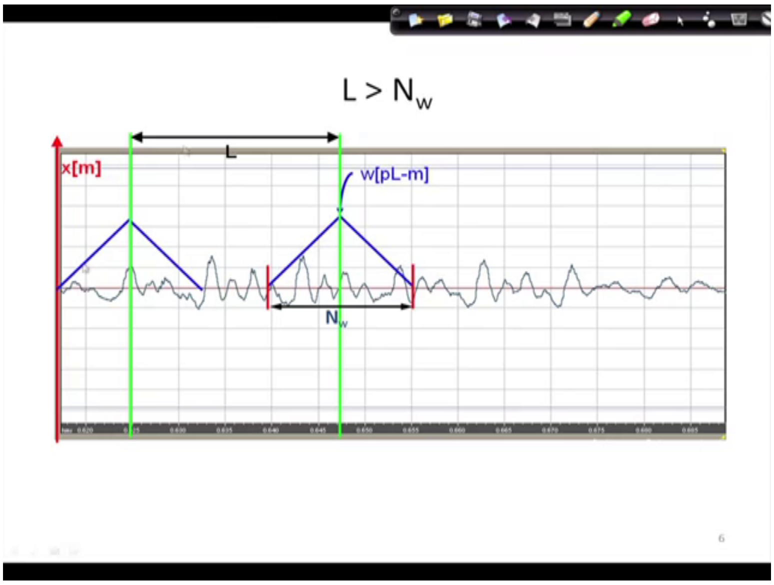

If you consider this slides. So, let us this is a triangular window triangle. So, triangular window length of the I just explain in board also because of the slides is. So, suppose I have a signal. So, I said STFT completely recovery if I shifted the signal by one sample ok.

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So, for every one sample it is, now I said lets shifted the window this decimation factor is L. So, let us this is my window. So, the length of the window here to here length of the window is N w. If my decimation factor means shifting of the window is greater than the Nw L is greater than the N w. So, next window may be come in here. So, I have shifted the window in here. So, next window come in here. So, what is happened for this portion of signal I do not know, I do not know this portion of the signal. So, complete recovery is not possible. So, the it is complete recovery is possible at least L should be less than N w ok.

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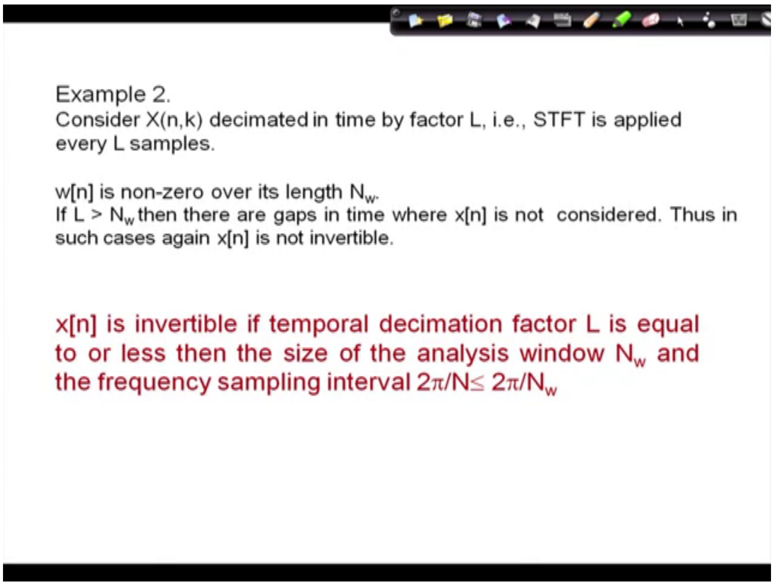

So, if you see in the red colour I have written x n is invertible if the temporal decimation factor L is equal to or less than the size of the analysis window. So, it should be at least L should be at least equal to, L should be Nw analyze window length or L must be less than to Nw this is one condition, second condition is that 2 pi by N must be either equal to B or greater than equal to, B must be greater than the 2 pi by n. So, number of channel if it is n so distance between the 2 channel is 2 pi by N this must be less than the bandwidth of the analysis window. So, there is a 2 condition is there STFT is completely invertible if L is less than equal to Nw.

So, length or equation decimation of time or shifting of the window is less than the length of the analysis window and the number of channel if it is n then the distance between the 2 channel which is 2 pi by N must be less than equal to the bandwidth of the analysis window. So, this is the 2 condition is there using these this 2 condition is satisfy, then it is completely invertible process. So, STFT synthesis is possible only for these 2 cases and there is a 2 procedure for STFT synthesis; one is call filter bank, summation method another is call overlap add method.

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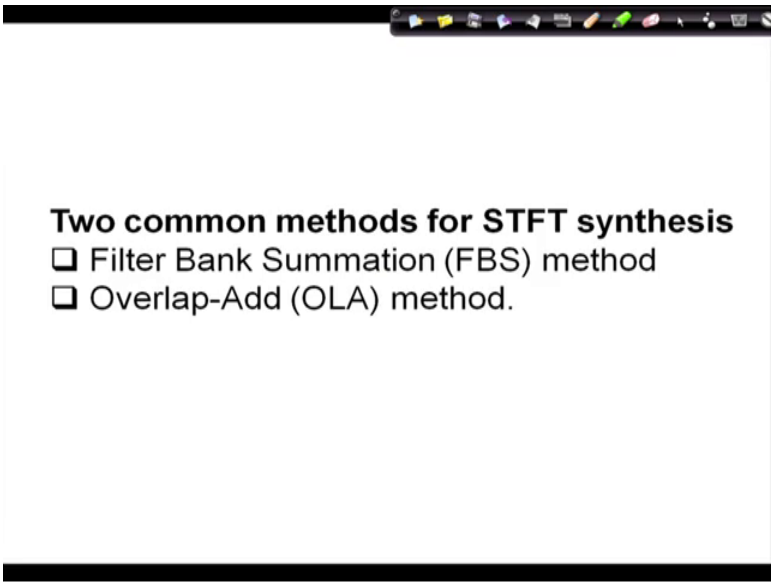

So, in this you can say in this lectures or their of this course I will discuss about the 2 methods one is call filter bank summation method another is call overlap add method. So, let us go one by one.

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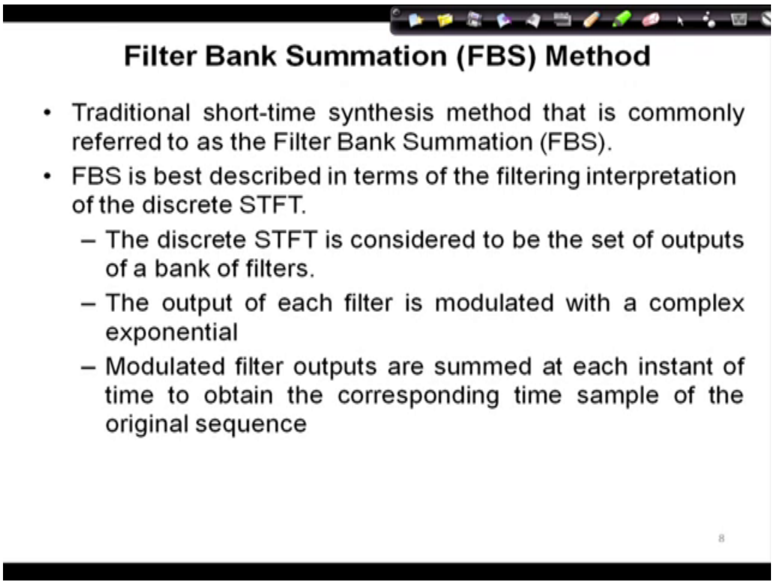

So, if you see filter bank summation method I have not. So, traditional short time synthesis method is a commonly referred as a filter bank summation method. So, FBS is best described in term of filtering interpretation of the discrete STFT. So, if you remember that STFT analysis block diagram.

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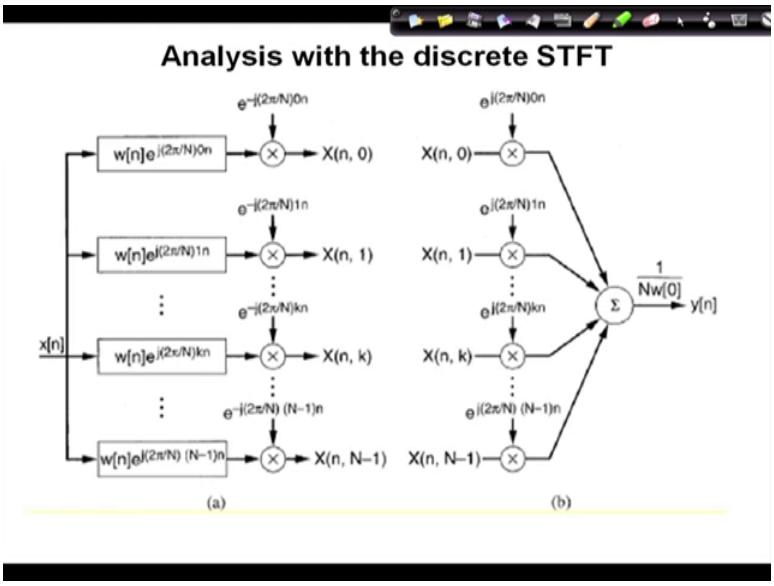

What we done we pass the signal through the window then we modulate the signal or you can say demodulate the signal or shifted the signal e to the power minus j 2 pi by N by 0. So, that is this is the synthesis equation this is the synthesis block portion, now if I want to analyze this thing. So, if I done demodulation I have to done again for every xn omega 0 I have to modulate it and take it sum and I get that the one by n into omega 0 w 0 I get back the signal yn.

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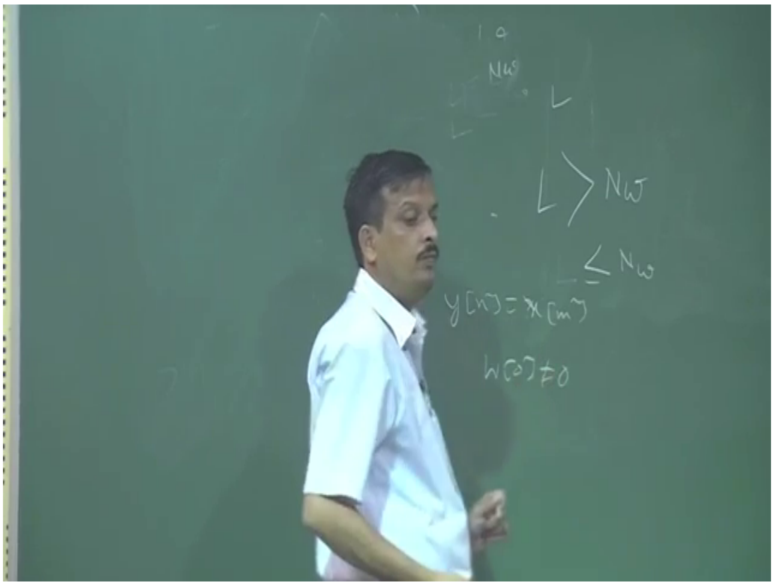

So, yn is equal to xn, if w 0, w 0 not equal to 0. So, that is the synthesis equation. So, this is FBS method STFT synthesis, let us go to the details maths what is there. So, in FBS method if I want to lets this is my frequency domain representation of nth window is X of nk. X of nk is my frequency domain representation of nth window, now if I take the IDFT I get the time domain signal. So, let us time domain signal is yn, then yn is equal to inverse Fourier transform of xnk so I can say 1 by N or I can say yn. So, in inverse Fourier transform what should I get instead of yn I can write yn, yn w, n minus m I will written back. So, instead of that I can say let us it is written that yn I have written and here I write w 0, that completely recover if the w 0 is not 0. So, let us I said w 0 I have write here. So,1 by N, N is the normalization factor into w 0 inverse Fourier transform if the inverse DFT k equal to 0 to N minus 1 length of the DFT X of nk into e to the power j 2 pi by N into nk inverse IDFT. So, this is the IDFT IDFT ok.

So, what is x nk? Xnk is nothing, but a Fourier transform which is n equal to minus infinity to infinity x of n w n minus m e to the power minus j 2 pi by N into nk or kn. So, if I replace this thing in here as it here I can replace it.

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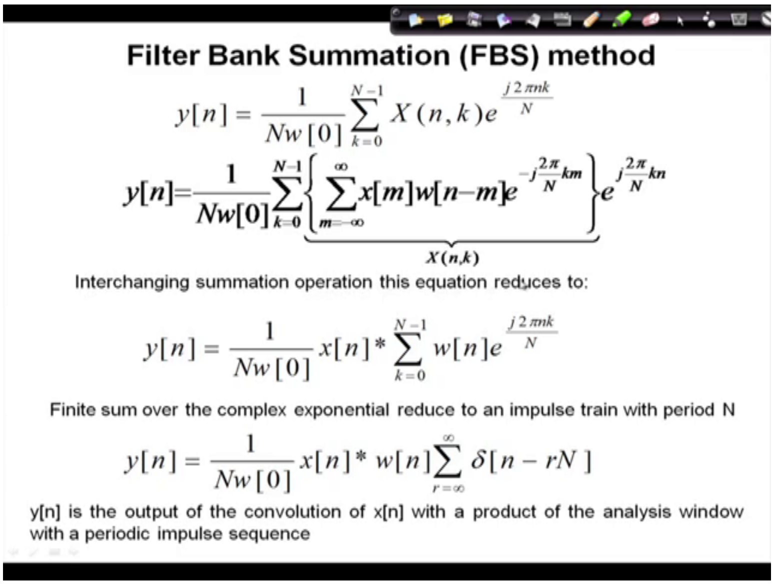

Now, if I interchange the summation operation this equation this equation reduce to one by nw 0 x of n convolution with k equal to 0 to n minus 1 w n e to the power j 2 pi by nk. So, I can write after putting value I can write w yn is equal to 1 by N w 0 ok.

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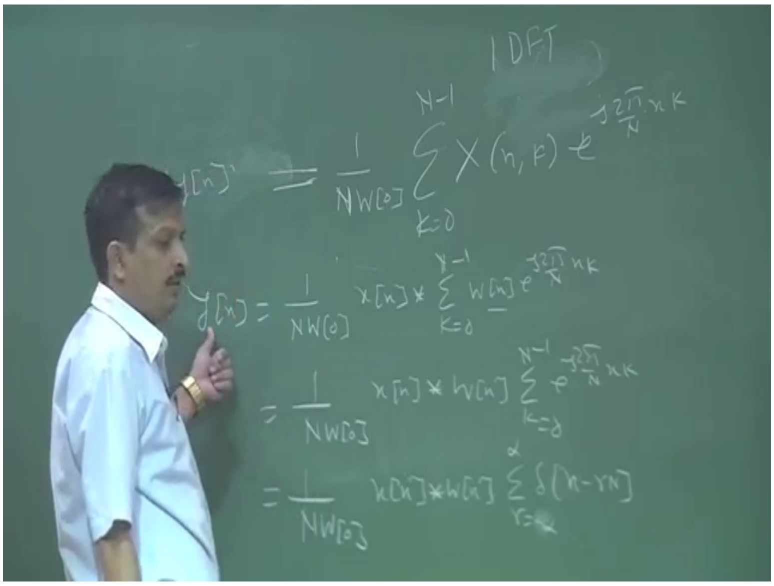

W 0 x of n convolved with k equal to 0 to N minus 1 w n e to the power j 2 pi by N into nk, if it is that then I can say what is this I can change this w n I can get back in here. So, I can say 1 by n w 0 into x of n convolved this w n outside and put that k equal to 0 to n minus 1 e to the power j 2 pi by N nk. So, what is this finite sum over the complex exponential finite sum over the complex exponential so that reduce to is nothing, but a impulse strength with period n. So, it is nothing, but a impulse strength with period n then I can write 1 by N into w 0 is equal to xn convolved with a w n into I can say let us r equal to infinity, minus infinity to infinity infinite sum del function of r n minus n minus r into n period is n r multiply by n. So, r equal to infinite sum or period n impulse strength with period n. So, this is the impulse strength with a period n. So, I can say yn is the output of the convolution xn this is the constant. So, I can say yn is the output of the convolution of xn with a product of analysis window and impulse strength.

So, ultimately yn is nothing, but a xn convolved with window function product of window function and impulse strength. So, what is the meaning product of window function and impulse strength ok.

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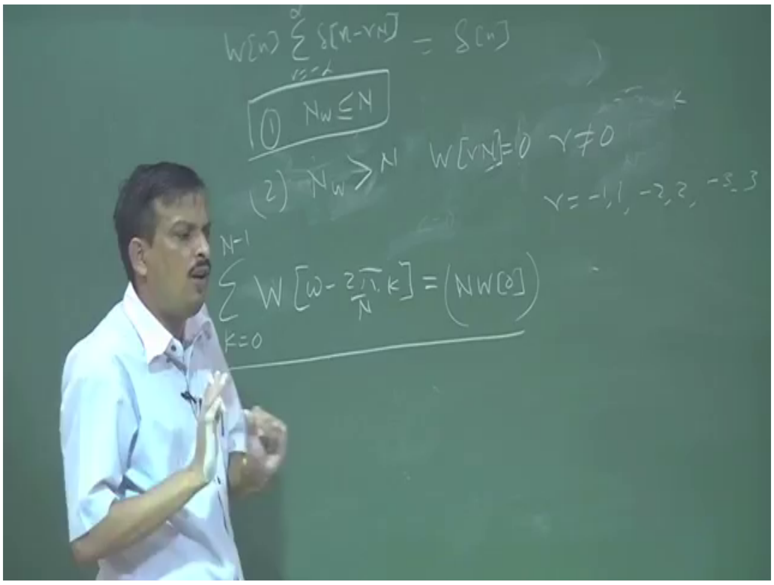

So, I can say w n into r equal to minus infinity to infinity del of n minus r n this is equal to delta n if this is equal to delta n, if window length N w is less than equal to the analysis length and this is true because if I want y n should be xn then this must be w 0. So, w 0 means delta n so this if it is w 0 then w 0, w 0 will be cancel then I can say yn equal to xn or I can say this I, if I say this is this is equal to w n into w 0, w 0 then I can say this replace by w n, w 0, w w o cancel then I get it or not. So, I can say this function, this w n product of w window analysis window and the impulse strength must be delta n if N w is less than equal to N w is less than equal to the analysis the length of the DFT. So, what is the physically meaning so wn is 1 ok.

So, this must be less than the analysis window because it is product of r into N. So, in that case this is the single case second case if N w is greater than equal to N or is greater than greater than n. So, another thing is N w is greater than N, then also it is possible, but if w r N is equal to 0 or r not equal to 0; that means, w r N equal to 0 for all r except 0 which is r equal to minus 1 1 because its minus infinity to infinity, minus 1 1, minus 2 2, minus 3 3. So, number one it is step follower delta N if it is N w is less than the N. Second one if every N w is greater than n then I have to ensure that w r into n, n is the length of the analysis window must be 0 for all r except r equal to 0. So, if I take this next instead of time domain if I take the so this is the constant of FBS analysis. So, if I take the frequency analysis of this then what I will get k equal to 0 to N minus 1, which is nothing, but a w. So, this is W capital W omega minus 2 pi by N into k is equal to delta n nothing, but a N into w 0 or not ok.

So, this if it is this then satisfy this condition then if I see this what it state that this is nothing, but a constant w 0 is not 0 summation of all frequency component if I take the expression state that frequency response of the analysis filter should sum to a constant across the entire bandwidth. So, this state that the frequency response of the analysis filter should sum to a constant across the entire bandwidth, this is constant is equal to constant. So, the summation of all frequency response of analysis filter must be a constant. So, next class I will describe the pictographically this one ok.

Thank you.