Digital Speech Processing

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Lecture - 24
Lattice Formulations of Linear Prediction

So, in last class, we have talk about that covariance method.

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For linear predictions and there first will explain that auto-correlation methods for linear predictions. So, now we are discussing about the lattices formulation of linear predictions. So, if you see in whether it is auto-correlation or covariance, we try to solve that matrix phi into alpha or I can say phi i k into alpha is equal to phi 0 k. We try to solve that matrix.

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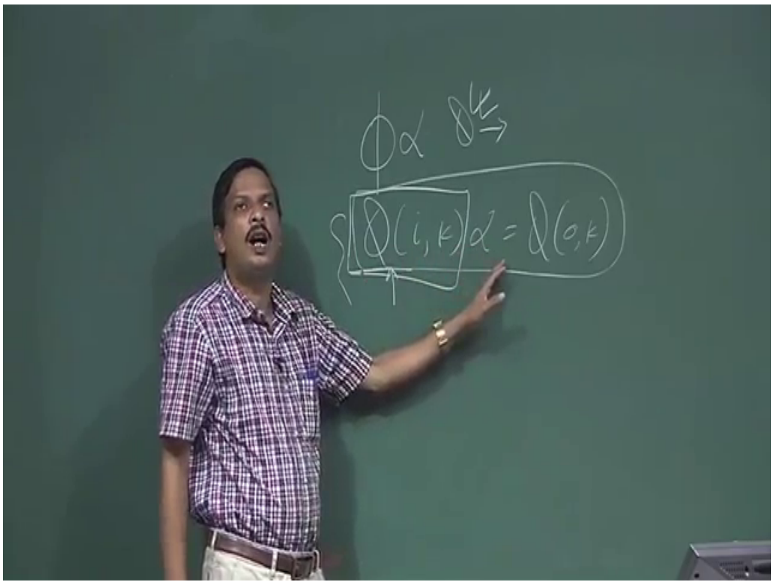

So, how do you solve that? We have calculated that phi. Phi is nothing but a correlation value in case of convolving. In case of covariance matrix, we calculate the correlation value of phi and then, we try to solve this linear equation. So, whether it is your auto-correlation methods or coral covariance methods, then a two step. First step is to compute this phi using correlation for covariance method using auto-correlation for auto-correlation methods. Next try to solve this peak linear equation p number of linear equation to find out the value of alpha.

So, I can say auto correlation methods and covariance methods both are two step solutions.

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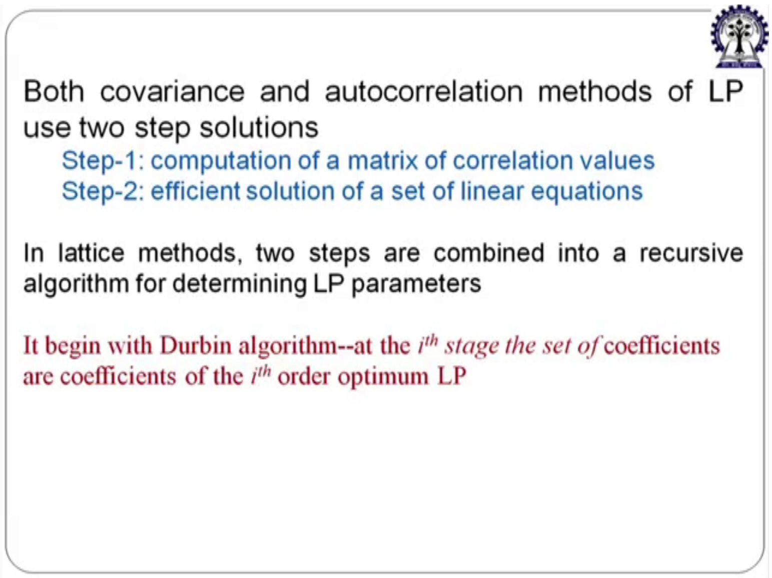

Step 1, computation of matrix of correlation value and then, efficient solution of set of linear equation. Now, this lattices method says that I can combine these two steps that says that I do not want to use that auto correlation, compute correlation and solve the linear equation in two separate steps. So, I want that. Can I combine these two steps? Yes, it is possible in case of lattices formulation combining these two steps and use a simple step. How it is possible?

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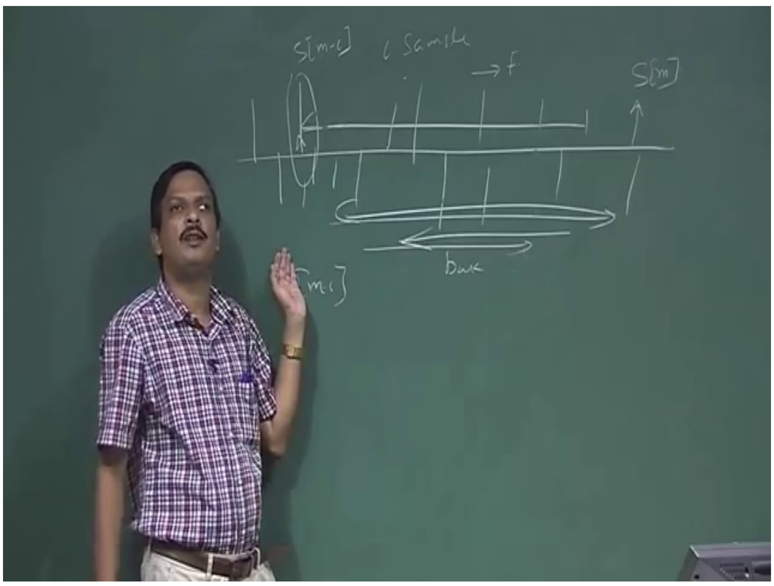

Now, first I have a speech signal m number of that is a speech signal that is a sample, lot of sample is there. So, there lot of sample are there in a equation. So, this is A S m and this; this sample is let S m minus i. So, i can say using this previous i sample, I want to predict this S m sample. So, using this previous i sample, I want to predict this S m sample. So, if I do that, then it is called forward prediction. I can say it is a forward prediction.

Similarly, I can predict less this S m minus 1 sample this sample based on that this previous sample. So, I can predict S m based on the i number of previous sample this side or I can predict this sample based on m number of sample i number of sample this side. So, either I can predict S m cross from S m minus i sample from this side or I can predict S m minus i from i sample in this side. If I predict this one, then the prediction is this direction. This is called backward prediction.

So, there is a forward prediction; there is a backward prediction. I can do both ways. So, next how we can do that I am not explaining the slide again. So, what is that linear prediction equation? If the system is linearly predictable, then equation of the system that H z.

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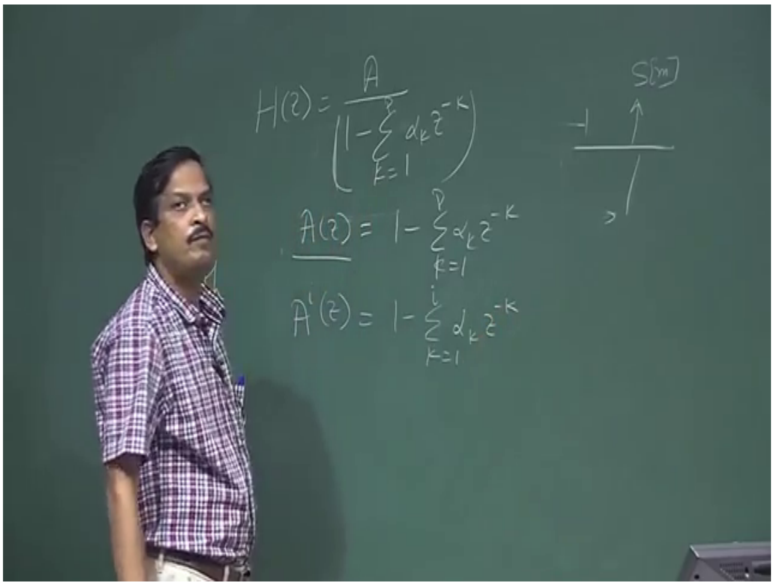

Transformation of the system is gain is A1 minus K equal to 1 to P A K or alpha K i can say z to the power minus K. I can say A K or alpha K whatever now if I say that what is the predic. So, this one is the error filter. So, this is A z. So, I can say A z let us this one is

A z. A z is 1 minus K equal to 1 to P alpha K z to the power minus K. This A z is called error filter. A z is the error filter. I want to use i th order predictor or say the ith order predictor, then I can say A i z is nothing but A1 minus K equal to 1 to i. It is a pth order predictor. If I say it is ith order predictor, it is nothing but A alpha K z to the power minus K, ok.

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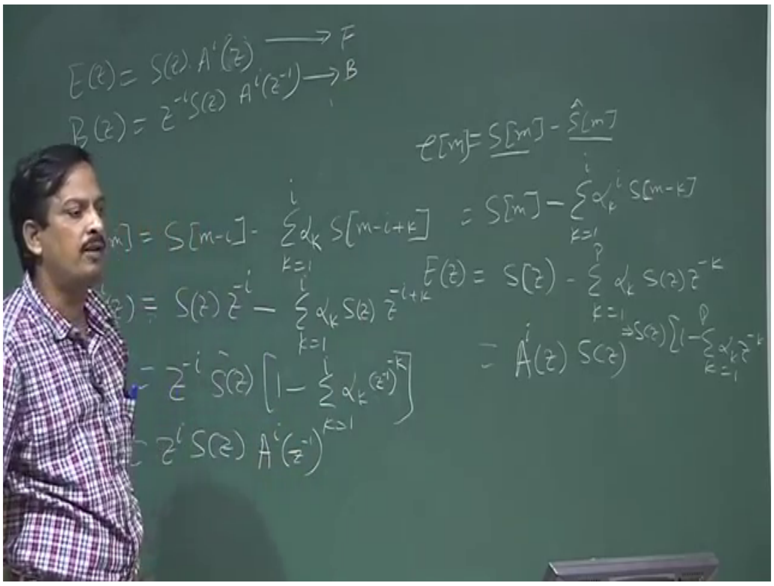

So, what is forward prediction? Forward prediction is says that A s m or I can say that if I have predicted error forward prediction error E m is nothing but a original signal. Original sample minus estimated sample is the prediction error. So, if this is my original sample and this is my predicted sample, so this is nothing but s m minus what is the predictor sample. This 1 k equal to 1 to i alpha k i s of m minus in z domain. I can say E z is nothing but s z. S z minus k equal to 1 to P alpha k s z into z to the power minus K or I can say this is nothing but A z into A z into s z or I can say i z into s z. If I take s z, so this is if I the take the s z, then s z into 1 minus k equal to 1 to p alpha K x to the power minus K, ok.

So, I can say forward prediction error E z is nothing but s z into A i z. Now, I want to say the backward prediction error. So, I want to estimate s m minus i from previous i sample. So, I can say s m minus i is the original sample minus estimated sample k equal to 1 to i alpha k s of m minus i plus k. I want to predict.

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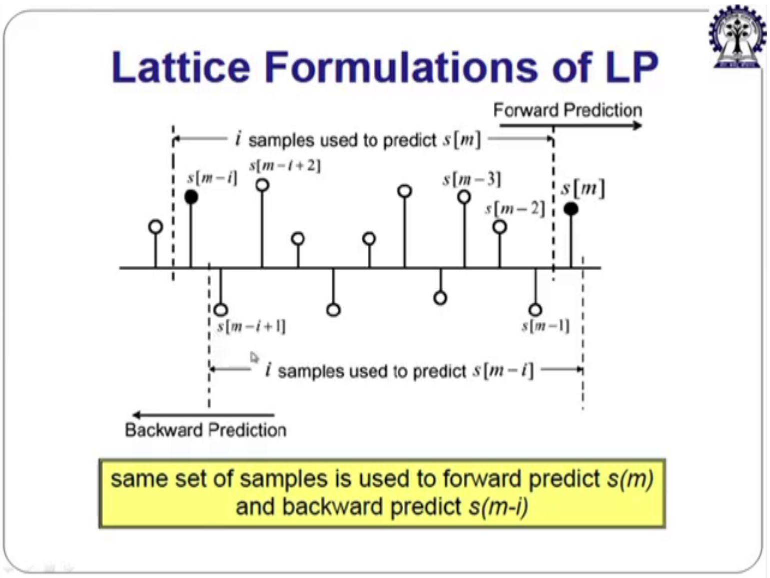

If I see this, I want to predict this s m minus ith sample from the previous sample. So, if it is that, then I can say this is nothing but bi or bm, sorry bm. So, bm that original sample minus estimated sample value is the prediction error. So, I can say b if it is ith order predictor, so I can say b iz is equal to i, take the z domain s z into z to the power minus i minus k equal to 1 to i alpha k sz into z to the power minus i plus k z domain.

So, I can say it is nothing but a z to the power minus i if I take and sz if I take out there, it is 1 minus k equal to 1 to i alpha k z to the power k. If I write z to the power k is nothing but a z to the power minus if I say z to the power k is positive. So, I can say z to the power k is nothing but a z to the power minus 1 to minus k. So, in that case I can say this is nothing but a z to the power i s z into b i z to the power minus 1 or I can say it is nothing but i, sorry ai z is nothing but z. So, it is 1 minus k equal 1 to i alpha k z to the power minus k. So, here instead of z to the power minus k a minus ki z to the power minus 1 inside, I can say it is nothing but a i z to the power minus 1. So, I can say bz is nothing but a z to the power minus i s z into a i z to the power minus 1.

So, this is forward prediction error and this is backward prediction error. If it is that, then forward prediction error, backward prediction error, you now think about the levinson recursion. What are the levinson recursion said in, levinson recursion I am not explaining in the board. It is written in the slides if you see.

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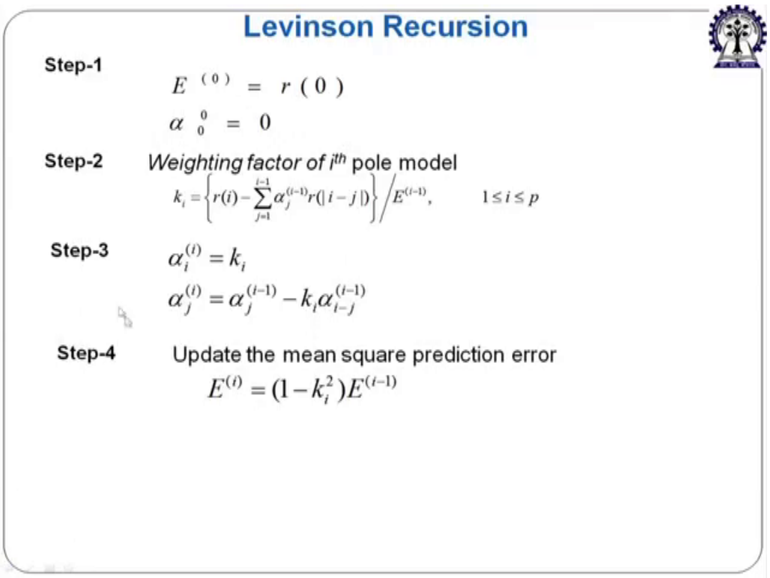

Step 1, E to the power zero order predictor is nothing but a r 0. So, 0 0 alpha 0 0 is equal to 0, then you know that there are three iteration and one is weight factor ith pole model ki, then alpha i ki is alpha i i equal to ki, then alpha ji is equal to iteration and then, update the mean square error.

So, those are the fourth prediction is used in levinson recursion. So, I can say that let levinson recursion take this 1 alpha i i is equal to ki.

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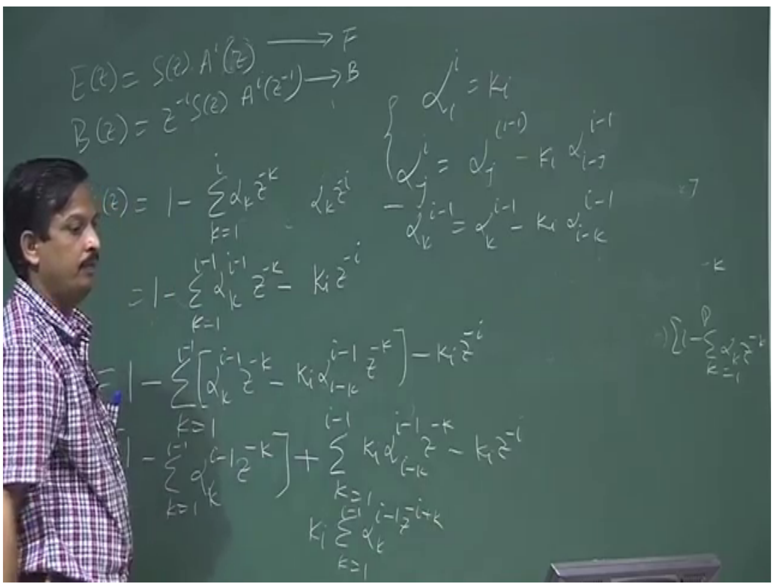

Similarly, alpha j i is equal to alpha j i minus 1. Levinson recursion said that current state can be estimated from the previous iteration. So, it is previous iteration alpha value minus ki into alpha i minus j i alpha i to the power minus 1, ok.

So, that is levinson recursion. Now, think about in here what A i z is. A iz is nothing but 1 minus k equal to 1 to i alpha k z to the power minus k. I can write this one as A1 minus k equal to 1 to i alpha k alpha k i minus 1 previous iteration alpha value of previous iteration z to the power minus k minus z to the power minus k minus k i into z to the power minus i. This is i minus 1. So, I can say k equal to 1 to i, I can say k equal to 1 to i minus 1 alpha k i minus 1 predictority. So, it is not i th i minus 1th iteration z to the power minus k minus ki into z to the power minus i. So, z if I do it, the sum k equal to 1 to i, then this will be sum together. So, it will be same as this one, ok.

So, I just from the summation term only 1 alpha k z to the power minus i term ith term, I have taken out. So, this is nothing but A1 minus k equal to 1 to i minus 1 alpha k i minus 1 z to the power minus k minus k i alpha i minus k i minus 1 z to the power minus k. I can write this as 1 minus ki z to the power minus i. Why I write this one? I write alpha k i minus 1 from here alpha k i minus 1, I can write in term of alpha k i minus 1 minus ki into k i minus j. Sorry, this is alpha k i minus k into i minus 1 z to the power minus k. So, alpha k instead of here ji, I put instead of jk. So, alpha k i minus 1 equal to alpha k i minus 1 minus ki into alpha k i minus k i minus 1. That is why I write i minus ki minus 1 into z to the power minus k, sorry z to the power minus k minus k is here.

So, it is now if I do that 1 minus I can put that value 1 minus k equal to 1 to i minus 1 alpha k i minus 1 z to the power minus k. This is one term. I can write another term is if I take that this sign, this is minus minus 1 is a plus. So, I can say plus k equal to 1 to i minus 1 ki, ki alpha i minus k i minus 1 z to the power minus k minus ki z to the power minus i. If I see this term 1 minus k equal to 1 to i minus 1 alpha k i minus 1 z to the power minus k, it is nothing but A i minus 1 or naught A i minus 1 and then, if I push same thing in here also, if I say this is one term. Second term is this one. So, if the second term if I say ki ki and ki. So, I can say ki is here, then I can say k equal to 1 to i minus 1 alpha k i minus 1 z to the power minus i plus k or I will do it here.

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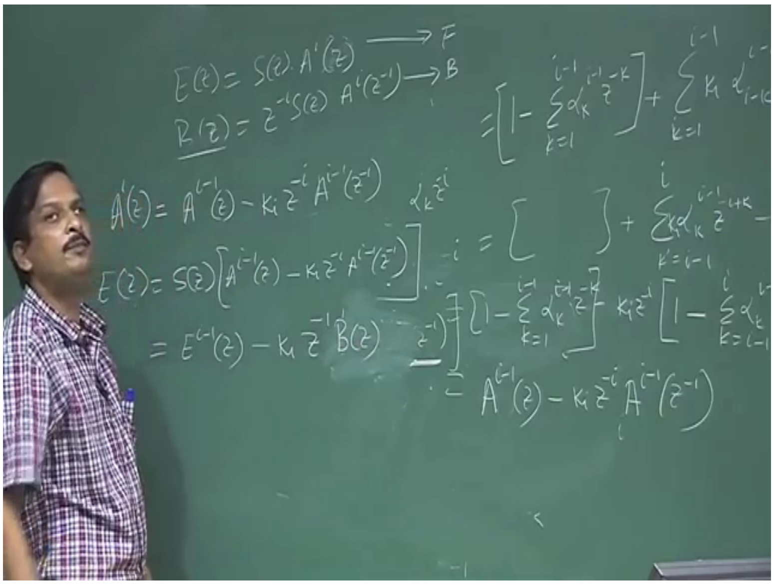

So, I can write from here that is equal to 1 minus k equal to 1 to i minus 1 alpha k i minus 1 z to the power minus k. This is one equation. This part, this part and another equation is minus minus plus plus ki ki. I put the sum also here. So, summation of i is equal to k equal to 1 to i minus 1. This sum minus minus plus ki alpha i minus k i minus 1 z to the power minus k minus ki z to the power minus i ki z to the power minus i.

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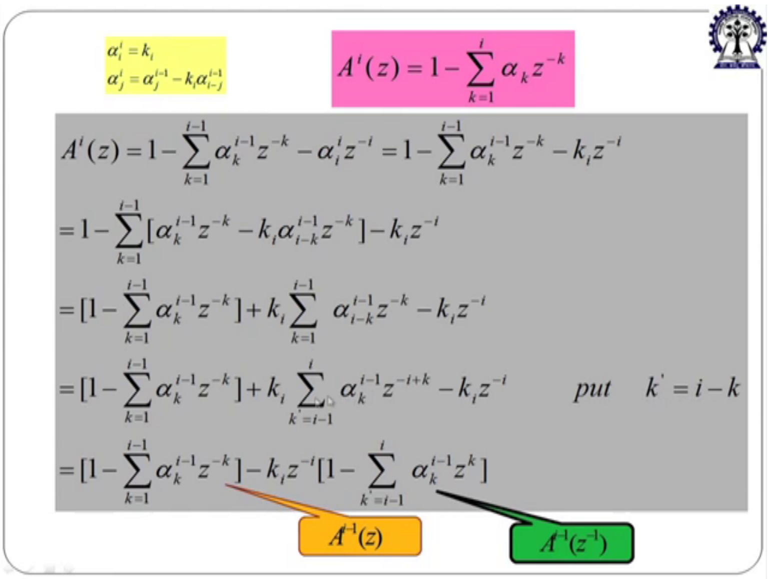

Now, if this here if I say that k dash is equal to i minus k, k dust is equal to i minus k. So, if I put k dust is equal to i minus k, so I can say this is k dust is equal to i minus k ki k equal to 1 to i minus 1 and k dust is equal to i minus k. So, k dust is equal to i minus 1 to i if you see here. So, ki k dust is equal to i minus 1 to i alpha k i minus 1. So, i minus k is equal to since this there is A i minus k, so I can say k dash is equal to i minus k if it is k dust is i minus k. So, what is k? K is nothing but A k dust. So, if it is k varies from 1 to i minus 1, k varies from 1 to i minus 1. So, if I instead of k if I put it, this term is there class instead of k i, put k dust.

So, I can say k dust is equal to k minus 1 or k dust, sorry k dust is equal to i minus 1 to i alpha k. Ki will be there, alpha k i minus 1, ki into alpha k i minus 1 into z to the power minus k; so z to the power minus i plus k minus ki z to the power minus i. So, if I say this term, this term will be same. Then, this term will be ki. If I take minus sign here, then this will become beside ki z to the power minus i, if I take common, then what will happen is, 1 minus this term 1 minus k equal to i minus 1 i alpha k i minus 1 z to the power plus and this term. So, finally this will be 1 minus k equal to 1 to i minus 1 alpha k i minus 1 z to the power minus k minus this term.

So, the two term if you see the slides, there are two terms. One is this one and another is this one. So, if I say this is i minus 1th order predictor, so I can say this is nothing but A i minus 1 z minus ki z to the power minus i into since it is z to the power plus k, then I can say it is nothing but A i minus 1 z to the power minus 1 A i minus 1 z to the power minus 1. So, I can say that prediction predicted the error filter A iz can be expressed as A i minus 1 z previous iteration i minus 1 z minus ki z to the power minus i, A i minus 1 z to the power minus 1. If it is that, then what is forward prediction errors? So, I can say ez or I can say ez is nothing but A i. So, instead of A i, I can put this one. So, sz into A i minus 1 z minus ki z to the power minus i A i minus 1 z to the power minus 1. I just put the value of A iz. So, I can say it is nothing but sz into A i minus 1 z. So, it is e i minus 1 z minus ki into z to the power z to the power minus 1 A i minus 1 or I can say z to the power minus 1 digit. What is digit? It is z to the power minus i sz into A i z to the power minus 1.

So, if it is c ki z to the power minus i, z to the power if I want to write in term of bz, I have to write z to the power minus 1 into z to the power i minus 1. I take one outside. So, i minus 1 and minus 1, then I can write A i minus 1 j to the power minus 1. So, if you see these term, they are nothing but bz. So, I can say it is nothing but A z to the power minus 1 if it is bz. So, I can say the forward predict foreigner in time domain.

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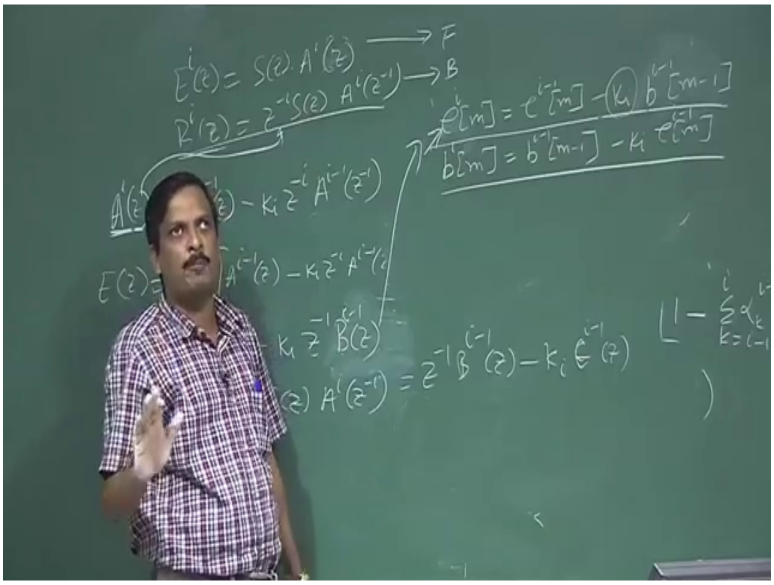

If I express in time demand this terminology, I can say em or I can write em time domain ez em e to the e i minus 1 m. So, I can say it is e i m is e i minus 1 m forward prediction error minus ki into z to the power minus 1. It is there into be i minus z. So, it is be i minus z. So, it is bi minus 1 bi minus 1 m. So, if it is z to the power minus 1 is here, I can write minus 1 here. So, I can say here or I can write clearly bi minus 1 m minus 1 because z to the power minus 1 is there. So, b z b i minus 1 bi minus 1 m 1 sample delay bi minus m minus 1 sample delay m minus 1. So, this is the forward prediction error.

Similarly, if you said B iz if it is B iz, B iz is equal to if I want to replace this ziz in here, then what I will get B iz is equal to z to the power minus i is z into A i j to the power minus 1. So, now I can put some Aiz value instead of z to the power minus 1. I can put here. If I put that, then I get z to the power minus 1 b i minus 1 z minus ki e i minus 1 z.

If I put this B zi to value this, this A z value in here instead of z to the power minus 1. So, from basic question I can get b m is nothing but A b m is nothing but A be i minus 1. So, B i m, I have order prediction m minus 1 z to the power minus 1 is there minus ki into e, that means e m i minus 1. So, this is called forward prediction error; this is called backward prediction error. So, I can easily say forward prediction error ith itteration, forward prediction error is equal to i minus 1th iteration forward prediction error minus ki into bi minus 1 m minus 1, where k is that parker partial reflection coefficient, ok.

So, I can say the forward prediction error can be expressed in top of previous forward prediction error and backward prediction error. Similarly, backward prediction error can be expressed in top of previous backward prediction error and forward previous forward prediction error.

So, next class we want to discuss about how we draw the signal flow diagram, what should be the pictographic or signal flow diagram of these two equations, ok.

Thank you.