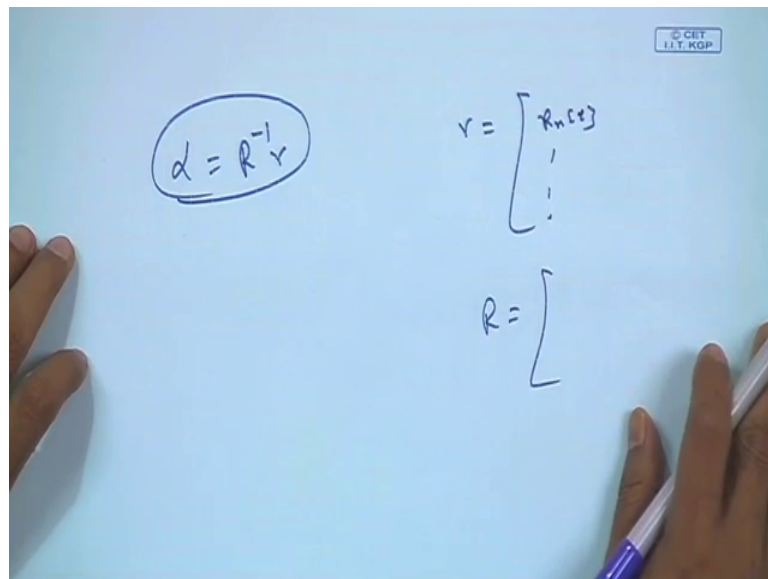


Digital Speech Processing
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Lecture - 23
Autocorrelation Method Of LPC Analysis (Contd.)

So last class; we are here that α is equal to R inverse; small r .

(Refer Slide Time: 00:25)


$$\alpha = R^{-1} r$$
$$r = \begin{bmatrix} r_n(c) \\ 1 \\ \vdots \end{bmatrix}$$
$$R = \begin{bmatrix} \end{bmatrix}$$

So, I can say the small r ; the small R is nothing but a small R is equal to R_{n1} , R_{n2} like that and R is nothing but a this matrix. So, R is this matrix and small R_n is this matrix; so, this matrix and this matrix. Now, we have to solve this; how to solve this? Solving is done based on the Levinson and Durbin methods. So, what is this method? Basically I am not details describe the method, but let us go through that.

(Refer Slide Time: 00:59)

The set of optimum predictor coefficients satisfy

$$\sum_{k=1}^p \alpha_k R_n[i-k] - R_n[i] = 0 \quad 1 \leq i \leq p$$

with minimum mean-squared prediction error of

$$R_n[0] - \sum_{k=1}^p \alpha_k R_n[k] = E^{(p)}$$

So, basically these equation can be solved to set up optimize predictor coefficient; satisfy if I solve this basically I am optimize the alpha value will satisfy these two equation.

So, these two equation derived; so, if I write down that these two equation in matrix form; the first equation is give this matrix and second equation if I included the minimum prediction error; that means, if I estimation is very correct, then it should produce the minimum prediction error. So, minimum prediction error equation it should satisfy. So, if I put this two equation in a matrix form it will look like this.

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$$\begin{bmatrix} R_n[0] & R_n[1] & R_n[2] & \dots & R_n[p] \\ R_n[1] & R_n[0] & R_n[1] & \dots & R_n[p-1] \\ R_n[2] & R_n[1] & R_n[0] & \dots & R_n[p-2] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_n[p-1] & R_n[p-2] & R_n[p-3] & \dots & R_n[0] \end{bmatrix} \begin{bmatrix} 1 \\ -\alpha_1^{(p)} \\ -\alpha_2^{(p)} \\ \vdots \\ -\alpha_p^{(p)} \end{bmatrix} = \begin{bmatrix} E_n^{(p)} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Expanded matrix is still Toeplitz and It can be solved iteratively by incorporating new correlation value at each iteration and solving for next higher order predictor in terms of new correlation value and previous predictor

i^{th} order solution can be derived from $(i-1)^{st}$ order solution

given $\alpha^{(i-1)}$, the solution to $R_n^{(i-1)} \alpha^{(i-1)} = E_n^{(i-1)}$

we derive solution to $R_n^{(i)} \alpha^{(i)} = E_n^{(i)}$

1 enp; because some matrix that this equation to many matrix form. Now, if I see I want to solve this matrix; it shows that if I use the Levinson recursion; that means, that means the i th order solution can be derived from i minus first order solution. So, suppose I have a P is my level of prediction; P is the number of prediction that I want the order of the predictor is p . So, I can say I can iteratively predict the alpha value and I want that if I iteratively predict it; then at p th order will give me the optimum solution or p th iteration give me the optimum solution.

So, in Levinson recursion I said we can say that I can predict the current i value or current this matrix equation; current value from the previous or i th order solution. So, what is the i minus 1th order solution?

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$$R_n^{i-1} \alpha^{i-1} = E_n^{i-1} \Rightarrow R_n^i \alpha^i = E_n^i$$

$$k_i = \frac{y^{i-1}}{E^{i-1}} = \frac{R_n^i[i] - \sum_{j=1}^{i-1} \alpha_j^{i-1} R_n^i[i-j]}{E^{i-1}} \quad \text{--- (1)}$$

$$E_n^i = E_n^{i-1} - k_i y^{i-1} = E_n^{i-1} [1 - k_i^2] \quad \text{--- (2)}$$

R_n^i ; i minus 1 alpha i minus 1; is equal to E_n i minus 1 if I see this matrix. So, from here we can derive the solution for R_n^i ; i alpha i is equal to E_n i this we can derive.

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The $(i-1)^{\text{st}}$ solution can be expressed as:

$$\begin{bmatrix} R_n[0] & R_n[1] & R_n[2] & \dots & R_n[i-1] \\ R_n[1] & R_n[0] & R_n[1] & \dots & R_n[i-2] \\ R_n[2] & R_n[1] & R_n[0] & \dots & R_n[i-3] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_n[i-1] & R_n[i-2] & R_n[i-3] & \dots & R_n[0] \end{bmatrix} \begin{bmatrix} 1 \\ -\alpha_1^{(i-1)} \\ -\alpha_2^{(i-1)} \\ \vdots \\ -\alpha_{i-1}^{(i-1)} \end{bmatrix} = \begin{bmatrix} E_n^{i-1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

How is derived? That is the details explanation that matrix I can write this way and then I can append 0 vector at alpha multi; this is the matrix, you can see that this ride and you can find out this is the nothing but a mathematics; nothing gains in the mathematics. So, I can say gamma minus i to the power one is equal to this and then I can say this is the toeplitz matrix; special symmetry, we can reverse the order of the equation we can reverse it and then once you revert you can combined into set of multiplication factor R n this is minus and this is minus both side.

So, it remains same and then we choose i minus 1. So, that the vector this K i right side has a only a single non zero entry. So, K i it is the wide vector of the predictor is nothing but a gamma i minus 1; divided by E i minus 1 where it is nothing but a R n i minus j equal to 1 to i minus 1 alpha j; i minus 1 R n; i minus j, divided by E of i minus 1; this is the equation number 1 for autocorrelation method solution ki.

Now, if it is this is ki; then what is the E n i is nothing but a; if I see E n i, I can write down E n i is nothing but a R 0 minus alpha K R n K ik. So, if I use that value then E n i becomes E n i minus 1 minus K i gamma i minus 1; which is nothing but a E n i minus 1; 1 minus K i square.

So, ith K i is nothing but this R n i minus. So, R ni means R n is the autocorrelation coefficient; ith autocorrelation coefficient minus j into 1 to i minus 1, alpha j i minus 1 R

n minus j divided by E minus 1; this is the equation number 1, this is the equation number 2; then I can put that value and this K_i is called partial correlation coefficient.

(Refer Slide Time: 05:58)

The first element of the right hand side vector is now

$$E_n^{(i)} = E_n^{(i-1)} - k_i \gamma^{(i-1)} = E_n^{(i-1)} (1 - k_i^2)$$

The k_i parameters are called Partial Correlation (PARCOR) coefficients

So the vector of order i^{th} predictor coefficients is

$$\begin{bmatrix} 1 \\ -\alpha_1^{(i-1)} \\ -\alpha_2^{(i-1)} \\ \vdots \\ -\alpha_{i-1}^{(i-1)} \\ -\alpha_i^{(i)} \end{bmatrix} = \begin{bmatrix} 1 \\ -\alpha_1^{(i-1)} \\ -\alpha_2^{(i-1)} \\ \vdots \\ -\alpha_{i-1}^{(i-1)} \\ 0 \end{bmatrix} - k_i \begin{bmatrix} 0 \\ -\alpha_1^{(i-1)} \\ -\alpha_2^{(i-1)} \\ \vdots \\ -\alpha_{i-1}^{(i-1)} \\ 1 \end{bmatrix}$$

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$$\alpha_{ij}^{(i)} = \alpha_j^{(i-1)} - k_i \alpha_{i-j}^{(i-1)}$$

$$\alpha_i^{(i)} = k_i$$

A. The final solution for order p is:

$$\alpha_j = \alpha_j^{(p)} \quad 1 \leq j \leq p$$

B. with prediction error

$$E_n^{(p)} = E_n[0] \prod_{m=1}^p (1 - k_m^2) = R_n[0] \prod_{m=1}^p (1 - k_m^2)$$

C. If we use normalized autocorrelation coefficients:

$$r_n[k] = \frac{R_n[k]}{R_n[0]}$$

D. normalized prediction error

$$v^{(i)} = \prod_{m=1}^i (1 - k_m^2) \quad 0 \leq v^{(i)} \leq 1 \quad -1 \leq k_i \leq 1$$

Then we can solve for α_j and this α_i K_i this is the forth equation. So, final solution for a order P if my LPC order is p .

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$$\alpha_j = \alpha_j^P$$

$$E_n^{(P)} = E_n^{(0)} \prod_{n=1}^P [1 - K_n^2]$$

$\alpha_1, \alpha_2, \alpha_3, \alpha_4$

So, alpha j is equal to alpha j P or I can say k; my alpha j P is the my last set of alpha value or optimal set of alpha value alpha 1, alpha 2, alpha 3, alpha 4; optimal set of alpha value with a prediction error E n P is equal to E 0, E n 0 energy n equal to 1 to P 1 minus km square.

So if I write down this four equation implement this four equation in a softwares; then I can calculate that alpha value and I can calculate the prediction error. So, if you see the next slides; so, there is a equation.

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$$K_i = \frac{R_n[i] - \sum_{j=1}^{i-1} \alpha_j^{(i-1)} R_n[i-j]}{E_n^{i-1}}$$

$$E_n^i = E_n^{i-1} [1 - K_i^2]$$

$$\alpha_j^i = \alpha_j^{i-1} - K_i \alpha_{i-j}^{i-1}$$

$$(\alpha_j^i = K_i)$$

$\alpha = \alpha_j^P \quad 1 \leq j \leq P$

So, if I say here; if I write down the four equation one is K_i is equal to; if I write down the equation K_i is equal to R_{n-i} minus j equal to 1 to i minus 1; α_j i minus 1, R_{n-i} minus j divided by E_n or E_{i-1} ; what is E_n ? E_n is nothing but a E_{n-1} into $1 - K_i$ square. So, this is equation number 1; this is equation number 2; then I can say α_j is equal to α_{j-1} minus K_i α_{i-1} minus j ; i minus 1 and α_j is equal to k_i ; this is the 3, this is the 4 equation.

So, now suppose I have a speech segment and I want to find out this value; K_i E_n and ultimately I have to find out this value. So, at the end alpha value is nothing but a α_j P ; the P is the order of the α_j P ; where j is also 1 equal to j P ; j varies from 1 to P . So, what I want? I want a set of α_1 , α_2 , α_3 , α_4 , α_p , but those set of alpha value can be extracted in iterative method; α_1 , 1; α_1 , 2; α_1 , 3 to α_1 P . So, α_1 P is my final answer; so, using this four equation this I can do. So, how I can do it? I just take up that let us take a simple example.

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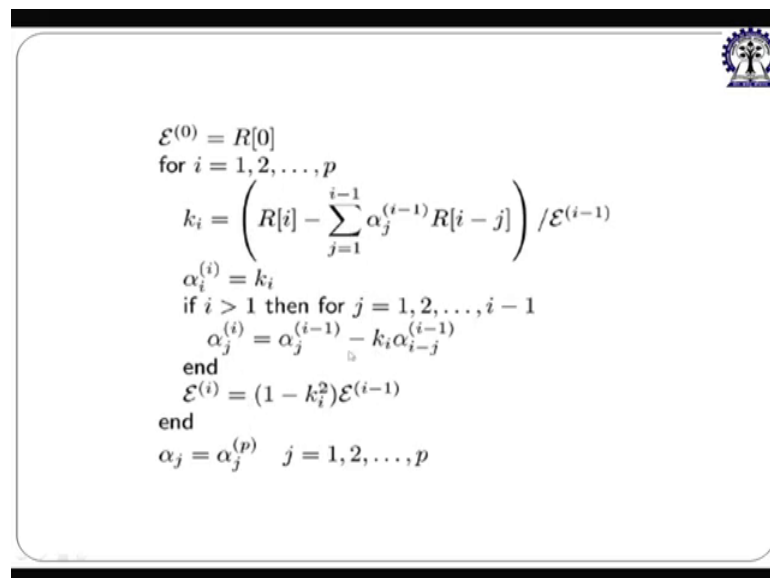
Handwritten mathematical derivation on a blue background. The derivation shows the calculation of K_1 for $P=3$. It starts with $E^0 = R(0)$ and $P=3$. Then it shows the formula for K_i : $K_i = \frac{R_n[i] - \sum_{j=1}^{i-1} R_n[i-j] \alpha_j}{E_n}$. For $i=1$, the sum is zero, so $K_1 = \frac{R_n[1]}{E_n}$. Then it shows $E_n = R_n[0]$ and finally $K_1 = \frac{R_n[1]}{R_n[0]}$. There is a small logo in the top right corner that says "© CET I.I.T. KGP".

Let us; I want to find out the order of the predictor P is equal to 3; so, i is varies from 1 to 3 and j is varies from 1 to 3. Now, if I see k_i ; so, what is R_i is; sorry i is varies from 1 to P and j is varies from 1 to p . So, if I see; I want to find out K_1 ; if I use this equation I want to find out K_1 . So, what is k ? K_1 is equal to R_{n-1} ; R_{n-1} minus j is equal to 1 to i minus 1; R_{n-i} minus j .

Now, here I say I is equal to 1; so, j is equal to 1. So, if it is j is equal to 1; then i 1 minus 1 is 0. So, this will not be equated that; so, this R 1 divided by E n i minus 1; E 0 j minus 1 is 0; E 0. So, what is E 0? This is the energy which is nothing but a R 0. So, I can say R n 1 divided by R n 0; now I can find out K 2; K 1; I find out, then what is E n i; 1 is equal to E n 0 into 1 minus K 1 square.

So, I get the value of K 1 here; I can put the value K 1 here and find out E 1 n; then what is alpha j i? If you see alpha j i here I put; i equal to 1. So, it is alpha 1; 0 minus K 1, alpha 0 0; alpha 0 0 is equal to alpha j i 1, 1; alpha 1 0; alpha minus K 1; 0 0, I can get the K alpha 1 and alpha 1, 1; then I can say alpha 1, 1 equal to K 1. So, alpha 1, 1 is equal to K 1; so, this way; this is implemented in here.

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$$\mathcal{E}^{(0)} = R[0]$$

for  $i = 1, 2, \dots, p$ 

$$k_i = \left( R[i] - \sum_{j=1}^{i-1} \alpha_j^{(i-1)} R[i-j] \right) / \mathcal{E}^{(i-1)}$$


$$\alpha_i^{(i)} = k_i$$

if  $i > 1$  then for  $j = 1, 2, \dots, i-1$ 

$$\alpha_j^{(i)} = \alpha_j^{(i-1)} - k_i \alpha_{i-j}^{(i-1)}$$

end

$$\mathcal{E}^{(i)} = (1 - k_i^2) \mathcal{E}^{(i-1)}$$

end

$$\alpha_j = \alpha_j^{(p)} \quad j = 1, 2, \dots, p$$


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If you see; this is the implementation the program is here. So, you can write the program using this four equation. Now, sometime you may find out that is called normalize autocorrelation; so, normalize, if you see the autocorrelation; the value of reflection coefficient or K i or alpha i; both are depends on R value.

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Handwritten notes on a blue background showing mathematical derivations for normalized autocorrelation and prediction error.

Top left: K_i and L_i with a horizontal line below each.

Top right: R and a list of equations:

$$\left. \begin{aligned} R(1) &\rightarrow r(1) = \frac{R(1)}{R(0)} \\ R(2) &\rightarrow r(2) = \frac{R(2)}{R(0)} \\ R(3) &\rightarrow r(3) = \frac{R(3)}{R(0)} \end{aligned} \right\}$$

Bottom left: γ and a list of equations:

$$\left. \begin{aligned} K_i &= \frac{R(i)}{R(0)} \\ E_i &= 1 - \sum_{m=1}^i K_m r_m \end{aligned} \right\}$$

Bottom right: $E_0 = 1$ and a formula for E_i :

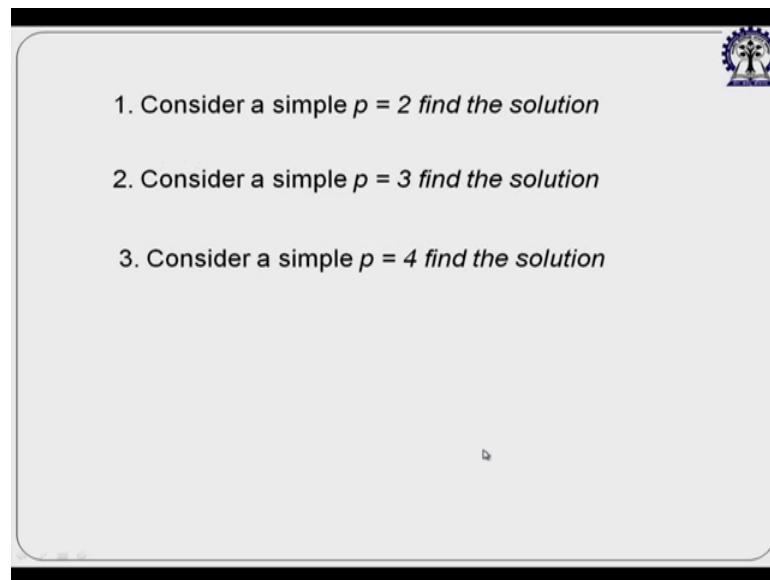
$$E_i = 1 - \sum_{m=1}^i K_m r_m, 1 \leq i \leq P$$

Autocorrelation; value of autocorrelation; so, sometime if you find that value of if it is magnitude its change of the speech signal, then may be R value will be change and autocorrelation coefficient may slightly changed. So, what I want? I want normalized autocorrelation that with respect to R_0 ; I can normalize that correlation value. So, suppose my correlation value is 1; R_2 , R_3 .

Now, I can normalize R_1 with respect to R_0 ; because R_0 is the energy of the signal. So, I can say R instead of R_1 , I can use small r_1 which is nothing but R_1 by R_0 . R_2 small r_2 is nothing but a R_1 by R_0 ; R_2 by R_0 . Similarly, R_3 small r_3 ; R_3 by R_0 ; so those are called normalized then if I use this small R to the extract the value of K_i and E_i ; E_i and α_{ij} , then I can say this is normalized autocorrelation. So, if I say what is the normalized prediction error? If it is normalized, then prediction error E_0 is equal to 1 because it R_0 y R_0 . So, it is nothing but a m equal to 1 to i ; 1 minus $K_i r_m$ square I varies from 1 to P .

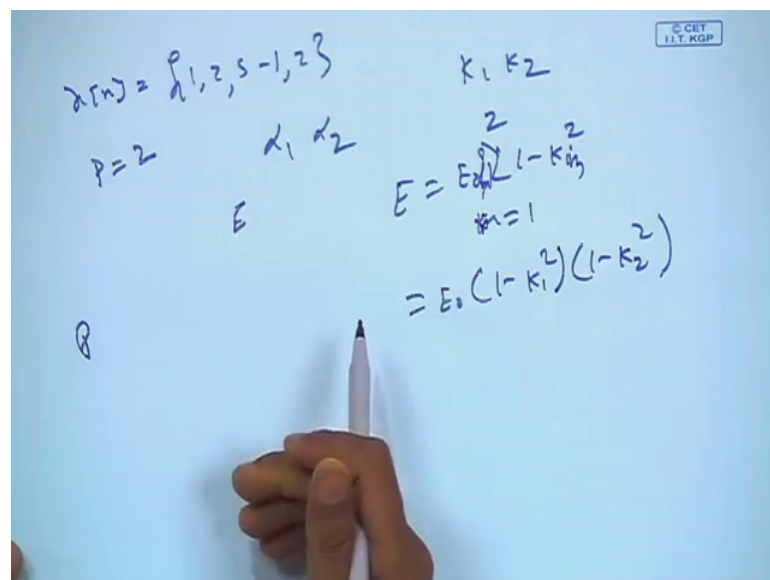
So this is the normalized autocorrelation; so, this is the program you can implement it in find out whether it is happen or not.

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Then that is the task I have given; consider a sample P is equal; second order P is equal to find the solution. So, I can give you let us x_n is equal to I give you not speech signal.

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That is 1, 2, 5 minus 1, 2 like that and then I told you that for P is equal to 2; find out that α_1 and α_2 and find out the autocorrelation error; E .

So, you can find out K_1, K_2 ; so, I can say E is equal to nothing but E_0 into $1 - K_i^2$ where i varies from; I can say 1 to m square; where m equal to 1 to order 2. So, it

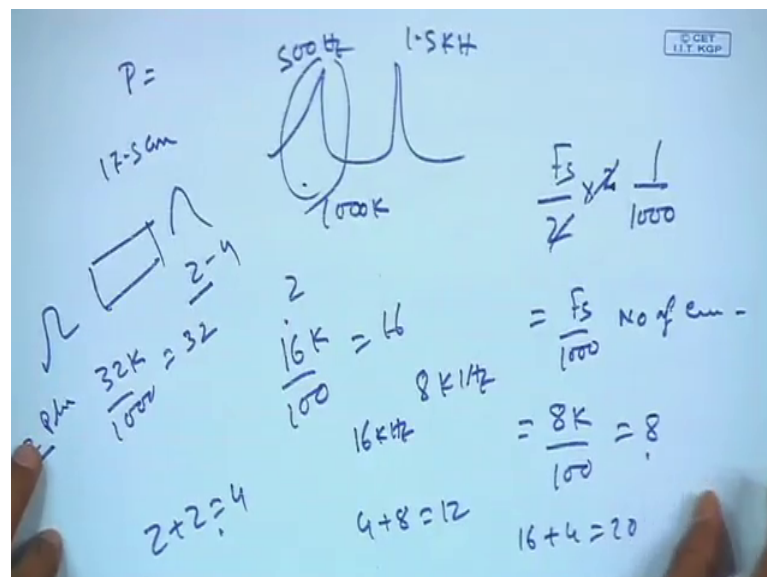
is nothing but a E^0 into $1 - K_1^2$ into $1 - K_2^2$; and I can find out α_1 to α_2 using this equation in hand made without programming.

So, if P value are very less without programming I can find out that α_1 to α_2 . So, those kind of questions can be you expect in that examination paper or I can write a program and run as real speech signal and find out the that α value and E value for autocorrelation function, using the autocorrelation coefficient. So, those are the autocorrelation solution.

Now, how do you define the order of the P? So, I say prediction order. So, how you which factor the P is depended; prediction order I have take a signal which is 8 kilo hertz sampling rate I have take a signal which is 16 kilo hertz sample; I have take a signal which is 32 kilo hertz sample. So, make different sampling rate; I can sample the speech signal.

Now, what kind of order I should use to get that correct coefficient value; if it is true that if I increase the order, your error will be less. So, I cannot take infinite order. So, what kind of order is optimum; so, in what basis I can decide the order?

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So, if you see that in tube model for formant; every formant are separated by 1 kilo hertz; 1 kilo hertz signal. So, if you see; if you remember in the first first class; the first tube

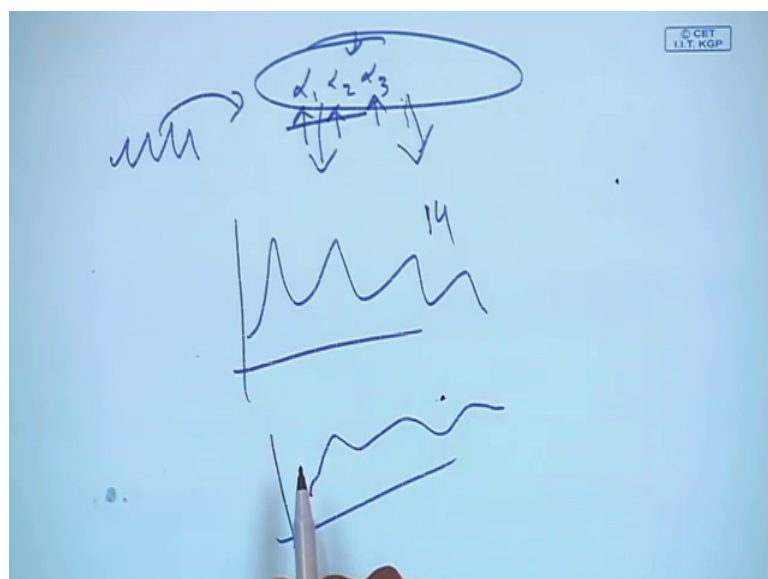
model uniform tube model; if tube length is 17.5 centimetre, first formant is 500 hertz; next formant is 1.5 kilo hertz.

So, they are separated by 1000 hertz; that means, 1 kilo hertz; so, I can say that if it is my F_s is my sampling frequency. So, what is the basement frequency? F_s by 2. How many complex pole is required to realize one formant? 2; complex conjugate pole is required to realize one formant; so, it is 2. So, F_s by 2 into 2 pole; how many? Every 1000 hertz signal; so, I can say F_s by 2 into 2 by 1000; 2; 2 cancel. So, I can say F_s by 1000 number of pole; number of complex conjugate pole; complex pole not conjugate, conjugate then it is will be have; so, complex pole.

So, if it is 8 kilo hertz; then 8 K divided by 1000; so, I can say 8; if it is 16 kilo hertz then 16 K divided by 1000; 16. If it is 32 kilo hertz 32 K divided by 1000; so, it is 32. So, those are for formant; that means, those are for tube only. Next is there is a glottis and there is a radiation laws; for glottis 2 to 2 poles and for radiation laws; 2 to 4 poles. So, if I say radiation 2 pole and glottis 2 pole; so, 2 plus 2 is equal to 4 pole; 4 plus 8; 12 pole, if my signal is sampled at 8 kilo hertz.

If my signal is sampled by 16 kilo hertz; then I can say 16 plus 4; 20; 16 plus 20 or 21 or 22 I can say; 16 plus 6 also; if I glottis if the radiation laws is some used by 4 pole. So, order of that LPC analysis; it is not arbitrary, it depends on the sampling frequency. So, based on the sampling frequency; I can take the order and find out the LPC analysis.

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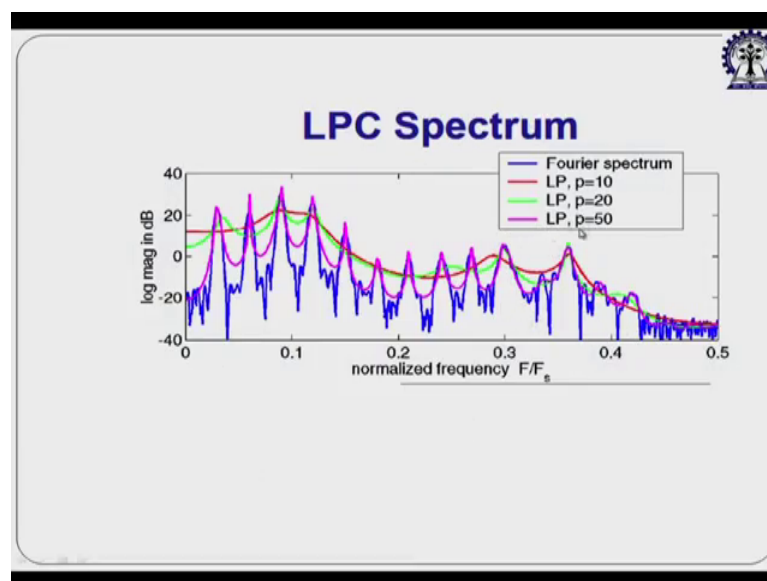


Now, there is another question is there LPC analysis; then if I have the speech signal; I extract that filter LPC analysis give me the α_1 , α_2 , α_3 value. Those actually characterize the mobile track; now forget about this part I will come later on then if those coefficients are represent; you can say the pole position.

So, formant those are the represent the actual pole position. So, I can say if I take the frequency task form of those coefficient; I should get that LPC spectrum which will give me the resonant frequency; have you understand or not? So, if I say F_s by 2; number of pole I said now those coefficient are represent the pole position of the speech signal. Now if I take the spectrum or the frequency analysis of this LPC coefficient; this would give the LPC spectra; they should represent the peak at forward frequency.

Now, if order of the analysis is reduced then this should be. So, suppose I required let us 14 order I make it 12 order. So, some of the formant will combine together and give me a some broad kind of structure. So, if you see in this picture

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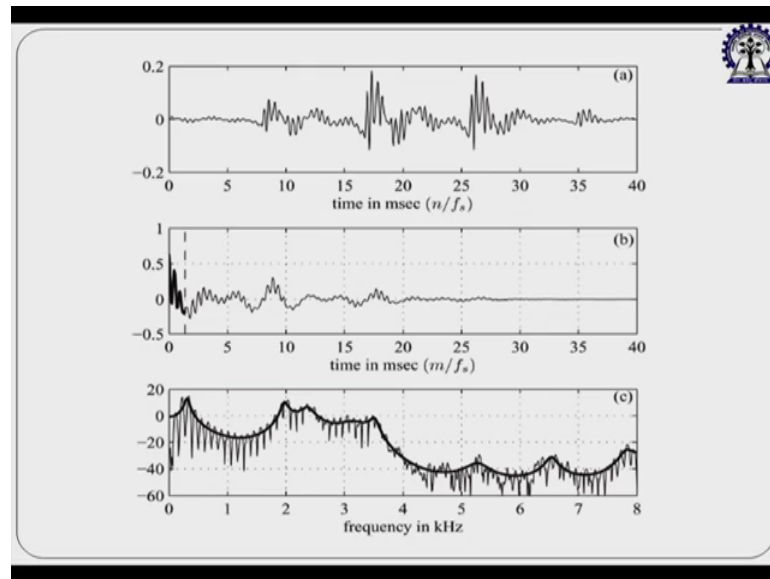


If my LPC order is increased is more or less; copy the actual blue colour is the actual spectrum. And if you see the red colour why the LPC analysis is 10; so, there is a lot of formants are there. So, I can see that; it is roughly estimate that spectral envelope, if I increase the LPC order; the number of variation is increases. So, it accurately increase the; it accurately copy the spectrum envelope. So, depending on my requirement if I want the smooth spectrum; I do not want this lot of variation, then I can use LPC order

reduce the LPC order if I want that I want to exactly copy the spectrum, then I can increase the LPC order.

So, using this I can also draw the spectrogram also this is the real life example of LPC spectrum.

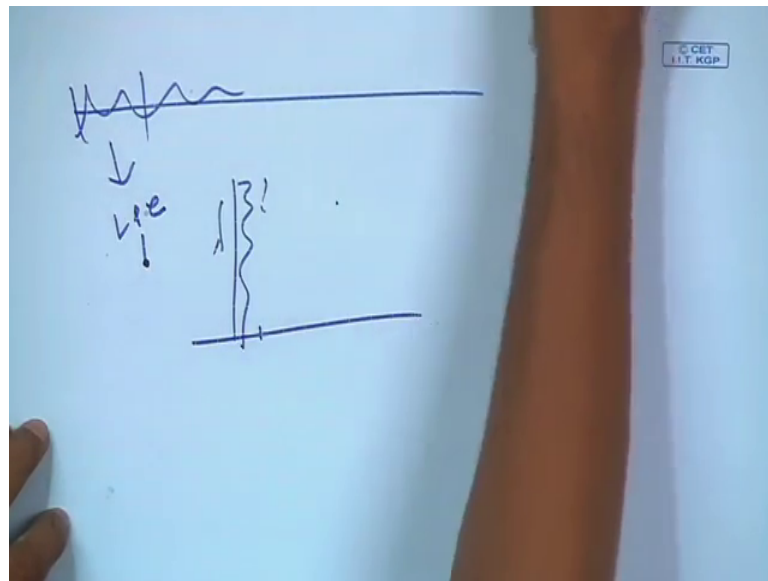
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If you see how many formants are there 1, 2, 3, 4, 5, 6, 7; now if I say what is the possible order of the LPC analysis?. So, I can say there is a 7 formant. So, I can say 7 into 2; each formant require two complex pole. So, it is nothing but a 14 order LPC analysis.

So, this can be explained; this is called LPC spectrum and then there is a LPC spectrogram can; this is the frequency analysis equation, you can go through that slides I am not details describe this frequency analysis equation and this is the LPC spectrogram, because here if you see the formants are much more clear; this is done, how it is plotted?

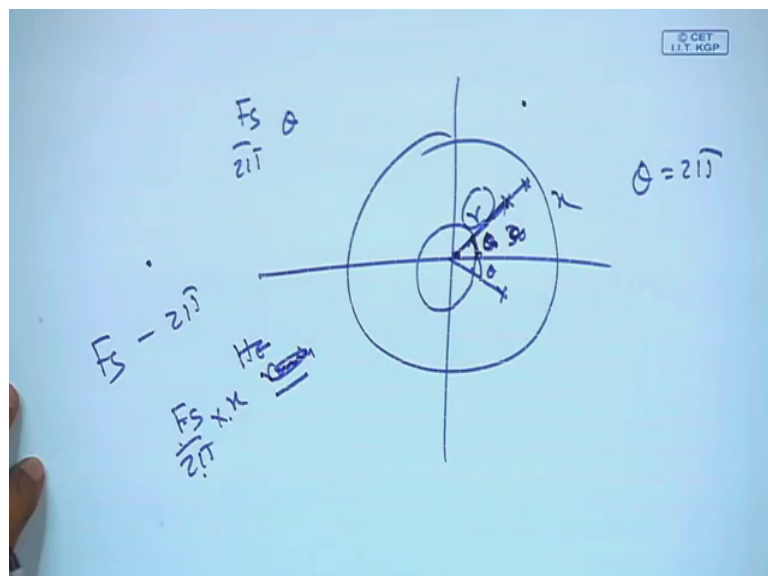
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Suppose you have a speech signal; you take a window, find out the LPC coefficient take the spectrogram of the LPC spectrum of the draw the magnitude spectrum of the LPC coefficient. Then for that time plot this in this frequency scale; again shift it and plot it and that way you get the LPC spectrogram.

So, the LPC spectrogram now formant; suppose that this I will discuss later on. Also I have discuss in that the during the tube analysis that formant position R E to the power j theta. So, frequency of the formant is given by F_s by 2π into theta.

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So, if I say every complex conjugate pole indicate one formant; if it is close to unit circle this is called formant frequency it contribute to the formant frequency because energy will be very high.

So, if I say complex conjugate pole. So, suppose this is my unit circle and if there is a formant if there is a pole angle is theta. So, the system is real; so there will be a complex conjugate pole minus theta here and this is r. So, r which if it is close to unit circle the bandwidth r will be increase and bandwidth will be increased; if it is close to 0 the bandwidth will be 0. So, if you see the formant frequency is determined by this angle theta and the formant bandwidth it determined by R; which is already explained in tube modelling.

So, if F_s is my sampling frequency; so, what is the normalized frequency of the digital signal? Is 2π is equal to F_s . So, what is the value of the theta; theta maximum value is 2π and which is equal to F_s . So, if the theta is let us 30 degree or this radian; if it is radian let us say it is x radian then I can say F_s by 2π into x is the radian is the frequency, if it is hertz then I can convert F_s is in hertz. So, I can say F_s by 2π into x hertz is my formant frequency. Suppose I have a pole I do not know whether the example is here or not; no example is not there.

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$\theta = 30^\circ = \frac{\pi}{3}$
 $F_s = 16 \text{ kHz}$

$\frac{F_s \times \frac{\pi}{3}}{2\pi} = \frac{f_s}{\theta} = \frac{16k}{6} = 1 \text{ kHz}$

$\text{BW} = \frac{F_s}{\pi} \cdot -\log(|z|)$
 $z = r \cdot e^{j\theta}$

So, suppose the theta is equal to let us 30 degree or let us pi by 3. So, what is the formant frequency? So, let us say F_s is equal to 16 kilo hertz. So, if it is 16 kilo hertz then F_s by

2π into π by 3. So, F_s by 6 is nothing but a 16 K divided by 6; it is 1 kilo hertz by
 formant frequency and formant bandwidth is nothing but a F_s by π into \log of this is Z_r
 \log of r ; you know that; \log of r what is r ? r is nothing but a absolute value. So, if it is
 complex pole $a + jb$; then I can say root over of $a^2 + b^2$ or mod of if it
 is pole is $Z_K \text{ mod of } Z_K$; give me the r . So, I can say π by F_s by π into minus \log of
 $\text{abs } z_k$ or $\text{mod of } z_k$.

So, I can find if the r value I know, if I know the sampling frequency if π I know. So,
 you can got the I get the formant bandwidth. Or vice versa if I know the formant
 bandwidth, I can find out the value of r ; if I know the formant frequency I can find out
 the value of θ then I can derive the transfer function of that thing. Because Z is
 nothing but a r into e to the power $j\theta$ or you can use that $r\theta$ equation in tube
 model. So, both way if I know the formant bandwidth and formant frequency; I can
 derive the transfer function; linear LTI transfer functions. Or if I know that pole position
 or pole value or θ and r ; I can derive the formant frequency and bandwidth.

So, this is autocorrelation method I have described and all other things I have described.
 Then there is a other methods also that if you see that there is a covariance methods for
 linear prediction. So, covariance methods is also one another methods instead of
 autocorrelation, I can find out the covariance method to detect the; to find out the LPC
 coefficient. So, what you want? We want the same things same solution of that matrix
 solution of this matrix; this we have already derived.

Now, that difference is that; in this matrix that I have not taken the outside the window
 signal.

(Refer Slide Time: 30:04)

$$\phi \alpha = \psi$$
$$\alpha = \phi^{-1} \psi$$
$$\phi = A D A^T$$
$$\phi \alpha = \psi$$

Is 0 if I take it then problem will come; so what we will take? The key difference model method is that limit of summation include the term before m equal to 0. So, if the order is P instead of 0; I will consider the previous sample also and here also I compute previous sample also I have to consider. So, if the order is P ; P number of previous sample I have to consider.

So, tapering of the window does not matter here the window does not matter here whatever window function I can use, but I should say that P number of previous sample is required in this analysis. Then same simple things; I have to solve this matrix equation. So, ϕ you can say this ψ into α is equal to let us this one; then I can say α is equal to ϕ inverse or I can rate this is not ϕ ; how to solve this?

Now, this can be solved called Cholesky decomposition methods who is that; let ϕ is equal to matrix $A D A^T$ and ϕ into α is equal to let us this one. If ϕ is this matrix; where A is equal to lower triangular matrix and first in the main diagonal and D is the diagonal matrix and A^T is the upper triangular matrix. So, I can say; so instead of let us solve for P equal to 4; order is equal to 4.

(Refer Slide Time: 32:02)

$$P=4$$

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ A_{21} & 1 & 0 & 0 \\ A_{31} & A_{32} & 1 & 0 \\ A_{41} & A_{42} & A_{43} & 1 \end{bmatrix} \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$$

$$d_1 = D_{11} \quad A_{21} = \frac{D_{21}}{d_1}$$

So, I can get phi equation is phi 1, 1; phi 1, 2, phi 1, 3, phi 1, 4; phi 2, 1, phi 2, 2 phi 2, 3; phi 2, 4; phi 3 1, phi 3 2, phi 3 3, phi 3 4 phi; 4 1, phi 4 2, phi 4 3, phi 4, 4. This matrix let us equal to what triangular matrix; that is diagonal element is 1; so, 1 then it is A 2 1, A 3 1, A 4, 1; then I can say put 1; I can say A 3, 2 A 4 2; then I can put one; I can say A put 3; then I put 1 here; rest are 0, 0, 0, 0, 0, 0 this into let us diagonal element d 1, d 2, d 3, d 7; 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0. So, diagonal element and a transpose if it is lower triangular is the transpose is upper triangular matrix.

So, this is this; now if I solve this, then I can get solve this matrix, if I solve this d 1 is nothing but a phi 1, 1. Similarly A 2 1 is nothing but A phi 2 1; divided by d 1; A 3 1.

(Refer Slide Time: 34:01)

Solve matrix

$$d_j = \phi_j - \sum_{k=1}^{j-1} A_{jk}^2 d_k$$

$$A_{j1} = \phi_{j1} / d_1$$

Else

$$A_{ij} = \phi_{ij} - \sum_{k=1}^{j-1} A_{ik} d_k A_{jk} / d_j$$

$$d_1 = \phi_{11}$$

$$A_{21} d_1 = \phi_{21} \rightarrow A_{21} = \frac{\phi_{21}}{d_1}$$

$$A_{31} d_1 = \phi_{31} \rightarrow A_{31} = \frac{\phi_{31}}{d_1}$$

$$A_{41} d_1 = \phi_{41} \rightarrow A_{41} = \frac{\phi_{41}}{d_1}$$

$$d_2 = \phi_{22} - A_{21}^2 d_1$$

$$A_{32} d_2 = \phi_{32} - A_{31} d_1 A_{21} \rightarrow A_{32} = \frac{\phi_{32} - A_{31} d_1 A_{21}}{d_2}$$

$$A_{42} d_2 = \phi_{42} - A_{41} d_1 A_{21} \rightarrow A_{42} = \frac{\phi_{42} - A_{41} d_1 A_{21}}{d_2}$$

iterate procedure to solve for d_3, A_{43}, d_4

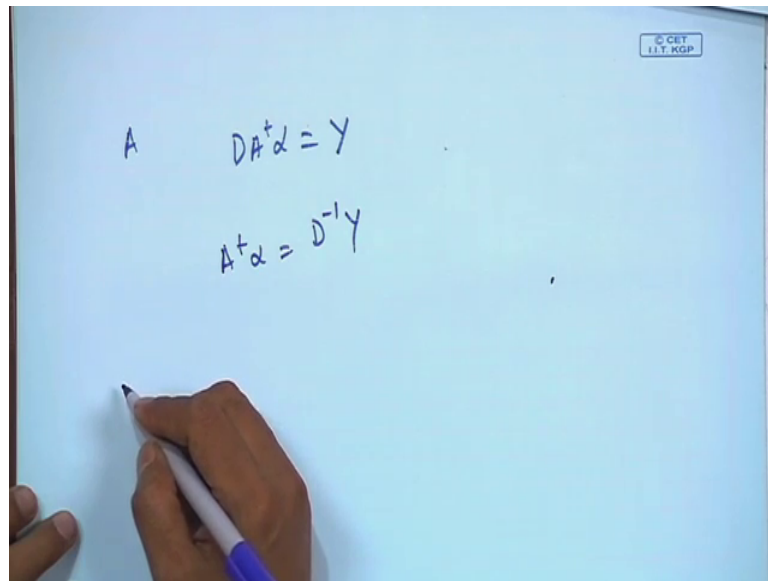
```

d1=phi11
for(i=1; i<=p; i++)
{
    A_i1=phi_i1/d1
}
for(j=2; j<=p; j++)
{
    d_j=phi_jj - sum_{k=1}^{j-1} A_jk^2 d_k
    for(i=j+1; i<=p; i++)
    {
        A_ij=phi_ij - sum_{k=1}^{j-1} A_ik d_k A_jk / d_j
    }
}
    
```

If you see that A_{31} ; you do it ϕ_{31} by d_1 , A_{41} ; ϕ_{41} , by d_1 . Now, if I solve for d_2 , it will come ϕ_{22} ; this will be ϕ_{22} ; there is a some typing mistake. So, ϕ_{22} ; minus A_{21}^2 square into d_1 ; this will be ϕ_{22} ; all are ϕ this is ϕ . So, I can write down the program, I can generalize this d equation and generalize the A equation and write down that program. So, I can find out the d and A value.

Now, once I get the d and A value; I find out d and A value I get in terms of other d and A value. So, d_1 in terms of ϕ value; I get it, now if I see ADA^T ; α is equal to let us this one; instead of A ; this one I write A into Y . So, I can say $D A^T \alpha$; $D A^T \alpha$ is equal to it is Y matrix.

(Refer Slide Time: 35:18)



A

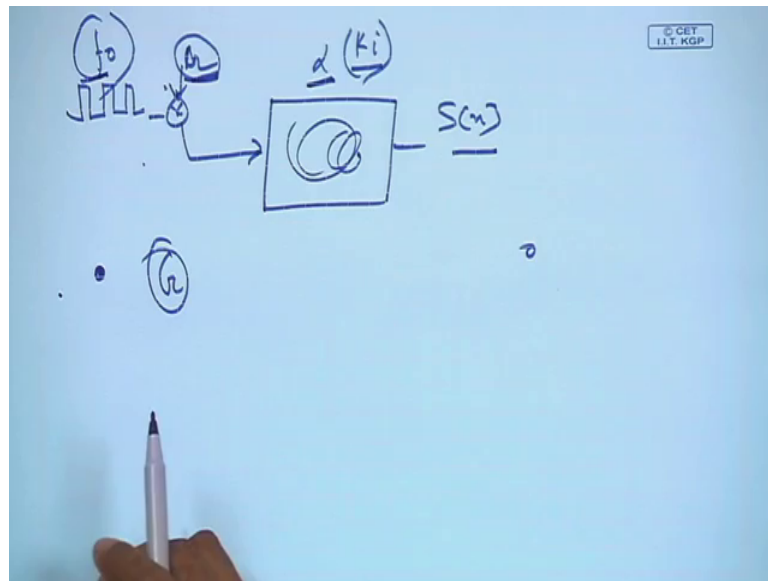
$$A^T D A \alpha = Y$$
$$A^T \alpha = D^{-1} Y$$

So, I can write down this one again; $A^T D A \alpha = Y$ and this one and then again I solve and generalize this one; for any order, this is the generalization; I can write down this matrix, I get Y value. Once I get that Y value; so, this is $D A^T A \alpha = Y$. So, I can say $A^T A \alpha = D^{-1} Y$. So, D^{-1} I can easily calculate; then I can say $A^T A \alpha = D^{-1} Y$. So, I put that A value; I put the $A^T A$ value matrix; D^{-1} and Y , then I can get that $\alpha_4, \alpha_3, \alpha_2, \alpha_1$. So, it is reverse order we are getting; we can write down the equation generalize program.

So, I can using this matrix decomposition technique; I also can calculate the α value; if I know the ϕ value. Because ϕ value is nothing but a ϕ_{ik} is nothing but a this one using the window. Only difference is that in matrix; this covariance matrix at n equal to 0 this signal is not 0; I required the P number of previous sample. Because this restrict me; so, this is called autocorrelation methods. Then there is a other techniques also that is called lattice methods; that we will discuss in the next class.

Let us discuss in one thing which is there; which is important which is called a gain.

(Refer Slide Time: 37:15)



So, what I am saying in LPC analysis; we can say if I apply the; if I know the alpha value or K value, I can design this filter and if I know the voice signal is nothing but a impulse. So, I generate the impulse based on that F_0 value of the voice signal and multiply the gain; that is G and then if I can pass through this filter; I can generate the speech signal.

So, if it is LPC decoder; the simple LPC encoder and decoder. So, if I want to transmit this signal from this point to this point; using simple LPC encoding and decoding, what I want? I want to send that F_0 value; Z value, alpha value or K_i value in the receiver end. And from F_0 value, Z value and alpha value; I can generate the speech signal. So, F_0 is extracted based on the; if I get the speech signal, I can extract that F_0 value.

So, next I can use K_i value; I can extract that K_i value using the autocorrelation technique or covariance technique. Then next I have to know the G value; what is the G value? Gain value. So, how do you calculate the g? If you see; I can calculate easily I can calculate the computational model gain; gain of the signal.

(Refer Slide Time: 38:47)

$$H(z) = \frac{G}{1 - \sum_{k=1}^P \alpha_k z^{-k}} \quad f[m]$$

$$H(z) - H(z) \sum_{k=1}^P \alpha_k z^{-k} = G$$

$$H(z) = \sum_{k=1}^P H(z) \alpha_k z^{-k} + G$$

$$h[m] = \sum_{k=1}^P \alpha_k h[m-k] + G \delta[m]$$

$$h[m] h[m-i] = \sum_{k=1}^P \alpha_k h[m-k] h[m-i] + G \delta[m] h[m-i]$$

If you see $H(z)$ is nothing but a G by 1 minus $\sum_{k=1}^P \alpha_k z^{-k}$ equal to 1 to P ; $\alpha_k z^{-k}$ to the power minus k . This is the LPC equation or I can say; in time domain if I multiply it this side $H(z)$ minus $H(z)$ into $\sum_{k=1}^P \alpha_k z^{-k}$ equal to 1 to P ; $\alpha_k z^{-k}$ to the power minus k is equal to G .

So, I can say $H(z)$ is equal to $H(z)$ into $\sum_{k=1}^P \alpha_k z^{-k}$ plus G . So, if it is $H(z)$ it is nothing but I can time domain $h[m]$ is nothing but a $\sum_{k=1}^P \alpha_k h[m-k]$ plus G ; if I say my input of the system is a delta function. So, I can say the G ; G is nothing but a gain with a delta function at m equal to 0 ; only G exists with the gain.

Now, if I multiply both side $h[m]$ multiply by $h[m-i]$. So, I can say $\sum_{k=1}^P \alpha_k h[m-k] h[m-i]$ plus $G \delta[m] h[m-i]$ or not. So, if it is that; so, what is or m let us m minus i ; multiply m minus i ; m minus i , multiply both side by m minus i .

(Refer Slide Time: 40:59)

$$R(i) = \sum_{k=1}^P \alpha_k R(i-k) + G \delta[i-m]$$

$$R(0) = \sum_{k=1}^P \alpha_k R(k) + G \cdot G$$

$$R(m) = \sum_{k=1}^P \alpha_k R(m-k) + G \delta[m]$$

$$G^2 = R(0) - \sum_{k=1}^P \alpha_k R(k)$$

Now, at i equal to 0 $h(m-i)$ is nothing but $h(m)$ multiplied by $h(m)$; K equal to 1 to P ; α_K into m equal to minus infinity to infinity $m-i$; I have taken minus infinity to infinity; $h(m-K)$ into $h(m)$; forget about this part, α_K into $h(m-K)$ into $h(m)$; plus G of $\delta(m-i)$ into $h(m)$; now $h(m)$ into $h(m)$ is nothing but autocorrelation $R(0)$; minus K equal to 1 to P α_K ; this is nothing but a $R(K)$ plus this is G and what is $\delta(m-i)$ and $h(m)$; at i equal to 0 $\delta(m)$ is only exists at m equal to 0. So, I can say this $h(m)$ is nothing but a K equal to 1 to P α_K ; $h(m-K)$ into plus G into $\delta(m)$.

Now, at m equal to 0 because $\delta(m)$ is only exists at m equal to 0. So, m equal to 0, I can say K equal to 1; this is nothing but a G . So, I can say G into G ; so, G square. So, I can say G square is nothing but a $R(0)$ minus K equal to 1 to P ; α_K ; r_K , K equal to 1 to P α_K ; r_K . If you know the α value; if I know the r_K value, if I know the $R(0)$ value; I can calculate G square; once I get the G square; I can calculate G value; is clear or not?

So, what about the speech segment I get; once I know the speech segment, I can calculate the α value, I can calculate the r value; $r(0)$ and r_K ; all r_K value, if the order of the predictor is P ; then I can calculate $r(1)$, $r(2)$, $r(3)$, $r(P)$ and I can calculate $\alpha(1)$, $\alpha(2)$, $\alpha(3)$, $\alpha(P)$. Then I calculate the G ; once I get the G square, G is nothing but a root over of that things. So, model gain; I can easily compute using this theory.

So, next class we will discuss about that which is very important lattice modelling for LPC extraction and also LPC parameter that extraction also implementation of pole 0 filter all those kind of things; we will discuss. So, LPC synthesis because that is very important; how do you synthesize the speech signal at decoder; once I know the K value and alpha value.

Thank you.