Digital Speech Processing
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Lecture - 22
Autocorrelation Method Of LPC Analysis

So in last class we have derived this equation.

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$$\begin{bmatrix} \phi_n[1,1] & \phi_n[1,2] & \phi_n[1,3] & \cdots & \phi_n[1,p] \\ \phi_n[2,1] & \phi_n[2,2] & \phi_n[2,3] & \cdots & \phi_n[2,p] \\ \phi_n[3,1] & \phi_n[3,2] & \phi_n[3,3] & \cdots & \phi_n[3,p] \\ \vdots & \vdots & \vdots & & \vdots \\ \phi_n[p,1] & \phi_n[p,2] & \phi_n[p,3] & \cdots & \phi_n[p,p] \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ . \\ \alpha_p \end{bmatrix} = \begin{bmatrix} \phi_n[1,0] \\ \phi_n[2,0] \\ \phi_n[3,0] \\ \vdots \\ \phi_n[p,0] \end{bmatrix}$$

The resulting covariance matrix is symmetric, but not Toeplitz, and can be solved efficiently by a set of techniques called Cholesky decomposition

If you see this equation we try to we have to solve for this alpha 1, alpha 2, and alpha 3 and alpha p. So, one of the method is called auto correlation method before I going to autocorrelation method.

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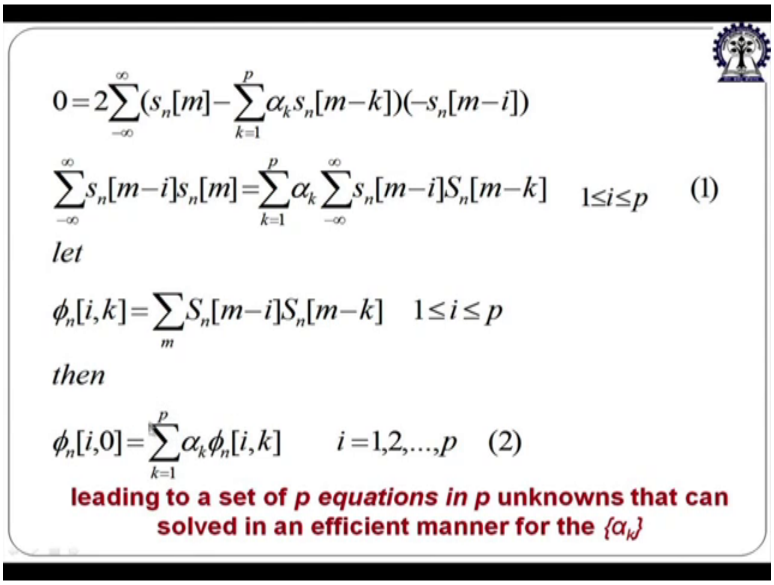

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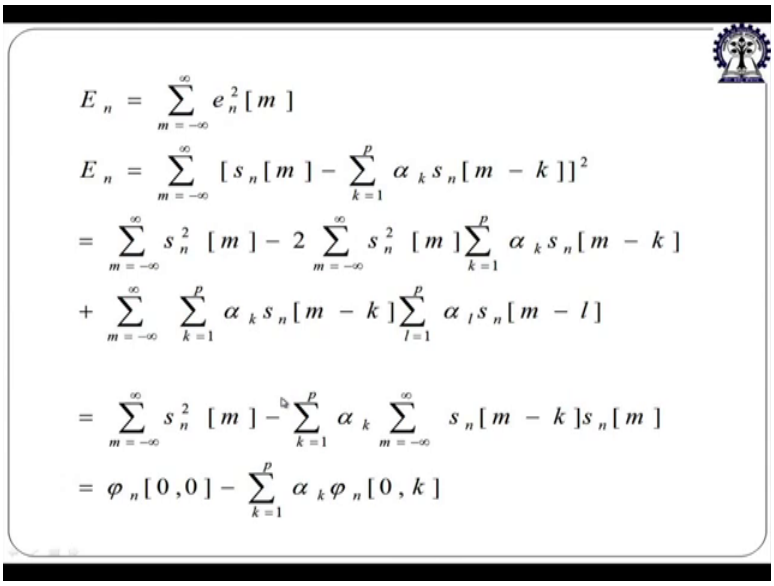

The functions that mean; mean square error minimization give me this equation, now another things I can say how to find out the error, what is the error function?

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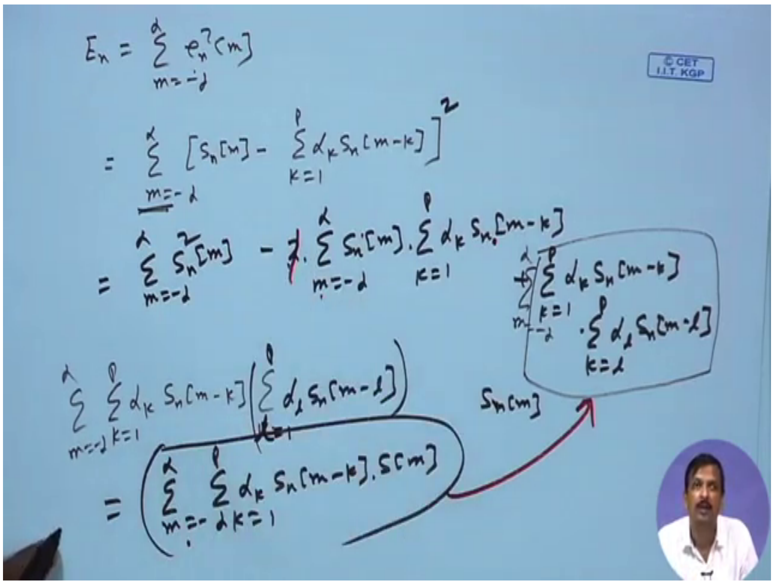

So, if you see e n is nothing but a m equal to minus infinity to infinity e n square m, error mean square this is the mean square error.

So, what is this? This is nothing but a m equal to minus infinity to infinity s n m minus k equal to 1 to p alpha k s of n m minus. So, if this is the mean square error square. So, if I execute the square a minus b square so s square m equal to minus infinity to, s of n m s square s square minus 2 into m equal to minus infinity to infinity s of n m a into b, k equal to 1 to p alpha k s of n m minus k plus k equal to 1 to p alpha k s n m minus k whole square m minus k.

What are the square square of this means k equal to l 2 p alpha l s of n m minus l, I can write square of this term I can write 2 summation term with k equal to 1 2 p and k equal to l 2 p multiplication x s l ok.

Now, if I root do that then, if you see what is this term, third term, what is this third term stand for? So, third term said that k equal to 1 to p, alpha k s of n m minus k . So, for the, that will be another-another summation which is infinite sum which is nothing but this infinite sum will be there. So, infinite sum will be here m equal to minus infinity to infinity. So, here m equal to minus infinity to infinity k equal to p m minus k another term is k equal l equal to 1 2 sorry l equal to 1 to p alpha l s n m minus l ok.

Now, if I see this is estimation of s m is only s m m. So, I can write this is is equal to m equal to minus infinity to infinity, k equal to 1 to p alpha k s of n m minus k into s m into s m. So, this term I can replace here so, if you see 2 into this n term 1 term so 2 will be cancel.

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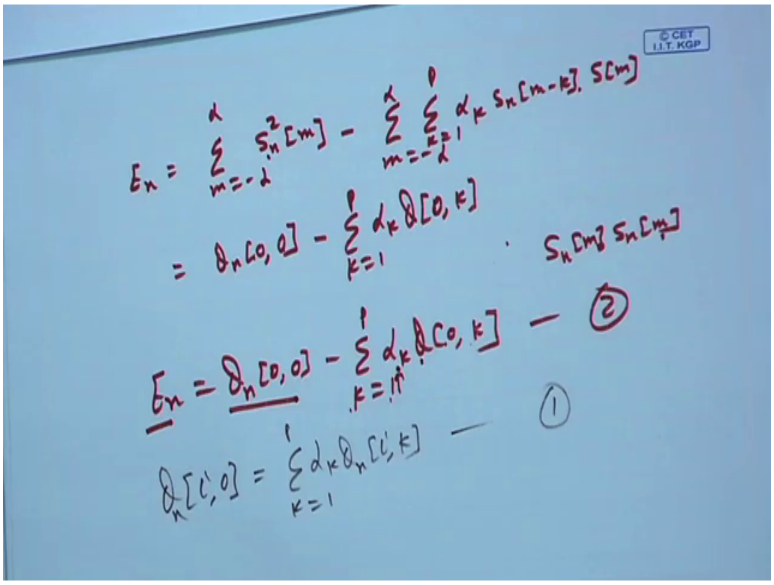

So, I can say that e n, e n is nothing but a e n is equal to m equal to minus infinity to infinity s n square, minus m equal to minus infinity to infinity k equal to 1 to p alpha k s of n m minus k into s m.

If it is that then I can say it is nothing but a phi n 0, 0 s n square means s n of m into s n of m so I can say I and k both are 0. So, both are 0 I can write minus k equal to 1 to p alpha k phi of 0, I is 0 and k is there phi of 0 k. So, I can say e n is nothing but a phi n 0, 0 minus k equal to 1 to p alpha k phi of 0 k. So, predication error, prediction error e n I can if I know the phi n matrix for 0, 0 minus k equal to 1 to p alpha k predicted coefficient into phi of 0 k.

So, this is 1 equation number you can say equation number 2 and first equation as we have derived phi of i 0, i 0 is equal to k equal to 1 to p alpha k phi of n I k this is 1. We have already derived so, using this 2 things I have to find out the set of value or I have to solve that matrix. So, first method is call autocorrelation methods. So, what is autocorrelation method?

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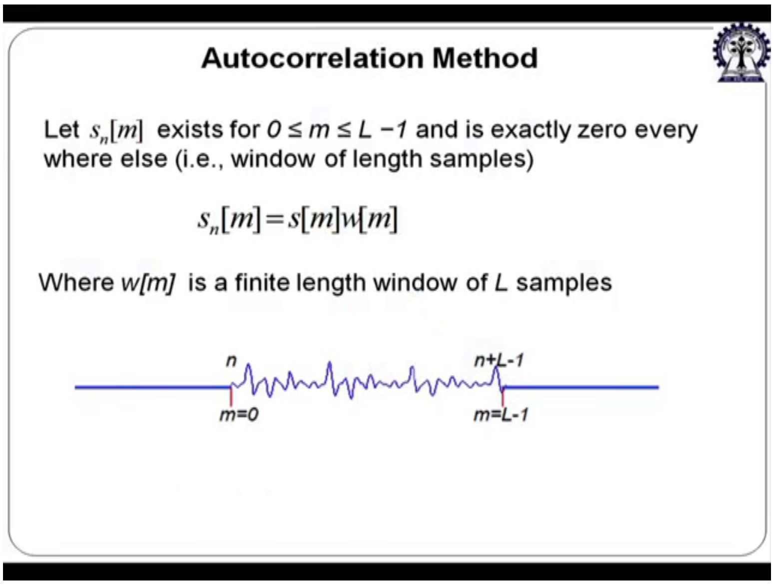

Now, before go to the autocorrelation methods that if you see, if I take the whole speech signal at a time and try to find out the estimate the value of alpha 1, alpha 2, alpha 3 then my purpose is not sub because.

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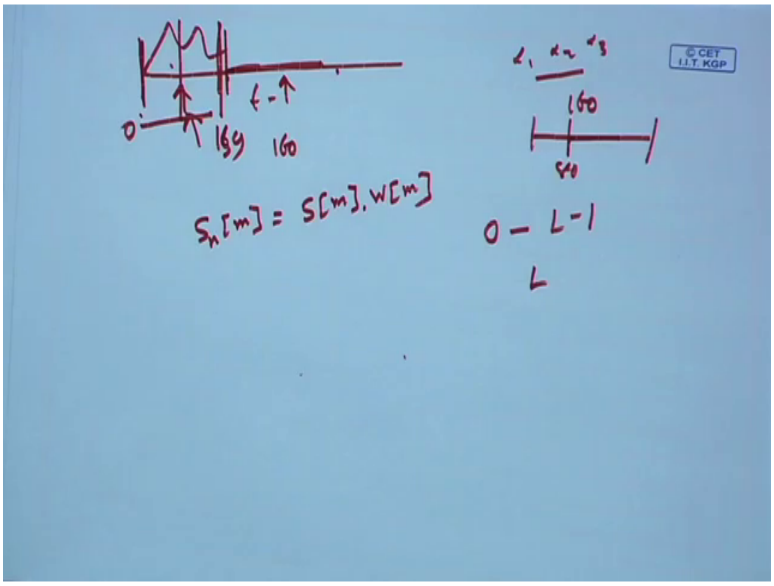

If you know a space signal is not a stationary signal. So, it can consist of different type of signal with respect to time, sometime it is voiced, sometime it is unvoiced, sometime it is sibilant. So, what I want, if I want if I take that all voiced sibilant silence at a time and try to find out the set of alpha 1, alpha 1, alpha 2, alpha 3 there is no use because whole signal is set and I do not whether this time I have to supply those impulse response and this time I have to supply the noise a I do not know that. So, since the speech is a time variance signal I try to modulate or try to take the signal for the small window where I can consider, let us this is a time invariance signal that we have already discuss in time domain methods.

So, that is why I take a small window from the long space signal and within that window I consider speech property is not changing. So, it is a time invariant signal if I take that then I can say the speech signal if you see this that the speech signal which I have taken s n m is nothing but a infinite series of s m multiply by a window function whose vicinity is between 0 to l minus 1. So, for the long speech signal I let this is the n th window. So, here m equal to 0 n th window. So, here m equal to l minus 1 where n plus this is n th window so, it is if it is starting point is n. So, suppose I have a this is the first window so, the from starting is 0 signal up to let us 160 sample, 159 sample, 160 sample period window ok.

Next, next window is start from 160 sample to that signal. So, I can say if it is frame rate is 10 millisecond then I can say over lap signal. So, first window may be I have taken 160 sample then shifted the window by 80 sample and I take the window from here to here. So, let us this is my n th window where I take m equal to 0 my window length is l. So, outside the window length the, I consider the signal is 0 there is a no signal. So, there is a speech signal outside the window length signal is 0 this, this is the very you can say this; this part is idea, I mean this by is the concept what is the concept in that autocorrelation things.

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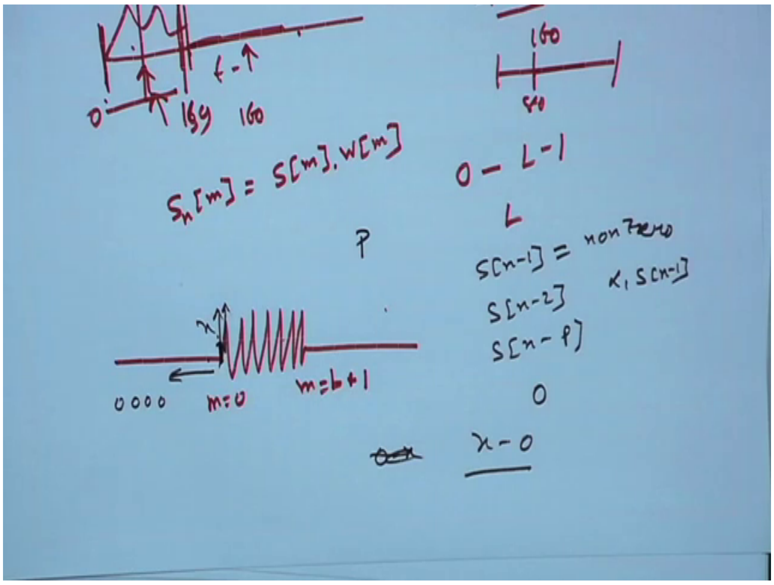

So, I can say I take a small window of signal outside the window the signal is 0, now if it is that outside the window window signal is 0 within the window signal is there. So, this is m equal to 0 this is m equal to l minus 1. Now, so if I want to predict I said the linear prediction means I am to predict the current sample from the previous sample. So, if I say the my I take the p point predictor alpha alpha alpha 1, alpha 2, alpha p. So, suppose I want to predict the first sample in here what is the pa previous sample all are 0.

So, from a 0 sample so, all s n minus 1, s n minus 2 all s n minus p all are 0. So, what is the linear combination of those 0, I get all is 0, but make first sample value is not 0. So, this is first nonzero sample value let us its value is something x. So, what is the total error 0 minus x or again sorry x minus 0 nothing I can predict. So, total signal, total sample itself is the error. So, I can say that at the beginning of the prediction the prediction error will be maximum.

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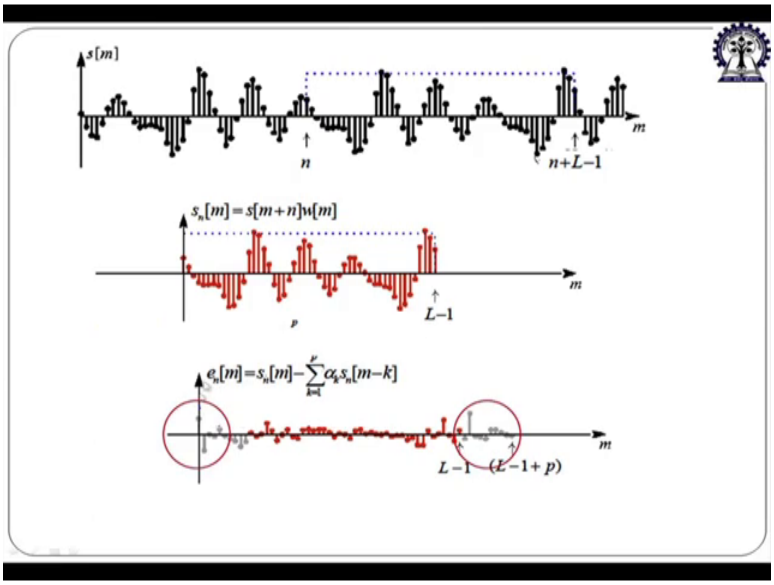

If I see the error signal at the beginning of the prediction, prediction error will be maximum because, if I say here my window is started this is the, my sample and outside the window signal if 0. So, because those samples are not there I forcefully make them 0 by windowing. So, once make them 0 I am estimating the first sample from the all 0 sample. So, I get the estimation 0. So, error the error is the sample 1 take the next sample I am estimating the next sample from previous p sample where only 1 sample is there which is s n minus 1 is nonzero. So, my prediction will be alpha 1 into s n minus 1. So, then also error will be maximum because this is not linear combination of all sample. So, once I move towards this inside the window my prediction error will be minimize, minimize, minimize ok.

Minimum, again when I want to exit from the window so, I am a l minus 1 sample, but I have p th order. So, from last sample if I, if I take the last sample is this 1 let us last sample is this one this one is the last sample.

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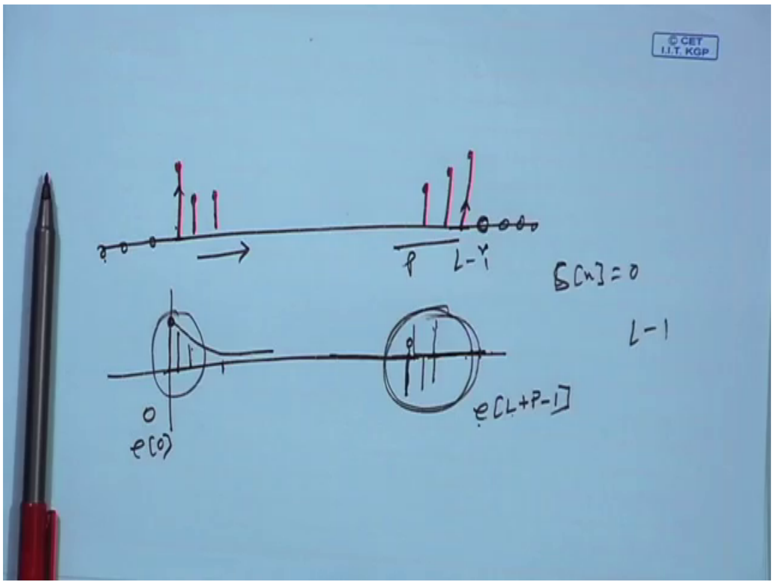

So, I can say this is my window this is the first sample, this is the second sample, this is the third sample, this is the last sample then this is the previous last sample this is the previous last sample. So, once I predict this side all are 0, all are this side also all are 0. So, one side predict this one from 0 in the error maximum error I will get, I want to predict this one all 0, but one sample is there there will be slightly reduce I want to predict this one from 2 nonzero slightly reduce.

So, once I go towards the window my error will be decreasing, decreasing, decreasing once I come in the end sample then what I want, I want to predict 0 this is my current sampling 0 from the previous p sample. So, current s n is 0. So, linear combination of previous p sample cannot be 0. So, I can say there will be a error which will be maximum. So, there will be a error in here also, all kinds of error may be may be positive, negative I do not know which kind of error it will be maximum kept it is will be negative error. So, error will be maximum at this point also. So, if I see the error error plot is started from 0, e 0 and I get e l plus p minus 1 because I have to go p sample this side also. So, my order of the my error, error, error length is e l plus p minus of because l minus 1 is this sample p order will be added here ok.

So, this is call l p c error. So, if I see the l p c error beginning of the window l p c error will be maximum and the end of the window l p c error will be maximum, if I met the signal windowed by a windows who any window, any kind of window you can say the window. So, outside the window the signal value is 0 I am not considering the sample value ok.

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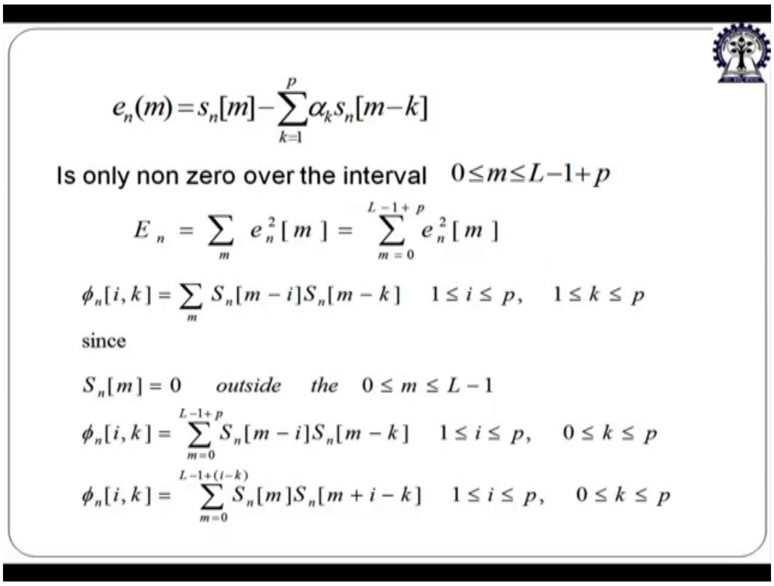

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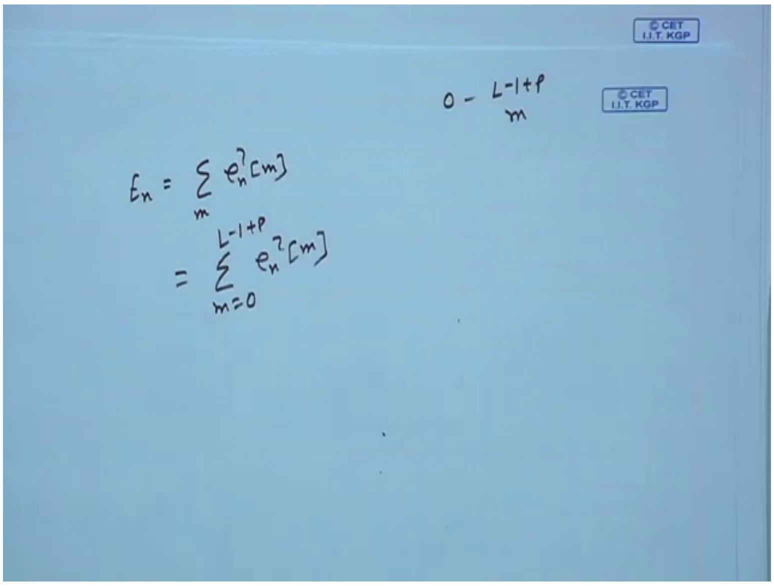

Now, if I do that then what what is do my equation. So, my equation e n is nothing but a m e n square m, now if you see m I do not have. So, where is the error error vicinity error can varies from 0 to l minus 1 plus p. So, there is a no infinite error series I can say this is varies from m equal to 0 to l 1 minus 1 plus p e n square m.

So, there is no infinite signal length, I was signal length whose varies error varies from l minus 1 plus p n square m.

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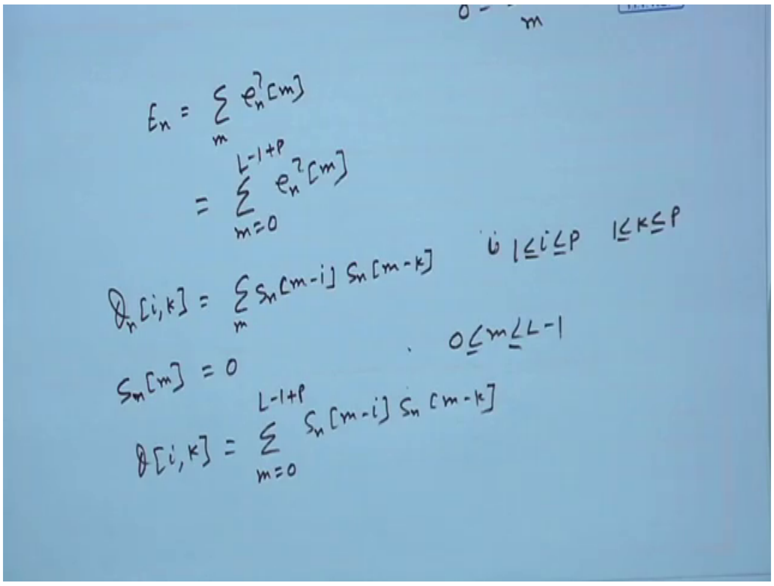

Now, I know phi n I k is equal to m s n, m minus I and s n m minus k where 1 I p 1, 1 to k p k varies from 1 to p I varies from 1 to p. So, now, s n m, s n m outside the window is equal to 0 which is only nonzero if m varies from l minus 1 So, I can say phi I k is nothing but a m is equal to 0 to l minus 1 plus p s n m minus I s n m minus k or not.

If it is that now if you see that this phi extraction of phi i k only possible if I see that this is my let us this take that s n minus.

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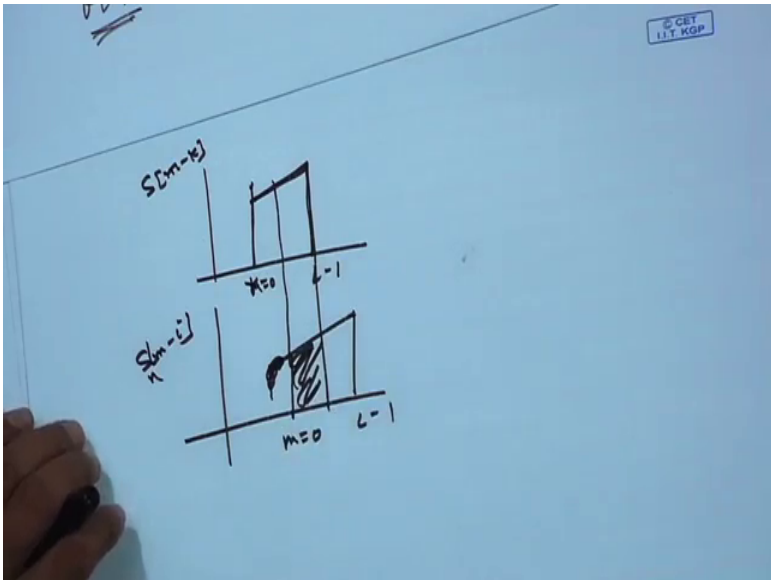

Let this is my window let this is the window of this is s n minus m minus k. So, k equal to 0 to so, this is, this is, the this is the n m minus k of window length m equal to 0 to l minus 1 and let us this 1 is my let us this one is my m equal to 0 to l minus 1 s m minus i. So, only the overlapping region will contribute to the calculation of value of phi I k.

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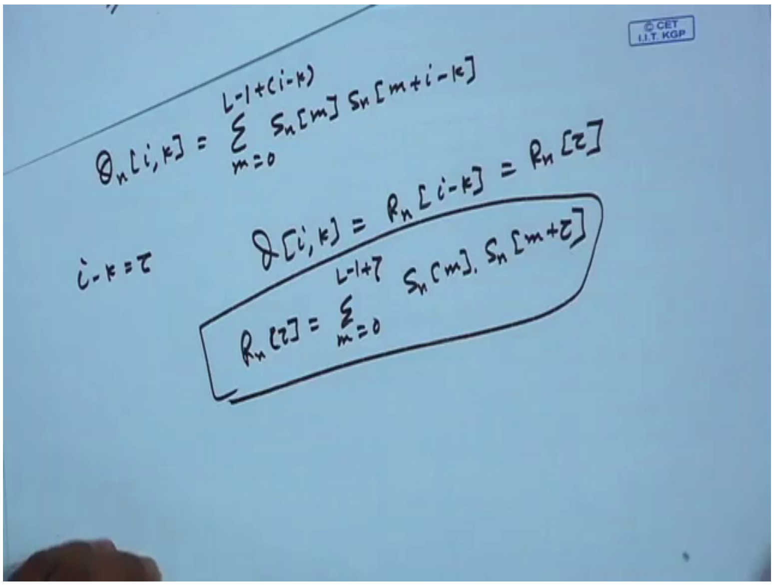

So, I can say that value that phi I phi n I k phi n I k I can rewrite m equal to 0 to l minus 1 plus I minus k s n m s n m plus I minus k.

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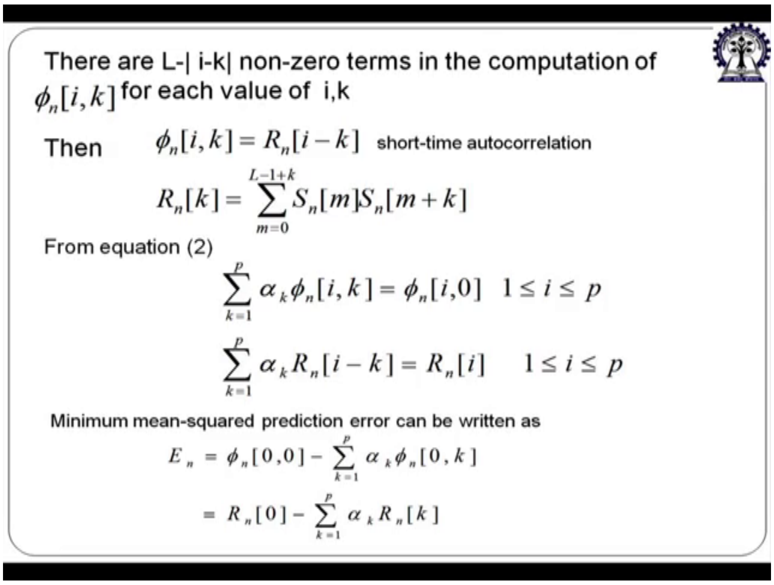

I can rewrite this 1 m minus so phi I k. So, if I see I minus k is equal to tau let us tau sample kind of tau time time. So, I can say phi I k is nothing but a R n I minus k is equal to R n tau. So, I can say R n tau is nothing but a m equal to 0 to l minus 1 plus tau s n m into s n m plus tau. So, if it is that, what is this? What is the correlation between the 2 signal correlation between the 2 signal.

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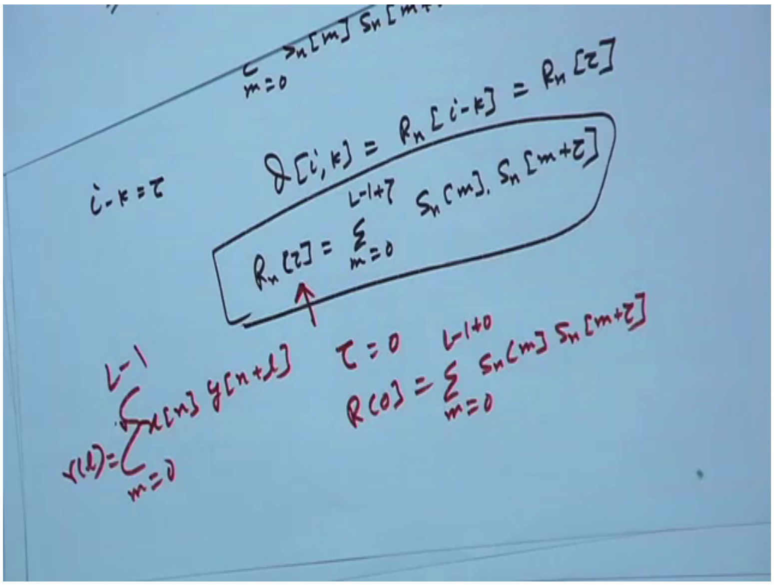

If it is x x n and y n plus l r l is equal to m equal to 0 to l minus 1.

So, I can say this is the correlation of tau R n tau. So, if it is tau equal to 0 then it will be r 0, is equal to m equal to 0 to 1 l minus 1 plus 0 s n m s n m plus tau, ok.

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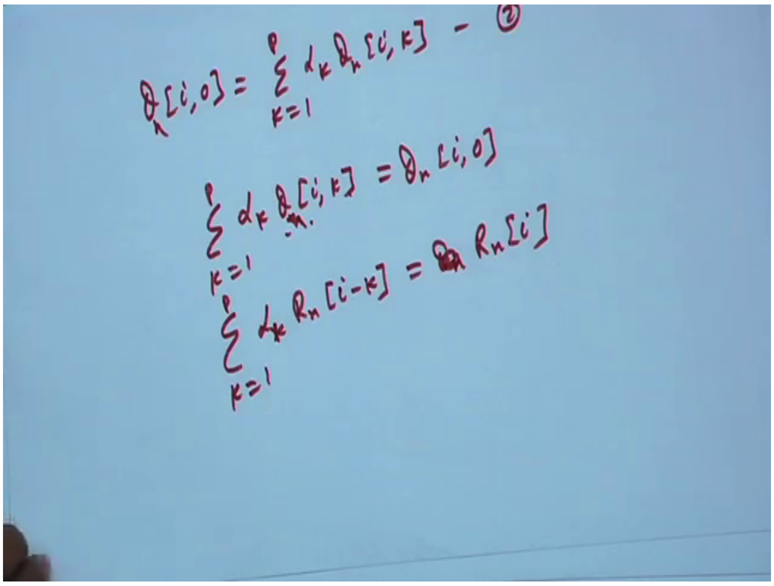

So, I can write down that let us from equation let us then you have take the equation 2 which is the equation 2, equation 2 is phi I 0 is equal to a equal to 1 to p alpha k phi n I k this is the equation number 2. So, we have already derived from here so now, from here if I write down then I can say k equal to 1 to p alpha k phi I k is nothing but a phi n I 0.

So, I can say this is nothing but a k equal to 1 to p instead of phi n I k I can write R n I minus k is equal to phi n I can say R n I minus k is k is 0. So, I can say R n I, now from the mean square error, if you see the mean square error equation we have derived.

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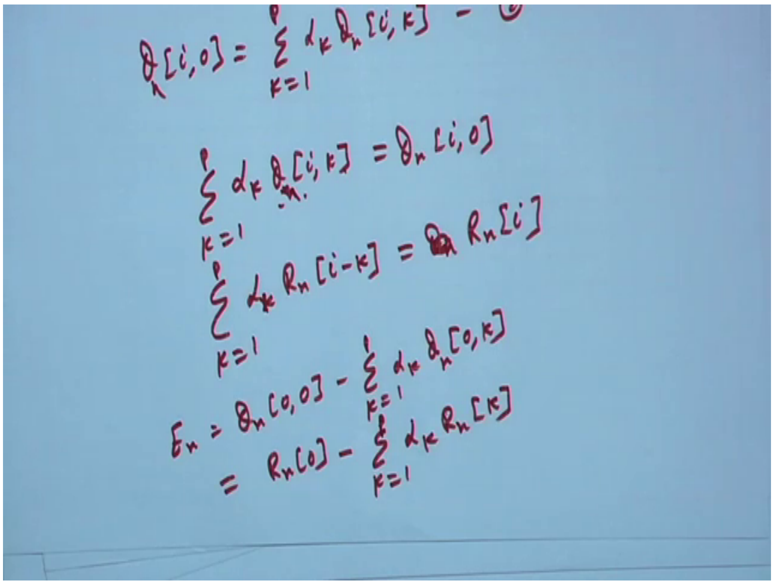

So, mean square error e n is nothing but a phi n 0 0 minus k equal to 1 to p alpha k phi n 0 k. So, that also can be written R n 0 so, phi n is replaced by R n which is autocorrelation, minus k equal to 1 to p alpha k R n k 1 I th 0 so k only.

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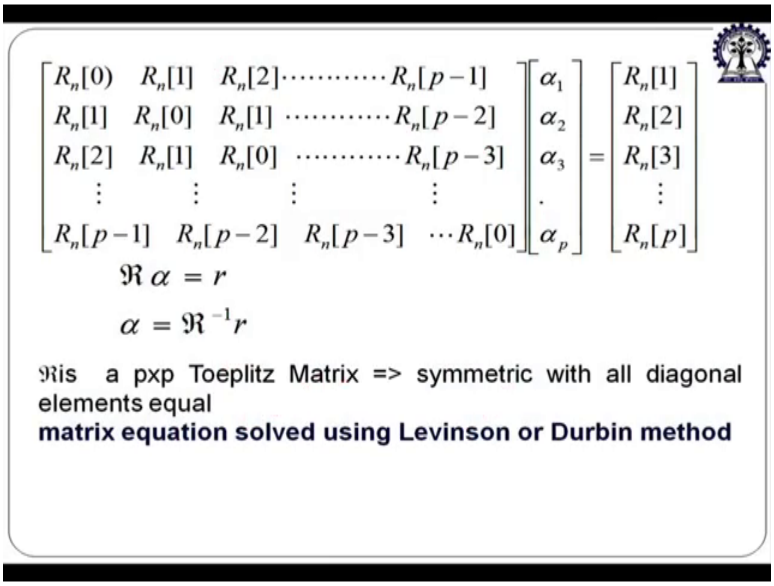

So, I can say e n is nothing but a R n 0. So, now, if it is R n 0 what is, so I can say the R n 0 so if I know the R n 0.

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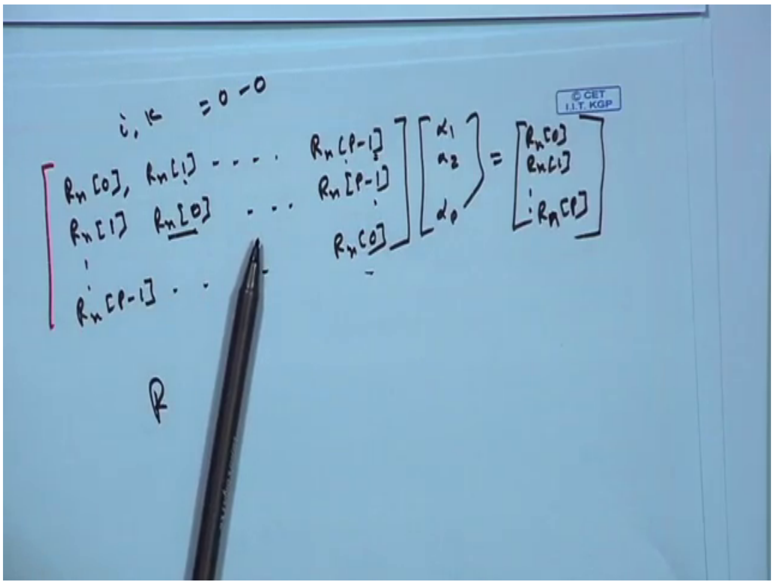

I can write down the matrix, this matrix from this equation, from this equation, from this equation I can write down the matrix. So, I can say R n 0. R n 1, dot dot dot dot R n p minus 1 this side R n 1 dot dot dot dot R n p minus 1, R n 0 dot dot dot R n p minus 1, equal to alpha 1, alpha 2, alpha p is equal to R n I so I can say R n 0, R n 1 dot dot dot dot r p R n p here is say here k I and k r there. So, I equal to 0, I k equal to 0 R n 0 minus 0 0 1 dot p minus 1 here I equal to 1 there it will be 0 p minus 1.

So, that way it will come. So, if it is that if I solve this. So, I can say r. So, if you see the r into alpha if it is capital r is this matrix into alpha is equal to r. So, alpha is equal to.

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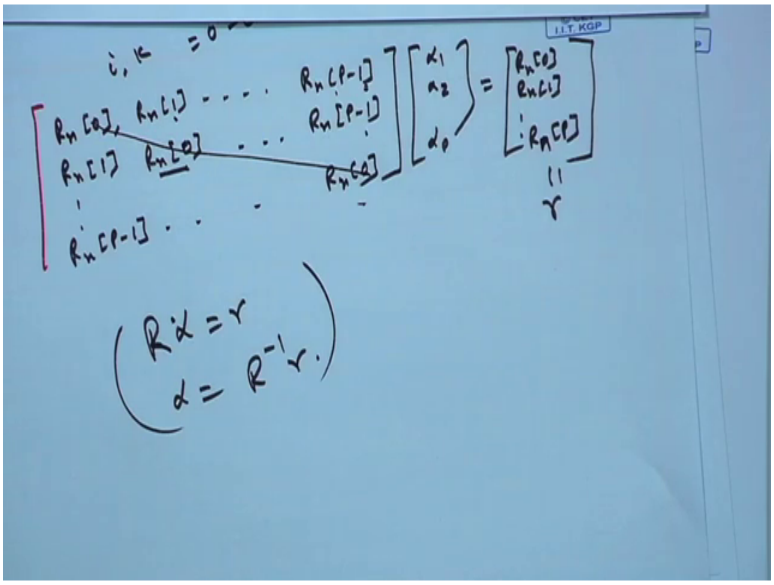

So, if the capital r is the matrix and r into alpha is equal to small r if it is this is call small r matrix then I can say alpha is equal capital r inverse small r I can solve. So, all diagonal element if you see the r and 0. So, this has a special type of matrix. So, symmetric with all diagonal elements are equal. So, matrix equation solves using then I can apply the Levison Durbin algorithm to solve this matrix equation. So, next class we try to discuss how we want to solve this matrix equation.

So, if you see that R n I have established here that R n is nothing but a autocorrelation. So, instead of phi I m I have to know R n 0.

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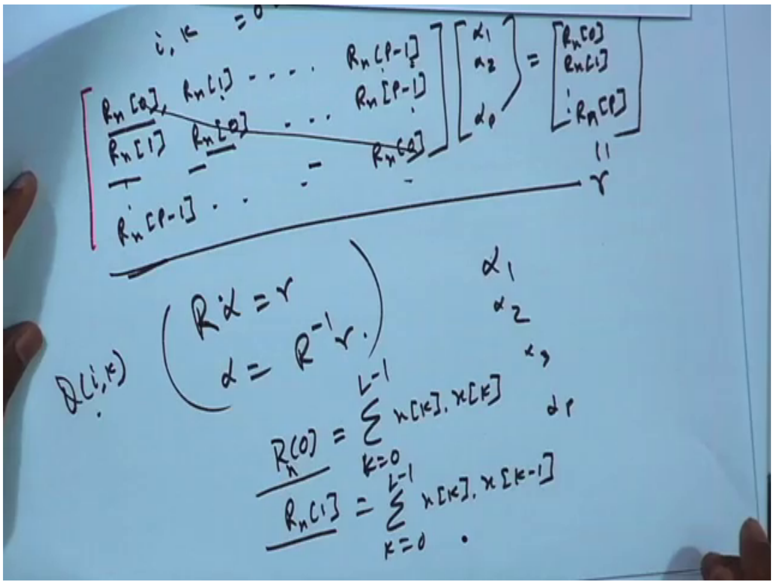

So, what is R n 0, R n 0 is nothing a first r 0, r 0 or R n 0 is nothing but a, if it is l number of sample. So, a let us k equal to 0 to l minus 1 x of k into x of k, now what is R n one is nothing but a k equal to 0 to l minus 1 x of k into x of k minus 1. So, I can calculate those autocorrelation R n values easily from the given signal and put in this matrix and using Levinson methods I can solve this matrix for value of alpha 1, alpha 2, alpha 3 and alpha p. So, instead of writing phi I k I have now writing in autocorrelation this method is called autocorrelation methods this l p c coefficient analysis.

So next class, I will try to derive those 3 Levinson recursion equation by which this matrix can be solved using this type of autocorrelation value, ok.

Thank you.