Digital Speech Processing
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Lecture - 21
Introduction to Linear Prediction

So, let us start with that new chapter which is called Analysis Synthesis of Pole-Zero model. Basically here we are dealing with that LPC analysis and LPC synthesis.

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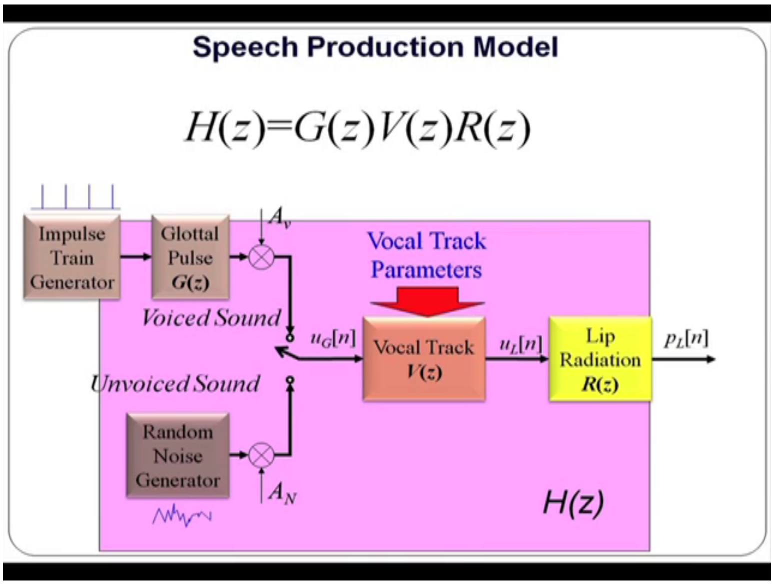

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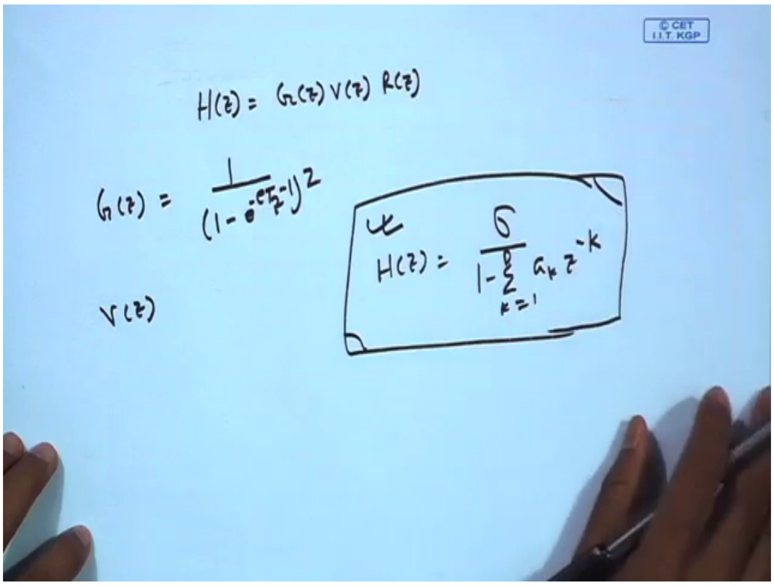

Now, if you see in the speech production system that H z is my as for the discussion in this speech production system; H z is the transfer function of the speech production system which is nothing but a G z glottal transfer function multiply by V z mobile track transfer function and radius and transfer function. So, this is the three transfer function which has to be multiplied to get that overall H z; if you see in the figure the overall H z is this.

So, if you see that how it is speech is model in digital domain less there is impulse generator; then there is glottal pulse modulator. Glottal transfer function then there is a gain; speech gain and if the speech is voiced, then it is connected though here, if it is a unvoiced you to connected to here which is random noise generator. And that u G N pass through the vocal track; find out the u L n; which is that output of the vocal track and to the lip radiation; I get the speech.

So, this is the vocal track you can say that LPC synthesis model or I can say; this is the vocal track; this is your transfer function how the vocal track is can be digitized or how a vocal track can be implement that block diagram. So, H z is nothing but a G z, V z and R z. If you remember that what is the G z?

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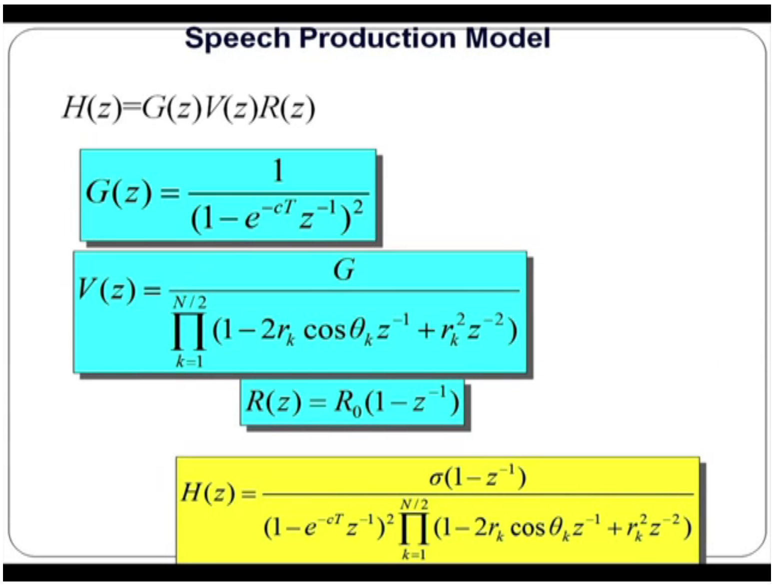

G z is nothing but a 2 pole; I can vocal track can be modal as a 2 pole 1 minus e minus c T z to the power minus 1 square and V z is already said that; if it is there is a N; n junction. So, there is a N by 2 k equal to 1 to N by 2 and this we have already derived a radiation is nothing but a R 0 into one minus z 2 the power minus 1. 1 pole is derived that radiation pattern; now if I multiply these three into get the H z.

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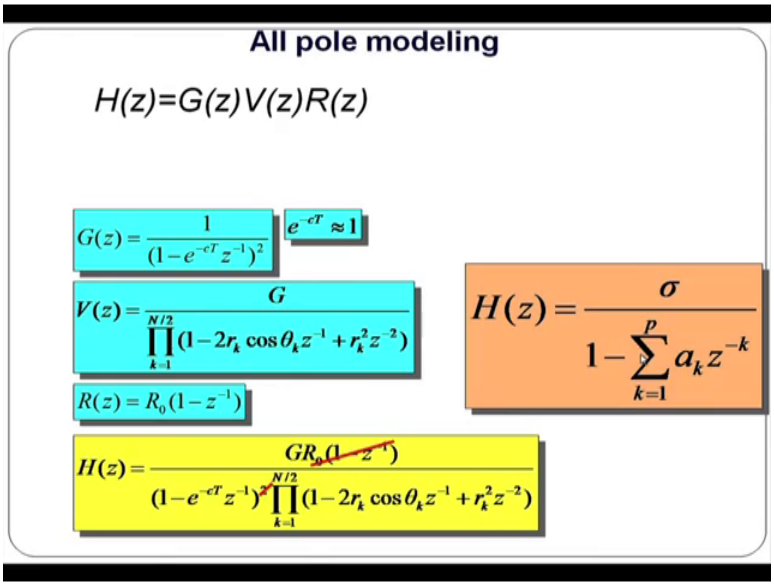

If you see that H z can be if I say that e to the power minus c T is equal to 1; c is the velocity of time T is the time period of the glottal in pulse, then I can get second glottal will be can cancel.

So, I can see the total H z is in the form of a all pole model. So, H z; I can express H z is nothing but a all pole model 1 minus k equal to 1 to a k z to the power minus k. So, this is the basis point why we do LPC analysis linear predictive analysis for voice. Since vocal track transformation can be modelized or can be model using a all pole model that is why that we have liner prediction can be possible in vocal track or you can in this speech production systems. Now, if I go to that is the linear prediction systems.

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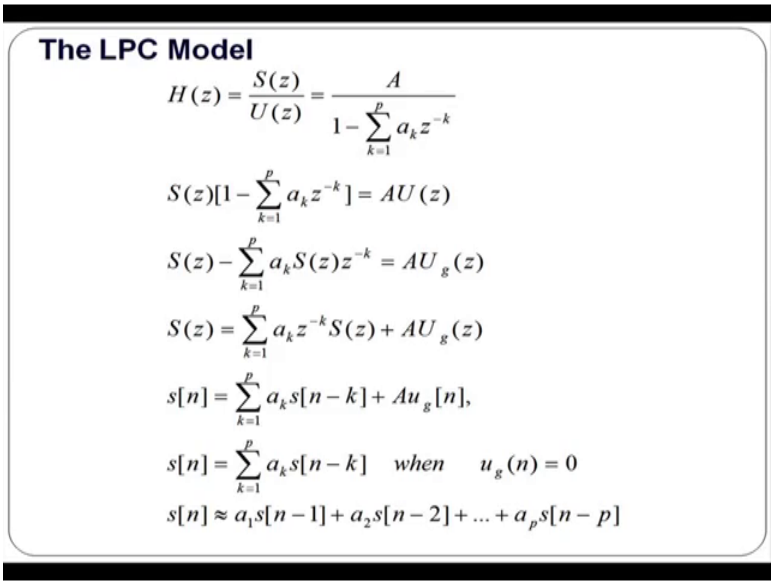

So, I can say H z; let us H z is nothing but a output speech H z; divided by input is u z.

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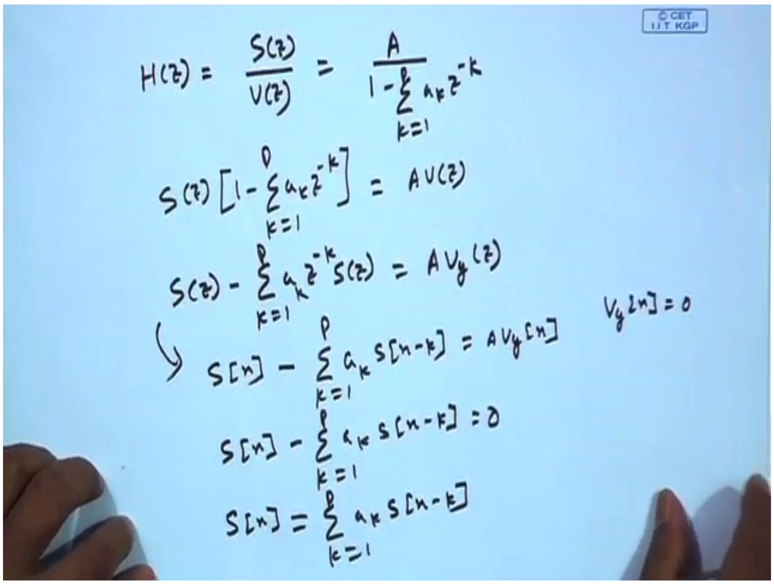

So, which can be model as A is the gain divided by 1 minus k equal to 1 to P a k; z to the power minus k. P is the number of pole which we have already studied.

Now, if it is that then I can say the S z is nothing but a S z into 1 minus k equal to 1 to P; a k; z to the power minus k can be equal to A into u z; u z is the input impulse. So, or I can write S z minus k equal to 1 to P a k; z to the power minus k is z equal to A into u G z which is the input of the vocal track; if it is voiced. So, if I see that the time domain function z domain it is like that. So, time domain S n is nothing but a S n minus k equal to 1 to P a k; S of N minus k is equal to a u G n. Now, if I consider that u G n is equal to 0 then I can say S n is nothing but a minus k equal to 1 to P a k S of N minus k is equal to 0 or I can say S n can be roughly equal to k equal to 1 to P a k; S of N minus k.

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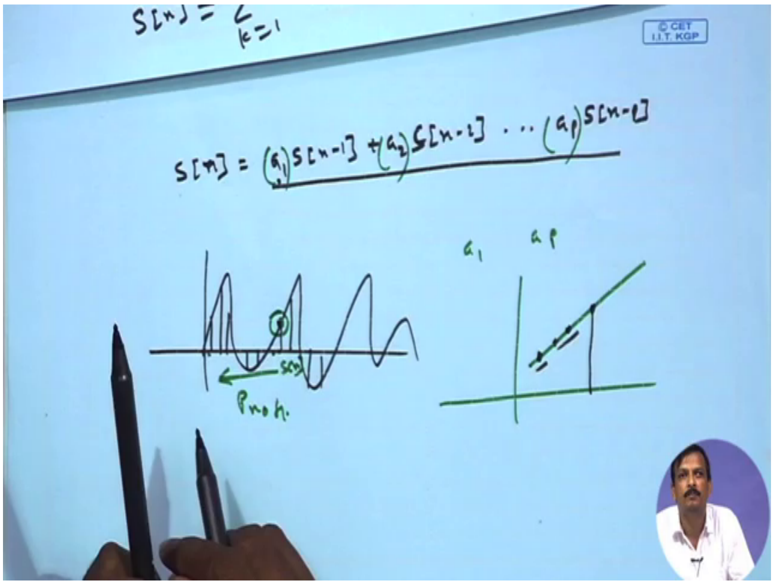

Now, if it is that; then what I get from this equation, if I say that S n is nothing but a linear combination of P b S sample N minus k; would a multiplication factor a k; I can say that for k equal to 1. So, a 1; S n minus 1 plus a 2 S n minus 2 dot dot dot dot a P into S n minus p. So, I can say that current speech sample can be a lead summation of P b a multiplication summation of P b a sample will be a multiplication factor a. So, I can say if you see the; this is the linear combination. So, I can say if suppose there is a P sample is like this and what is the P sample; this is the P signal those are the sample.

Now, I can say does a sample number; this sample or this sample can be predicted or can be generated from the previous P number of sample. So, if I get the previous P number of sample and the value of a 1 to a P, then I can say I can generate the current signal which is S n current sample S n. So, what is this? This is nothing but a linear prediction. So, why it is called linear prediction? Suppose there is a line if this is the line; if I know this point this point and this point, I can say I can predict this point by linear combination of the previous point with some coefficient factors.

So, this is called linear prediction; so, I can say I can predict current sample from previous P number of sample; with a corresponding factor a 1, a 2 and a P. So, what I get into here.

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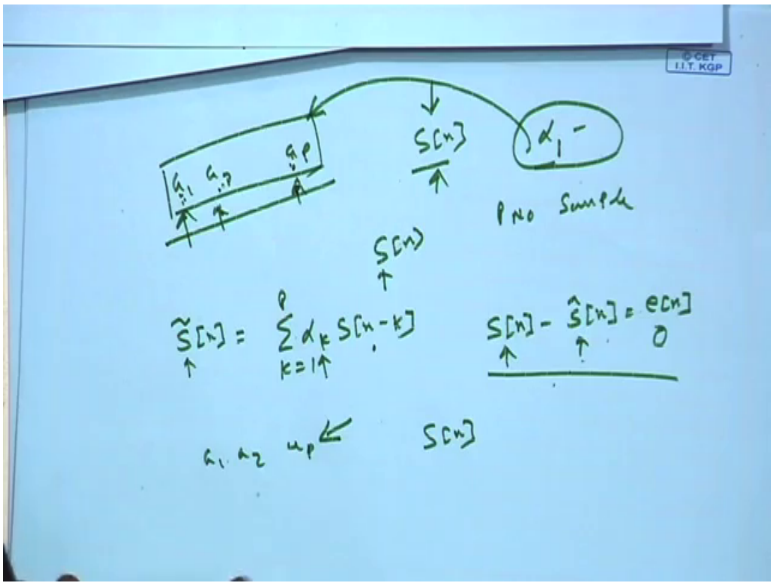

So, if I know a 1, a 2, a P; those are the call coefficient. So, those are the linear prediction coefficient I can say those are describing the properties of the speech signal and they will be multiply with the previous sample which can be implement easily by a delay. So, I can say this is nothing, but a filter; those are the coefficient of the filter and those filter if I design and all pole filter using this coefficient and if I pass the previous P number of sample; then the presence P signal can be predicted.

So, this is called linear prediction. So, there is a two problem; one is that I can generate S n or if I know a 1, a 2, a P. Suppose I do not know a 1, a 2, a P then I can say yes if I know the current sample and previous P number of sample; P number of sample then I am able to predict the value of a 1, a 2, a P for which the prediction is 100 percent correct; so, what is the prediction?

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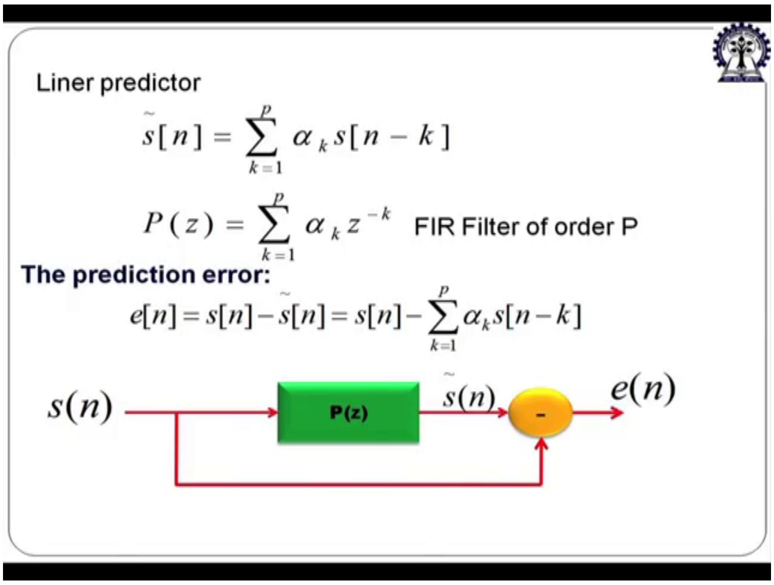

So, I can say let us my prediction signal is x cap N which is nothing but a k equal to 1 to P; alpha k, s of n minus k; these are the predicted signal. So, I can say that those are the predicted signal previous P number of sample from the previous P number of sample; I know the alpha k, I predicted the current sample which is s cap n.

Now, what is the prediction error? If I know the current sample then S n minus procured sample is my prediction error. So, s n minus s cap n is my prediction error. So, if I able to make that prediction error equal to 0; such that for the value of alpha k alpha value for whose value this prediction this error of the prediction is 0.

This my estimated signal and my original signal difference is 0, then I can say those set of alpha represented this a 1, a 2 and a p. So, I can say if I know those set of alpha I know the system; I can generate the system. So, there is a two kind of things; one is called analysis another is called synthesis. If I know a 1, a 2, a P and previous P number sample; I can synthesis the current signal or if I know the current signal and previous P number of sample; from the previous P number of sample I can predict the set of value of a for which the error will be 0,

So, one is called analysis; when you deriving the value of a 1, a 2 and a P is called LPC analysis; all pole analysis. When I know the value generating the signal S n; this is called LPC synthesis; I synthesize the signal let us describe in a block diagram in the error.

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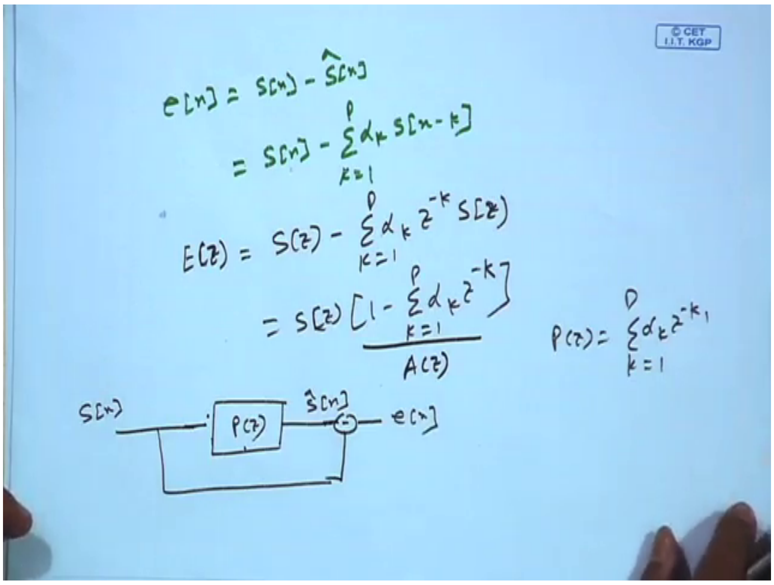

So, I can say prediction error e n is nothing but my current sample; minus my estimated sample. If I write down that; so, S n is the current sample minus k equal to 1 to P alpha k S of N minus k. So, I can say if I in the z domain e z; e z is nothing but S z minus k equal to 1 to P alpha k; z to the power minus k; S z. So, I can say S z 1 minus k equal to 1 to P alpha k z to the power minus k.

Let us I write alpha k into z to the power minus k; this portion is nothing, but a a z. So, a z; so, I can say this is my signal which is S n; if I pass through this signal through let us I write down this is called let us say estimation P z; I can get estimated signal which is S cap n. So, S n previous samples are pass through a estimator which is nothing, but a alpha k S n minus k; alpha k into z to the power minus k; P z is equal to nothing, but a alpha k. So, P z is nothing, but a all k equal to 1 to P alpha k; z to the power k. I get capital the estimated signal; now if I differentiate that those 2 signal; I get the error signal which is e n or I can system diagram, I can say from the beginning that S cap S n minus S cap N is my error.

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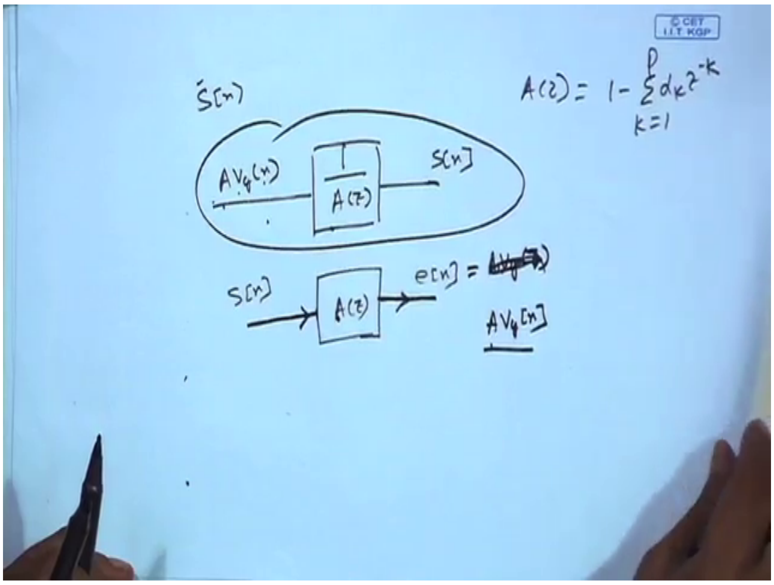

So, I can say that if I synthesize 1 by A z and if I pass the value of A u G n through this filter; I get signal speech signal S n.

Where S z is nothing but a 1 minus k equal to 1 to P alpha k z to the power minus k; this we have done; reversely if I know A z if I implement a z and pass the signal S n; I can get e n because e n is nothing but a S z into A z; So, I can say if I pass S z; S n through A z filter I get e n. So, I can say; I can get the error signal, if I implement A z and pass the presents P signal I get the error signal or which is nothing but A n is nothing but a A u G z; if there my estimation is very correct or u u N I can say; if it is N then I write A u G; n.

If my estimation is very correct; so, that only I as an error. So, if I do this one this is called analysis; if I do this one this is called synthesis; if I pass A u G n and implement 1 by A z; I can estimate S n.

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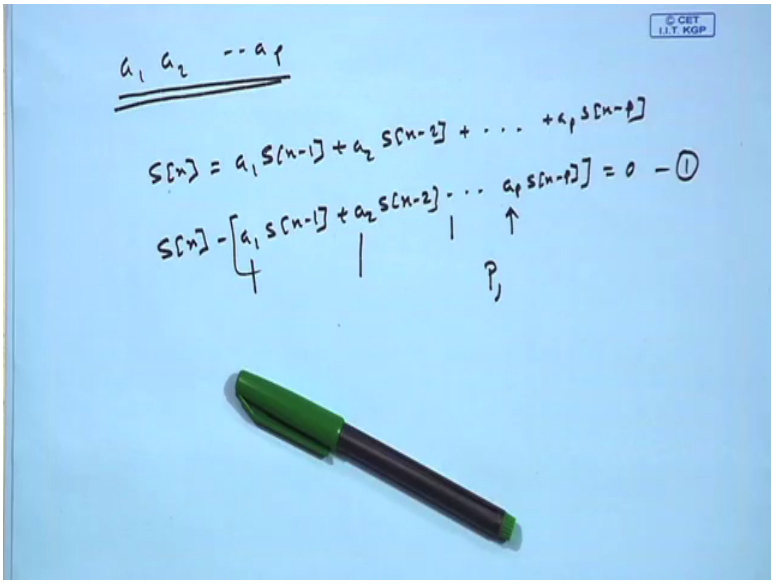

Now, what is the boil down the principle? Principle is that I am to either estimate the value of a 1, a 2, up to a P or if I know a 1, a 2, a P; I have to pass the A u G with 1 minus A z to get the signal synthesis. So, how do I estimate this values? If you see, I have only one equation; if you see this equation I have only one equation where I can say S n is equal to a 1; S of N minus 1 plus a 2; S of N minus 2 plus dot dot dot dot plus a P S of N minus p. So, I can write S n minus a 1; S of N minus 1 plus a 2; S of N minus 2 dot, dot, dot, dot; a P S of N minus P is equal to 0. So, this is single equation; how many unknown? a 1, a 2, a 3 up to a p.

So, in a P th order this equation I get P number of unknown but only single equation. So, if I have a 2 unknown how may equation I require to derived that unknown to equation?, But here I see there is a P number of unknown but only I have a single equation. The trick is that; how do I finite find out a set of solution for which this is equal to 0? So; that means, that if I know the previous N number of sample, some linear combination of those previous N number of sample; we will provide me the current sample.

So, suppose I know previous three sample; I know some set of combination of this previous three sample will provide me the current sample. So, some set of previous sample with the some multiplication, some kind of combination of this; we will provide with the current sample. Now, I have to find out which combination which multiplication factor with sample number 1, which multiplication factor with sample number 2, which multiplication sector of for sample number 3.

So, there may be a infinite set of solution; I do not know; a 1, a 2, a 3 can be with the set can we take the any value in infinite plane infinite set. So, for any value I can get this current sample but for particular some value the error will be 0. So, for a infinite set of solution; I have to find out the optimum set of solution or I can say find out the set of solution or find out the set of the way a 1, a 2, a 3 value a P value for which the either is 0.

So that means, actually I am minimizing the error to predict the value of a 1, a 2, a P up to a p. So, that is why it is called linear prediction; I am predicting. So, I can say the current sample is predicable from set of previous sample with a some linear combination, I can say if I know the current sample, if I know the previous P number of sample; I can find out for a value set of value of a 1, a P; this current sample can be properly estimated or error will be 0.

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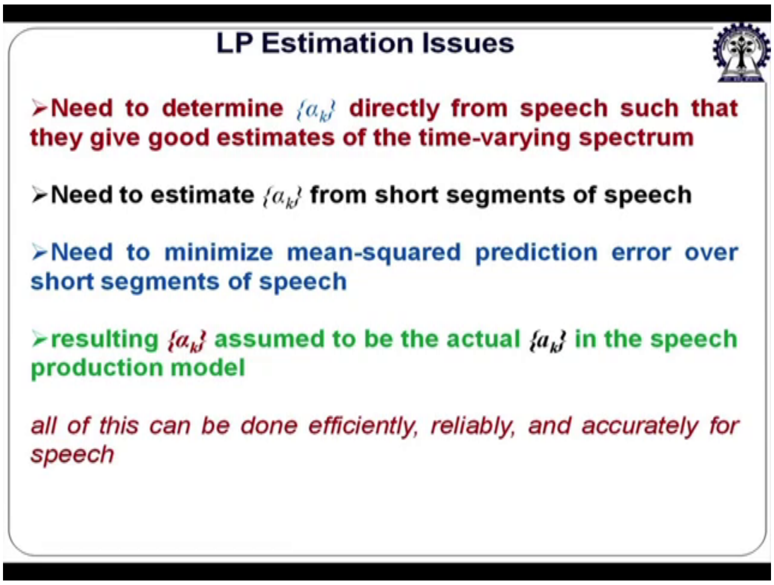

So, I have to minimize the error and find out the set of solution. So, I can say if my estimation of alpha k is let us I estimate that alpha k.

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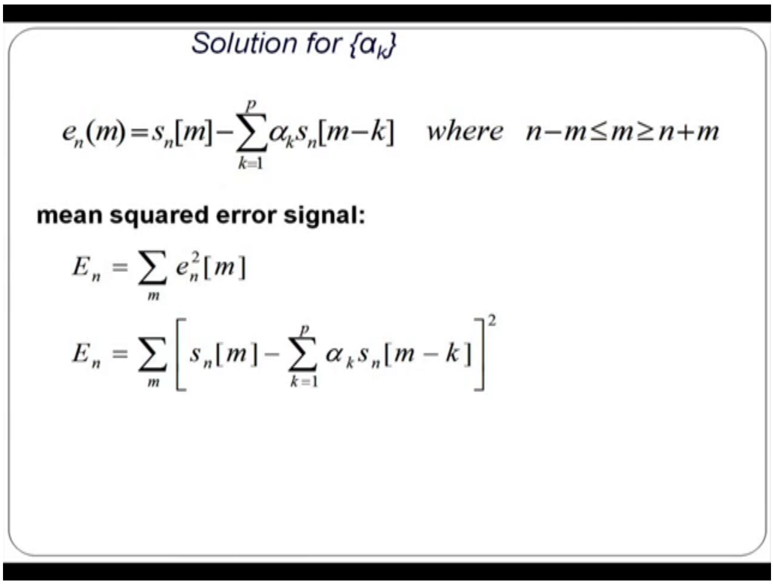

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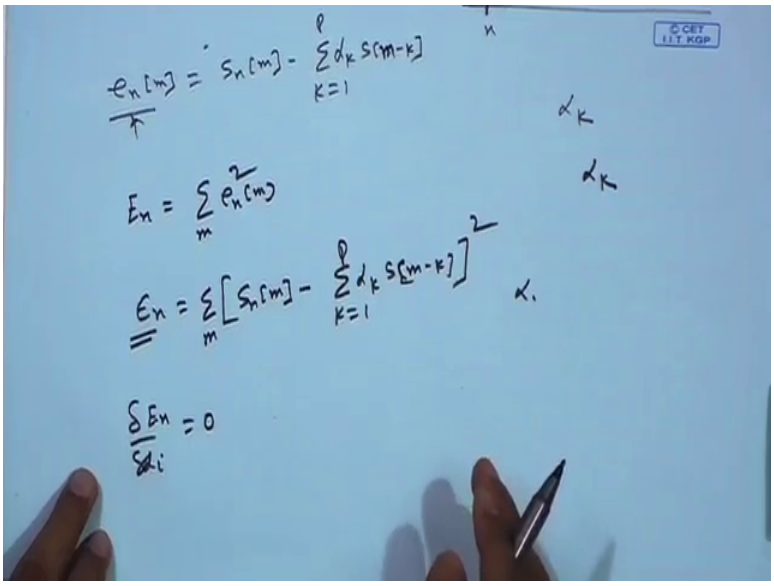

So, I can say e n error; there is a N number of signal speech signal I can take N N th position e n m is nothing but a S n m minus k equal to 1 to P; alpha k, S of N minus k. So, I can say; I have to estimate I know the error, so I have T find out the set of alpha k for which this error is minimum. So, what is the procedure to minimize the error? One of the procedure is mean square error minimization. So, I can say find out the mean square error. So, what is mean square error? e n is nothing but a m of e n m square; mean square. So, I can say e n is nothing, but a m; what is the error? S n m minus k equal to 1 to P alpha k; S of m minus k whole square; so, that is the error?

Now, what I have to know; I have to find out a set of value of alpha k for which this error is minimum; how do minimize the error? Let us take that you know that function minimization problem. So, minimize by setting what? I want to set d of e n by d alpha I is equal to 0. So, how do minimize? I have to minimize the function. So the first order differentiation with respect to for which value I for with respect to which value I minimize it; with those value is alpha. So, let us del e n by del alpha i; equal to 0, then I get the set of value of alpha k for which the function is minimum.

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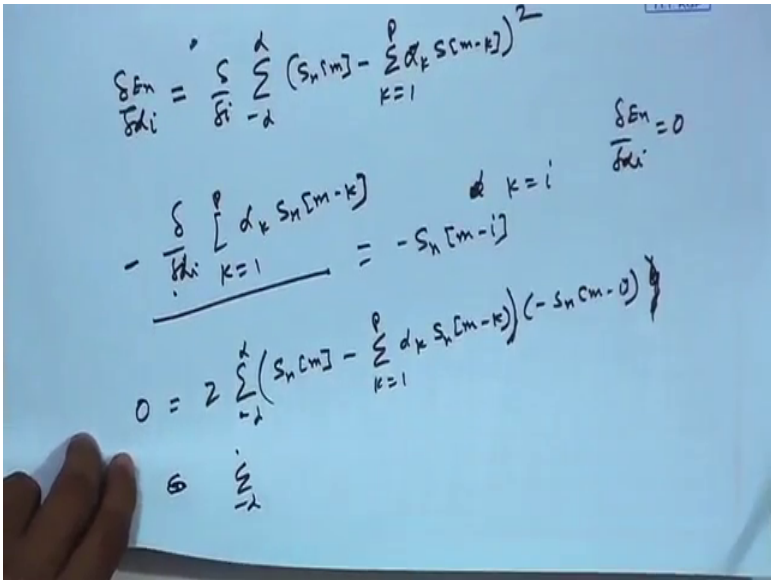

So, I can say del e n by del alpha i is equal to what? del of del alpha I minus infinity to infinity m is minus infinity to infinity S of N m minus k equal to 1 to P alpha k; S of m minus k square.

I just put the value of e n. So, if I take the derivative what I will get? I will get I am not deriving in paper; if you see the slides; if I take the derivative of this line derivative I get 2 into minus infinity.

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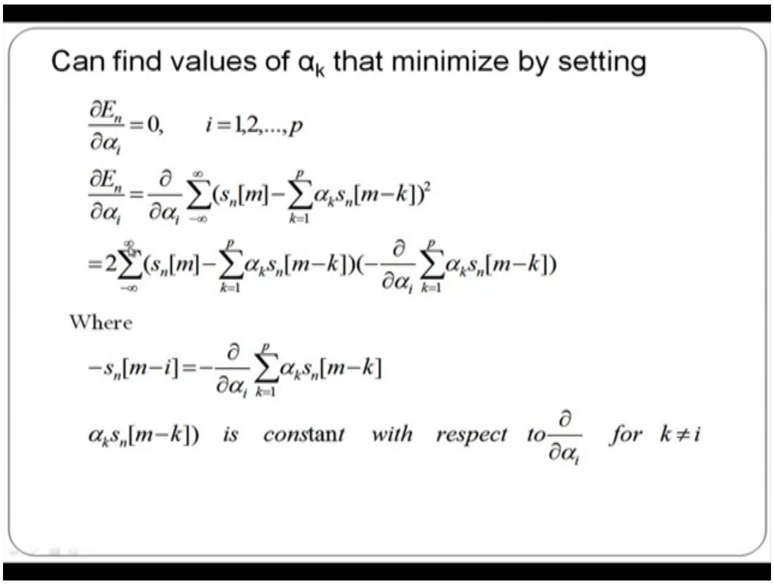

So, a minus b whole square or the S squares of 2 into whole function; again minus d by d alpha I k equal to 1 to P alpha k; S n m minus k or not. So, if it is this; now if you see this d by del i; del alpha i; k equal to 1 to P alpha k; S n m minus k if it is this. So, this derivative only exist when this k is equal to i; if k equal to i for that value this derivative exist other otherwise it is a constant. So, I can say this is nothing but a minus; so, it is minus. So, minus; S n m minus i; otherwise it is 0.

So, I can say the output of this one is nothing but a S of n m minus i. So, I can put that value in here; so, I can put 0 is equal to 2 of minus infinity to infinity S n m minus k equal to 1 to P alpha k S of N m minus k into minus S n m minus i. So, if it is that this equation because del e N by del alpha i is equal to 0; I put that value equal to 0. So, 0 is equal to this; so, I can say this is equal to this. So, I can say this there will be bracket in here also sorry bracket in here.

So, I can say phi; now if I just come out this side S n, so minus infinity to infinity; I will take other papers, let us take this one.

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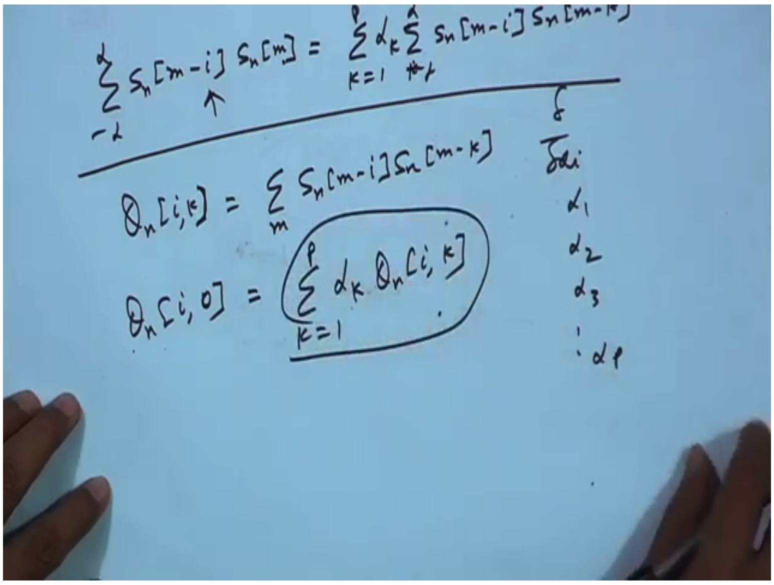

So, I can say S n m minus i; minus infinity to infinity into S n m is equal to k equal to 1 to P; alpha k k equal to minus infinity to infinity, S n m minus i into S n m minus k. Please just can do a do this; after I multiplying this factor with this two term then I can put it this side. So, I can minus infinity to infinity S n m minus i into S m; this negative I can take this side equal to k equal to 1 to P minus; infinity to infinity, S n this one; where i is equal to; i can varies from 1to P.

Because I am taking the derivative of del; del alpha i; where i with respect to first factor alpha 1 with respect to second factor respect to up to alpha P; that is why the i will be 1 to p. So, now once this is equation like this; this is my equation, then I can say I can write in matrix from phi n; i k. I can write is equal to let us like phi n; i k is equal to m S n m minus i; S n; m minus k.

If this is like this; then this is nothing but a phi n; i is there but k is 0; i is there; I can say k is 0 is equal to k equal to 1 to P; alpha k phi n; i k. So, after minimizing the function error function; I get phi n i 0 is equal to k equal to P alpha k; phi n; i k. So, I can see there is a leading to set of P equation in P unknown that can be solve in efficient manner. Now if I say; I write it in matrix form. So, what I will write? This form; so, I can write this side; if I want to write this side; how do I write this side? This side I can write; let us this one phi i k; k equal to 1 to P and i also varies from 1 to p.

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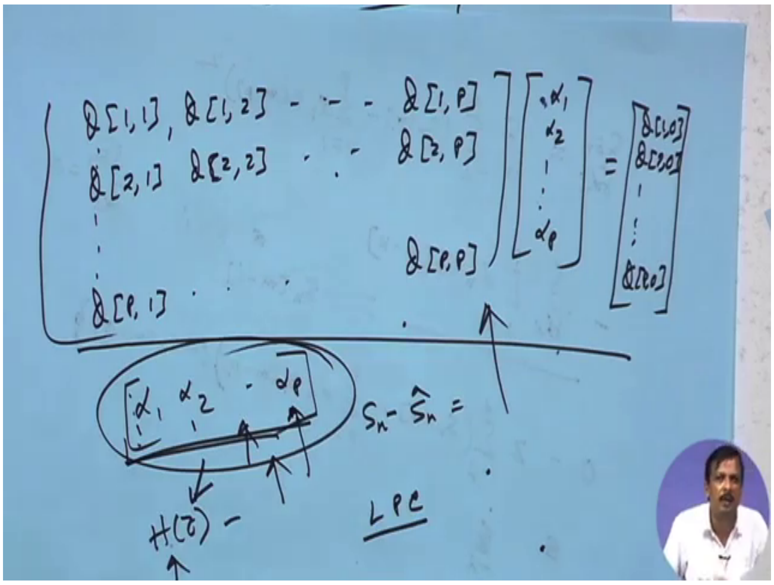

So, I can say phi let us 1, 1 phi 1, 2 dot, dot, dot, dot; phi I can say 1, P. Similarly, this side also phi 2, 1; phi 2, 2 dot, dot, dot phi 2 P; I can write dot, dot, dot, dot phi P 1; phi P, P with multiply this matrix has to be multiplied by alpha 1, alpha 2 dot, dot, dot, dot; alpha P this side I can write this matrix form is equal to k equal to 0. So, I can say phi 1, 0; phi 2, 0 dot, dot, dot, dot phi P, 0.

Now, if I able to solve this matrix I can get the value of alpha 1, alpha 2 and alpha P. Once I get the value of alpha 1, alpha 2 and alpha P for which my error is minimum; then I can say; using this set of alpha value, I can correctly estimate the signal; the difference between the my present signal and the estimated signal error will be minimum.

So, if the error is minimum that can I say; this set of alpha value actually representing the production system or representing H z; for that time. Since the speech is time varying signal; let us I take the time instant this instant I get the some speech value, for those speech value if I extract this alpha value; then I can say these alpha value can represent this H z or H z can be implemented using this set of alpha value; to generate the current signal.

So, both way I can synthesis or I can estimate. So, once I estimate this alpha value those set of alpha value represent that time the production system. So, those can be parameters for those speech events; those parameter is called LPC parameter; Linear Predicted Coefficient parameter alpha 1, alpha 2, alpha P are could the LPC coefficient.

Now, how to estimate this alpha value; I have to solve this matrix. There may be a number of method to solve this matrix; with the next class we will discuss one by one methods. So, I ultimately I have to solve this matrix, so if I want to solve this matrix there may be a number of method is available. So, using those methods; how efficiently I can solve this matrix so that I can estimate this value and that is my target.

So, first we will discuss about the autocorrelation method. So, there is a three kinds of method autocorrelation methods, matrix method; the co variance method and another one is called the latex filter methods. So, using these 3 methods; we will try to estimate the value of alpha 1, alpha 2 and alpha 3. So, next class we will do that.

Thank you.