Digital Speech Processing
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Lecture - 14
Uniform Tube Modeling Of Speech Processing Part – VI

So, last class, we derived that junction effect. If I consider the vocal track is nothing but a junction of several tube, and of the cross sectional area across the tube is different; what should be the junction effect we have discussed.

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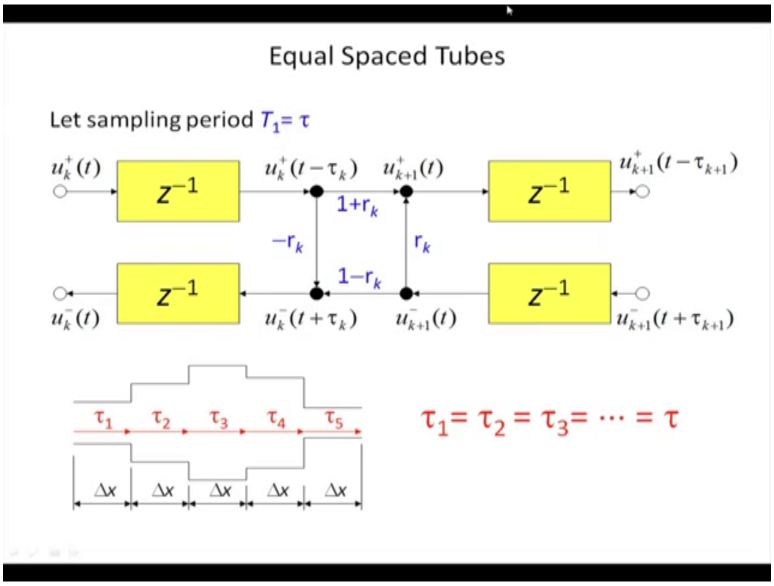

Now if I We said that if the each tube has a fixed length del x, then this tau the delay line tau can be represented by z to the power minus 1 because z to the power minus if the t one is equal to tau. So, this is the single junction k th junction signal flow diagram.

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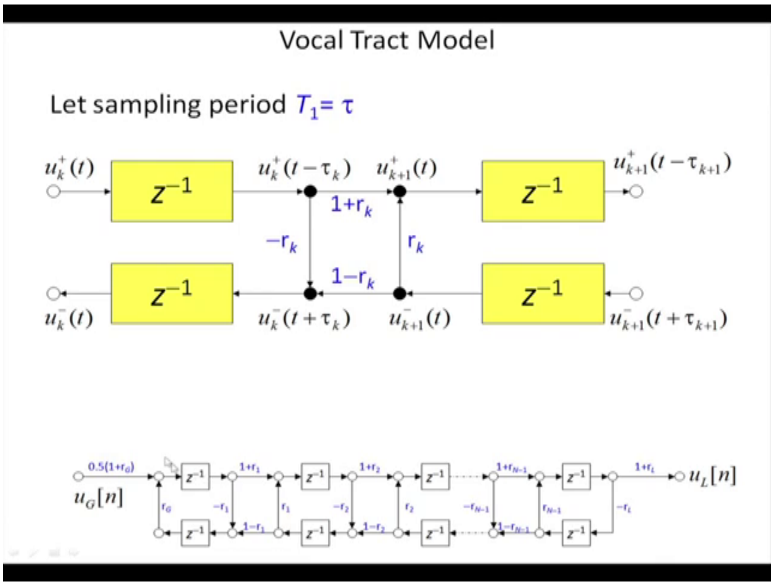

Now, if I consider the same if the tube has n number of section. So, suppose I have a tube length is l.

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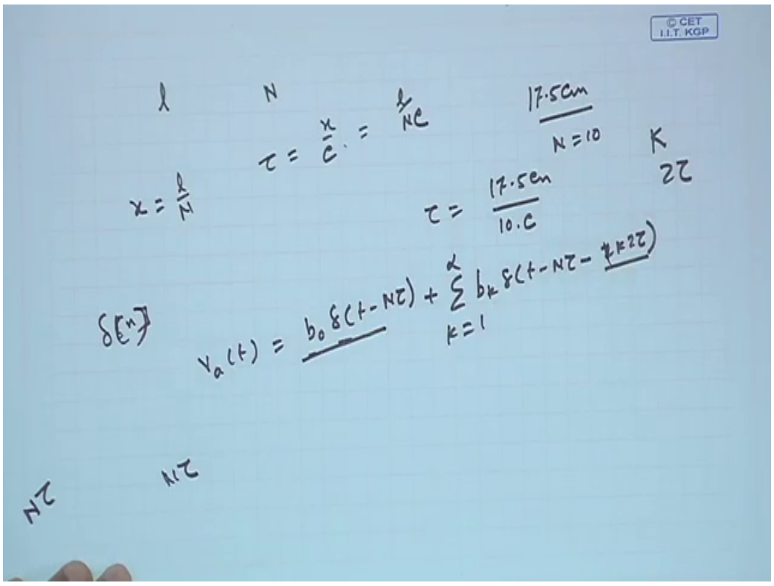

And I cut this tube in n number of section. So, the length of the each section x is equal length l by N. So, tau is nothing but a tau is nothing but a x by c.

So, it is nothing but a tau is nothing but a x is nothing but a l by NC. So, if I have a tube of 17.5 second 5 let say 5 centimeter tube long. And if I divided the tube n, n equal to 10 junction. Then the tau is nothing but a 17.5 centimeter divided by n into c, c is the speed of the sound. That much of delay is required. Now I come if my tube is n number of tube is there. So, there will be a n minus 1 are the junction 1 to n minus 1 number of junction. So, I can draw the equation like this way let us this is u G n, then the first junction, second junction, dot, dot, dot in a n minus 1 junction, and this is the and if you see the last one is the boundary condition at list. And this one is the boundary condition at glottis. So, it a r G it is half of 1 plus r G it is minus rl 1 plus rl I can draw the n 2 equivalent signal diagram. Signal flow diagram and is delay is represented by z to the power minus 1. Now if this is my tube let us u G n is nothing but a delta signal delta n delta n delta signal or delta n.

If it is delta signal then output if this is the my signal flow diagram, then output I should get bat at the output is nothing but a b 0 delta t minus n tau plus k equal to 1 to infinity bk delta t minus n tau minus 2 k 2 tau, why? So, if you see if I have a tube with n number of junction of length let us n the sec n section of junction. So, each section create a delay tau. So, n section create a delay n tau, but that is the first. First signal first delta that is why b 0 into delta n, but after all a signal the after the first round the delay will be there. So, what is the delay 2 tau is the round tip delay. So, this is the first arrival of the signal. Second will be arrival you know due to the back propagation. So, there will be a 2 tau delay.

So, earliest arrival is n tau, which is n number of tube is section delayed by tau. Next signal will come by 2 tau delay. So, I can (Refer Time: 04:23) next signal will be integer multiple of 2 tau delay. So, that is why I said 2 k sorry, k 2 tau k into 2 tau. K is the integer and 2 tau is the delay. Now if I consider to avoiding the alysing of digitization process, if I consider my sampling period t is equal to 2 tau.

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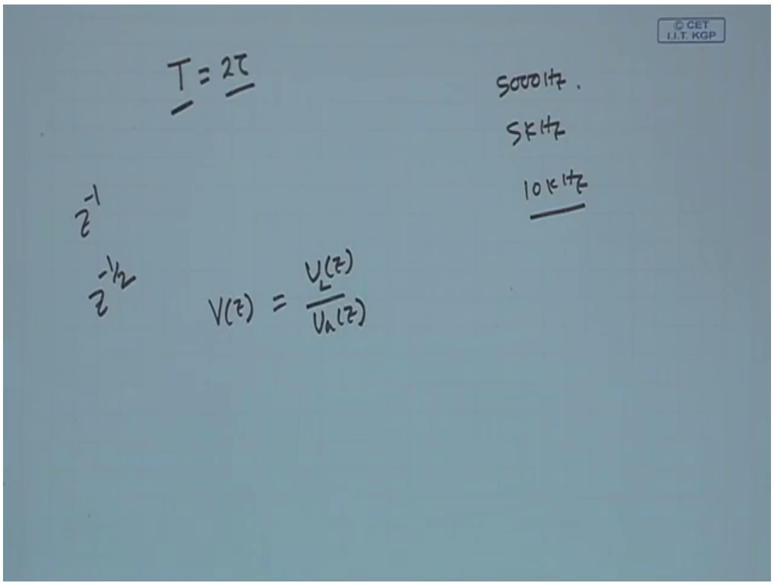

So, my sampling period t is equal to 2 tau. What I am saying suppose I want to create a band limited signal or let us p signal whose maximum frequency is 500 hertz, the 5 5 kilo hertz or 5 kilo hertz let us 5 kilo hertz is the maximum band width. What is the sampling frequency? 10 kilo hertz, 10 kilo hertz is the sampling frequency. So, band limited is 5 kilo hertz nyquist criteria is 10 kilo hertz, to if I if I want to reach the nyquist criteria in this 2 tau delay. So, 2 tau must be equal to the sample period. So, 2 tau is equal to sampling period if 2 tau is equal to sample period then how do I derive this V z?

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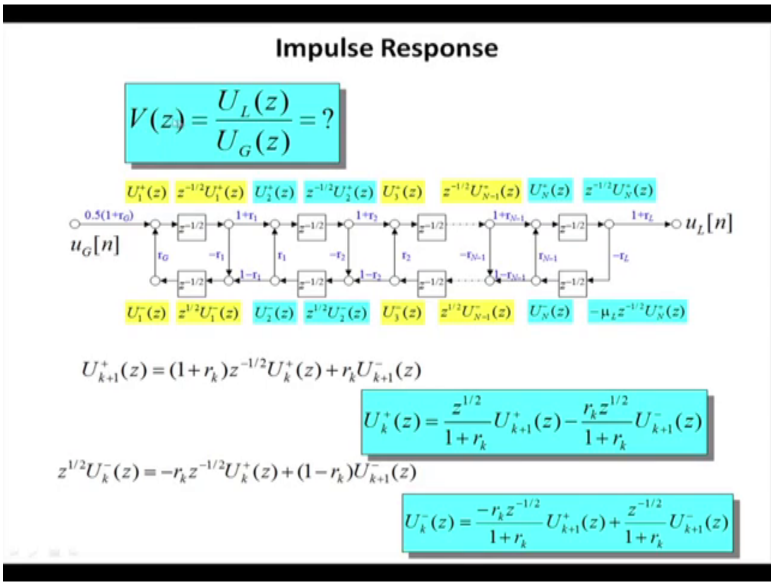

If we if I consider this is a z domain vz which is nothing but a u L z divided by ug z. And if z to the power minus 1 now z to the power minus 1 is single sample delay. Now my sample delay is t is equal to 2 tau. So, I can say instead of z to the power minus it will minus 1 to z to the power minus half. So, all the z value will be replaced by z to the power minus half, z to the power minus half, z to the power minus half, z to the power minus half. So, I get this signal flow diagram. Once I get this signal flow diagram can I derive the vz transfer function of the tube which is nothing but a output which is uz u L z divided by u G z z domain. So, u G z I know it is nothing but a impulse or gotal or gotal response. Now what is ulz? So, I could derive this transfer function.

So, how do I derive it using a signal flow diagram I can derive it. So, let us there is a procedural the for derive it. So, let us use some procedure of it is a some simplest this may be the simplest form procedure for deriving he transfer function of vz, which is nothing but the u L z divided by u G z. How do we derived it? If I consider each one of the junction is acts as a lattice, if you see the symmetrical, this is r 1, this is r 2, this is r n minus 1. But all are symmetrical. Now let us I draw in here because for our understanding if I draw it here it will be very clear to you.

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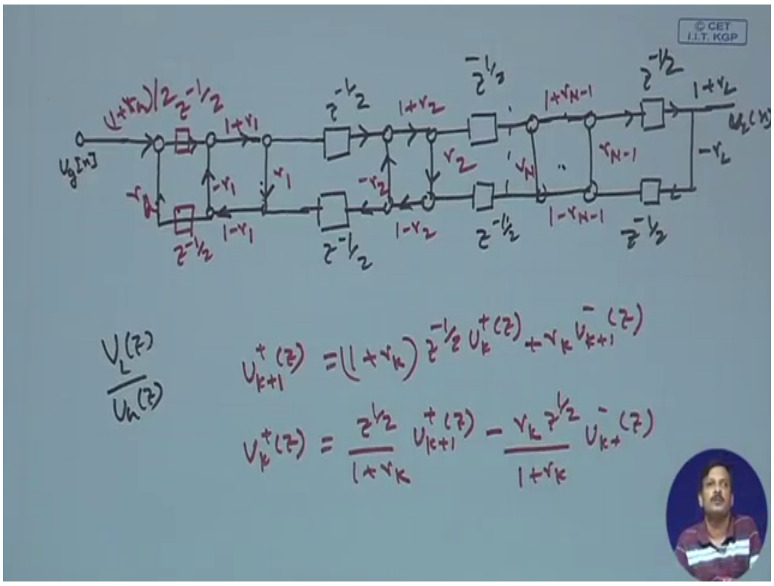

Now let us this is nothing but a my u G n coming to the first junction, let us first junction is here which is nothing but a I will first draw the junction and then I write down the signal flows. This is this way, this is this way, this is this way and this is this way. So, this I write in the red pen. So, this is nothing but a 1 minus r 1, this is nothing but a 1 plus r 1, this is R 1 this is minus R 1 ok.

Now, I have a u G. So, I have a ug is here u G n is coming to here. And then there is a terminal at glottis which is nothing but a. So, there will be z to the power. So, let us there will be z to the power minus 1 delay, z to the power minus half here will be delay, z to the power minus half and this will be r G. And this will be 1 plus r G divided by 2. And then the same junction I can continue there will be a delay z to the power minus half there will be a delay z to the power minus half and again there will be a junction. So, junction let us signal flow diagram. This is this, this is this, this is this, again this will be 1 minus R 2 this will be 1 plus R 2 this is R 2 this is minus r 2.

Then I can say there will be again delay again delay z to the power minus half z to the power minus half. Again there will be a junction let us this is nth junction n minus 1 junction, there will be a junction this is the let us n minus 1 junction. So, there will be a lot of junction in here. So, this if it is n minus 1 junction this is 1 plus r n minus 1, this is 1 minus R N minus 1 this is R N minus 1 this is R N minus 1. And at the end lip there will be boundary condition. So, there will be a z to the power minus 1 then put the boundary condition which is like this, z to the power minus half z to the power minus half and this is nothing but a minus rl this is 1 plus rl I get vln, this is output.

So, I have to find out you if it is z domain the uz u L z divided by ugz, that I have to find out. So, how do we find out let us calculate let us calculate stage by stage. So, if I say any u. So, you know that U k plus 1 plus z is nothing but a 1 plus rk, that this oh only we derived 1 plus R K z to the power minus half ukz U k plus z plus 1 z. Plus R K into U k plus 1 minus z. That we have derived last day. U k plus 1 z the volume velocity which is injected in the k plus 1 tube is nothing but this one. From here I can find out what is U k plus z is nothing but in term of k plus 1 tube. So, it is nothing but a.

So, z to the power minus half if I say it will come z to the power plus half divided by 1 plus R K into U k plus 1 plus z minus this will be come this side. So, minus R K z to the power half divided by 1 plus R K U k plus 1 minus z. So, this is you can say U k plus z U k plus at kth tube U k plus z is nothing but this one. Similarly I can find out what is U k minus z.

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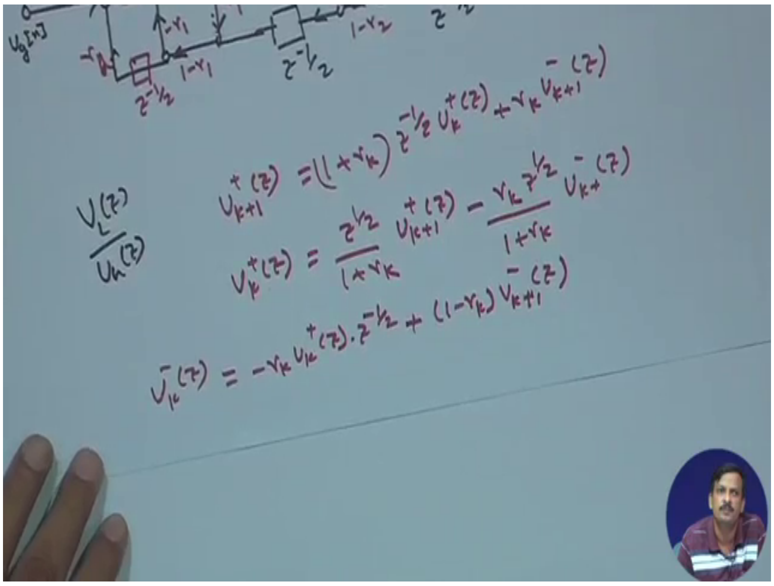

So, what is U k minus z? Already we have done U k minus z is nothing but a minus R K into U k minus R K into U k plus z into z to the power minus half, plus 1 minus R K U k plus 1 minus z. This we have already done, already derived.

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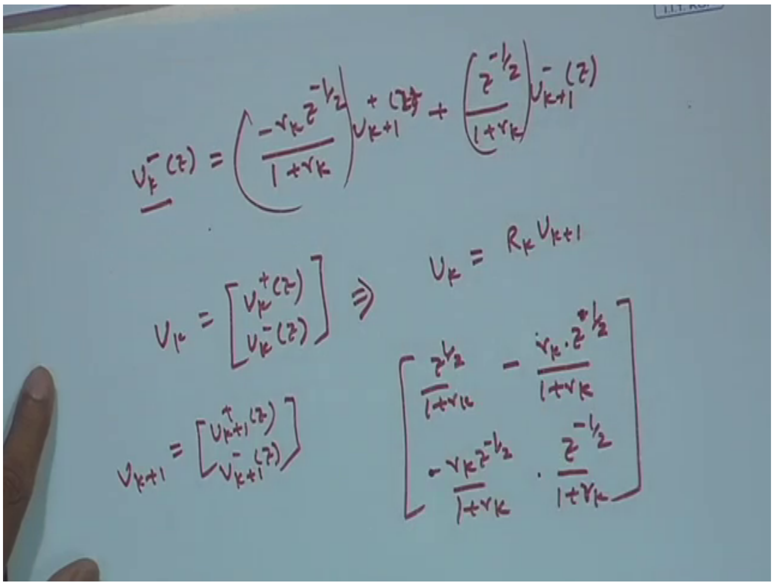

So, from here I can say U k minus z is nothing but a minus R K z to the power minus half divided by 1 plus R K U k plus 1 plus z, z plus z to the power minus half 1 plus R K U k plus 1 minus z, I can do that k plus 1. So, I this I represent U k plus z U k minus z in term of U k plus 1 z U k ba ba U k pla U k plus 1 plus z U k plus 1 minus z.

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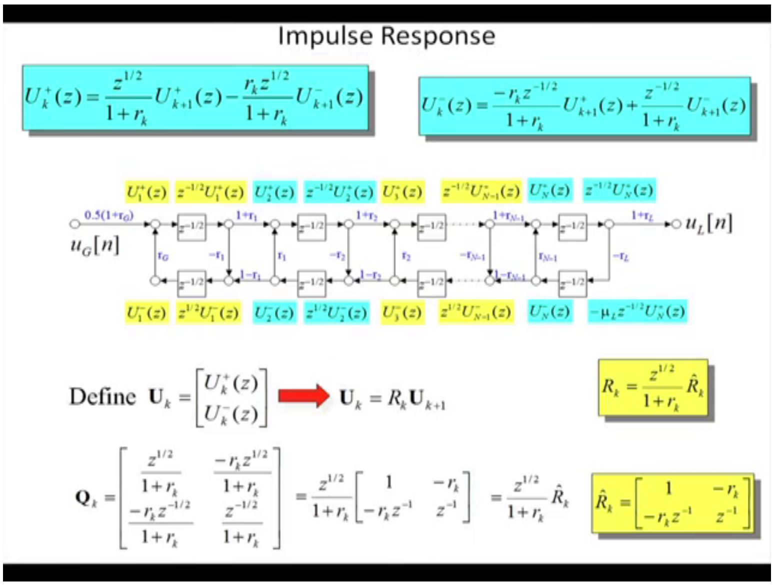

Now once I do that, then if you see here. This is my U k plus equation and this is my U k minus equation. Once I do that now if you see that if I say matrix this this this all us U k 1. So, u 1 u 1 plus u 1 minus the u 1 minus z u 1 plus u 1 minus z you can see that blue and yellow. So, those are the equation. Now if I say let us U k is nothing but a consist of 2 plus U k plus z and U k minus z. Whole U k is nothing but a forward wave and backward wave, and from here I can write U k is nothing but a R K into U k plus 1.

If I write that if you see this is the 2 equation U k minus and U k plus U k. So, if it is U k plus 1 is nothing but a U k plus 1 plus z and U k plus 1 minus z. This has to be multiply by this matrix equation what is the matrix? Z to the power half divided by 1 plus R K minus R K into z to the power minus half a my z to the power half divided by 1 plus rk. And another one is minus minus R K z to the power minus half divided by 1 plus rk. And z to the power minus half divided by 1 plus R k. So, those coefficients I write in matrix form. If I write this matrix from then I can say I simplify this matrix z to the power minus half 1 plus R K I can write down here.

So, it is 1 minus R K divided by this one a 1 minus R K 1 minus R K minus R K z to the power minus 1 z to the power minus 1. Now if I see these matrix R K cap and this whole matrix qk is equal to rk. So, I can write R K is nothing but a z to the power minus half. So, qk is equal to R K I can write. So, R K is nothing but this one. Now if it is that U k equal to U k equal To R K pla into U k plus 1.

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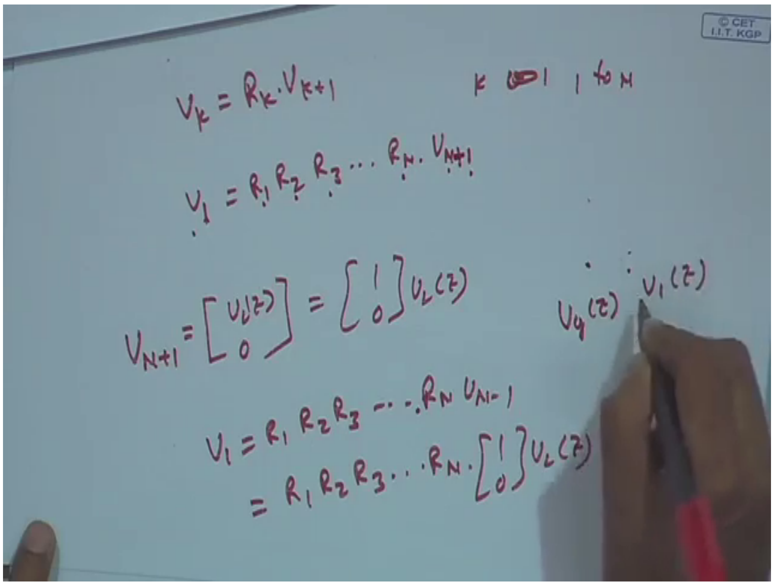

So, if I say k varies from 1 to N, that is 0 to 1 to N 1 to N if I k varies varies with the 1 to N, then what will happen? Then u 1 I can say is nothing but a R 1 R 2 R 3 dot dot dot R N into u n minus 1. Sorry u u n plus 1. I can write k varies from 1 to N let us if it is k varies from 1 to N u 1 is nothing but a R 1 R 2 R 3 dot dot dot r n into un plus 1. Now if you see that this diagram there is a nth n minus 1 symmetry junction. So, this is 1 2 and this is n minus 1. Here the junction is not symmetry in rl cases only problem is that there is a no backward wave in here. Let us I put a backward wave with 0. So, there will be a I can put a backward wave here which is nothing but a 0. So, it is 1 minus rm this is 1 plus rm 1 plus rl let us rl is equal to rn. So, it is 1 plus R N and this is minus R N if this is plus R N this is minus R N I can write I can write. So, see this here in the slides if you see I added this block.

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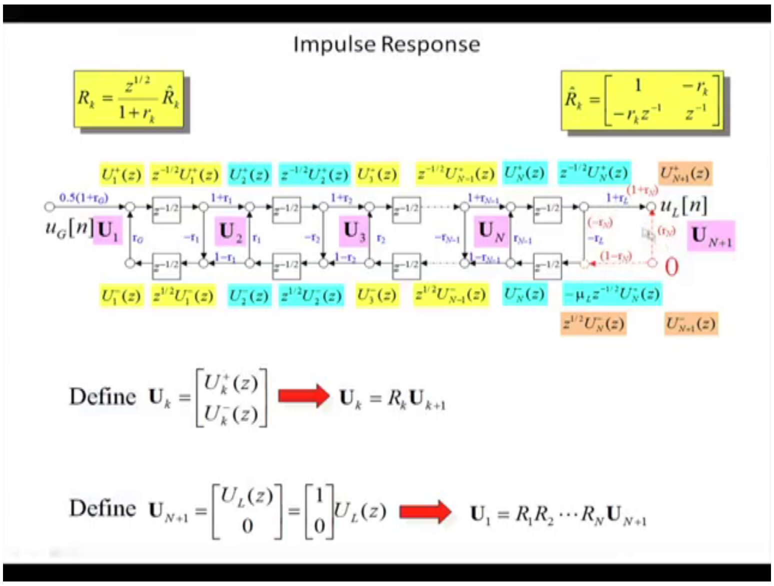

So now, there is a n number of block instead of n minus 1 block; I can get n number of block with input here backward wave is 0. So, I can write u n plus 1 un plus 1 is equal to nothing but a U L z and 0. Because here u L z is only 0 is added with l ulz. So, U L z plus and minus wave. So, I can write it is nothing but a 1 0 U L z. And u 1 equal to R 1 R 2 R 3 dot, dot, dot, dot, R N u n minus 1. So, it is nothing but a R 1 R 2 R 3 dot dot dot R N into 1 0 ulz.

Now u 1 I get now I have to find out what is the relation between u G z and u 1 z. So, I have to consider the boundary condition at the glottis. So, what is ug relation between the u G z and U L z u G z.

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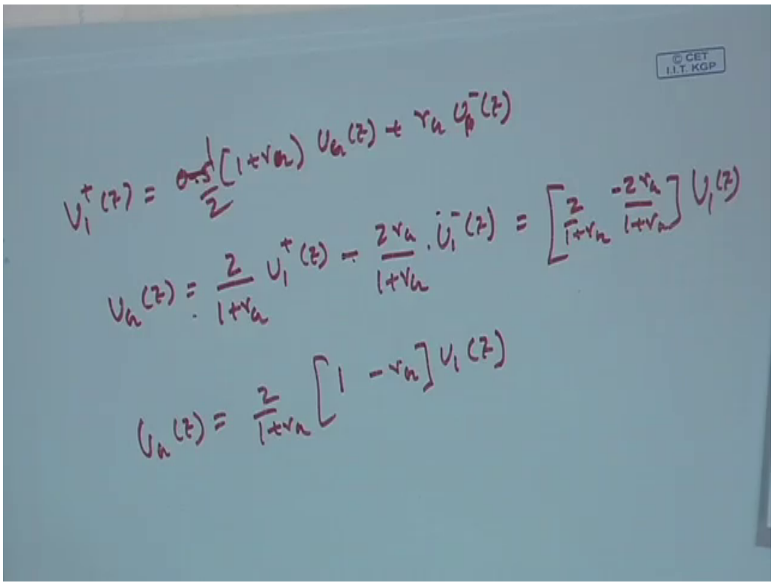

So, you 1 plus z is equal to 0.5 1 plus r G 1 plus r G u G z plus r G ug u 1 minus z I can write. So, I can say u G z is equal to 2 by 1 plus r G. So, 0 point half means or you can write 0.5 by 1 plus r G 2. This is 2 2 by 1 plus r G because this is half only. 2 by 1 plus r G u 1 plus z minus 2 r G by 1 plus r G u 1 minus z. So, I can say it is nothing but a 2 coefficient 2 by 1 plus r G one and another is 2 r G divided by 1 plus r G into u 1 z. So, u G z is nothing but a 2 by 1 plus r G it is constant it is nothing but a 1; this is minus I think this is minus minus. So, it is minus r G into U 1 z.

So, I can write u 1 is equal to R 1, R 2, R 3, R 4, R N into 1 0 ulz. So, I can write u G z.

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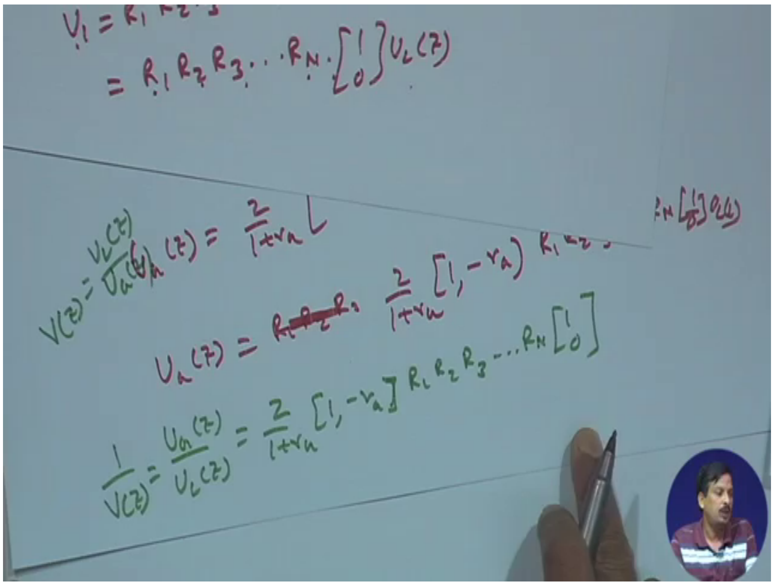

Is equal to R 1, R 2, R 3, sorry, u G z equal to 2 by 1 plus r G 1 minus r G. So, instead of u 1 z I can write R 1, R 2, R 3, dot, dot, dot, R N 1 0 u L z. Now I get u G z divided by u L z is equal to 2 by 1 plus r G 1 minus r G R 1 R 2 R 3 dot dot dot dot R N into 1 0. I can say that. So, which is nothing but a 1 by vz 1 by the transfer function. So, V z I want to find out V z which V z is equal to u L z divided by u G z. So, it is nothing but a 1 by vz. So, 1 by V z I can write in term of 2 by 1 plus r G 1 minus r G R 1, R 2, R 3, R N, 1 0.

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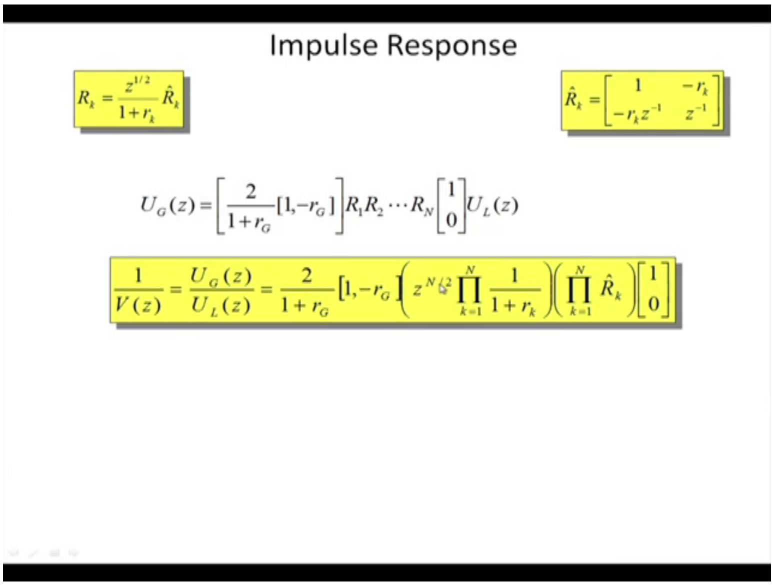

Now if I write that then if I put the value of R 1 and R 2 the equation will becomes 2 by 1 plus r G 1 minus r G z to the power n by 2. So, if you see the R K is z to the power half. So, I can say if it is n.

So, it is z to the power n by 2 k equal to 1 to n 1 by 1 plus R K. R 1 z to the power minus half for R 2 z to the power minus z to the power half for R 3 z to the if it is R N product of z to the power half z to the power half z to the power half. So, n number of So, it is nothing but a z to the power n by 2 k equal to 1 to n 1 plus 1 plus R K k equal to 1 to n cap R K cap. So, R K is like the matrix is this and 1 0. So now, if I say my n is equal to 2 then I can find out the impulse or transfer function of the tube model. So, if I simplify this thing for 2 tube.

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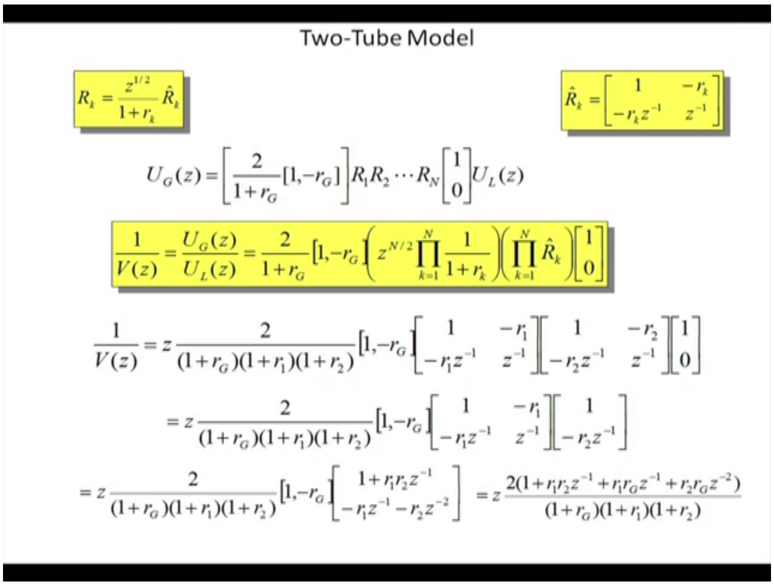

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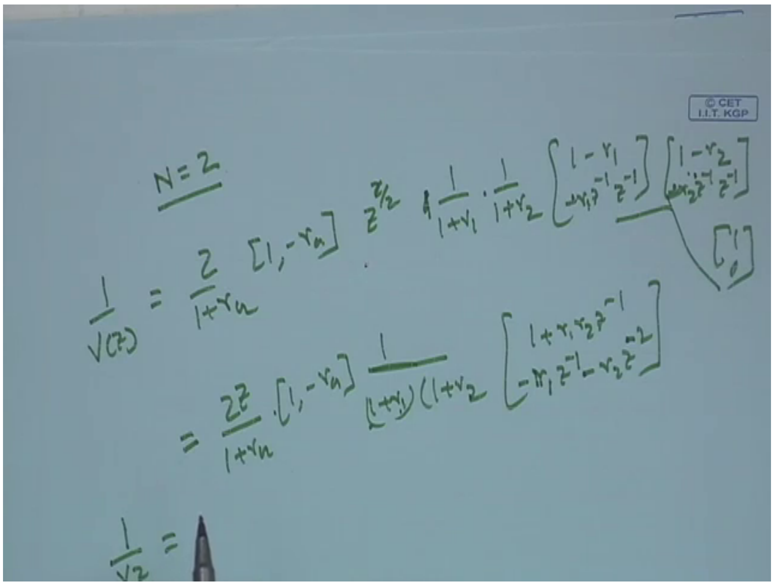

If I consider that my length of tube whole vocal chord is vocal track is simulated using 2 tube. So, if it is 2 tube. So, n is equal to 2, if I put n equal to 2 then 1 by V z will become 2 by 1 plus r G will be same, this will be 1 minus r G will be there. So, there is no effect on here. Now is z to the power n by 2 n equal to 2. So, 2 by 2. Then product of k equal to 1 to n k equal to 1 to n means if n equal to 2. So, k instead of k equal to 1 to n I can write 1 by 1 plus R 1 into 1 by 1 plus R 2. Then I can write k equal to 2 in this equation. So, k equal to 2 k equal to 1 1 minus R 1 1.

So, I can write 1 minus R 1 another one is minus R 1 z to the power minus 1 z to the power minus 1 into 1 minus upto minus upto z to the power minus 1 z to the power minus 1 then 1 0. If I evaluate it it will come. So, z to the power half z. So, z 2 z divided by 1 plus r G into 1 minus r G. This will be 1 by 1 plus R 1 into 1 plus R 2 then I can matrix multiply this in this one. So, this one is 1 plus R 1 R 2 z to the power minus 1 1 minus R 1 z to the power minus 1 minus R 2 z to the power minus 2. If I evaluate this matrix will be 1 0, I can say first evaluate this I 2 then come to the here this side. So, if I write down the equation then whole equation 1 by V z is nothing but a I can multiply this thing. 1 plus r R 1 1 plus R 2 and this thing I can multiply together and I can find out.

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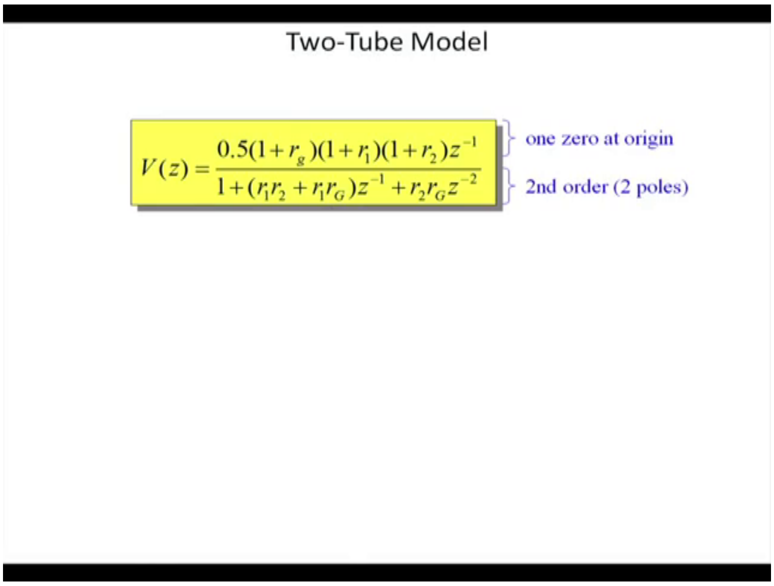

So, V z 1 by V z and then V z will becomes if I derive these things and then the V z will become, V z will be 0.5 1 plus r G into 1 plus R 1 into 1 plus R 2 z to the power minus 1 divided by 1 plus R 1 R 2 plus R 1 r G z to the power minus 1 plus R 2 r G z to the power minus 2.

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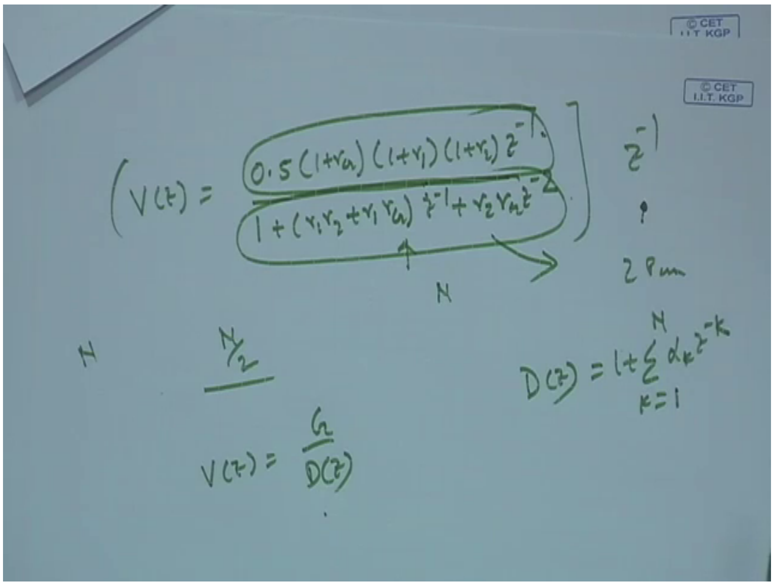

This will be the 2 tube vocal track model. If you see this transfer function I have a 0 at origin z to the power minus 1, but I have a 2 pole this is second order equation.

So, I have a second 2 second order you can say 2 pole or it is a second order equation. So, I have a single 0, but 2 tube model I have a 0 at center 1 0 and 2 pole. So, at origin 1 0 and 2 pole. So, if I have a n tube model I can say n by 2 0 at origin and nth this equation become n model. So, I can say then we n number of pole this equation become nth order. So, I can say this whole V z can be same this is all 0 are in center and this is all pole model. So, I can write V z is nothing but a G by dz. Where dz is nothing but a pole model. So, if it is if it is nth pole nth order. So, it is nothing but a 1 plus k equal to 1 to n alpha k z to the power minus k. So, kth pole n nth pole will be the n number of pole will be there. So, I can write down the equation 1, 1 plus k equal to 1 to n alpha k z to the power minus k and n by 2 0 at origin.

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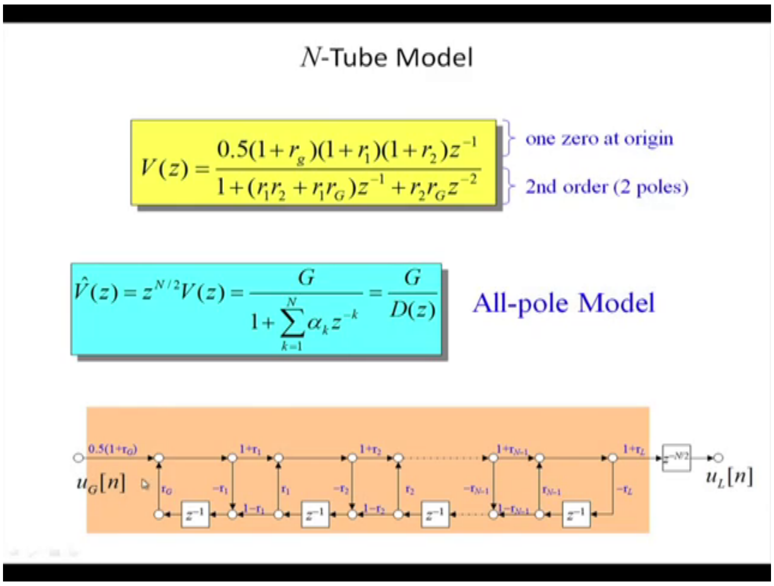

So, instead of writing this the all 0 model z to the power minus 1 minus have been here z to the power minus half in here I can write this dz in a signal flow diagram as a all pole model, and z equal to this V z z to the power minus.

So, all pole model then it will be z to the power minus n by 2. So, I can write down the signal flow diagram in all pole model. So, I can say mathematically it is proved that a vocal track transfer function can be simplify as a all pole digital filter. All pole digital filter I can simplify it. So, if I able to simplify all pole digital filter and if I know alpha k, if I know alpha k which is nothing but in term of R 1 R 2 and the R 1 R 2 r G I have to know. So, what is R 1, R 2 all are reflection coefficient. If I know all reflection coefficient I can simulate this d z. So, it is impossible to implement this dz using digital filter. So now, I can say.

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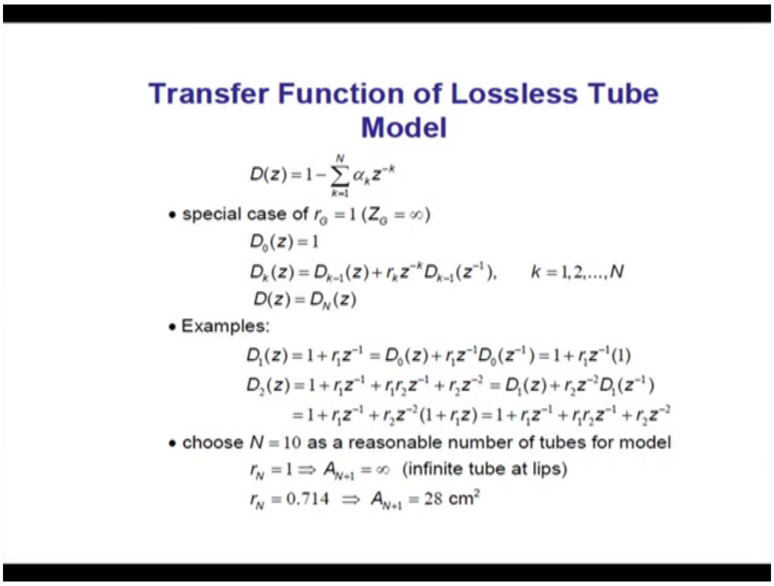

D z; d z is nothing but a there is a error in the slides please correct it. D z is nothing but a 1 plus k equal to 1 to n alpha k z to the power minus k. So, let us r G is equal to 1; that means, the let us there is no gotal impedance r G, r G is equal to 1.

Thank you.